Lecture 3

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Definition 1. A *walk* in a graph $G = (V, E)$ is a sequence of vertices $(v_0, ..., v_k)$ such that $(v_{i-1}, v_i) \in E$ for all $i = 1...k$.

Definition 2. A *Euler walk* is a walk that spans every edge exactly once, i.e., $E = \{(v_{i-1}, v_i) | i \in \{1...k\}\}$.

Remark. A walk is the image of a morphism $P_k \to G$. A tour is the image of a morphism $C_k \to G$.

Definition 3. Let $G = (V, E)$. Vertices u, v are *connected* if $\exists$ a walk that starts at u and ends at v.

Remark. We say G is *connected* if u is connected to v for $u, v \in V$.

Theorem 1. Let G be a connected graph. Then, G has an Euler tour iff. the degree of every vertex is even.

Proof. Prove by contradiction. Assume there is no Euler tour. Take longest tour and remove edges. Result still has all even degrees. So, there is a tour that shares n vertex with the longest tour. Combine to get a longer tour and a contradiction. $\square$

Definition 4. The *connected components* of a graph are its maximal connected subgraphs.

Remark. Connectivity is an Equivalence relation. Equivalence connected components are the subgraphs induced on the equivalence classes of connectivity.

Definition 5. Say $u, v \in V$ are *2-connected* if they remain connected even if I remove any one vertex other than u, v.

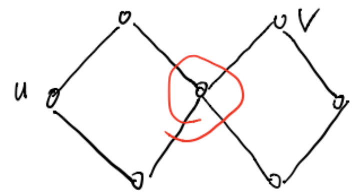

Figure 1: This graph is not 2-connected

Definition 6. Say G are *2-connected* if u, v are 2-connected for all $u, v \in V$.

Definition 7. Say G are *k-connected* if $G \setminus S$ is conencted for any subset $S \subset V$ such that $|S| < k$.