Q1).

Consider a 2-class classification task in the 3-dimensional space, where the two classes, ω_1 and ω_2, are modeled by Gaussian distributions with means $m_1 = [0, 0, 0]^T$ and $m_2 = [0.5, 0.5, 0.5]^T$, respectively. Assume the two classes to be equiprobable. The covariance matrix for both distributions is

$$S = \begin{bmatrix} 0.8 & 0.01 & 0.01 \\ 0.01 & 0.2 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

Given the point $x = [0.1, 0.5, 0.1]^T$, classify x (1) according to the Euclidean distance classifier and (2) according to the Mahalanobis distance classifier. Comment on the results.

Q2).

Consider the 2-dimensional pdf

$$p(x) = P_1 p(x|1) + P_2 p(x|2)$$

where $p(x|j), j = 1, 2$ are normal distributions with means $m_1 = [1, 1]^T$ and $m_2 = [3, 3]^T$ and covariance matrices

$$S_1 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}, \quad S_2 = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

with $\sigma_1^2 = 0.1$, $\sigma_2^2 = 0.2$, $\sigma_{12} = -0.08$, $\sigma^2 = 0.1$.

Generate and plot a set X consisting of $N = 500$ points that stem from $p(x)$ for (i) $P_1 = P_2 = 0.5$, and (ii) for $P_1 = 0.85$, $P_2 = 0.15$; and (iii) experiment by changing the parameters σ_1^2, σ_2^2, σ_{12}, σ^2 of the covariance matrices and the mixing probabilities P_1 and P_2.