

Gradient Descent Algorithm

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

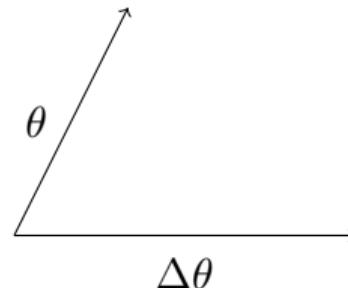
vector of parameters,
say, randomly initialized

$$\theta = [w, b]$$

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$$\begin{array}{c} \curvearrowright \theta = [w, b] \\ \curvearrowright \Delta\theta = [\Delta w, \Delta b] \end{array}$$

change in the
values of w, b

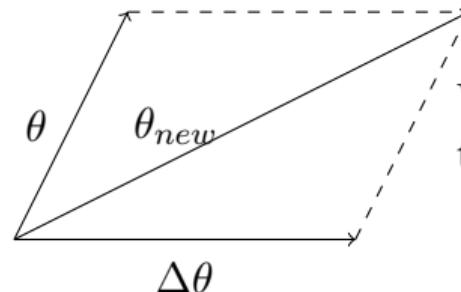


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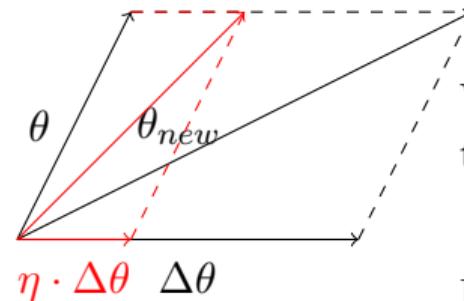
We moved in the direction of $\Delta\theta$

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$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$



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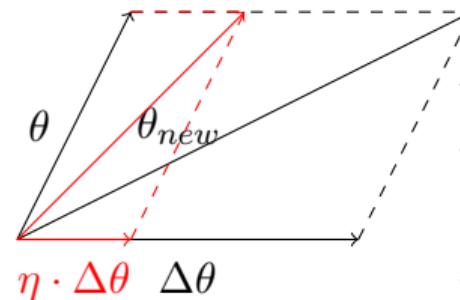
Let us be a bit conservative: move only by a small amount η

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Question: What is the right $\Delta\theta$ to use?

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient
- In other words, move in a direction opposite to the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$

$$b_{t+1} = b_t - \eta \nabla b_t$$

where, $\nabla w_t = \frac{\partial \mathcal{L}(w, b)}{\partial w}$ at $w = w_t, b = b_t$, $\nabla b_t = \frac{\partial \mathcal{L}(w, b)}{\partial b}$ at $w = w_t, b = b_t$

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So we now have a more principled way of moving in the w - b plane than our “guess work” algorithm

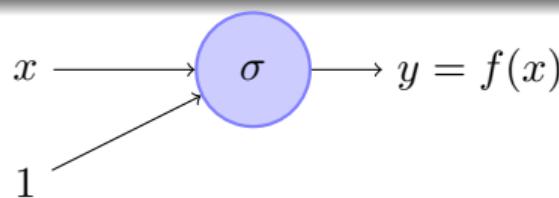
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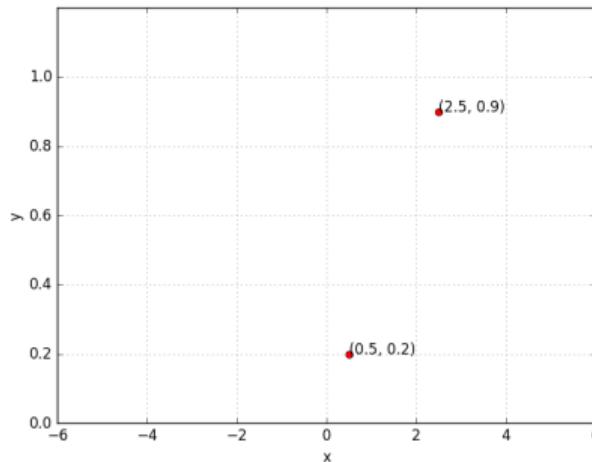
Algorithm 1: gradient_descent()

```
t ← 0;  
max_iterations ← 1000;  
while  $t < max\_iterations$  do  
     $w_{t+1} \leftarrow w_t - \eta \nabla w_t$ ;  
     $b_{t+1} \leftarrow b_t - \eta \nabla b_t$ ;  
end
```

- To see this algorithm in practice let us first derive ∇w and ∇b for our toy neural network

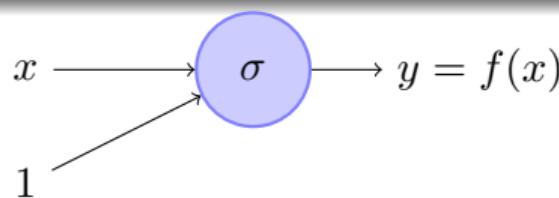


$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

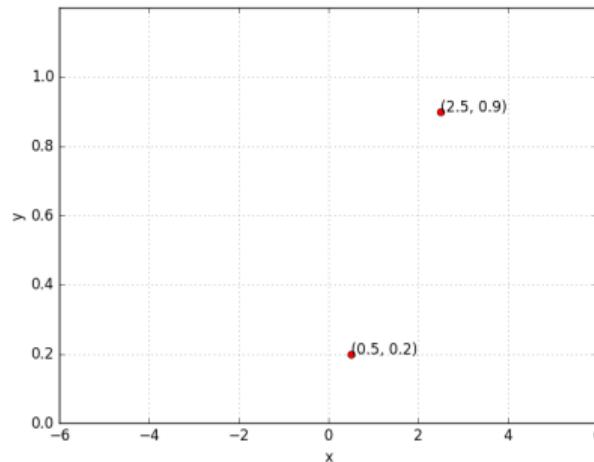


Let's assume there is only 1 point to fit
 (x, y)

$$\mathcal{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$



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$$\nabla_w = \frac{\partial \mathcal{L}(w, b)}{\partial w} = \frac{\partial}{\partial w} \left[\frac{1}{2} * (f(x) - y)^2 \right]$$

$$\nabla w = \frac{\partial}{\partial w} \left[\frac{1}{2} * (f(x) - y)^2 \right]$$

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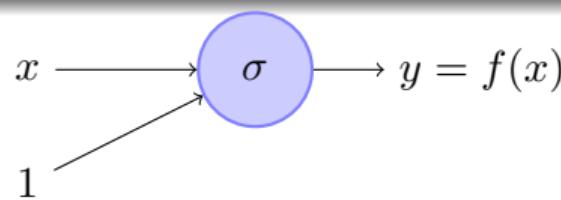
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&= f(x) * (1 - f(x)) * x
\end{aligned}$$

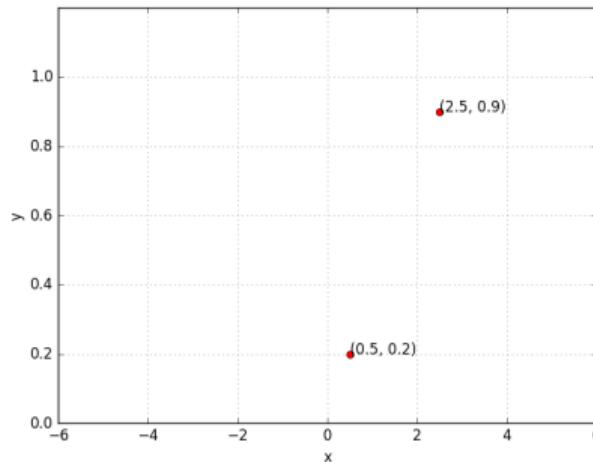
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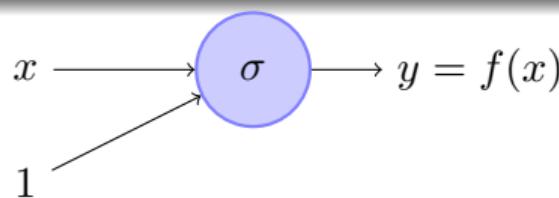
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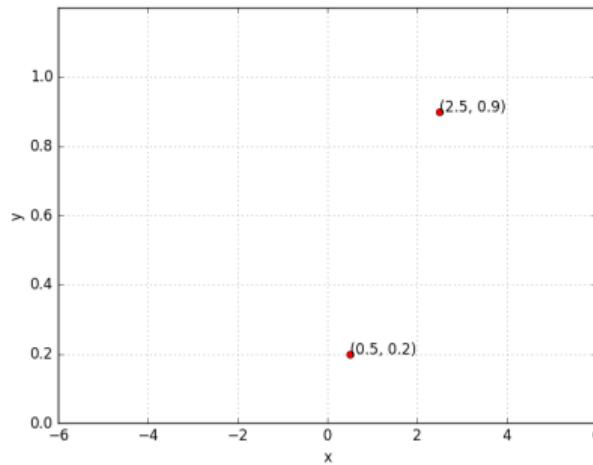
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y) , we have,



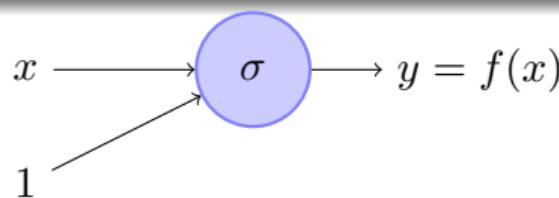


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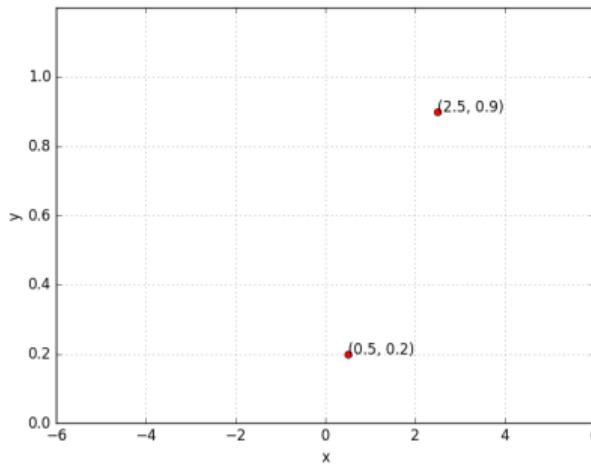


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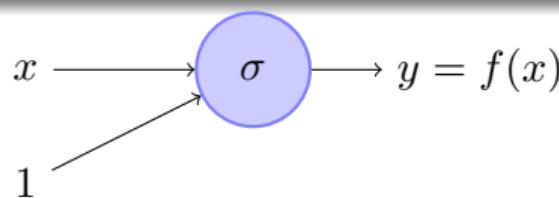
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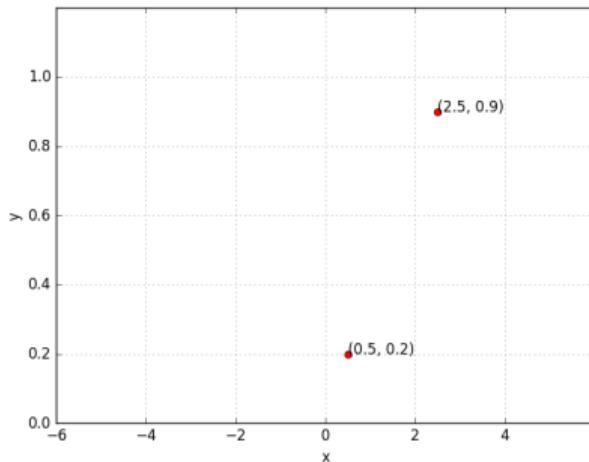
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For two points,



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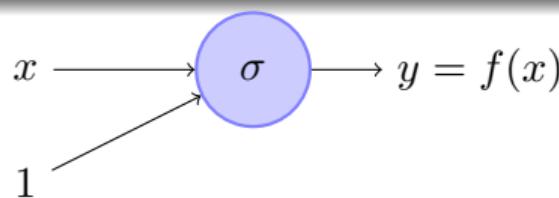


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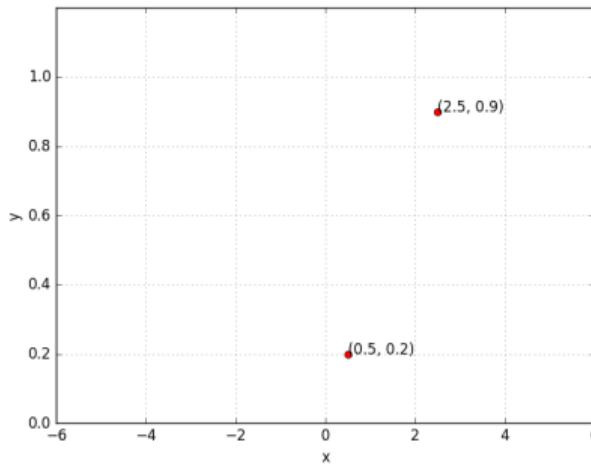
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$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$



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$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$

$$\nabla b = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

```

[X = [0.5, 2.5]
Y = [0.2, 0.9]

def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
    err = 0.0
    for x,y in zip(X,Y) :
        fx = f(w,b,x)
        err += 0.5 * (fx - y) ** 2
    return err

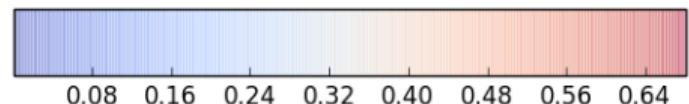
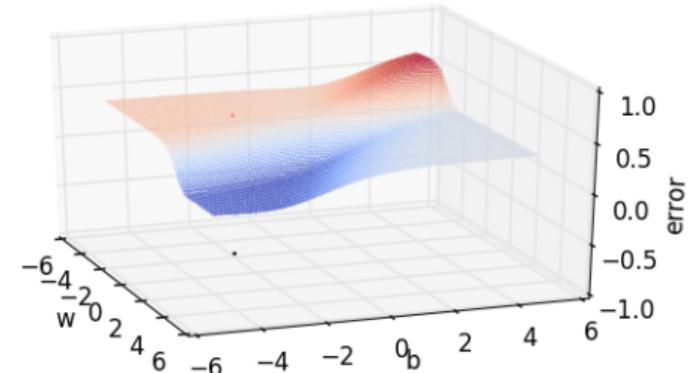
def grad_b(w,b,x,y) :
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def grad_w(w,b,x,y) :
    fx = f(w,b,x)
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def do_gradient_descent() :
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)
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Gradient descent on the error surface



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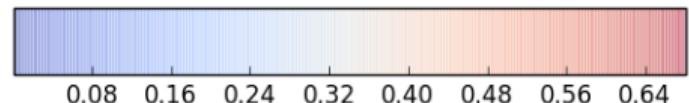
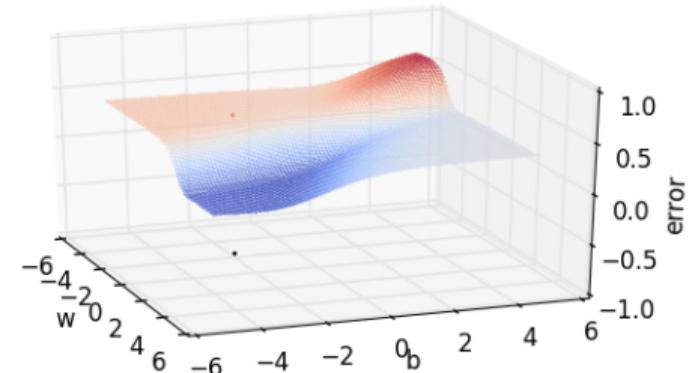
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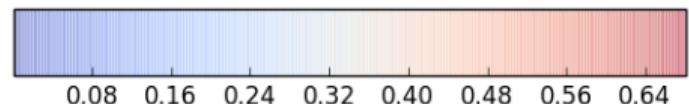
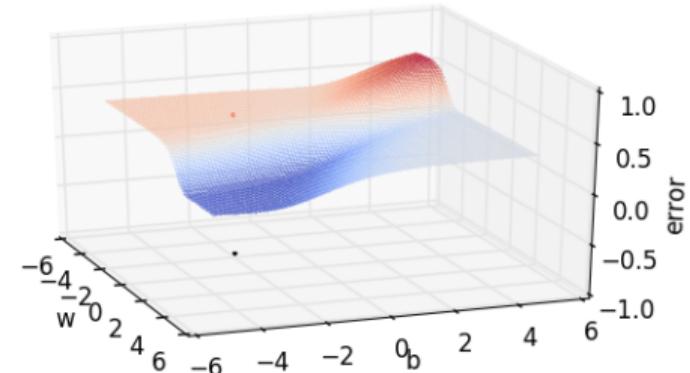
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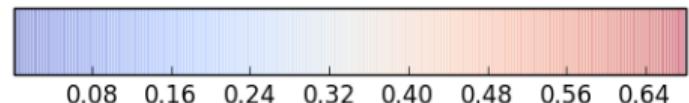
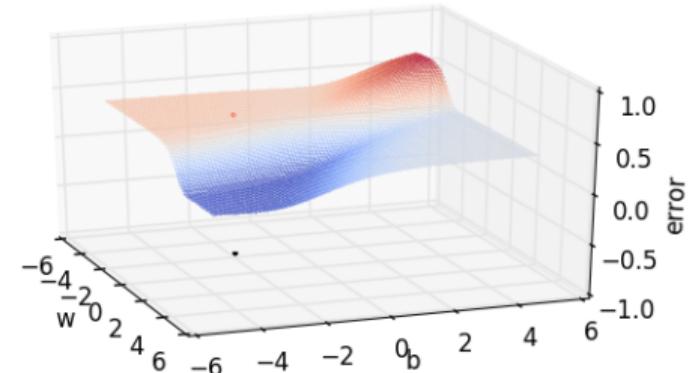
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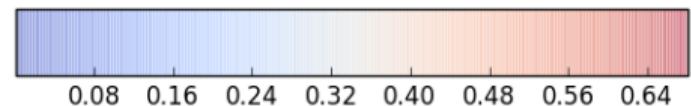
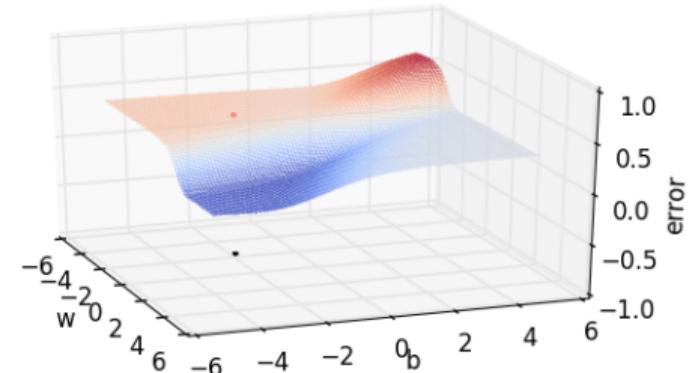
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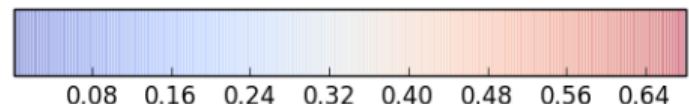
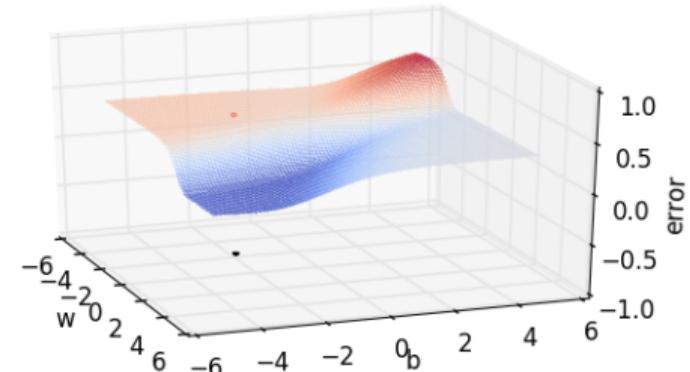
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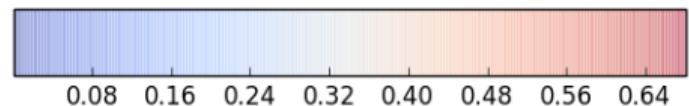
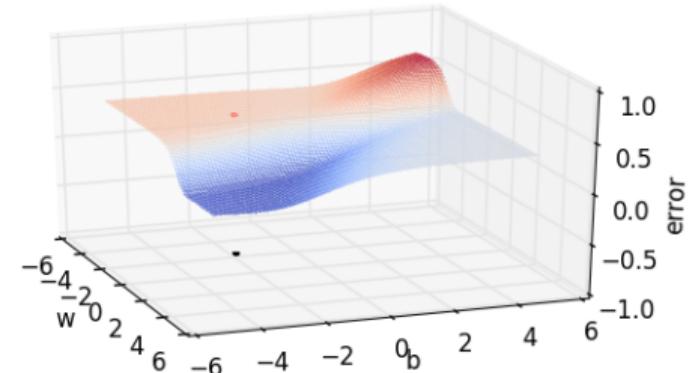
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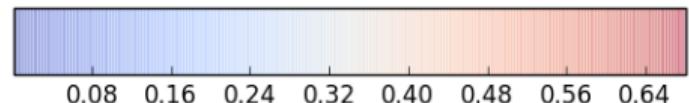
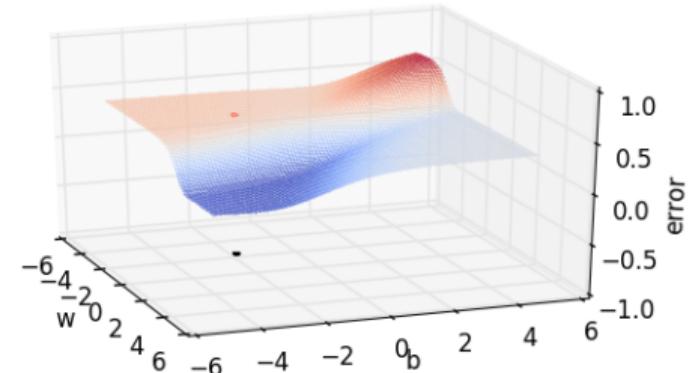
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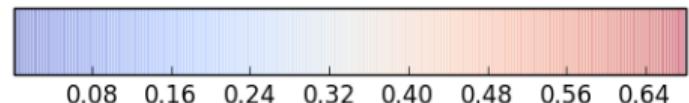
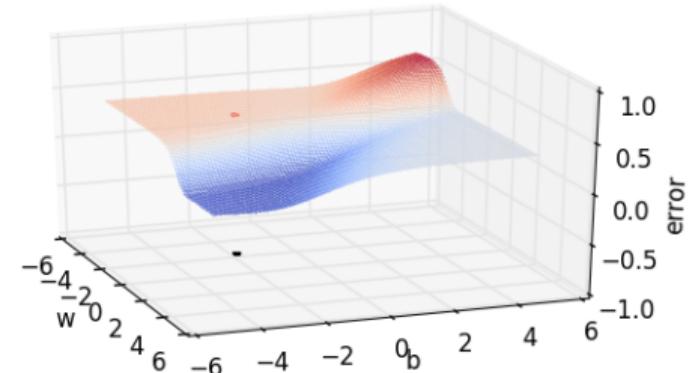
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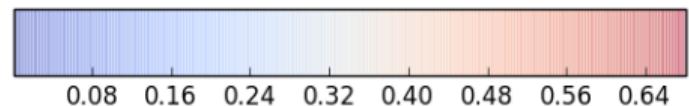
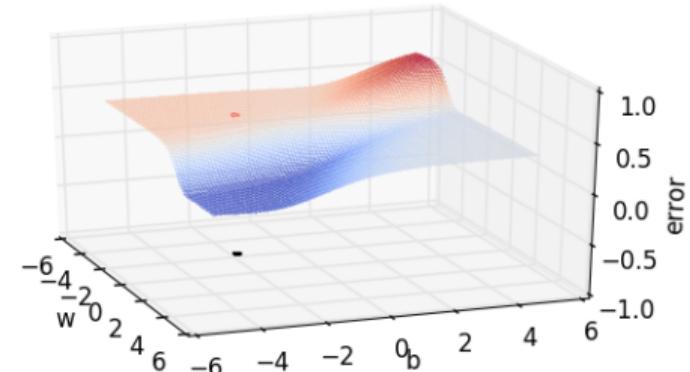
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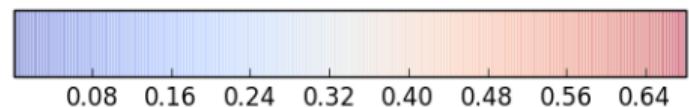
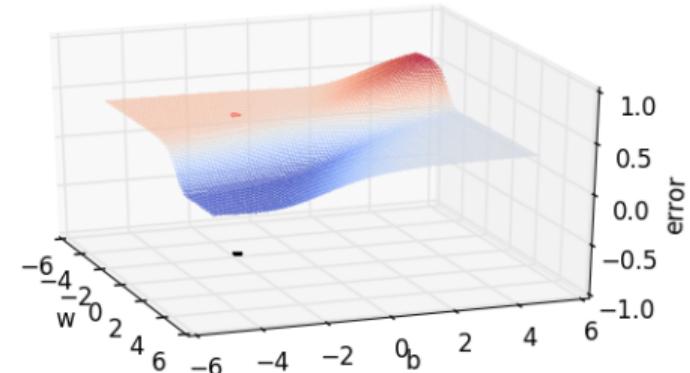
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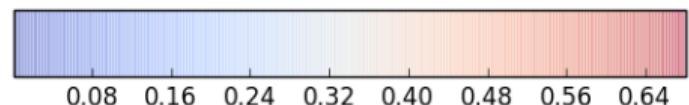
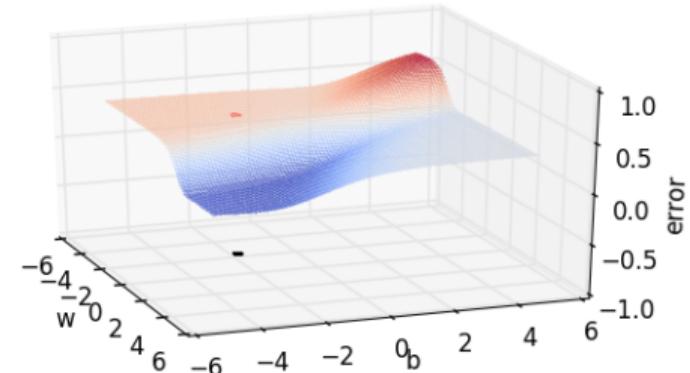
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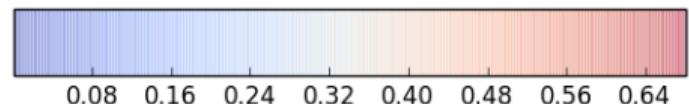
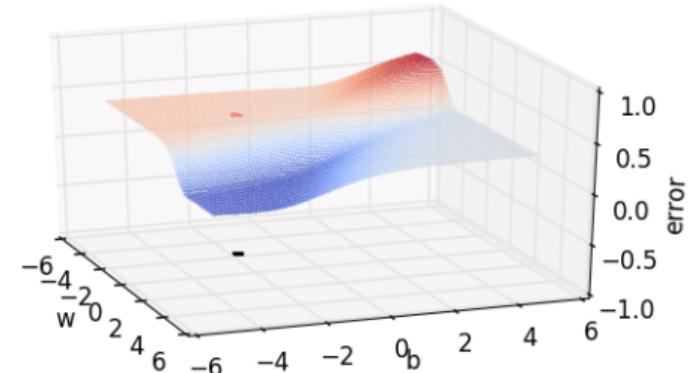
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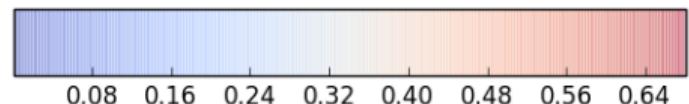
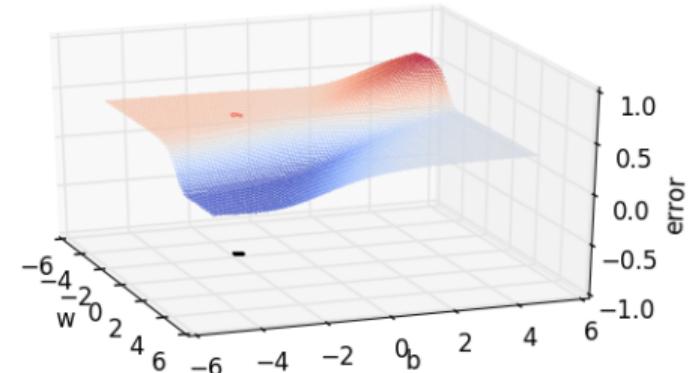
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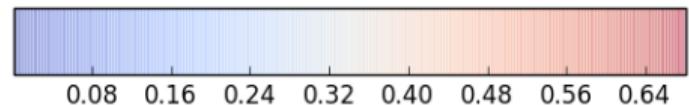
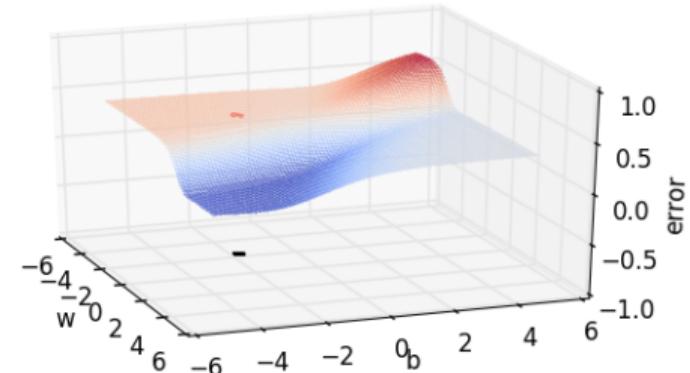
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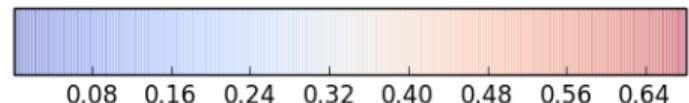
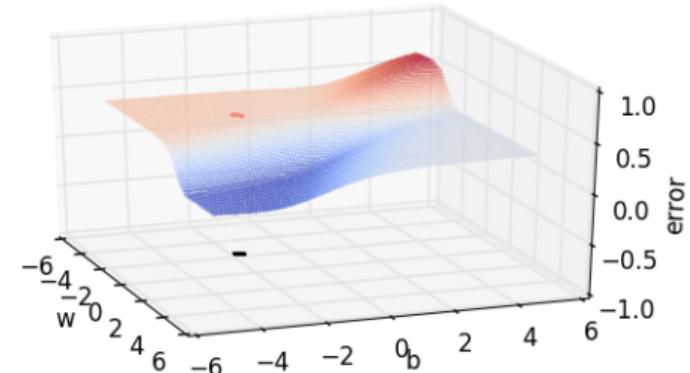
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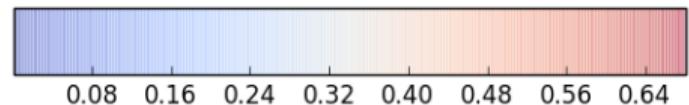
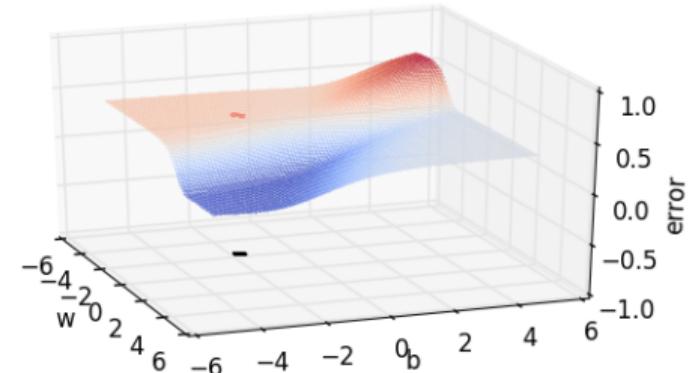
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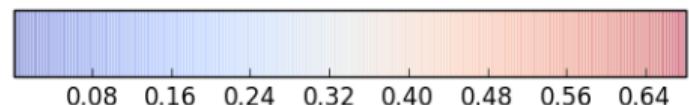
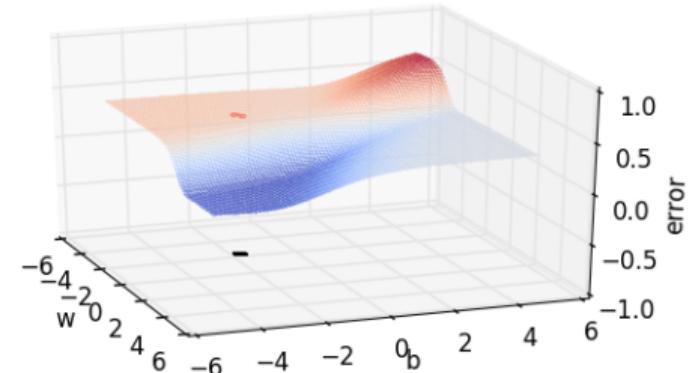
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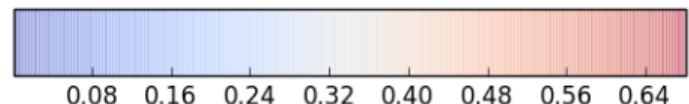
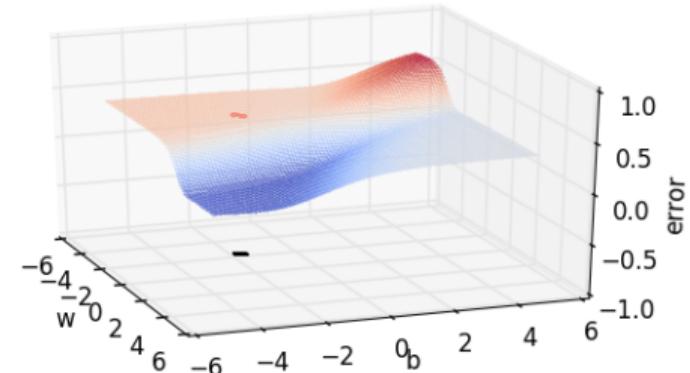
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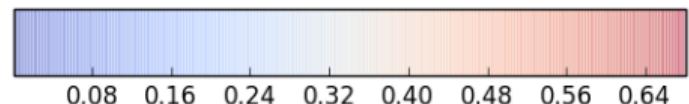
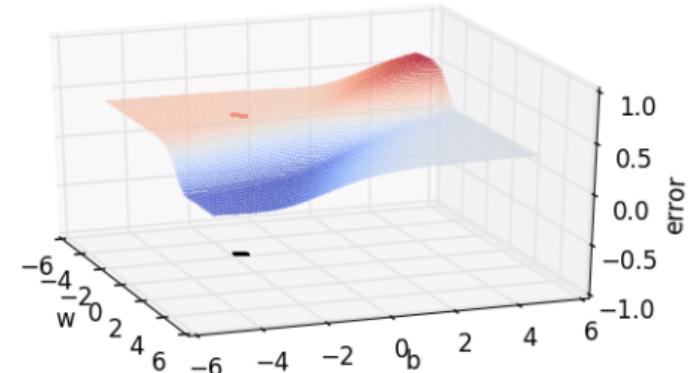
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Gradient descent on the error surface



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[X = [0.5, 2.5]
Y = [0.2, 0.9]

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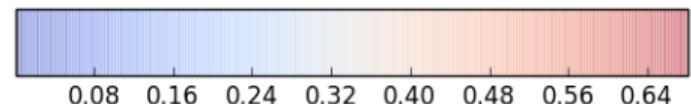
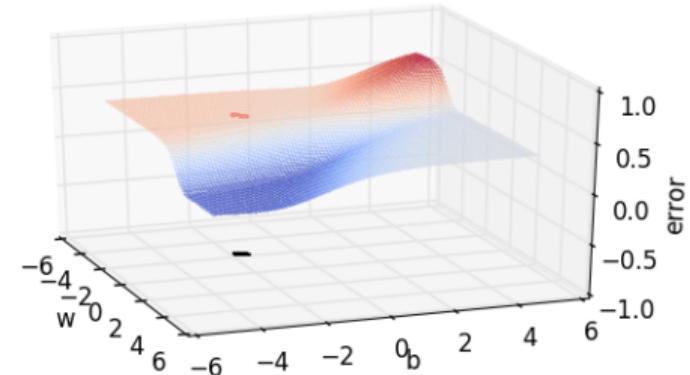
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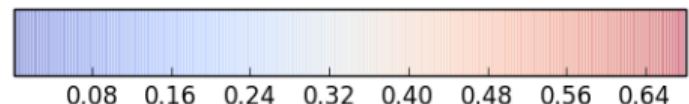
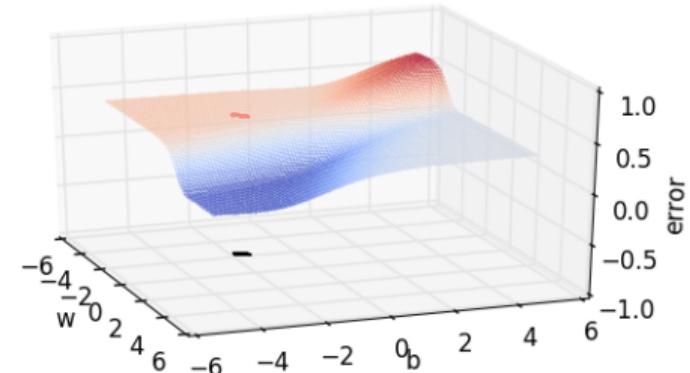
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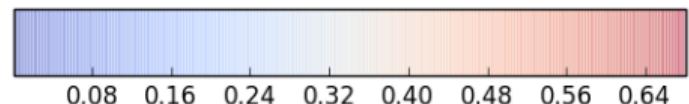
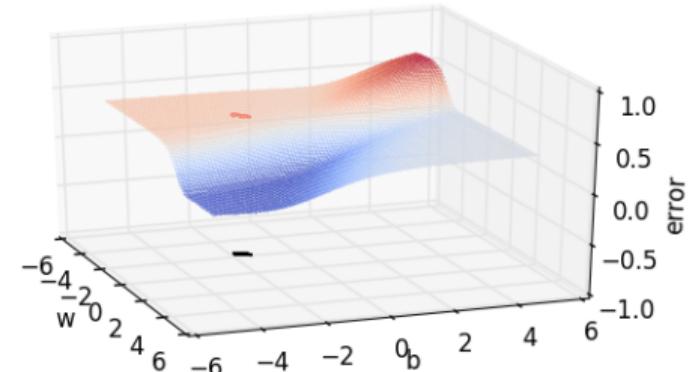
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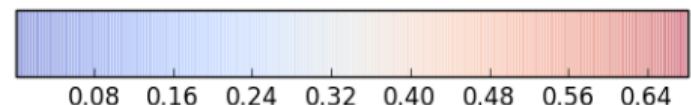
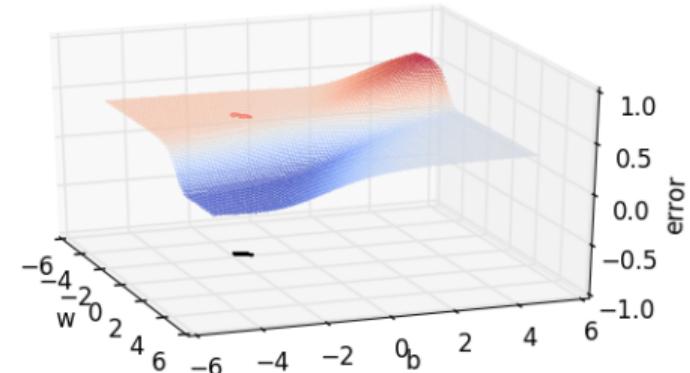
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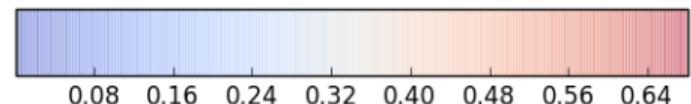
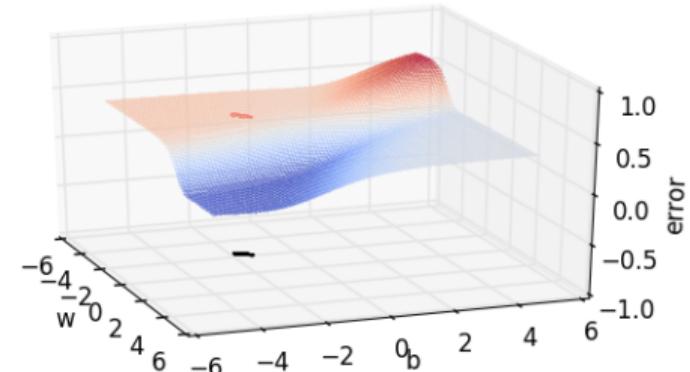
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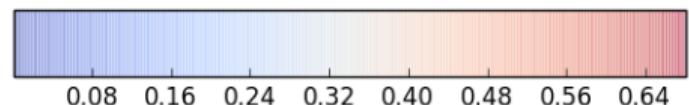
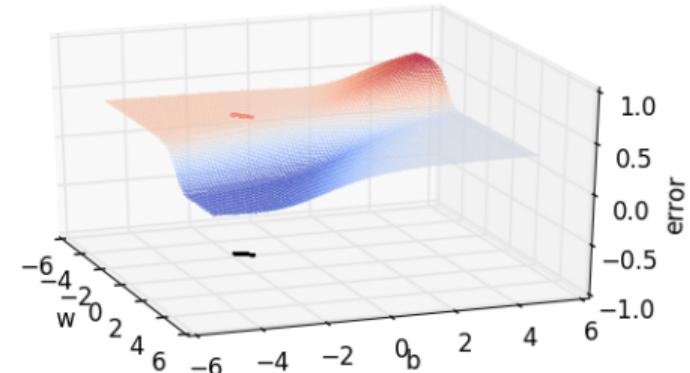
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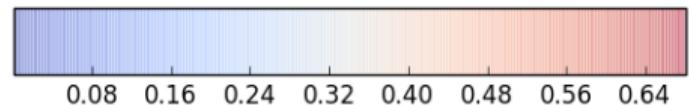
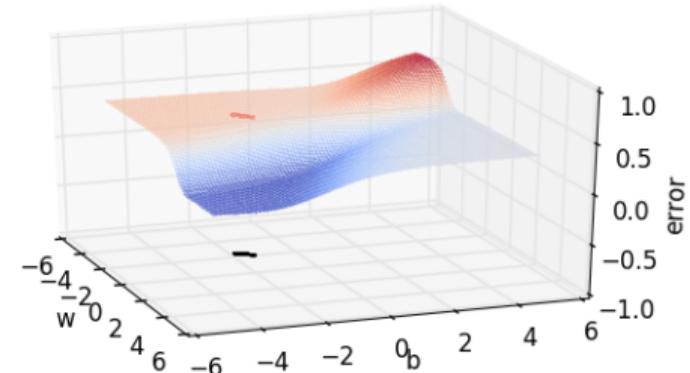
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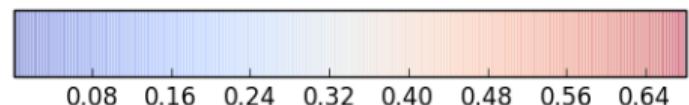
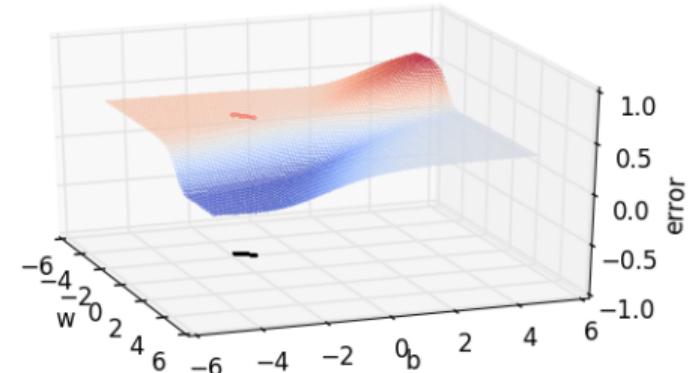
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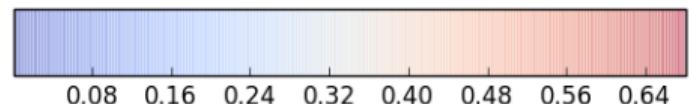
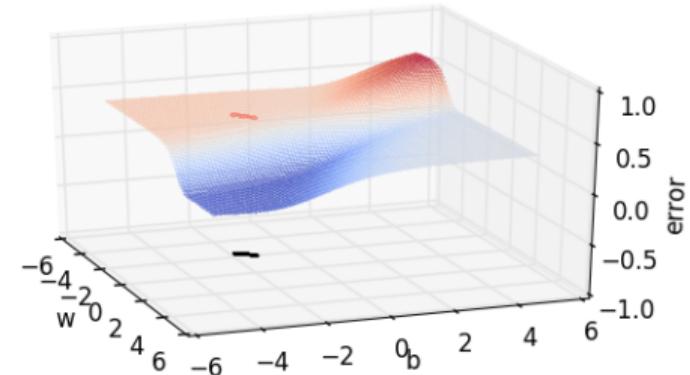
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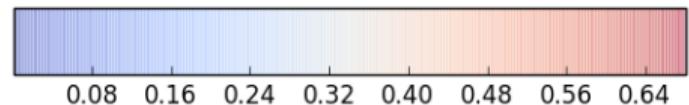
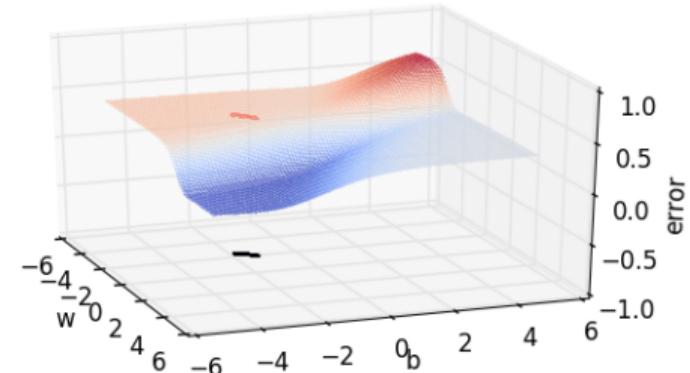
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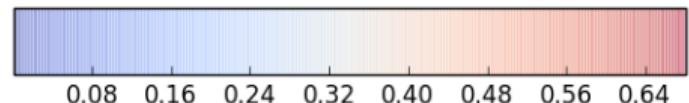
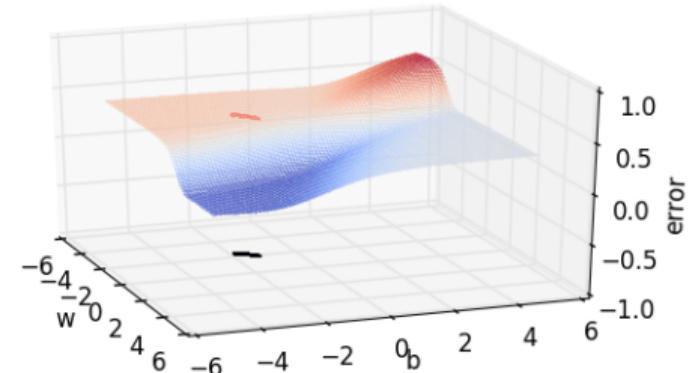
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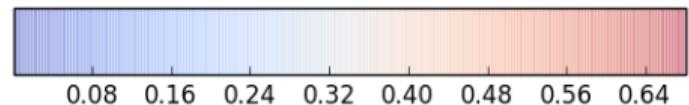
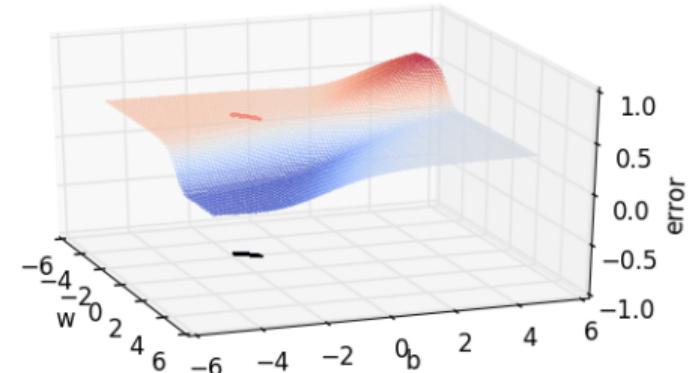
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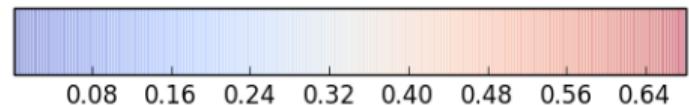
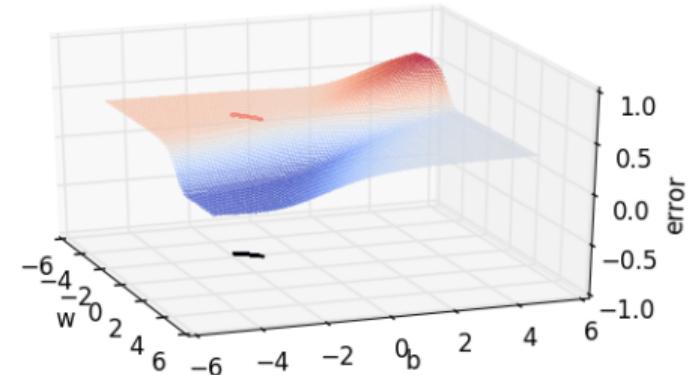
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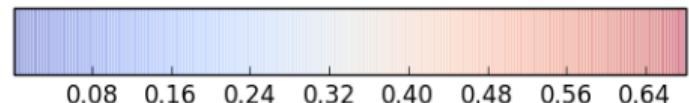
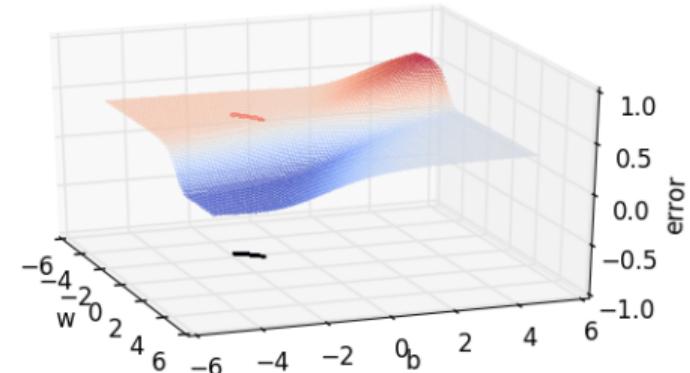
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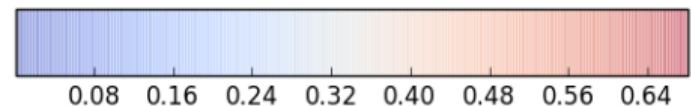
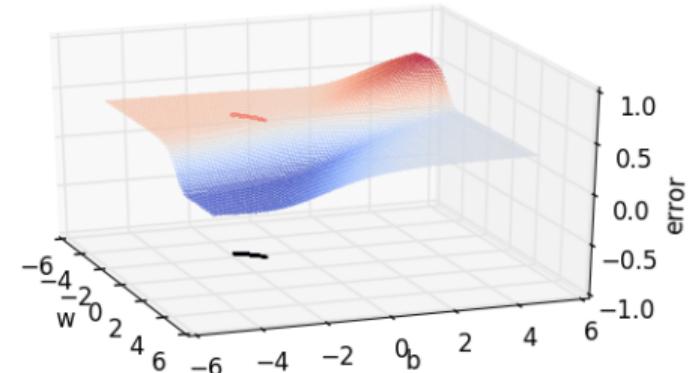
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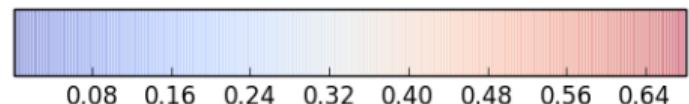
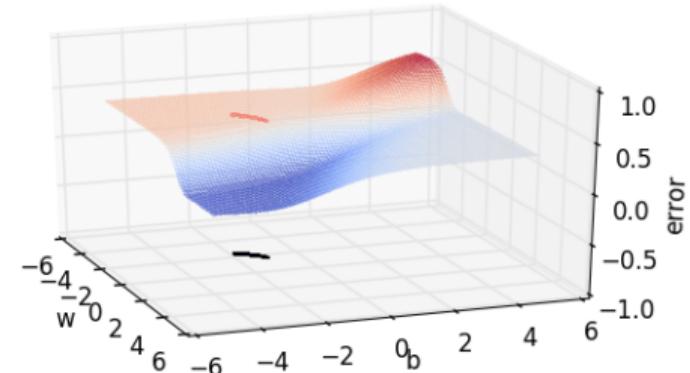
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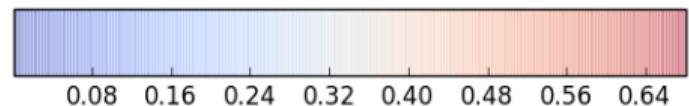
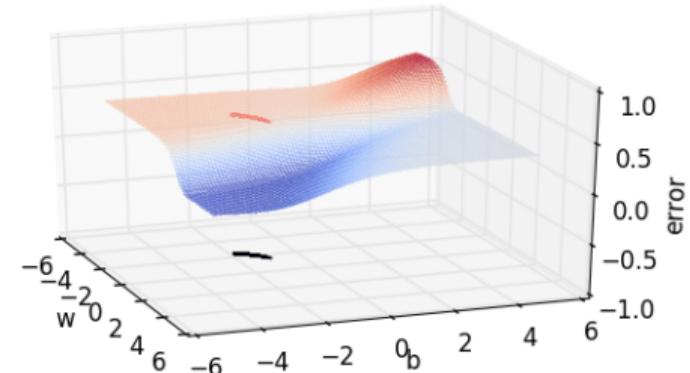
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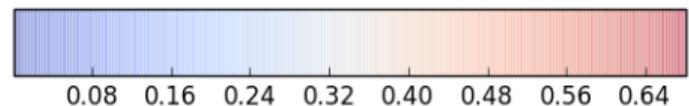
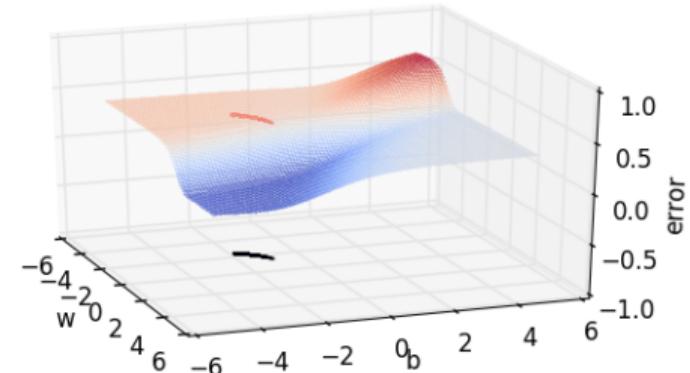
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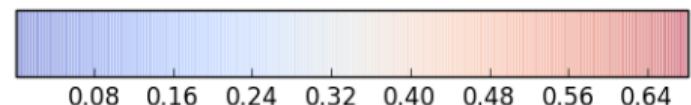
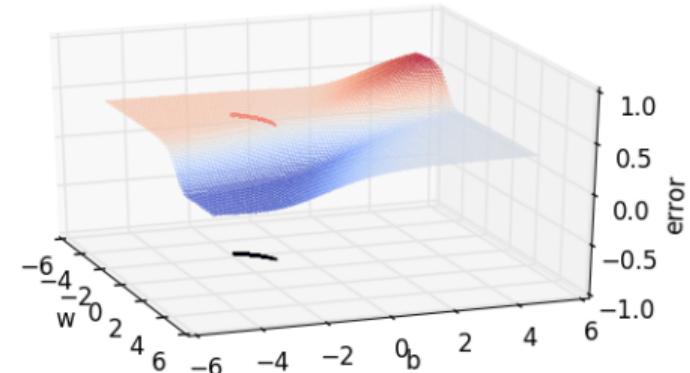
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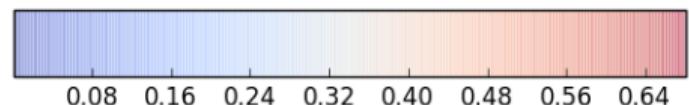
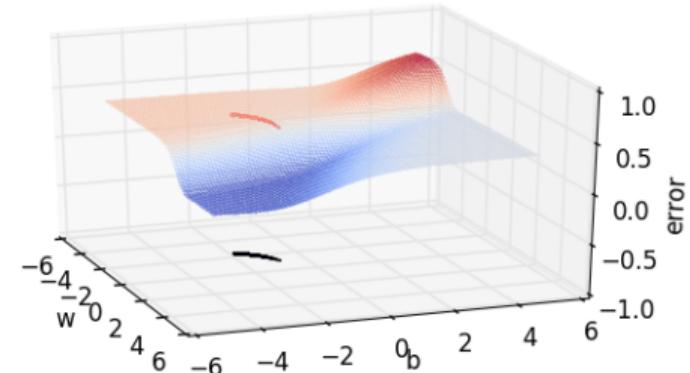
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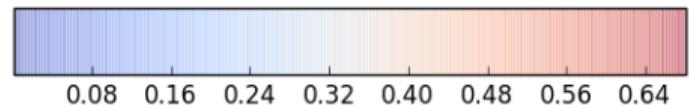
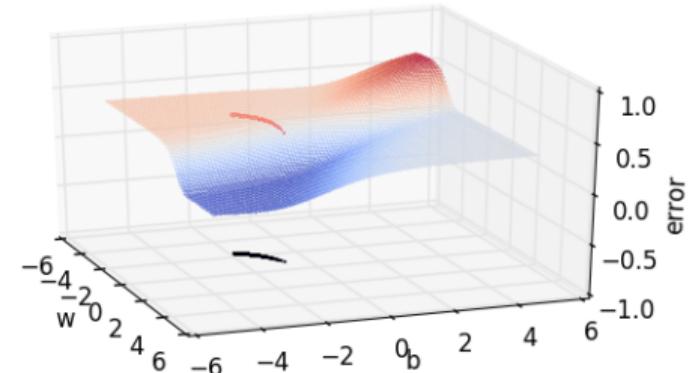
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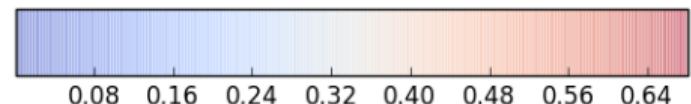
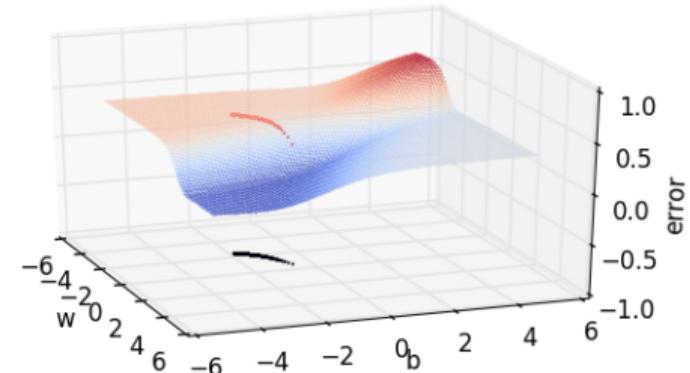
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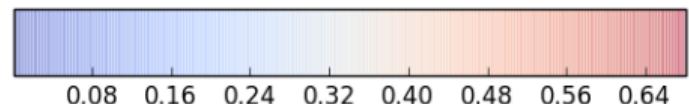
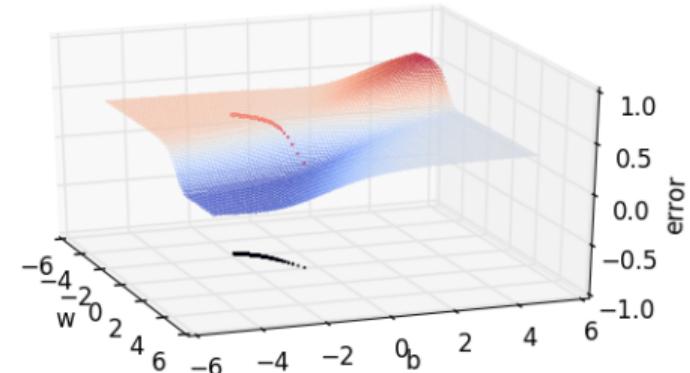
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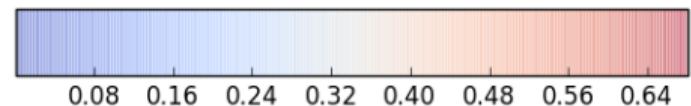
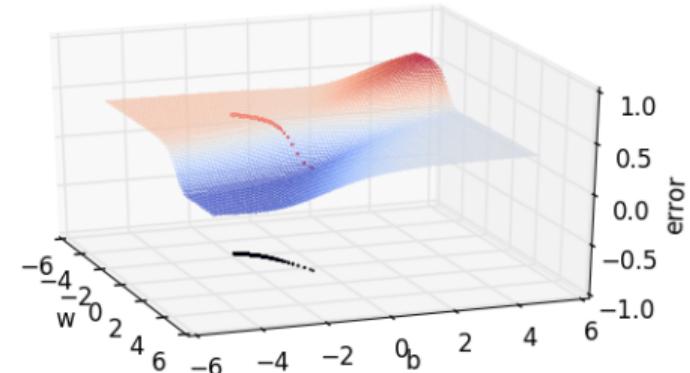
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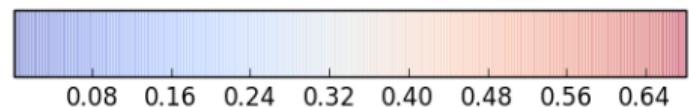
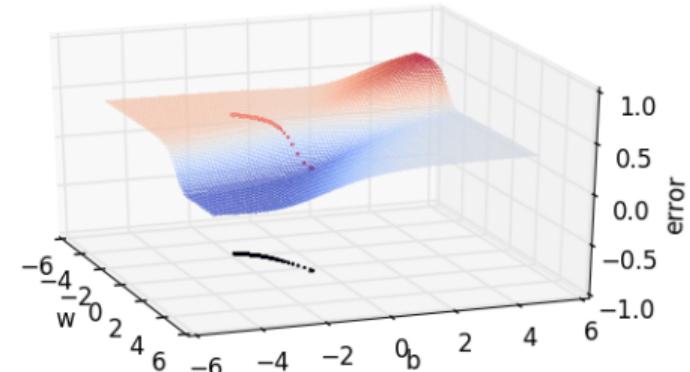
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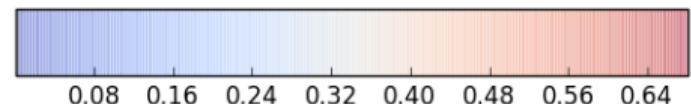
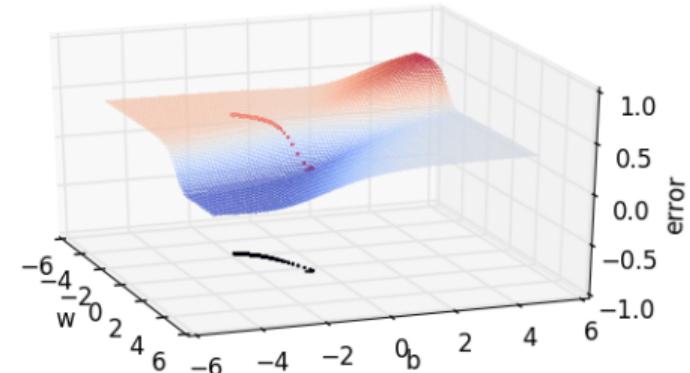
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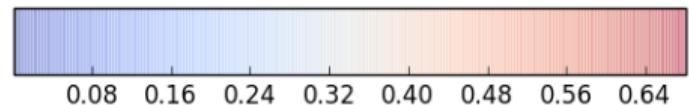
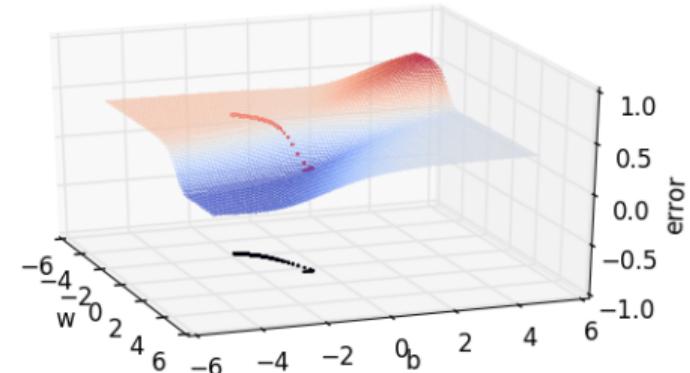
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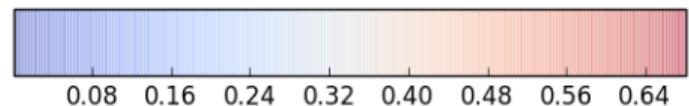
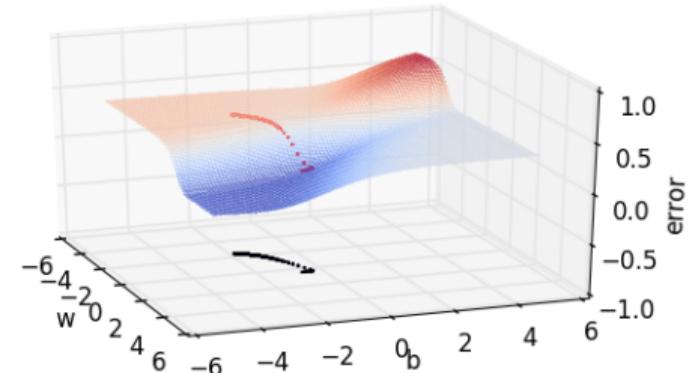
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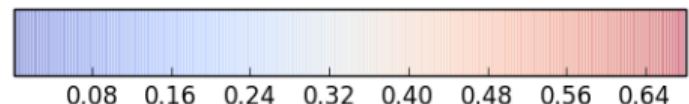
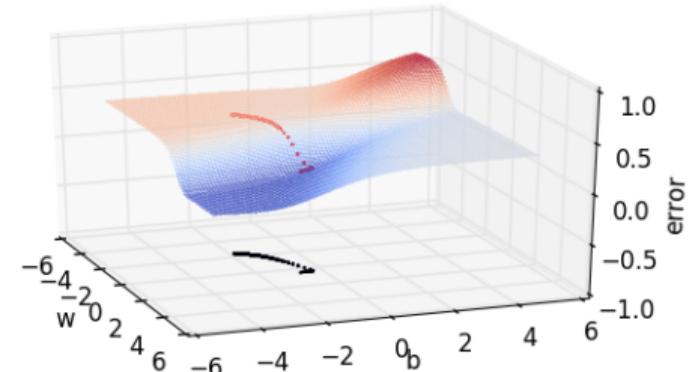
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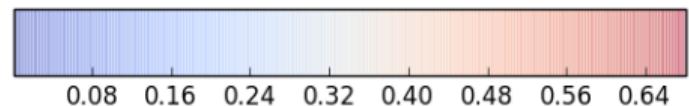
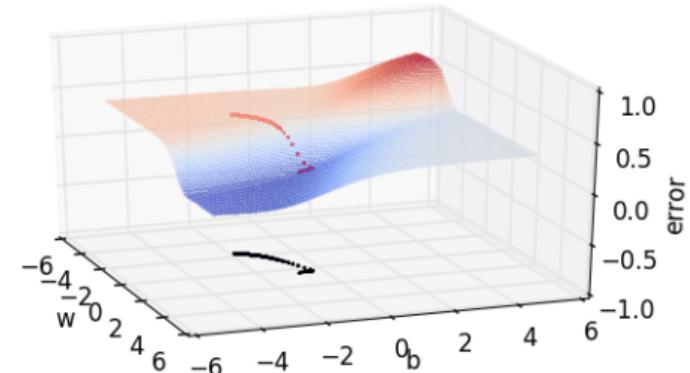
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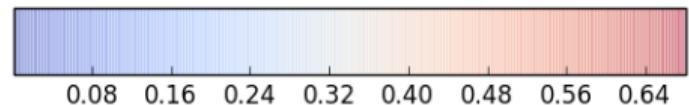
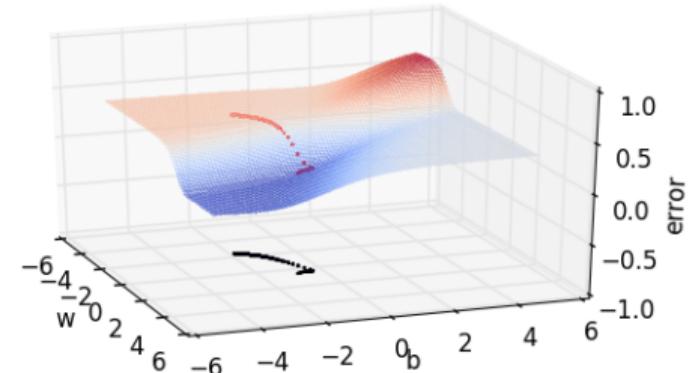
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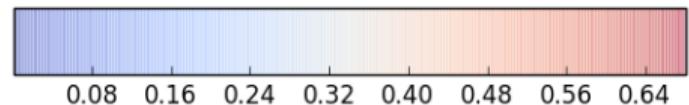
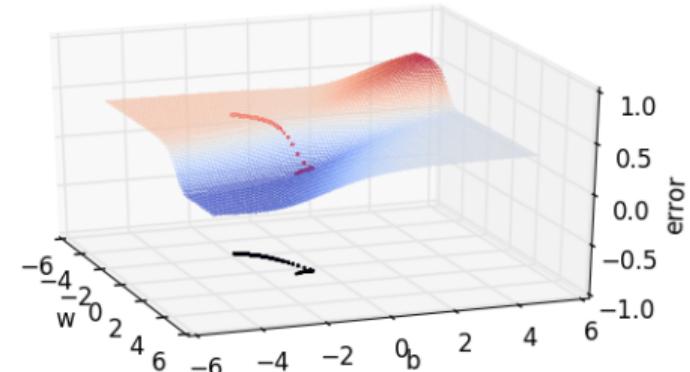
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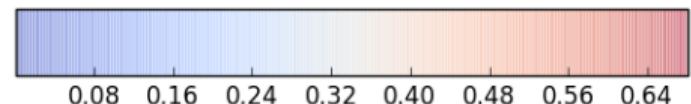
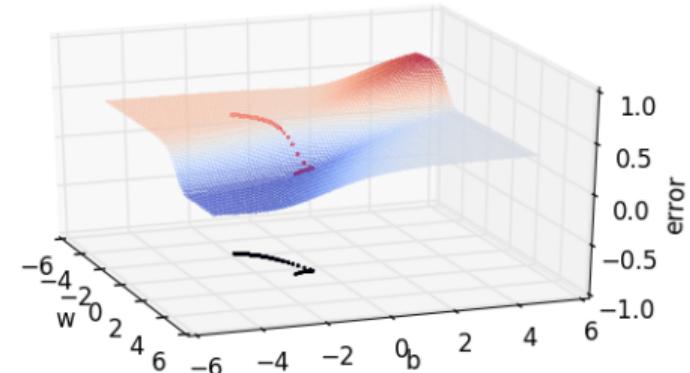
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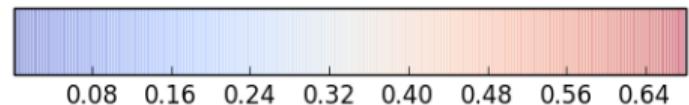
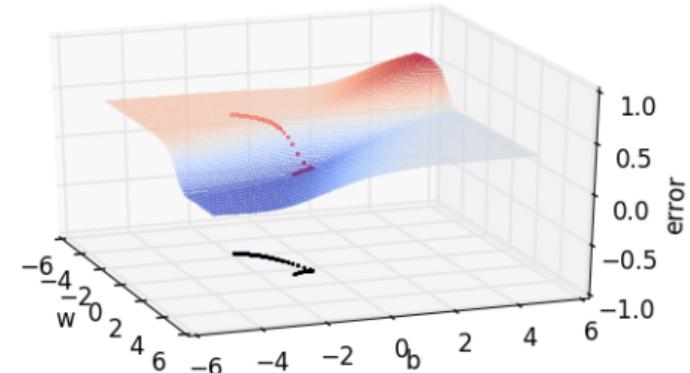
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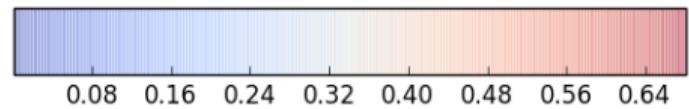
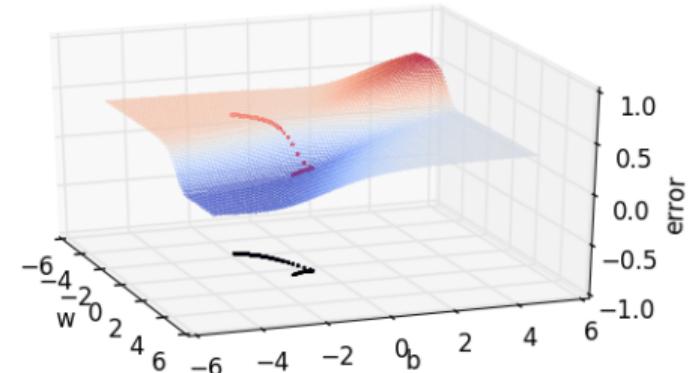
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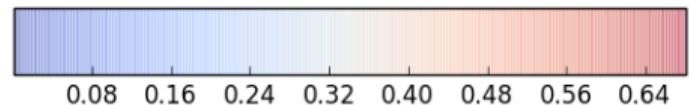
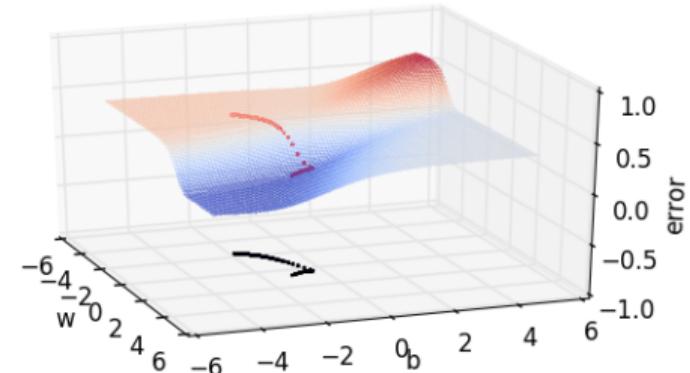
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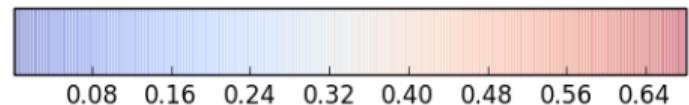
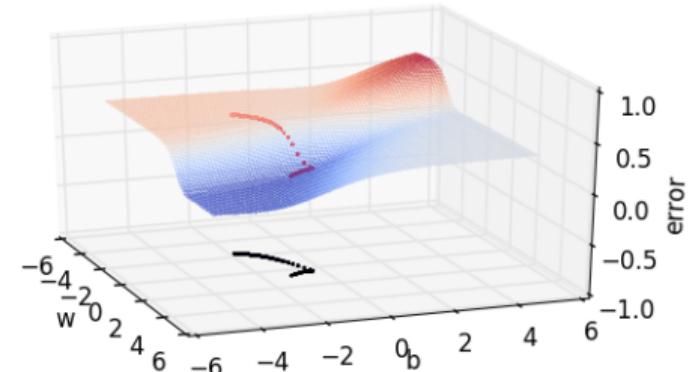
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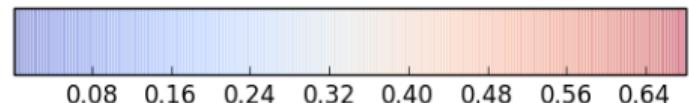
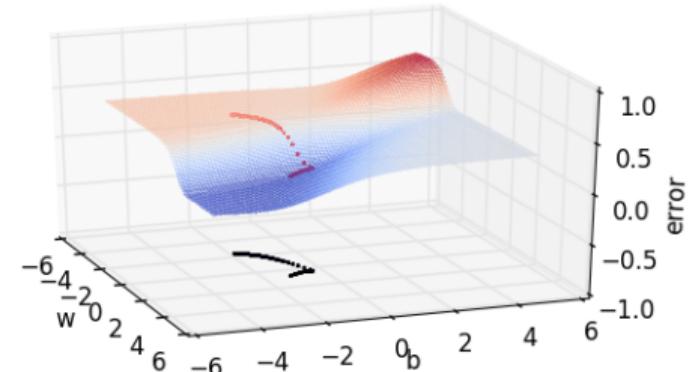
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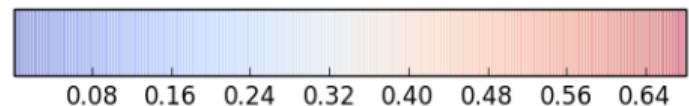
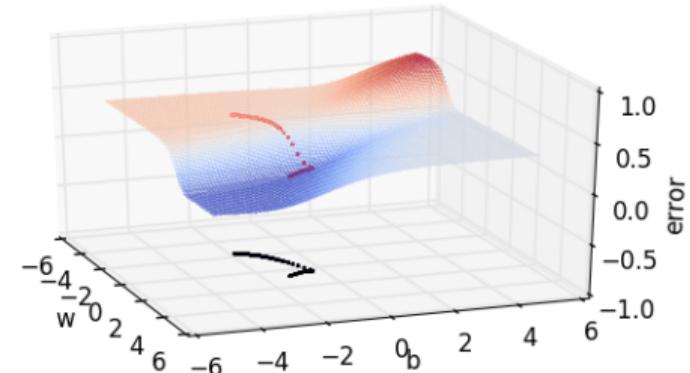
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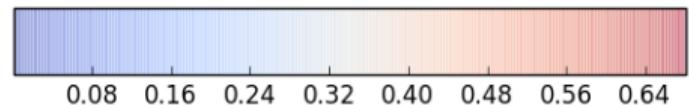
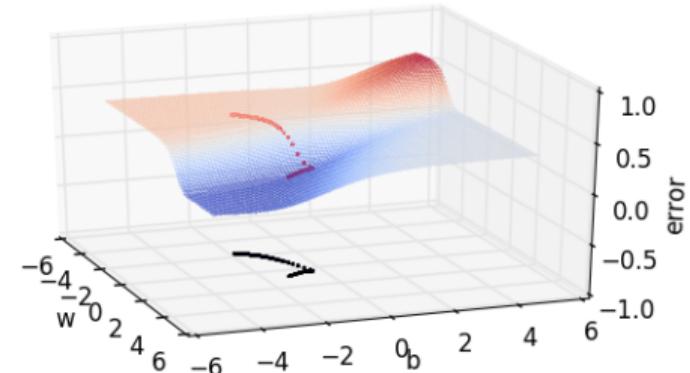
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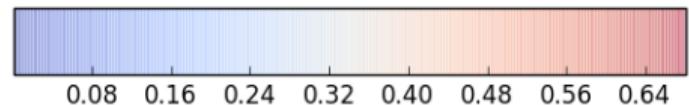
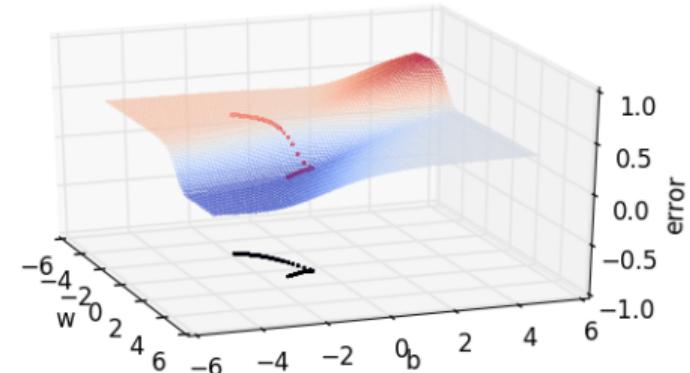
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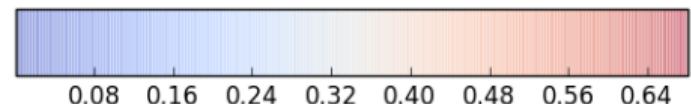
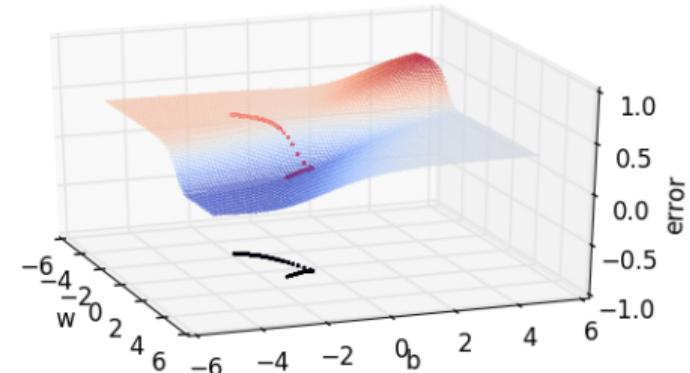
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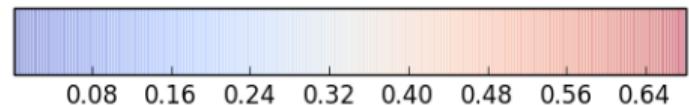
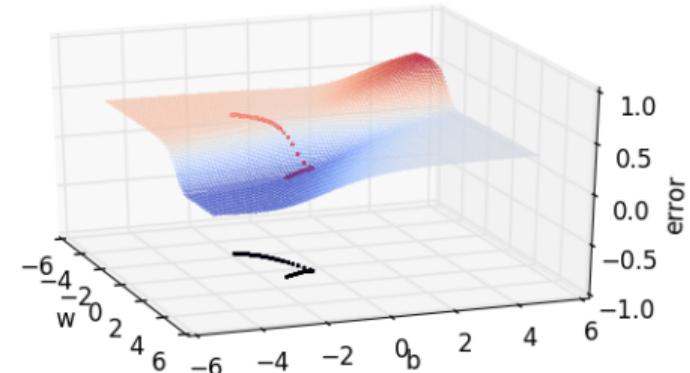
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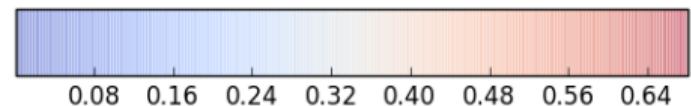
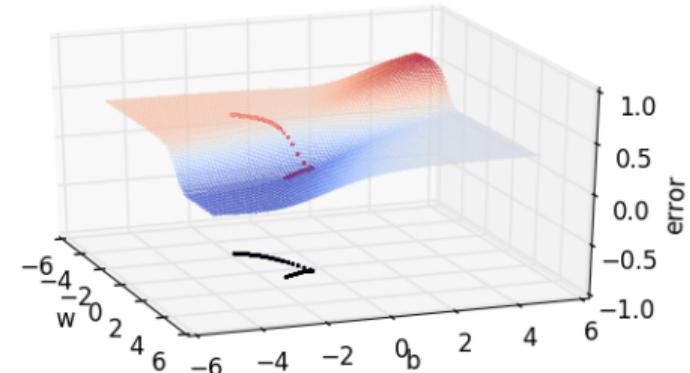
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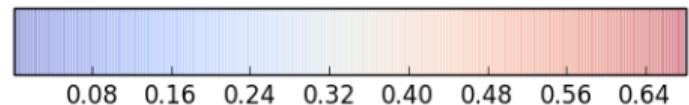
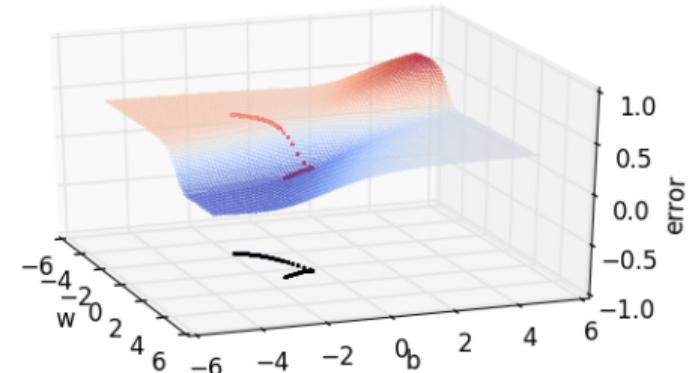
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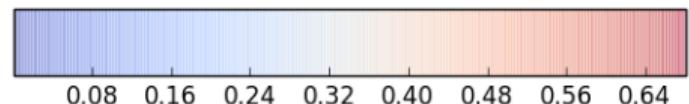
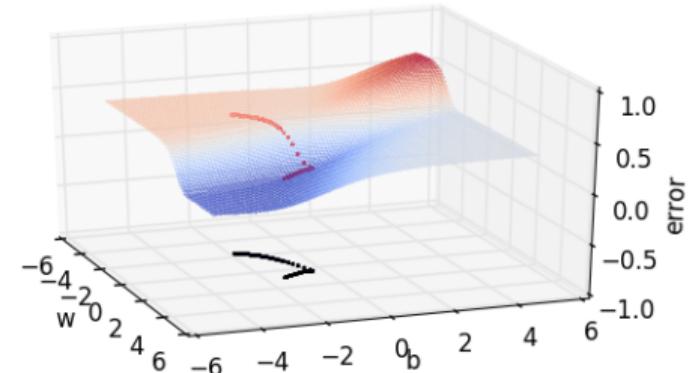
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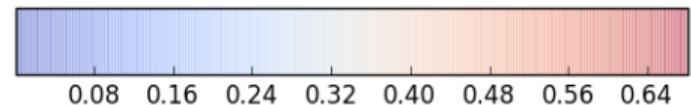
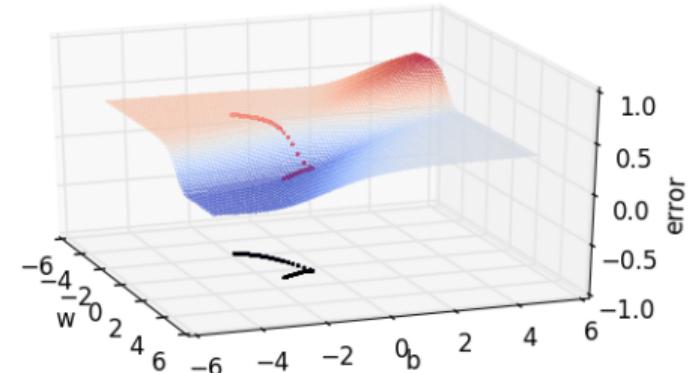
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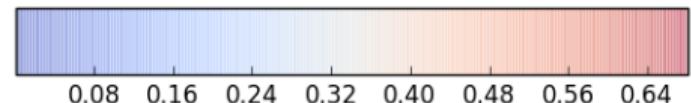
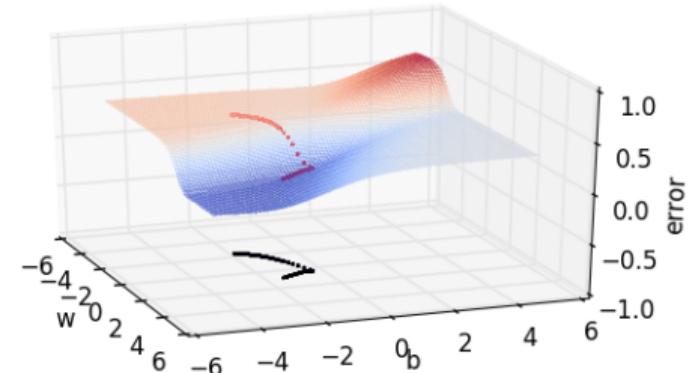
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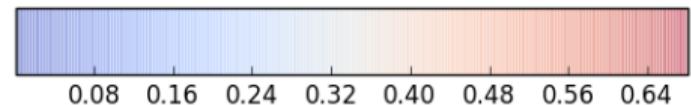
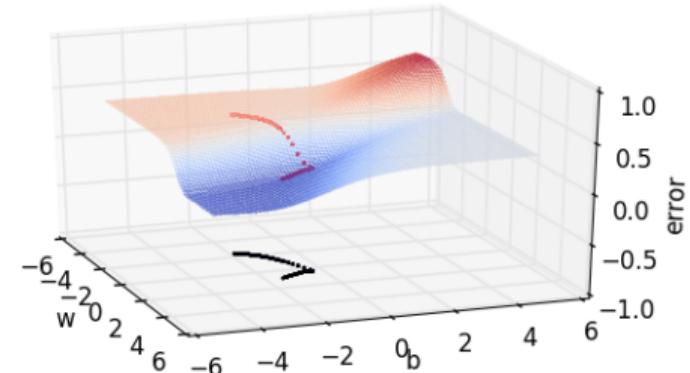
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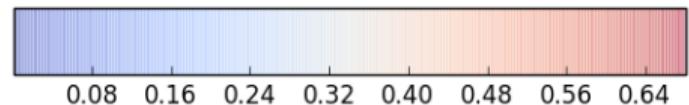
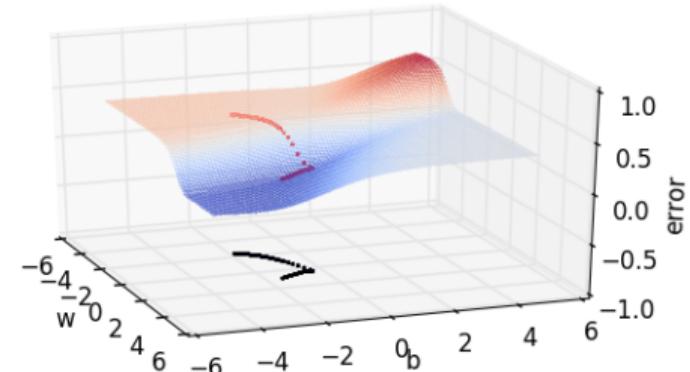
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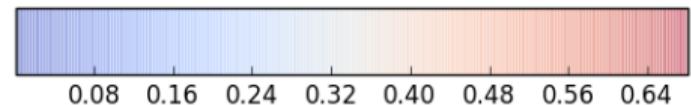
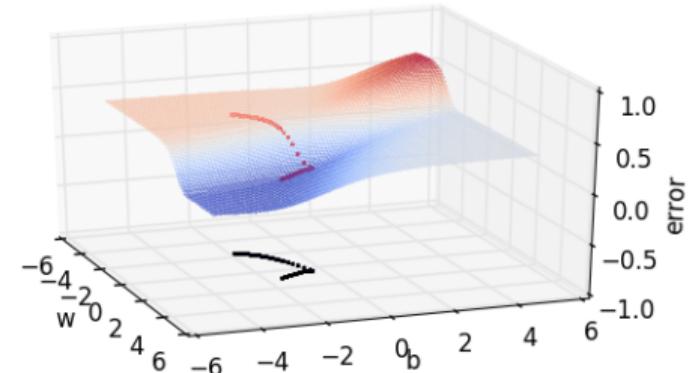
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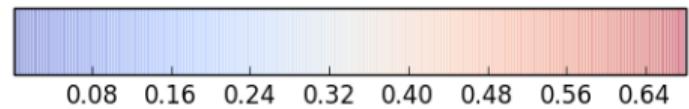
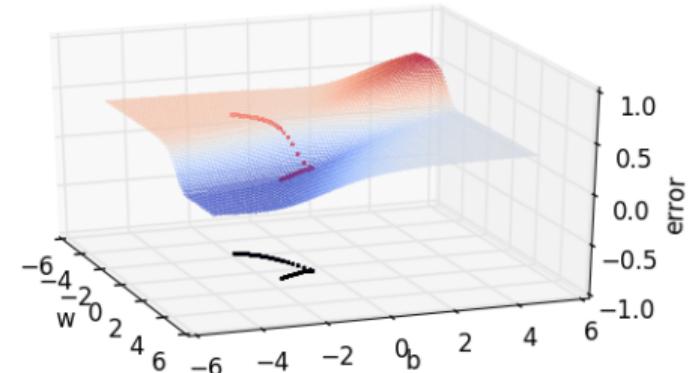
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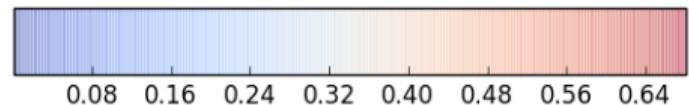
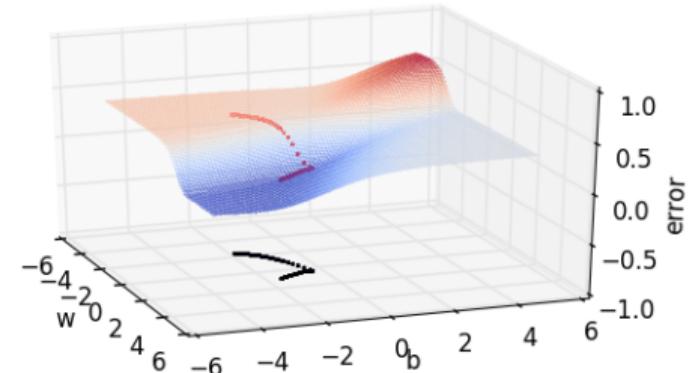
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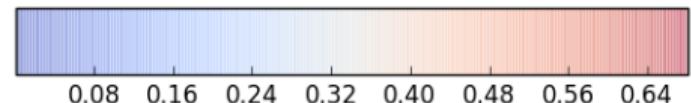
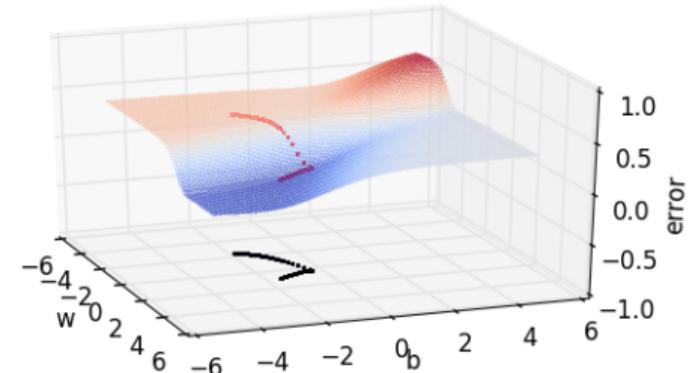
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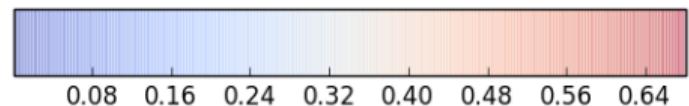
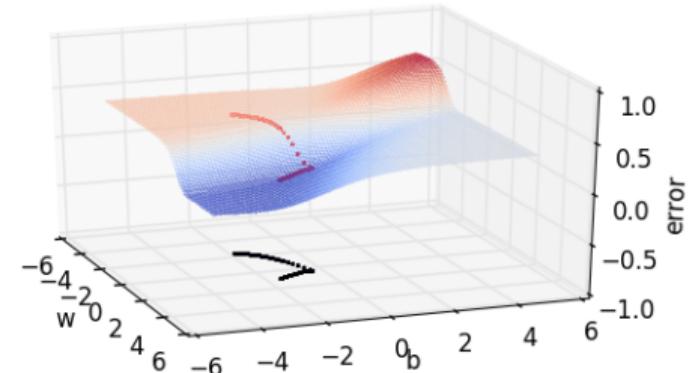
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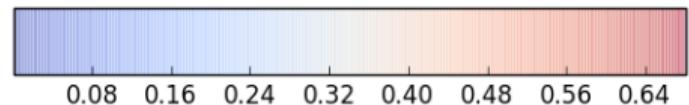
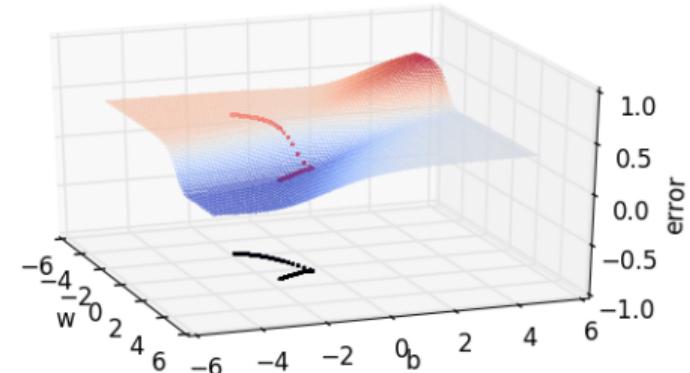
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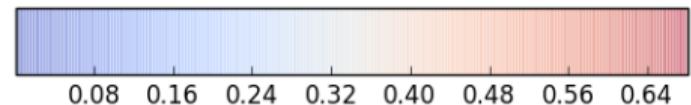
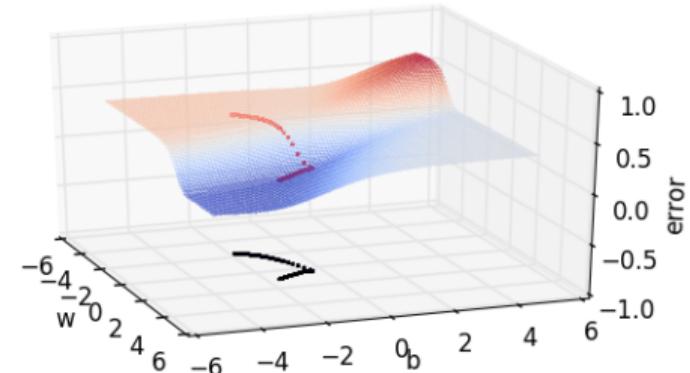
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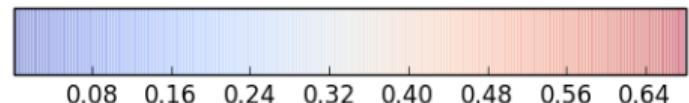
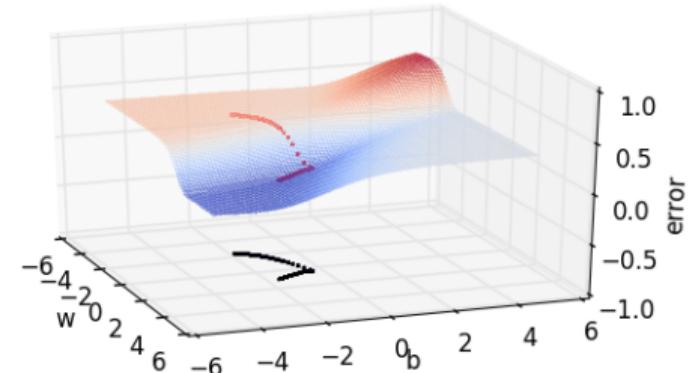
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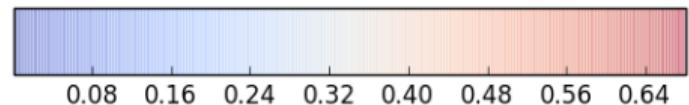
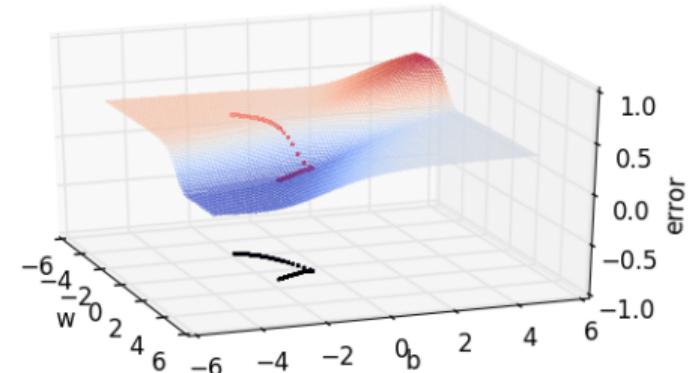
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Gradient descent on the error surface



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Y = [0.2, 0.9]

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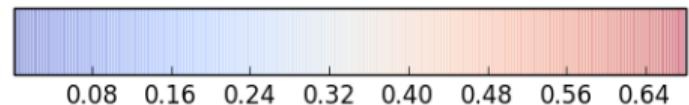
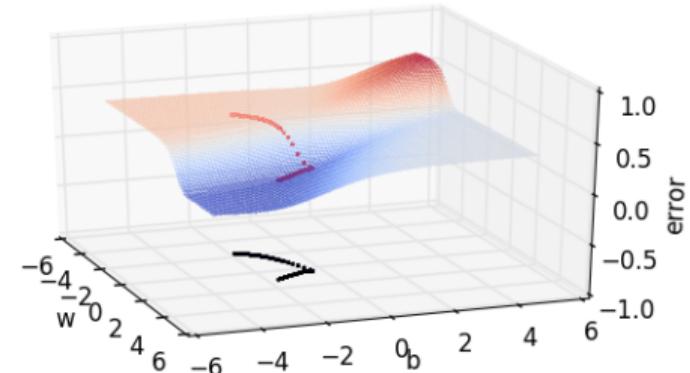
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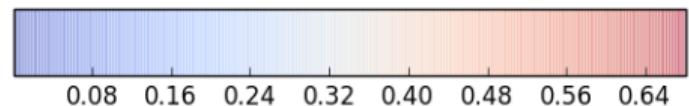
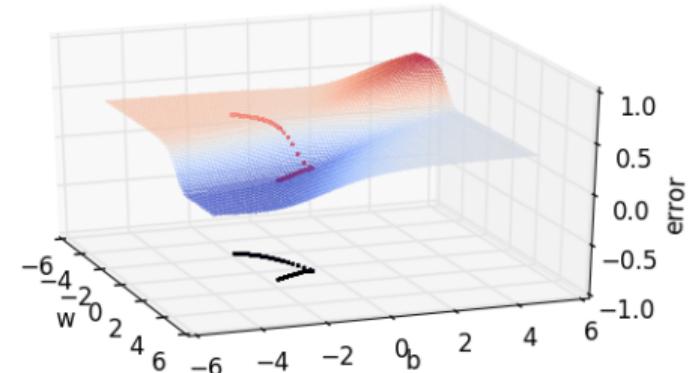
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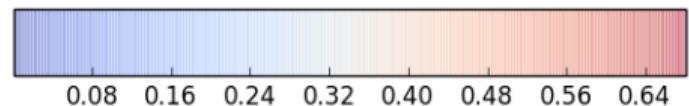
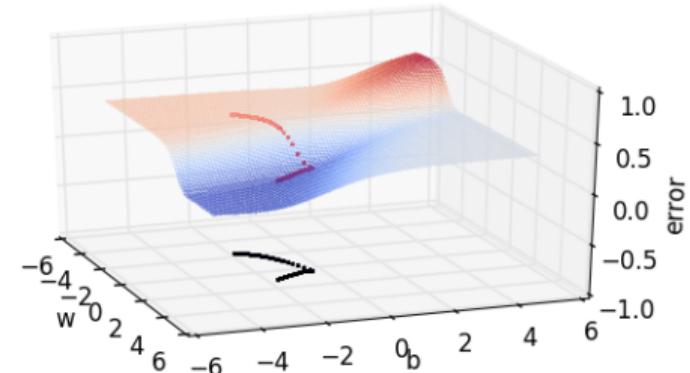
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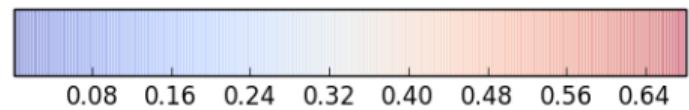
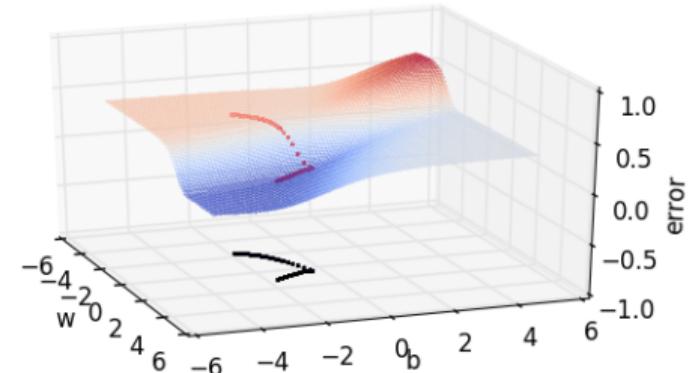
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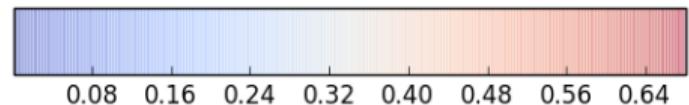
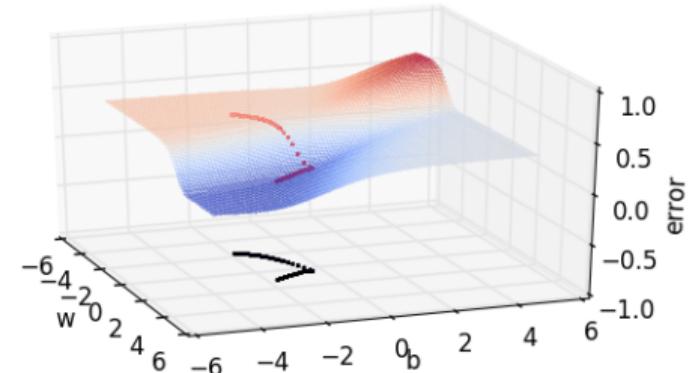
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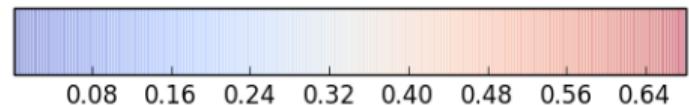
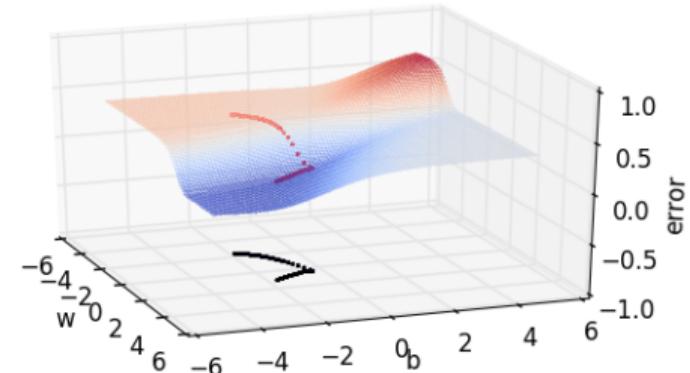
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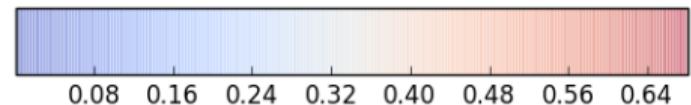
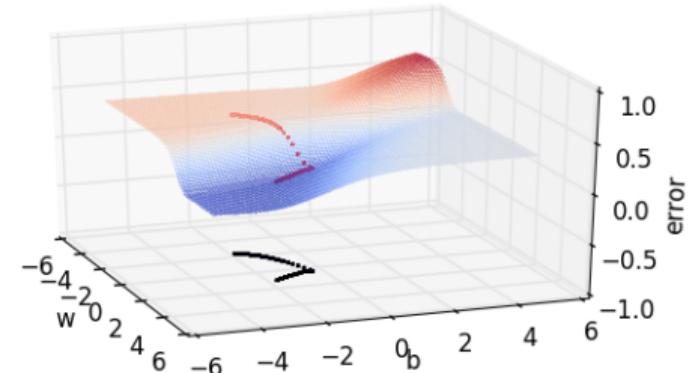
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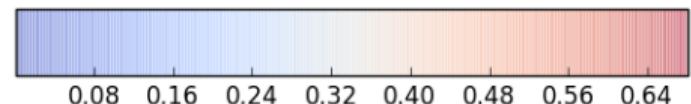
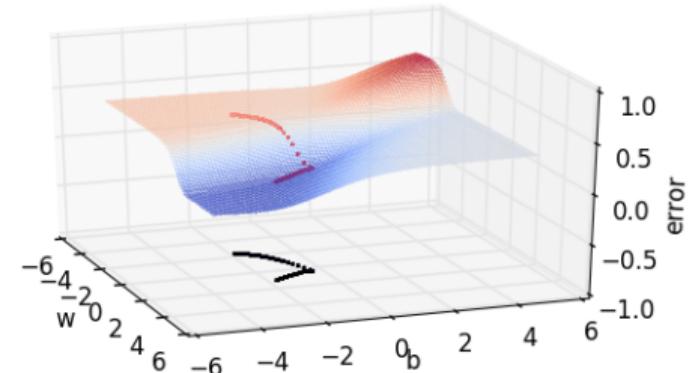
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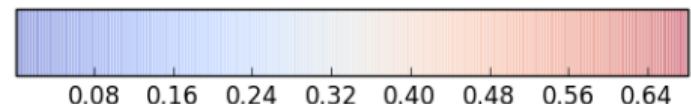
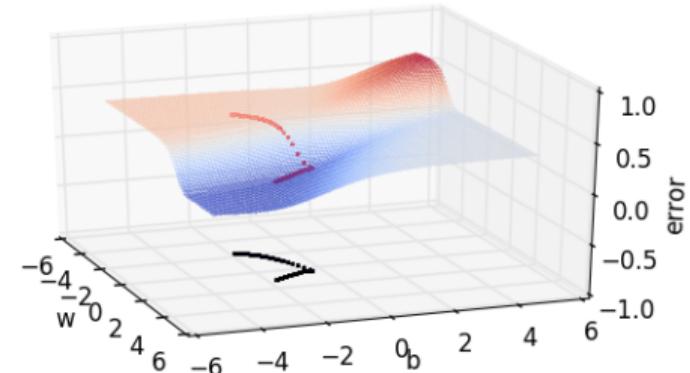
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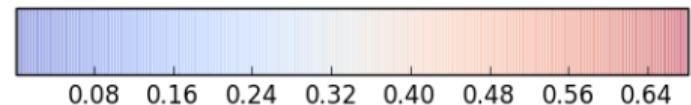
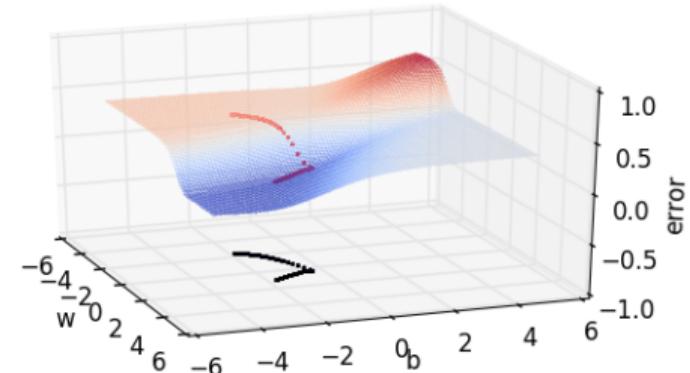
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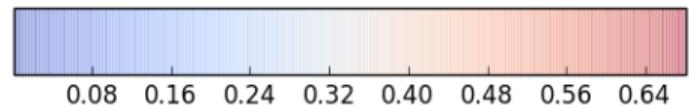
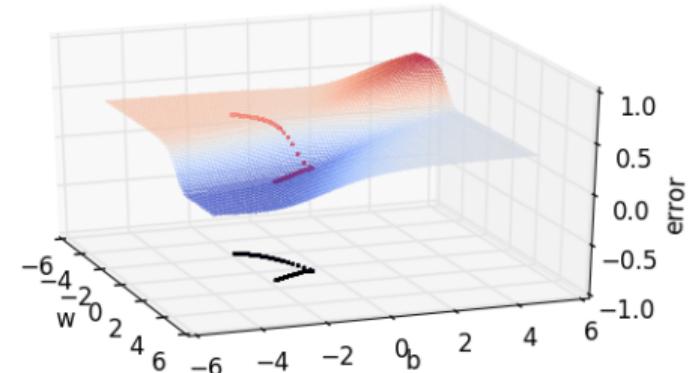
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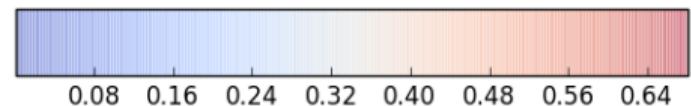
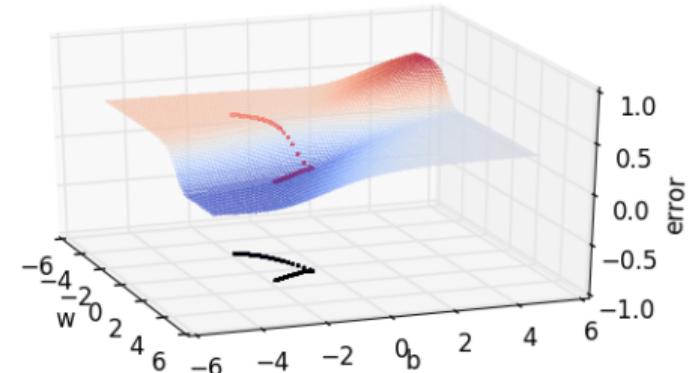
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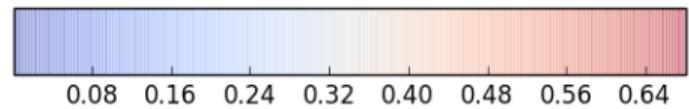
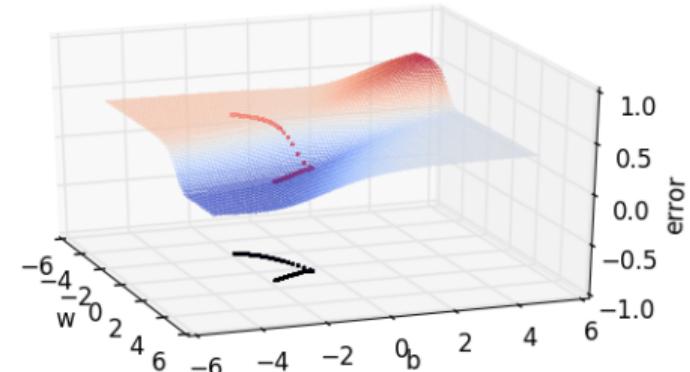
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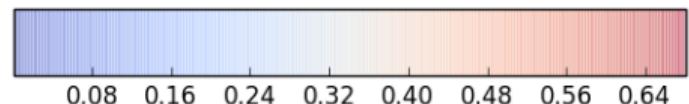
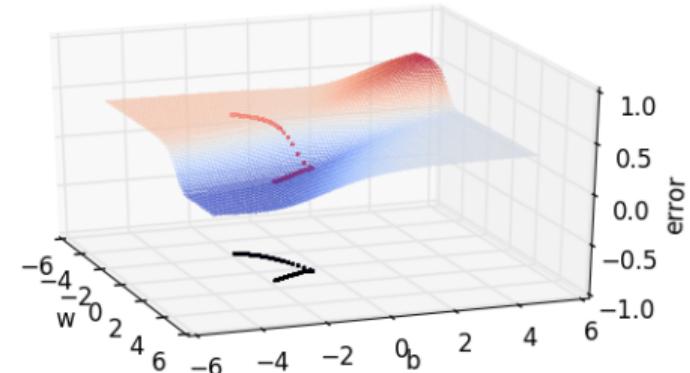
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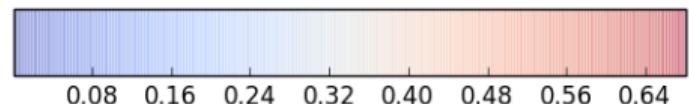
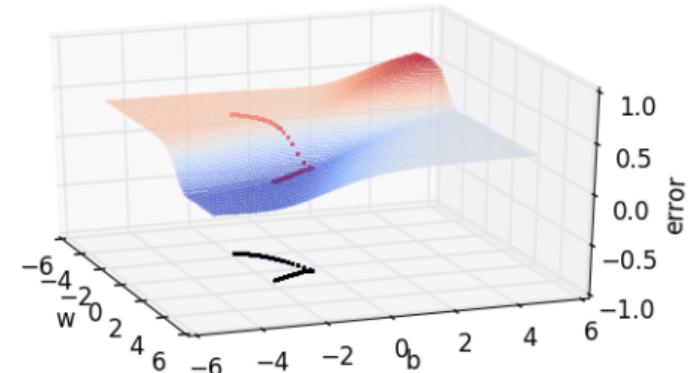
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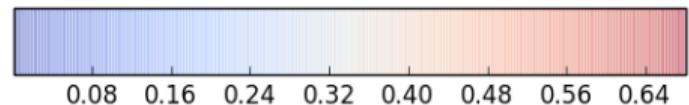
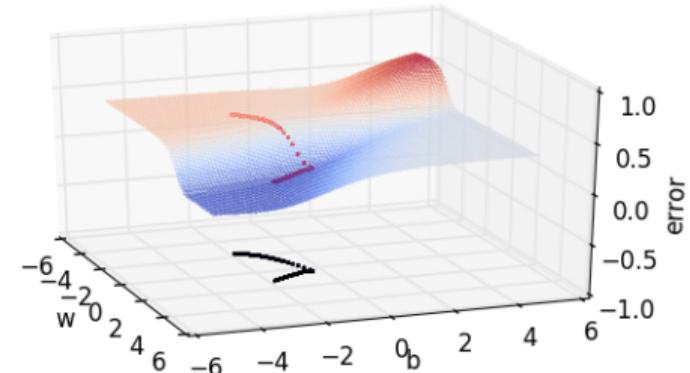
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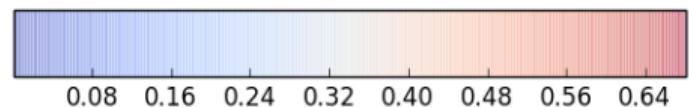
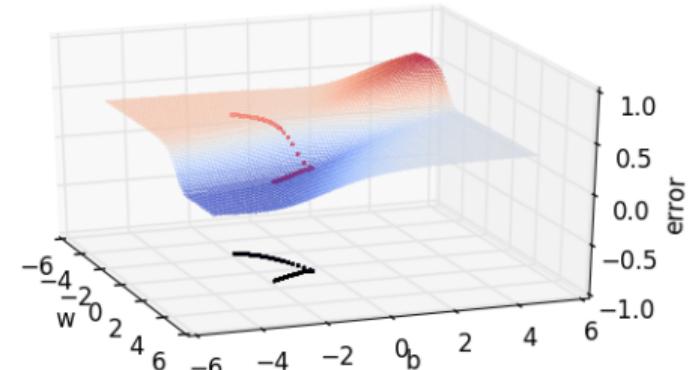
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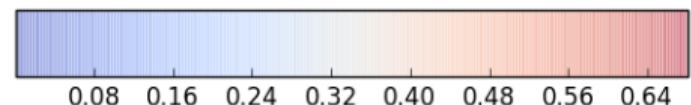
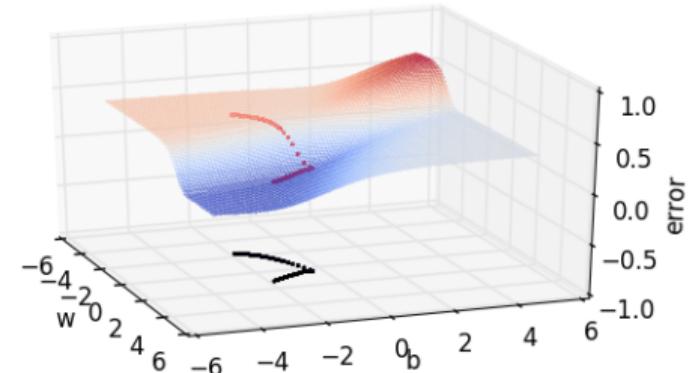
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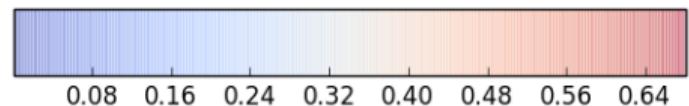
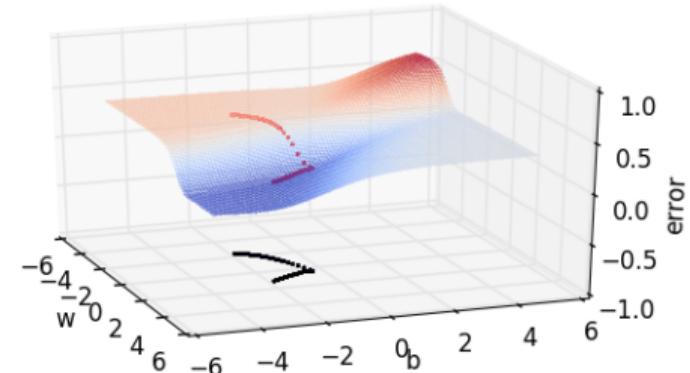
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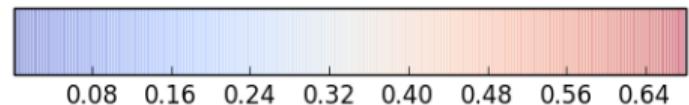
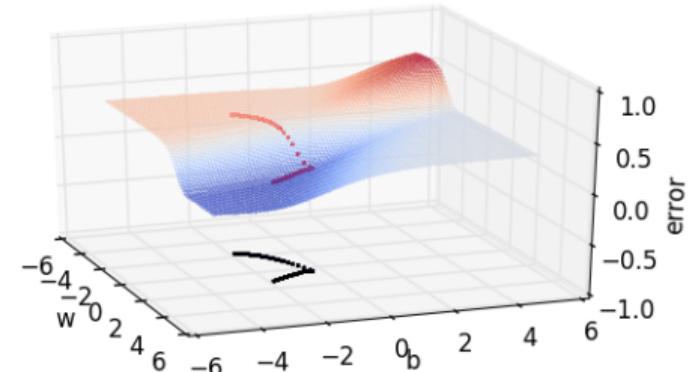
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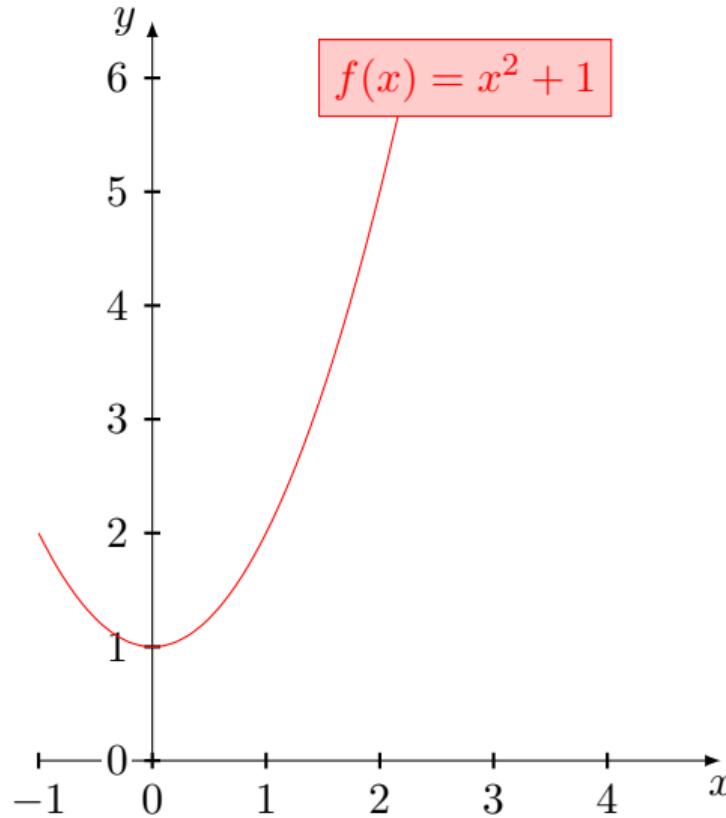
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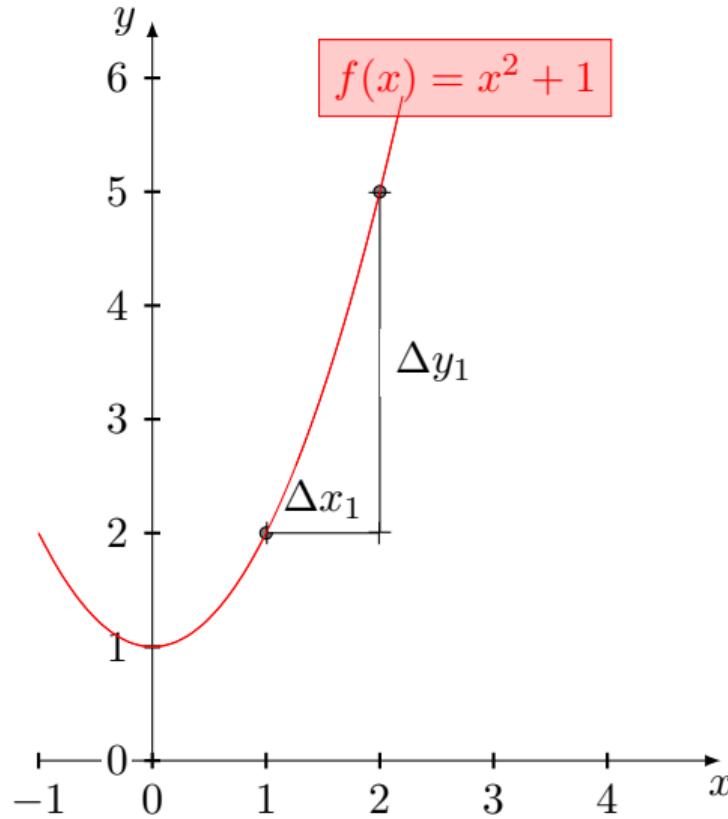
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Gradient descent on the error surface

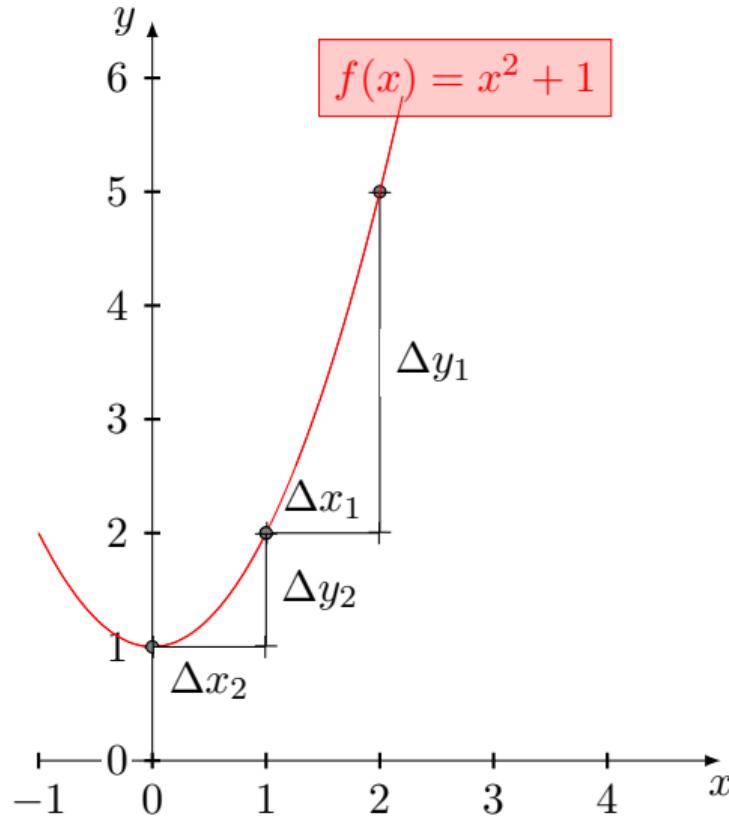




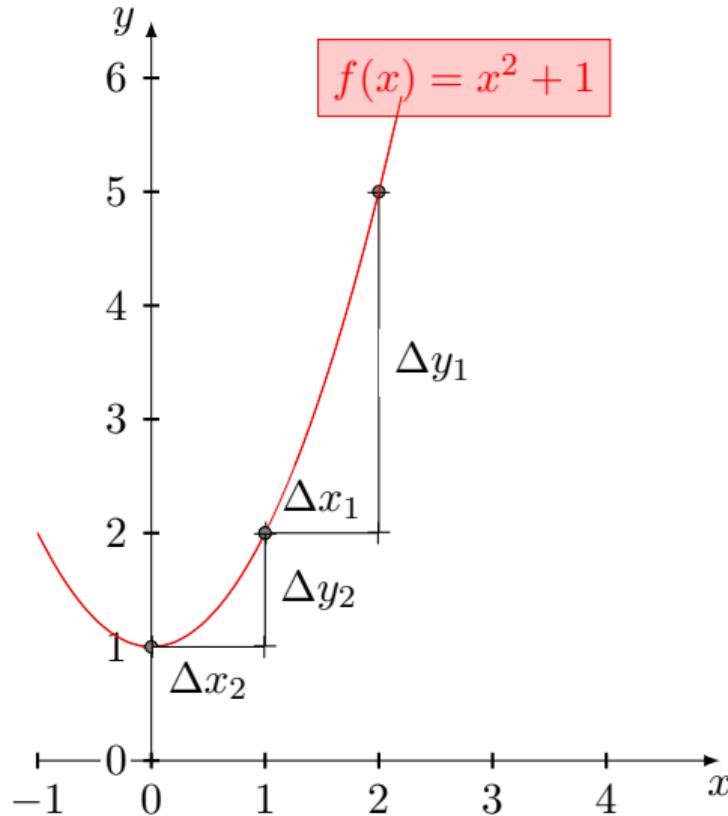
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- Recall that our weight updates are proportional to the gradient $w = w - \eta \nabla w$
- Hence in the areas where the curve is gentle the updates are small whereas in the areas where the curve is steep the updates are large