

Q1] Given data:-

To apply perceptron learning algorithm we need to initiate weight vector & bias term.

Therefore, initial weight vector: $w = [1, 1]$

Bias term: $b = 0$, $x_1 + x_2 = 0$

learning rate: $\alpha = 1$

| data | x_1 | x_2 | class |
|------|-------|-------|-------|
| 1 | 1 | 1 | +1 |
| -1 | -1 | -1 | -1 |
| 0 | 0 | 0.5 | -1 |
| 0.1 | 0.1 | 0.5 | -1 |
| 0.2 | 0.2 | 0.2 | +1 |
| 0.9 | 0.9 | 0.5 | +1 |

$$y_{in} = w_1 x_1 + b = w_1 x_1 + w_2 x_2 + b$$

$$\text{Condition} \rightarrow y = \begin{cases} 1 & \text{if } y_{in} \geq 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$$

$$\begin{aligned} \Delta w_1 &= \alpha t x_1, & \Delta b &= \alpha t \\ \Delta w_2 &= \alpha t x_2 \end{aligned}$$

Now apply the perceptron algorithm to give data & count the no of update until all sample are classified correctly.

Iteration I

| x_1 | x_2 | class (t) | y_{in} | y | Δw_1 | Δw_2 | Δb | w_1, w_2, b |
|-------|-------|--------------|----------|-----|--------------|--------------|------------|---------------|
| 1 | 1 | +1 | 2 | +1 | 0 | 0 | 0 | 1, 1, 0 |
| -1 | -1 | -1 | -2 | -1 | 0 | 0 | 0 | 1, 1, 0 |
| 0 | 0.5 | -1 | 0.5 | +1 | 0 | -0.5 | -1 | 1, 0.5, -1 |
| 0.1 | 0.5 | -1 | -0.65 | -1 | 0 | 0 | 0 | 1, 0.5, -1 |
| 0.2 | 0.2 | +1 | -0.7 | -1 | 0.2 | 0.2 | 1 | 1.2, 0.7, 0 |
| 0.9 | 0.5 | +1 | 1.43 | +1 | 0 | 0 | 0 | 1.2, 0.7, 0 |

II] Iteration - II

| x_1 | x_2 | class | y_{in} | y | ΔW_1 | ΔW_2 | Δb | W_1 | W_2 | b |
|-------|-------|-------|----------|-----|--------------|--------------|------------|-------|-------|-----|
| 1 | 1 | +1 | 1.9 | +1 | 0 | 0 | 0 | 1.2 | 0.7 | 0 |
| -1 | -1 | -1 | 1.9 | -1 | 0 | 0 | 0 | 1.2 | 0.7 | 0 |
| 0 | 0.5 | -1 | 0.35 | +1 | 0 | -0.5 | -1 | 1.2 | 0.2 | -1 |
| 0.1 | 0.5 | -1 | -0.78 | -1 | 0 | 0 | 0 | 1.2 | 0.2 | -1 |
| 0.2 | 0.2 | +1 | -0.72 | -1 | 0.2 | 0.2 | 1 | 1.4 | 0.4 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |

III] Iteration - III

| x_1 | x_2 | class | y_{in} | y | ΔW_1 | ΔW_2 | Δb | W_1 | W_2 | b |
|-------|-------|-------|----------|-----|--------------|--------------|------------|-------|-------|-----|
| 1 | 1 | +1 | 1.8 | +1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |
| -1 | -1 | -1 | -1.8 | -1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |
| 0 | 0.5 | -1 | 0.2 | +1 | 0 | -0.5 | -1 | 1.4 | -0.1 | -1 |
| 0.1 | 0.5 | -1 | -0.81 | -1 | 0 | 0 | 0 | 1.4 | -0.1 | -1 |
| 0.2 | 0.2 | +1 | -0.74 | -1 | 0.2 | 0.2 | 1 | 1.6 | 0.1 | 0 |
| 0.9 | 0.5 | +1 | 1.49 | +1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |

IV] Iteration - IV

| x_1 | x_2 | class | y_{in} | y | ΔW_1 | ΔW_2 | Δb | W_1 | W_2 | b |
|-------|-------|-------|----------|-----|--------------|--------------|------------|-------|-------|-----|
| 1 | 1 | +1 | 1.7 | +1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |
| -1 | -1 | -1 | -1.7 | -1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |
| 0 | 0.5 | -1 | 0.05 | +1 | 0 | -0.5 | -1 | 1.6 | -0.4 | -1 |
| 0.1 | 0.5 | -1 | -1.04 | -1 | 0 | 0 | 0 | 1.6 | -0.4 | -1 |
| 0.2 | 0.2 | +1 | -0.76 | -1 | 0.2 | 0.2 | 1 | 1.8 | -0.2 | 0 |
| 0.9 | 0.5 | +1 | 1.52 | +1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |

V] Iteration - V

| x_1 | x_2 | class | y_{in} | y | ΔW_1 | ΔW_2 | Δb | W_1 | W_2 | b |
|-------|-------|-------|----------|-----|--------------|--------------|------------|-------|-------|-----|
| 1 | 1 | +1 | 1.6 | +1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| -1 | -1 | -1 | -1.6 | -1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| 0 | 0.5 | -1 | -0.1 | -1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| 0.1 | 0.5 | -1 | 0.08 | +1 | -0.1 | -0.5 | -1 | 1.7 | -0.7 | -1 |
| 0.2 | 0.2 | +1 | -0.8 | -1 | 0.2 | 0.2 | 1 | 1.9 | -0.5 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |

VI] Iteration VI

| x_1 | x_2 | class | y y_{oh} | y | ΔW_1 | ΔW_2 | Δb | w_1 | w_2 | b |
|-------|-------|-------|-----------------|-----|--------------|--------------|------------|-------|-------|-----|
| 1 | 1 | +1 | 1.4 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| -1 | -1 | -1 | -1.4 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0 | 0.5 | -1 | -0.25 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.1 | 0.5 | -1 | -0.06 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.2 | 0.2 | +1 | 0.28 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |

Task 1] → The perceptron learning algorithm converged in 6 iteration.

Task 2] → The final weight vector of the decision boundary is $w = [1.9, -0.5]$
 $\therefore w_1 x_1 + w_2 x_2 = 0 \Rightarrow 1.9x_1 - 0.5x_2 = 0$

We can see that $1.9x_1 - 0.5x_2 = 0$ line separate the two classes.

