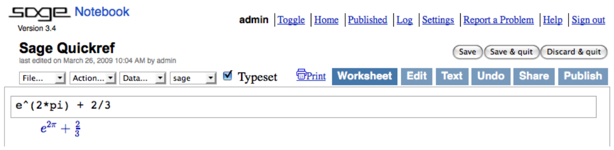
Sage Quick Reference

William Stein (based on work of P. Jipsen)

Notebook



Evaluate cell: ⟨shift-enter⟩

Evaluate cell creating new cell: ⟨alt-enter⟩

Split cell: ⟨control-;⟩

Join cells: ⟨control-backspace⟩

Insert math cell: click blue line between cells

Insert text/HTML cell: shift-click blue line between cells

Delete cell: delete content then backspace

Command line

com⟨tab⟩ complete *command*

`*`*bar*`*?` list command names containing "bar"

command`?`⟨tab⟩ shows documentation

command`??`⟨tab⟩ shows source code

`a.`⟨tab⟩ shows methods for object `a` (more: `dir(a)`)

`a._`⟨tab⟩ shows hidden methods for object `a`

`search_doc("`*string or regexp*`")` fulltext search of docs

`search_src("`*string or regexp*`")` search source code

`_` is previous output

Numbers

Integers: $\mathbf{Z}$ = `ZZ` e.g. `-2 -1 0 1 10^100`

Rationals: $\mathbf{Q}$ = `QQ` e.g. `1/2 1/1000 314/100 -2/1`

Reals: $\mathbf{R} \approx$ `RR` e.g. `.5 0.001 3.14 1.23e10000`

Complex: $\mathbf{C} \approx$ `CC` e.g. `CC(1,1) CC(2.5,-3)`

Double precision: `RDF` and `CDF` e.g. `CDF(2.1,3)`

Mod n: $\mathbf{Z}/n\mathbf{Z}$ = `Zmod` e.g. `Mod(2,3) Zmod(3)(2)`

Finite fields: $\mathbf{F}_q$ = `GF` e.g. `GF(3)(2) GF(9,"a").0`

Polynomials: $R[x,y]$ e.g. `S.<x,y>=QQ[] x+2*y^3`

Series: $R[[t]]$ e.g. `S.<t>=QQ[[]] 1/2+2*t+O(t^2)`

p-adic numbers: $\mathbf{Z}_p \approx$`Zp`, $\mathbf{Q}_p \approx$`Qp` e.g. `2+3*5+O(5^2)`

Algebraic closure: $\overline{\mathbf{Q}}$ = `QQbar` e.g. `QQbar(2^(1/5))`

Interval arithmetic: `RIF` e.g. `sage: RIF((1,1.00001))`

Number field: `R.<x>=QQ[];K.<a>=NumberField(x^3+x+1)`

Arithmetic

ab = `a*b` $\frac{a}{b}$ = `a/b` a^b = `a^b` $\sqrt{x}$ = `sqrt(x)`

$\sqrt[n]{x}$ = `x^(1/n)` $|x|$ = `abs(x)` $\log_b(x)$ = `log(x,b)`

Sums: $\sum_{i=k}^{n} f(i)$ = `sum(f(i) for i in (k..n))`

Products: $\prod_{i=k}^{n} f(i)$ = `prod(f(i) for i in (k..n))`

Constants and functions

Constants: π = `pi` e = `e` i = `i` ∞ = `oo`

ϕ = `golden_ratio` γ = `euler_gamma`

Approximate: `pi.n(digits=18)` = 3.14159265358979324

Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp` ...

Python function: `def f(x): return x^2`

Interactive functions

Put `@interact` before function (vars determine controls)

```
@interact
def f(n=[0..4], s=(1..5), c=Color("red")):
  var("x");show(plot(sin(n+x^s),-pi,pi,color=c))
```

Symbolic expressions

Define new symbolic variables: `var("t u v y z")`

Symbolic function: e.g. $f(x) = x^2$ `f(x)=x^2`

Relations: `f==g f<=g f>=g f<g f>g`

Solve $f = g$: `solve(f(x)==g(x), x)`

`solve([f(x,y)==0, g(x,y)==0], x,y)`

`factor(...)` `expand(...)` `(...).simplify_...`

`find_root(f(x), a, b)` find $x \in [a,b]$ s.t. $f(x) \approx 0$

Calculus

$\lim_{x\to a} f(x)$ = `limit(f(x), x=a)`

$\frac{d}{dx}(f(x))$ = `diff(f(x),x)`

$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`

`diff` = `differentiate` = `derivative`

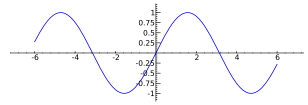
$\int f(x)dx$ = `integral(f(x),x)`

$\int_a^b f(x)dx$ = `integral(f(x),x,a,b)`

$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)`

Taylor polynomial, deg n about a: `taylor(f(x),x,`a`,`n`)`

2D graphics



`line([(`x_1`,`y_1`),...,(`x_n`,`y_n`)],`*options*`)`

`polygon([(`x_1`,`y_1`),...,(`x_n`,`y_n`)],`*options*`)`

`circle((`x`,`y`),`r`,`*options*`)`

`text("txt",(`x`,`y`),`*options*`)`

options as in `plot.options`, e.g. `thickness=`*pixel*,

`rgbcolor=(`r`,`g`,`b`)`, `hue=`h where $0 \le r,b,g,h \le 1$

`show(`*graphic*, *options*`)`

use `figsize=[w,h]` to adjust size

use `aspect_ratio=`*number* to adjust aspect ratio

`plot(f(`x`),(`x, $x_{\min}$, $x_{\max}$`),`*options*`)`

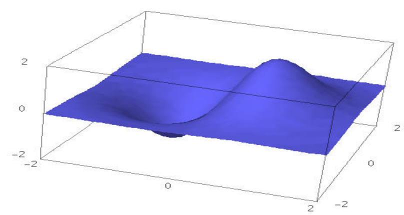
`parametric_plot((f(`t`),g(`t`)),(`t, $t_{\min}$, $t_{\max}$`),`*options*`)`

`polar_plot(f(`t`),(`t, $t_{\min}$, $t_{\max}$`),`*options*`)`

combine: `circle((1,1),1)+line([(0,0),(2,2)])`

`animate(`*list of graphics, options*`).show(delay=20)`

3D graphics



`line3d([(`x_1`,`y_1`,`z_1`),...,(`x_n`,`y_n`,`z_n`)],`*options*`)`

`sphere((`x`,`y`,`z`),`r`,`*options*`)`

`text3d("txt", (`x`,`y`,`z`),` *options*`)`

`tetrahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`cube((`x`,`y`,`z`),`*size*`,`*options*`)`

`octahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`dodecahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`icosahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`plot3d(f(`x,y`),(`x, x_b, x_e`), (`y, y_b, y_e`),`*options*`)`

`parametric_plot3d((f,g,h),(`t, t_b, t_e`),`*options*`)`

`parametric_plot3d((f(`u,v`),g(`u,v`),h(`u,v`)),`
`(`u, u_b, u_e`),(`v, v_b, v_e`),`*options*`)`

options: `aspect_ratio=`$[1,1,1]$, `color="red"`
`opacity=0.5, figsize=6, viewer="tachyon"`

Discrete math

$\lfloor x \rfloor$ = `floor(x)` $\lceil x \rceil$ = `ceil(x)`

Remainder of n divided by k = `n%k` $k|n$ iff `n%k==0`

$n!$ = `factorial(n)` $\binom{x}{m}$ = `binomial(x,m)`

$\phi(n)$ = `euler_phi(n)`

Strings: e.g. `s = "Hello" = "He"+'llo'`

`s[0]="H"`   `s[-1]="o"`   `s[1:3]="el"`   `s[3:]="lo"`

Lists: e.g. `[1,"Hello",x] = []+[1,"Hello"]+[x]`

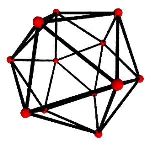
Tuples: e.g. `(1,"Hello",x)`   (immutable)

Sets: e.g. $\{1, 2, 1, a\}$ = `Set([1,2,1,"a"])` $(= \{1, 2, a\})$

List comprehension ≈ set builder notation, e.g.

$\{f(x) : x \in X, x > 0\}$ = `Set([f(x) for x in X if x>0])`

Graph theory



Graph: `G = Graph({0:[1,2,3], 2:[4]})`

Directed Graph: `DiGraph(`*dictionary*`)`

Graph families: `graphs.`⟨tab⟩

Invariants: `G.chromatic_polynomial()`, `G.is_planar()`

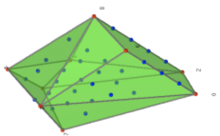
Paths: `G.shortest_path()`

Visualize: `G.plot()`, `G.plot3d()`

Automorphisms: `G.automorphism_group()`,

`G1.is_isomorphic(G2)`, `G1.is_subgraph(G2)`

Combinatorics



Integer sequences: `sloane_find(`*list*`)`, `sloane.`⟨tab⟩

Partitions: `P=Partitions(`n`)`   `P.count()`

Combinations: `C=Combinations(`*list*`)`   `C.list()`

Cartesian product: `CartesianProduct(P,C)`

Tableau: `Tableau([[1,2,3],[4,5]])`

Words: `W=Words("abc"); W("aabca")`

Posets: `Poset([[1,2],[4],[3],[4],[]])`

Root systems: `RootSystem(["A",3])`

Crystals: `CrystalOfTableaux(["A",3], shape=[3,2])`

Lattice Polytopes: `A=random_matrix(ZZ,3,6,x=7)`

`L=LatticePolytope(A)`   `L.npoints()`   `L.plot3d()`

Matrix algebra

$\begin{pmatrix}1\\2\end{pmatrix}$ = `vector([1,2])`

$\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}$ = `matrix(QQ,[[1,2],[3,4]], sparse=False)`

$\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}$ = `matrix(QQ,2,3,[1,2,3, 4,5,6])`

$\begin{vmatrix}1 & 2\\3 & 4\end{vmatrix}$ = `det(matrix(QQ,[[1,2],[3,4]]))`

Av = `A*v` A^{-1} = `A^-1` A^t = `A.transpose()`

Solve $Ax = v$: `A\v` or `A.solve_right(v)`

Solve $xA = v$: `A.solve_left(v)`

Reduced row echelon form: `A.echelon_form()`

Rank and nullity: `A.rank()`   `A.nullity()`

Hessenberg form: `A.hessenberg_form()`

Characteristic polynomial: `A.charpoly()`

Eigenvalues: `A.eigenvalues()`

Eigenvectors: `A.eigenvectors_right()` (also left)

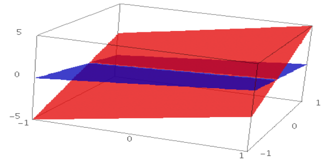
Gram-Schmidt: `A.gram_schmidt()`

Visualize: `A.plot()`

LLL reduction: `matrix(ZZ,...).LLL()`

Hermite form: `matrix(ZZ,...).hermite_form()`

Linear algebra



Vector space K^n = `K^n` e.g. `QQ^3`   `RR^2`   `CC^4`

Subspace: `span(vectors, `*field*` )`

E.g., `span([[1,2,3], [2,3,5]], QQ)`

Kernel: `A.right_kernel()` (also left)

Sum and intersection: `V + W` and `V.intersection(W)`

Basis: `V.basis()`

Basis matrix: `V.basis_matrix()`

Restrict matrix to subspace: `A.restrict(V)`

Vector in terms of basis: `V.coordinates(`*vector*`)`

Numerical mathematics

Packages: `import numpy, scipy, cvxopt`

Minimization: `var("x y z")`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Number theory

Primes: `prime_range(n,m)`, `is_prime`, `next_prime`

Factor: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`

Kronecker symbol: $\left(\frac{a}{b}\right)$ = `kronecker_symbol(`a,b`)`

Continued fractions: `continued_fraction(x)`

Bernoulli numbers: `bernoulli(n)`, `bernoulli_mod_p(p)`

Elliptic curves: `EllipticCurve(`$[a_1, a_2, a_3, a_4, a_6]$`)`

Dirichlet characters: `DirichletGroup(`N`)`

Modular forms: `ModularForms(`*level*, *weight*`)`

Modular symbols: `ModularSymbols(`*level*, *weight*, *sign*`)`

Brandt modules: `BrandtModule(`*level*, *weight*`)`

Modular abelian varieties: `J0(`N`)`, `J1(`N`)`

Group theory

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`

`SymmetricGroup(`n`)`, `AlternatingGroup(`n`)`

Abelian groups: `AbelianGroup([3,15])`

Matrix groups: `GL, SL, Sp, SU, GU, SO, GO`

Functions: `G.sylow_subgroup(p)`, `G.character_table()`,

`G.normal_subgroups()`, `G.cayley_graph()`

Noncommutative rings

Quaternions: `Q.<i,j,k> = QuaternionAlgebra(a,b)`

Free algebra: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

Python modules

`import` *module_name*

`module_name.`⟨tab⟩ and `help(module_name)`

Profiling and debugging

`time` *command*: show timing information

`timeit("command")`: accurately time command

`t = cputime(); cputime(t)`: elapsed CPU time

`t = walltime(); walltime(t)`: elapsed wall time

`%pdb`: turn on interactive debugger (command line only)

`%prun command`: profile command (command line only)

Sage Quick Reference (Basic Math)

Peter Jipsen, version 1.1

latest version at `wiki.sagemath.org/quickref`

Aim: map standard math notation to Sage commands

Notebook (and commandline)

Evaluate cell: ⟨shift-enter⟩

com⟨tab⟩ tries to complete *command*

command`?`⟨tab⟩ shows documentation

command`??`⟨tab⟩ shows source

`a.`⟨tab⟩ shows all methods for object `a` (more: `dir(a)`)

`search_doc('`*string or regexp*`')` shows links to docs

`search_src('`*string or regexp*`')` shows links to source

`lprint()` toggle LaTeX output mode

`version()` print version of Sage

Insert cell: click on blue line between cells

Delete cell: delete content then backspace

Numerical types

Integers: $\mathbb{Z}$ = `ZZ` e.g. `-2 -1 0 1 10^100`

Rationals: $\mathbb{Q}$ = `QQ` e.g. `1/2 1/1000 314/100 -42`

Decimals: $\mathbb{R} \approx$ `RR` e.g. `.5 0.001 3.14 -42.`

Complex: $\mathbb{C} \approx$ `CC` e.g. `1+i 2.5-3*i`

Basic constants and functions

Constants: π = `pi` e = `e` i = `i` ∞ = `oo`

Approximate: `pi.n(digits=18)` = 3.141592653589793 24

Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp`

ab = `a*b` $\frac{a}{b}$ = `a/b` a^b = `a^b` $\sqrt{x}$ = `sqrt(x)`

$\sqrt[n]{x}$ = `x^(1/n)` $|x|$ = `abs(x)` $\log_b(x)$ = `log(x,b)`

Symbolic variables: e.g. `t,u,v,y,z = var('t u v y z')`

Define function: e.g. $f(x) = x^2$ `f(x)=x^2`

or `f=lambda x: x^2` or `def f(x): return x^2`

Operations on expressions

`factor(...)` `expand(...)` `(...).simplify_...`

Symbolic equations: `f(x)==g(x)`

`_` is previous output

`_+a _-a _*a _/a` manipulates equation

Solve $f(x) = g(x)$: `solve(f(x)==g(x),x)`

`solve([f(x,y)==0, g(x,y)==0], x,y)`

`find_root(f(x), a, b)` find $x \in [a,b]$ s.t. $f(x) \approx 0$

$\sum_{i=k}^{n} f(i)$ = `sum([f(i) for i in [k..n]])`

$\prod_{i=k}^{n} f(i)$ = `prod([f(i) for i in [k..n]])`

Calculus

$\lim_{x\to a} f(x)$ = `limit(f(x), x=a)`

$\lim_{x\to a^-} f(x)$ = `limit(f(x), x=a, dir='minus')`

$\lim_{x\to a^+} f(x)$ = `limit(f(x), x=a, dir='plus')`

$\frac{d}{dx}(f(x))$ = `diff(f(x),x)`

$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`

`diff` = `differentiate` = `derivative`

$\int f(x)dx$ = `integral(f(x),x)`

`integral` = `integrate`

$\int_a^b f(x)dx$ = `integral(f(x),x,a,b)`

Taylor polynomial, deg n about a: `taylor(f(x),x,`a`,`n`)`

2d graphics

`line([(`x_1`,`y_1`),...,(`x_n`,`y_n`)],`*options*`)`

`polygon([(`x_1`,`y_1`),...,(`x_n`,`y_n`)],`*options*`)`

`circle((`x`,`y`),`r`,`*options*`)`

`text("txt",(`x`,`y`),`*options*`)`

options as in `plot.options`, e.g. `thickness=`*pixel*,

`rgbcolor=(`r`,`g`,`b`)`, `hue=`h where $0 \le r, b, g, h \le 1$

use option `figsize=[w,h]` to adjust aspect ratio

`plot(f(`x`),`$x_{\min}$`,`$x_{\max}$`,`*options*`)`

`parametric_plot((f(`t`),g(`t`)),`$t_{\min}$`,`$t_{\max}$`,`*options*`)`

`polar_plot(f(`t`),`$t_{\min}$`,`$t_{\max}$`,`*options*`)`

combine graphs: `circle((1,1),1)+line([(0,0),(2,2)])`

`animate(`*list of graphics objects, options*`).show(delay=20)`

3d graphics

`line3d([(`x_1`,`y_1`,`z_1`),...,(`x_n`,`y_n`,`z_n`)],`*options*`)`

`sphere((`x`,`y`,`z`),`r`,`*options*`)`

`tetrahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`cube((`x`,`y`,`z`),`*size*`,`*options*`)`

`octahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`dodecahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

`icosahedron((`x`,`y`,`z`),`*size*`,`*options*`)`

options e.g. `aspect_ratio=`$[1,1,1]$ `color='red' opacity`

`plot3d(f(`x,y`),[`x_b`,`x_e`],[`y_b`,`y_e`],`*options*`)`

add option `plot_points=[`m,n`]` or use `plot3d_adaptive`

`parametric_plot3d((f(`t`),g(`t`),h(`t`)),[`t_b`,`t_e`],`*options*`)`

`parametric_plot3d((f(`u,v`),g(`u,v`),h(`u,v`)),`
`[`u_b`,`u_e`],[`v_b`,`v_e`],`*options*`)`

use + to combine graphics objects

Discrete math

$\lfloor x \rfloor$ = `floor(x)` $\lceil x \rceil$ = `ceil(x)`

Remainder of n divided by k = `n%k` $k|n$ iff `n%k==0`

$n!$ = `factorial(n)` $\binom{x}{m}$ = `binomial(x,m)`

ϕ = `golden_ratio` $\phi(n)$ = `euler_phi(`n`)`

Strings: e.g. `s = 'Hello' = "Hello" = ""+"He"+'llo'`

`s[0]='H'` `s[-1]='o'` `s[1:3]='el'` `s[3:]='lo'`

Lists: e.g. `[1,'Hello',x] = []+[1,'Hello']+[x]`

Tuples: e.g. `(1,'Hello',x)` (immutable)

Sets: e.g. $\{1, 2, 1, a\}$ = `Set([1,2,1,'a'])` $(= \{1, 2, a\})$

List comprehension $\approx$ set builder notation, e.g.

$\{f(x) : x \in X, x > 0\}$ = `Set([f(x) for x in X if x>0])`

Linear algebra

$\begin{pmatrix}1\\2\end{pmatrix}$ = `vector([1,2])`

$\begin{pmatrix}1&2\\3&4\end{pmatrix}$ = `matrix([[1,2],[3,4]])`

$\begin{vmatrix}1&2\\3&4\end{vmatrix}$ = `det(matrix([[1,2],[3,4]]))`

Av = `A*v` A^{-1} = `A^-1` A^t = `A.transpose()`

methods: `nrows() ncols() nullity() rank() trace()`...

Sage modules and packages

`from` *module_name* `import *` (many preloaded)

e.g. `calculus coding combinat crypto functions games geometry graphs groups logic matrix numerical plot probability rings sets stats`

`sage.`*module_name*`.all.`⟨tab⟩ shows exported commands

Std packages: Maxima GP/PARI GAP Singular R Shell ...

Opt packages: Biopython Fricas(Axiom) Gnuplot Kash ...

`%`*package_name* then use package command syntax

`time` *command* to show timing information

Sage Quick Reference: Calculus

William Stein
Sage Version 3.4
http://wiki.sagemath.org/quickref

Builtin constants and functions

Constants: $\pi =$ `pi` $e =$ `e` $i =$ `I` $=$ `i`

$\infty =$ `oo` $=$ `infinity` NaN=`NaN` log(2) =`log2`

$\phi =$ `golden_ratio` $\gamma =$ `euler_gamma`

$0.915 \approx$ `catalan` $2.685 \approx$ `khinchin`

$0.660 \approx$ `twinprime` $0.261 \approx$ `merten` $1.902 \approx$ `brun`

Approximate: `pi.n(digits=18)` $= 3.14159265358979324$

Builtin functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp` ...

Defining symbolic expressions

Create symbolic variables:

`var("t u theta")` or `var("t,u,theta")`

Use `*` for multiplication and `^` for exponentiation:

$2x^5 + \sqrt{2}$ = `2*x^5 + sqrt(2)`

Typeset: `show(2*theta^5 + sqrt(2))` $\longrightarrow 2\theta^5 + \sqrt{2}$

Symbolic functions

Symbolic function (can integrate, differentiate, etc.):

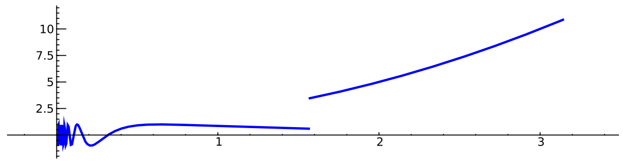
`f(a,b,theta) = a + b*theta^2`

Also, a "formal" function of theta:

`f = function('f',theta)`

Piecewise symbolic functions:

`Piecewise([[(0,pi/2),sin(1/x)],[(pi/2,pi),x^2+1]])`



Python functions

Defining:

```
def f(a, b, theta=1):
    c = a + b*theta^2
    return c
```

Inline functions:

```
    f = lambda a, b, theta = 1: a + b*theta^2
```

Simplifying and expanding

Below f must be symbolic (so **not** a Python function):

Simplify: `f.simplify_exp()`, `f.simplify_full()`, `f.simplify_log()`, `f.simplify_radical()`, `f.simplify_rational()`, `f.simplify_trig()`

Expand: `f.expand()`, `f.expand_rational()`

Equations

Relations: $f = g$: `f == g`, $f \neq g$: `f != g`,
$f \leq g$: `f <= g`, $f \geq g$: `f >= g`,
$f < g$: `f < g`, $f > g$: `f > g`

Solve $f = g$: `solve(f == g, x)`, and
`solve([f == 0, g == 0], x,y)`

`solve([x^2+y^2==1, (x-1)^2+y^2==1],x,y)`

Solutions:

`S = solve(x^2+x+1==0, x, solution_dict=True)`

`S[0]["x"]` `S[1]["x"]` are the solutions

Exact roots: `(x^3+2*x+1).roots(x)`

Real roots: `(x^3+2*x+1).roots(x,ring=RR)`

Complex roots: `(x^3+2*x+1).roots(x,ring=CC)`

Factorization

Factored form: `(x^3-y^3).factor()`

List of (factor, exponent) pairs:

`(x^3-y^3).factor_list()`

Limits

$\lim_{x \to a} f(x) =$ `limit(f(x), x=a)`

`limit(sin(x)/x, x=0)`

$\lim_{x \to a^+} f(x) =$ `limit(f(x), x=a, dir='plus')`

`limit(1/x, x=0, dir='plus')`

$\lim_{x \to a^-} f(x) =$ `limit(f(x), x=a, dir='minus')`

`limit(1/x, x=0, dir='minus')`

Derivatives

$\frac{d}{dx}(f(x)) =$ `diff(f(x),x)` = `f.diff(x)`

$\frac{\partial}{\partial x}(f(x,y)) =$ `diff(f(x,y),x)`

`diff` = `differentiate` = `derivative`

`diff(x*y + sin(x^2) + e^(-x), x)`

Integrals

$\int f(x)dx =$ `integral(f,x)` = `f.integrate(x)`

`integral(x*cos(x^2), x)`

$\int_a^b f(x)dx =$ `integral(f,x,a,b)`

`integral(x*cos(x^2), x, 0, sqrt(pi))`

$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)[0]`

`numerical_integral(x*cos(x^2),0,1)[0]`

`assume(...)`: use if integration asks a question

`assume(x>0)`

Taylor and partial fraction expansion

Taylor polynomial, deg n about a:

`taylor(f,x,a,n)`$\approx c_0 + c_1(x-a) + \cdots + c_n(x-a)^n$

`taylor(sqrt(x+1), x, 0, 5)`

Partial fraction:

`(x^2/(x+1)^3).partial_fraction()`

Numerical roots and optimization

Numerical root: `f.find_root(a, b, x)`

`(x^2 - 2).find_root(1,2,x)`

Maximize: find (m, x_0) with $f(x_0) = m$ maximal

`f.find_maximum_on_interval(a, b, x)`

Minimize: find (m, x_0) with $f(x_0) = m$ minimal

`f.find_minimum_on_interval(a, b, x)`

Minimization: `minimize(f,` *start_point*`)`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Multivariable calculus

Gradient: `f.gradient()` or `f.gradient(`*vars*`)`

`(x^2+y^2).gradient([x,y])`

Hessian: `f.hessian()`

`(x^2+y^2).hessian()`

Jacobian matrix: `jacobian(f,` *vars*`)`

`jacobian(x^2 - 2*x*y, (x,y))`

Summing infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Not yet implemented, but you can use Maxima:

`s = 'sum (1/n^2,n,1,inf), simpsum'`

`SR(sage.calculus.calculus.maxima(s))` $\longrightarrow \pi^2/6$

Sage Quick Reference: Linear Algebra

Robert A. Beezer

Sage Version 4.8

http://wiki.sagemath.org/quickref

Based on work by Peter Jipsen, William Stein

Vector Constructions

Caution: First entry of a vector is numbered 0

`u = vector(QQ, [1, 3/2, -1])` length 3 over rationals

`v = vector(QQ, {2:4, 95:4, 210:0})`

  211 entries, nonzero in entry 4 and entry 95, sparse

Vector Operations

`u = vector(QQ, [1, 3/2, -1])`

`v = vector(ZZ, [1, 8, -2])`

`2*u - 3*v` linear combination

`u.dot_product(v)`

`u.cross_product(v)` order: `u`×`v`

`u.inner_product(v)` inner product matrix from parent

`u.pairwise_product(v)` vector as a result

`u.norm() == u.norm(2)` Euclidean norm

`u.norm(1)` sum of entries

`u.norm(Infinity)` maximum entry

`A.gram_schmidt()` converts the rows of matrix `A`

Matrix Constructions

Caution: Row, column numbering begins at 0

`A = matrix(ZZ, [[1,2],[3,4],[5,6]])`

  3×2 over the integers

`B = matrix(QQ, 2, [1,2,3,4,5,6])`

  2 rows from a list, so 2×3 over rationals

`C = matrix(CDF, 2, 2, [[5*I, 4*I], [I, 6]])`

  complex entries, 53-bit precision

`Z = matrix(QQ, 2, 2, 0)` zero matrix

`D = matrix(QQ, 2, 2, 8)`

  diagonal entries all 8, other entries zero

`E = block_matrix([[P,0],[1,R]])`, very flexible input

`II = identity_matrix(5)` 5×5 identity matrix

  $\mathrm{I} = \sqrt{-1}$, do not overwrite with matrix name

`J = jordan_block(-2,3)`

  3×3 matrix, -2 on diagonal, 1's on super-diagonal

`var('x y z'); K = matrix(SR, [[x,y+z],[0,x^2*z]])`

  symbolic expressions live in the ring `SR`

`L = matrix(ZZ, 20, 80, {(5,9):30, (15,77):-6})`

  20×80, two non-zero entries, sparse representation

Matrix Multiplication

`u = vector(QQ, [1,2,3]),  v = vector(QQ, [1,2])`

`A = matrix(QQ, [[1,2,3],[4,5,6]])`

`B = matrix(QQ, [[1,2],[3,4]])`

`u*A,  A*v,  B*A,  B^6,  B^(-3)` all possible

`B.iterates(v, 6)` produces $vB^0, vB^1, \ldots, vB^5$

  `rows = False` moves `v` to the right of matrix powers

`f(x)=x^2+5*x+3` then `f(B)` is possible

`B.exp()` matrix exponential, i.e. $\sum_{k=0}^{\infty} \frac{1}{k!} B^k$

Matrix Spaces

`M = MatrixSpace(QQ, 3, 4)` is space of 3×4 matrices

`A = M([1,2,3,4,5,6,7,8,9,10,11,12])`

  coerce list to element of `M`, a 3×4 matrix over `QQ`

`M.basis()`

`M.dimension()`

`M.zero_matrix()`

Matrix Operations

`5*A+2*B` linear combination

`A.inverse()`, `A^(-1)`, `~A`, singular is `ZeroDivisionError`

`A.transpose()`

`A.conjugate()` entry-by-entry complex conjugates

`A.conjugate_transpose()`

`A.antitranspose()` transpose + reverse orderings

`A.adjoint()` matrix of cofactors

`A.restrict(V)` restriction to invariant subspace `V`

Row Operations

Row Operations: (change matrix in place)

Caution: first row is numbered 0

`A.rescale_row(i,a)` `a*`(row `i`)

`A.add_multiple_of_row(i,j,a)` `a*`(row `j`) + row `i`

`A.swap_rows(i,j)`

Each has a column variant, `row`→`col`

For a new matrix, use e.g. `B = A.with_rescaled_row(i,a)`

Echelon Form

`A.rref()`, `A.echelon_form()`, `A.echelonize()`

Note: `rref()` promotes matrix to fraction field

`A = matrix(ZZ,[[4,2,1],[6,3,2]])`

`A.rref()` `A.echelon_form()`

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

`A.pivots()` indices of columns spanning column space

`A.pivot_rows()` indices of rows spanning row space

Pieces of Matrices

Caution: row, column numbering begins at 0

`A.nrows()`, `A.ncols()`

`A[i,j]` entry in row `i` and column `j`

`A[i]` row `i` as immutable Python tuple. Thus,

  Caution: OK: `A[2,3] = 8`, Error: `A[2][3] = 8`

`A.row(i)` returns row `i` as Sage vector

`A.column(j)` returns column `j` as Sage vector

`A.list()` returns single Python list, row-major order

`A.matrix_from_columns([8,2,8])`

  new matrix from columns in list, repeats OK

`A.matrix_from_rows([2,5,1])`

  new matrix from rows in list, out-of-order OK

`A.matrix_from_rows_and_columns([2,4,2],[3,1])`

  common to the rows and the columns

`A.rows()` all rows as a list of tuples

`A.columns()` all columns as a list of tuples

`A.submatrix(i,j,nr,nc)`

  start at entry `(i,j)`, use `nr` rows, `nc` cols

`A[2:4,1:7]`, `A[0:8:2,3::-1]` Python-style list slicing

Combining Matrices

`A.augment(B)` `A` in first columns, matrix `B` to the right

`A.stack(B)` `A` in top rows, `B` below; `B` can be a vector

`A.block_sum(B)` Diagonal, `A` upper left, `B` lower right

`A.tensor_product(B)` Multiples of `B`, arranged as in `A`

Scalar Functions on Matrices

`A.rank()`, `A.right_nullity()`

`A.left_nullity() == A.nullity()`

`A.determinant() == A.det()`

`A.permanent()`, `A.trace()`

`A.norm() == A.norm(2)` Euclidean norm

`A.norm(1)` largest column sum

`A.norm(Infinity)` largest row sum

`A.norm('frob')` Frobenius norm

Matrix Properties

`.is_zero()`; `.is_symmetric()`; `.is_hermitian()`;

`.is_square()`; `.is_orthogonal()`; `.is_unitary()`;

`.is_scalar()`; `.is_singular()`; `.is_invertible()`;

`.is_one()`; `.is_nilpotent()`; `.is_diagonalizable()`

Eigenvalues and Eigenvectors

Note: Contrast behavior for exact rings (QQ) vs. RDF, CDF

`A.charpoly('t')` no variable specified defaults to `x`
  `A.characteristic_polynomial() == A.charpoly()`
`A.fcp('t')` factored characteristic polynomial
`A.minpoly()` the minimum polynomial
  `A.minimal_polynomial() == A.minpoly()`
`A.eigenvalues()` unsorted list, with mutiplicities
`A.eigenvectors_left()` vectors on left, `_right` too
  Returns, per eigenvalue, a triple: `e`: eigenvalue;
  `V`: list of eigenspace basis vectors; `n`: multiplicity
`A.eigenmatrix_right()` vectors on right, `_left` too
  Returns pair: `D`: diagonal matrix with eigenvalues
  `P`: eigenvectors as columns (rows for left version)
    with zero columns if matrix not diagonalizable
Eigenspaces: see "Constructing Subspaces"

Decompositions

Note: availability depends on base ring of matrix,
  try RDF or CDF for numerical work, QQ for exact
  "unitary" is "orthogonal" in real case
`A.jordan_form(transformation=True)`
  returns a pair of matrices with: `A == P^(-1)*J*P`
    `J`: matrix of Jordan blocks for eigenvalues
    `P`: nonsingular matrix
`A.smith_form()` triple with: `D == U*A*V`
  `D`: elementary divisors on diagonal
  `U, V`: with unit determinant
`A.LU()` triple with: `P*A == L*U`
  `P`: a permutation matrix
  `L`: lower triangular matrix, `U`: upper triangular matrix
`A.QR()` pair with: `A == Q*R`
  `Q`: a unitary matrix, `R`: upper triangular matrix
`A.SVD()` triple with: `A == U*S*(V-conj-transpose)`
  `U`: a unitary matrix
  `S`: zero off the diagonal, dimensions same as `A`
  `V`: a unitary matrix
`A.schur()` pair with: `A == Q*T*(Q-conj-transpose)`
  `Q`: a unitary matrix
  `T`: upper-triangular matrix, maybe 2×2 diagonal blocks
`A.rational_form()`, aka Frobenius form
`A.symplectic_form()`
`A.hessenberg_form()`
`A.cholesky()` (needs work)

Solutions to Systems

`A.solve_right(B)` `_left` too
  is solution to `A*X = B`, where `X` is a vector **or** matrix
`A = matrix(QQ, [[1,2],[3,4]])`
`b = vector(QQ, [3,4])`, then `A\b` is solution `(-2, 5/2)`

Vector Spaces

`VectorSpace(QQ, 4)` dimension 4, rationals as field
`VectorSpace(RR, 4)` "field" is 53-bit precision reals
`VectorSpace(RealField(200), 4)`
  "field" has 200 bit precision
`CC^4` 4-dimensional, 53-bit precision complexes
`Y = VectorSpace(GF(7), 4)` finite
  `Y.list()` has $7^4 = 2401$ vectors

Vector Space Properties

`V.dimension()`
`V.basis()`
`V.echelonized_basis()`
`V.has_user_basis()` with non-canonical basis?
`V.is_subspace(W)` `True` if `W` is a subspace of `V`
`V.is_full()` rank equals degree (as module)?
`Y = GF(7)^4`, `T = Y.subspaces(2)`
  `T` is a generator object for 2-D subspaces of `Y`
  `[U for U in T]` is list of 2850 2-D subspaces of `Y`,
  or use `T.next()` to step through subspaces

Constructing Subspaces

`span([v1,v2,v3], QQ)` span of list of vectors over ring

For a matrix `A`, objects returned are
  vector spaces when base ring is a field
  modules when base ring is just a ring
`A.left_kernel() == A.kernel()` `right_` too
`A.row_space() == A.row_module()`
`A.column_space() == A.column_module()`
`A.eigenspaces_right()` vectors on right, `_left` too
  Pairs: eigenvalues with their right eigenspaces
`A.eigenspaces_right(format='galois')`
  One eigenspace per irreducible factor of char poly

If `V` and `W` are subspaces
`V.quotient(W)` quotient of `V` by subspace `W`
`V.intersection(W)` intersection of `V` and `W`
`V.direct_sum(W)` direct sum of `V` and `W`
`V.subspace([v1,v2,v3])` specify basis vectors in a list

Dense versus Sparse

Note: Algorithms may depend on representation
Vectors and matrices have two representations
  Dense: lists, and lists of lists
  Sparse: Python dictionaries
`.is_dense()`, `.is_sparse()` to check
`A.sparse_matrix()` returns sparse version of `A`
`A.dense_rows()` returns dense row vectors of `A`
Some commands have boolean `sparse` keyword

Rings

Note: Many algorithms depend on the base ring
`<object>.base_ring(R)` for vectors, matrices,...
  to determine the ring in use
`<object>.change_ring(R)` for vectors, matrices,...
  to change to the ring (or field), `R`
`R.is_ring()`, `R.is_field()`, `R.is_exact()`
Some common Sage rings and fields
  `ZZ` integers, ring
  `QQ` rationals, field
  `AA`, `QQbar` algebraic number fields, exact
  `RDF` real double field, inexact
  `CDF` complex double field, inexact
  `RR` 53-bit reals, inexact, not same as `RDF`
  `RealField(400)` 400-bit reals, inexact
  `CC`, `ComplexField(400)` complexes, too
  `RIF` real interval field
  `GF(2)` mod 2, field, specialized implementations
  `GF(p) == FiniteField(p)` `p` prime, field
  `Integers(6)` integers mod 6, ring only
  `CyclotomicField(7)` rationals with 7^{th} root of unity
  `QuadraticField(-5, 'x')` rationals with `x=`$\sqrt{-5}$
  `SR` ring of symbolic expressions

Vector Spaces versus Modules

Module "is" a vector space over a ring, rather than a field
Many commands above apply to modules
Some "vectors" are really module elements

More Help

"tab-completion" on partial commands
"tab-completion" on `<object.>` for all relevant methods
`<command>?` for summary and examples
`<command>??` for complete source code

Sage Quick Reference:
Elementary Number Theory

William Stein

Sage Version 3.4

http://wiki.sagemath.org/quickref

Everywhere $m, n, a, b, etc.$ are elements of `ZZ`

`ZZ` = $\mathbf{Z}$ = all integers

Integers

$\dots, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots$

n divided by m has *remainder* `n % m`

`gcd(n,m)`, `gcd(`*list*`)`

extended gcd $g = sa + tb = \gcd(a, b)$: `g,s,t=xgcd(a,b)`

`lcm(n,m)`, `lcm(`*list*`)`

binomial coefficient $\binom{m}{n}$ = `binomial(m,n)`

digits in a given base: `n.digits(`*base*`)`

number of digits: `n.ndigits(`*base*`)`

(*base* is optional and defaults to 10)

divides $n \mid m$: `n.divides(m)` if $nk = m$ some k

divisors – all d with $d \mid n$: `n.divisors()`

factorial – $n!$ = `n.factorial()`

Prime Numbers

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, \dots$

factorization: `factor(n)`

primality testing: `is_prime(n)`, `is_pseudoprime(n)`

prime power testing: `is_prime_power(n)`

$\pi(x) = \#\{p : p \le x \text{ is prime}\}$ = `prime_pi(x)`

set of prime numbers: `Primes()`

$\{p : m \le p < n \text{ and } p \text{ prime}\}$ =`prime_range(m,n)`

prime powers: `prime_powers(m,n)`

first n primes: `primes_first_n(n)`

next and previous primes: `next_prime(n)`, `previous_prime(n)`, `next_probable_prime(n)`

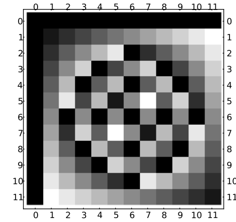
prime powers: `next_prime_power(n)`, `pevious_prime_power(n)`

Lucas-Lehmer test for primality of $2^p - 1$

```
def is_prime_lucas_lehmer(p):
    s = Mod(4, 2^p - 1)
    for i in range(3, p+1): s = s^2 - 2
    return s == 0
```

Modular Arithmetic and Congruences

`k=12; m = matrix(ZZ, k, [(i*j)%k for i in [0..k-1] for j in [0..k-1]]); m.plot(cmap='gray')`



Euler's $\phi(n)$ function: `euler_phi(n)`

Kronecker symbol $\left(\frac{a}{b}\right)$ = `kronecker_symbol(a,b)`

Quadratic residues: `quadratic_residues(n)`

Quadratic non-residues: `quadratic_residues(n)`

ring $\mathbf{Z}/n\mathbf{Z}$ = `Zmod(n)` = `IntegerModRing(n)`

a modulo n as element of $\mathbf{Z}/n\mathbf{Z}$: `Mod(a, n)`

primitive root modulo n = `primitive_root(n)`

inverse of $n \pmod{m}$: `n.inverse_mod(m)`

power $a^n \pmod{m}$: `power_mod(a, n, m)`

Chinese remainder theorem: `x = crt(a,b,m,n)`

finds x with $x \equiv a \pmod{m}$ and $x \equiv b \pmod{n}$

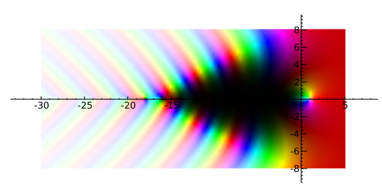
discrete log: `log(Mod(6,7), Mod(3,7))`

order of $a \pmod{n}$ =`Mod(a,n).multiplicative_order()`

square root of $a \pmod{n}$ =`Mod(a,n).sqrt()`

Special Functions

`complex_plot(zeta, (-30,5), (-8,8))`



$\zeta(s) = \prod_p \frac{1}{1-p^{-s}} = \sum \frac{1}{n^s}$ = `zeta(s)`

$\mathrm{Li}(x) = \int_2^x \frac{1}{\log(t)} dt$ = `Li(x)`

$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt$ = `gamma(s)`

Continued Fractions

`continued_fraction(pi)`

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cdots}}}}$$

continued fraction: `c=continued_fraction(x,` *bits*`)`

convergents: `c.convergents()`

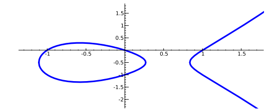
convergent numerator p_n = `c.pn(n)`

convergent denominator q_n = `c.qn(n)`

value: `c.value()`

Elliptic Curves

`EllipticCurve([0,0,1,-1,0]).plot(plot_points=300,thickness=3)`



`E = EllipticCurve(`$[a_1, a_2, a_3, a_4, a_6]$`)`

$$y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$$

conductor N of E =`E.conductor()`

discriminant Δ of E =`E.discriminant()`

rank of E = `E.rank()`

free generators for $E(\mathbf{Q})$ = `E.gens()`

j-invariant = `E.j_invariant()`

$N_p = \#\{\text{solutions to } E \text{ modulo } p\}$ = `E.Np(`*prime*`)`

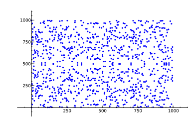
$a_p = p + 1 - N_p$ =`E.ap(`*prime*`)`

$L(E, s) = \sum \frac{a_n}{n^s}$ = `E.lseries()`

$\mathrm{ord}_{s=1} L(E, s)$ = `E.analytic_rank()`

Elliptic Curves Modulo p

`EllipticCurve(GF(997), [0,0,1,-1,0]).plot()`



`E = EllipticCurve(GF(p),` $[a_1, a_2, a_3, a_4, a_6]$`)`

$\#E(\mathbf{F}_p)$ = `E.cardinality()`

generators for $E(\mathbf{F}_p)$ = `E.gens()`

$E(\mathbf{F}_p)$ =`E.points()`

Sage Quick Reference: Abstract Algebra

B. Balof, T. W. Judson, D. Perkinson, R. Potluri

version 1.0, Sage Version 5.0.1

latest version: `http://wiki.sagemath.org/quickref`

Based on work by P. Jipsen, W. Stein, R. Beezer

Basic Help

com⟨`tab`⟩ complete *command*

`a.`⟨`tab`⟩ all methods for object `a`

`<command>?` for summary and examples

`<command>??` for complete source code

`*`*foo*`*?` list all commands containing *foo*

`_` underscore gives the previous output

`www.sagemath.org/doc/reference` online reference

`www.sagemath.org/doc/tutorial` online tutorial

`load foo.sage` load commands from the file `foo.sage`

`attach foo.sage`
  loads changes to `foo.sage` automatically

Lists

`L = [2,17,3,17]` an ordered list

`L[i]` the ith element of L
  Note: lists begin with the 0th element

`L.append(x)` adds x to L

`L.remove(x)` removes x from L

`L[i:j]` the i-th through $(j-1)$-th element of L

`range(a)` list of integers from 0 to $a-1$

`range(a,b)` list of integers from a to $b-1$

`[a..b]` list of integers from a to b

`range(a,b,c)`
  every c-th integer starting at a and less than b

`len(L)` length of L

`M = [i^2 for i in range(13)]`
  list of squares of integers 0 through 12

`N = [i^2 for i in range(13) if is_prime(i)]`
  list of squares of prime integers between 0 and 12

`M + N` the concatenation of lists `M` and `N`

`sorted(L)` a sorted version of `L` (`L` is not changed)

`L.sort()` sorts `L` (`L` is changed)

`set(L)` an unordered list of unique elements

Programming Examples

Print the squares of the integers $0,\ldots,14$:

```
for i in range(15):
    print i^2
```

Print the squares of those integers in $\{0,\ldots,14\}$ that are relatively prime to 15:

```
for i in range(13):
    if gcd(i,15)==1:
       print i^2
```

Preliminary Operations

`a = 3; b = 14`

`gcd(a,b)` greatest common divisor a, b

`xgcd(a,b)`
  triple (d,s,t) where $d = sa + tb$ and $d = \gcd(a,b)$

`next_prime(a)` next prime after a

`previous_prime(a)` prime before a

`prime_range(a,b)` primes p such that $a \le p < b$

`is_prime(a)` is a prime?

`b % a` the remainder of b upon division by a

`a.divides(b)` does a divide b?

Group Constructions

Permutation multiplication is left-to-right.

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
  perm. group with generators $(1,2,3)(4,5)$ and $(3,4)$

`G = PermutationGroup(["(1,2,3)(4,5)","(3,4)"])`
  alternative syntax for defining a permutation group

`S = SymmetricGroup(4)` the symmetric group, S_4

`A = AlternatingGroup(4)` alternating group, A_4

`D = DihedralGroup(5)` dihedral group of order 10

`Ab = AbelianGroup([0,2,6])` the group $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_6$

`Ab.0, Ab.1, Ab.2` the generators of `Ab`

`a,b,c = Ab.gens()`
  shorthand for `a = Ab.0; b = Ab.1; c = Ab.2`

`C = CyclicPermutationGroup(5)`

`Integers(8)` the group $\mathbb{Z}_8$

`GL(3,QQ)` general linear group of 3×3 matrices

`m = matrix(QQ,[[1,2],[3,4]])`

`n = matrix(QQ,[[0,1],[1,0]])`

`MatrixGroup([m,n])`
  the (infinite) matrix group with generators `m` and `n`

`u = S([(1,2),(3,4)]); v = S((2,3,4))` elements of `S`

`S.subgroup([u,v])`
  the subgroup of `S` generated by `u` and `v`

`S.quotient(A)` the quotient group `S/A`

`A.cartesian_product(D)` the group `A×D`

`A.intersection(D)` the intersection of groups `A` and `D`

`D.conjugate(v)` the group $\mathtt{v}^{-1}\mathtt{Dv}$

`S.sylow_subgroup(2)` a Sylow 2-subgroup of `S`

`D.center()` the center of `D`

`S.centralizer(u)` the centralizer of `x` in `S`

`S.centralizer(D)` the centralizer of `D` in `S`

`S.normalizer(u)` the normalizer of `x` in `S`

`S.normalizer(D)` the normalizer of `D` in `S`

`S.stabilizer(3)` subgroup of `S` fixing 3

Group Operations

`S = SymmetricGroup(4); A = AlternatingGroup(4)`

`S.order()` the number of elements of `S`

`S.gens()` generators of `S`

`S.list()` the elements of `S`

`S.random_element()` a random element of `S`

`u*v` the product of elements `u` and `v` of `S`

`v^(-1)*u^3*v` the element $\mathtt{v}^{-1}\mathtt{u}^3\mathtt{v}$ of `S`

`u.order()` the order of `u`

`S.subgroups()` the subgroups of `S`

`S.normal_subgroups()` the normal subgroups of `S`

`A.cayley_table()` the multiplication table for `A`

`u in S` is `u` an element of `S`?

`u.word_problem(S.gens())`
  write `u` as a product of the generators of `S`

`A.is_abelian()` is `A` abelian?

`A.is_cyclic()` is `A` cyclic?

`A.is_simple()` is `A` simple?

`A.is_transitive()` is `A` transitive?

`A.is_subgroup(S)` is `A` a subgroup of `S`?

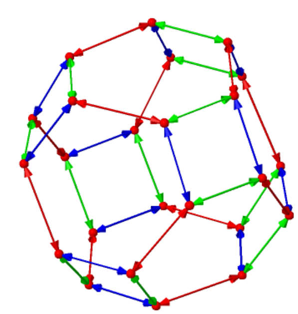
`A.is_normal(S)` is `A` a normal subgroup of `S`?

`S.cosets(A)` the right cosets of `A` in `S`

`S.cosets(A,'left')` the left cosets of `A` in `S`

`g = S.cayley_graph()` Cayley graph of `S`

`g.show3d(color_by_label=True, edge_size=0.01,`
  `vertex_size=0.03)` see below:



Ring and Field Constructions

`ZZ` integral domain of integers, $\mathbb{Z}$
`Integers(7)` ring of integers mod 7, $\mathbb{Z}_7$
`QQ` field of rational numbers, $\mathbb{Q}$
`RR` field of real numbers, $\mathbb{R}$
`CC` field of complex numbers, $\mathbb{C}$
`RDF` real double field, inexact
`CDF` complex double field, inexact
`RR` 53-bit reals, inexact, not same as `RDF`
`RealField(400)` 400-bit reals, inexact
`ComplexField(400)` complexes, too
`ZZ[I]` the ring of Gaussian integers
`QuadraticField(7)` the quadratic field, $\mathbb{Q}(\sqrt{7})$
`CyclotomicField(7)`
smallest field containing $\mathbb{Q}$ and the zeros of $x^7 - 1$
`AA, QQbar` field of algebraic numbers, $\overline{\mathbb{Q}}$
`FiniteField(7)` the field $\mathbb{Z}_7$
`F.<a> = FiniteField(7^3)`
finite field in a of size 7^3, $\mathrm{GF}(7^3)$
`SR` ring of symbolic expressions
`M.<a>=QQ[sqrt(3)]` the field $\mathbb{Q}[\sqrt{3}]$, with $a = \sqrt{3}$.
`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
the field $\mathbb{Q}[\sqrt{3}, \sqrt{5}]$ with $a = \sqrt{3}$ and $b = \sqrt{5}$.
`z = polygen(QQ,'z'); K = NumberField(x^2 - 2,'s')`
the number field in s with defining polynomial $x^2 - 2$
`s = K.0` set `s` equal to the generator of `K`
`D = ZZ[sqrt(3)]`
`D.fraction_field()`
field of fractions for the integral domain `D`

Ring Operations

Note: Operations may depend on the ring

`A = ZZ[I]; D = ZZ[sqrt(3)]` some rings
`A.is_ring()` is A a ring?
`A.is_field()` is A a field?
`A.is_commutative()` is A commutative?
`A.is_integral_domain()`
`True` is A an integral domain?
`A.is_finite()` is A is finite?
`A.is_subring(D)` is A a subring of D?
`A.order()` the number of elements of A
`A.characteristic()` the characteristic of A
`A.zero()` the additive identity of A
`A.one()` the multiplicative identity of A
`A.is_exact()`
`False` if `A` uses a floating point representation
`a, b = D.gens(); r = a + b`
`r.parent()` the parent ring of r (in this case, `D`)
`r.is_unit()` is r a unit?

Polynomials

`R.<x> = ZZ[ ]` `R` is the polynomial ring $\mathbb{Z}[x]$
`R.<x> = QQ[ ]; R = PolynomialRing(QQ,'x'); R = QQ['x']`
`R` is the polynomial ring $\mathbb{Q}[x]$
`S.<z> = Integers(8)[ ]` `S` is the polynomial ring $\mathbb{Z}_8[z]$
`S.<s, t> = QQ[ ]` `S` is the polynomial ring $\mathbb{Q}[s, t]$
`p = 4*x^3 + 8*x^2 - 20*x - 24`
a polynomial in `R` ($= \mathbb{Q}[x]$)
`p.is_irreducible()` is p irreducible over $\mathbb{Q}[x]$?
`q = p.factor()` factor p
`q.expand()` expand `q`
`p.subs(x=3)` evaluates p at $x = 3$
`R.ideal(p)` the ideal in R generated by p
`R.cyclotomic_polynomial(7)`
the cyclotomic polynomial $x^6+x^5+x^4+x^3+x^2+x+1$
`q = x^2-1`
`p.divides(q)` does p divide q?
`p.quo_rem(q)`
the quotient and remainder of `p` upon division by `q`
`gcd(p, q)` the greatest common divisor of p and q
`p.xgcd(q)` the extended gcd of p and q
`I = S.ideal([s*t+2,s^3-t^2])`
the ideal $(st + 2, s^3 - t^2)$ in `S` ($= \mathbb{Q}[s, t]$)
`S.quotient(I)` the quotient ring, S/I

Field Operations

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
`C.<c> = A.absolute_field()`
"flattens" a relative field extension
`A.relative_degree()`
the degree of the relative extension field
`A.absolute_degree()`
the degree of the absolute extension
r = a + b; `r.minpoly()`
the minimal polynomial of the field element `r`
`C.is_galois()` is `C` a Galois extension of Q?

Sage Quick Reference: Graph Theory
Steven Rafael Turner
Sage Version 4.7
http://wiki.sagemath.org/quickref

Constructing

Adjacency Mapping:
`G=Graph([GF(13), lambda i,j: conditions on i,j])`
Input is a list whose first item are vertices and the other is some adjacency function: [list of vertices, function]
Adjacency Lists:
`G=Graph({0:[1,2,3], 2:[4]})`
`G=Graph({0:{1:"x",2:"z",3:"a"}, 2:{5:"out"}})`
x, z, a, and out are labels for edges and be used as weights.
Adjacency Matrix:
`A = numpy.array([[0,1,1],[1,0,1],[1,1,0]])`
Don't forget to import numpy for the NumPy matrix or ndarray.
`M = Matrix([(....), (....), . . . ])`
Edge List with or without labels:
`G = Graph([(1,3,"Label"),(3,8,"Or"),(5,2)])`
Incidence Matrix:
`M = Matrix(2, [-1,0,0,0,1, 1,-1,0,0,0])`
Graph6 Or Sparse6 string
`G=':IgMoqoCUOqeb\n:I‘EDOAEQ?PccSsge\N\n'`
`graphs_list.from_sparse6(G)`
Above is a list of graphs using sparse6 strings.
NetworkX Graph
`g = networkx.Graph({0:[1,2,3], 2:[4]})`
`DiGraph(g)`
`g_2 = networkx.MultiGraph({0:[1,2,3], 2:[4]})`
`Graph(g_2)`
Don't forget to import networkx

Centrality Measures

`G.centrality_betweenness(normalized=False)`
`G.centrality_closeness(v=1)`
`G.centrality_degree()`

Graph Deletions and Additions

`G.add_cycle([vertices])`
`G.add_edge(edge)`
`G.add_edges(iterable of edges)`
`G.add_path`
`G.add_vertex(Name of isolated vertex)`
`G.add_vertices(iterable of vertices)`
`G.delete_edge( v_1, v_2, 'label')`
`G.delete_edges(iterable of edges)`
`G.delete_multiedge(v_1, v_2)`
`G.delete_vertex(v_1)`
`G.delete_vertices(iterable of vertices)`
`G.merge_vertices([vertices])`

Connectivity and Cuts

`G.is_connected()`
`G.edge_connectivity()`
`G.edge_cut(source, sink`
`G.blocks_and_cut_vertices()`
`G.max_cut()`
`G.edge_disjoint_paths(v1,v2, method='LP')`
This method can us LP (Linear Programming) or FF (Ford-Fulkerson)
`vertex_disjoint_paths(v1,v2)`
`G.flow(1,2)`
There are many options to this function please check the documentation.

Conversions

`G.to_directed()`
`G.to_undirected()`
`G.sparse6_string()`
`G.graph6_string()`

Products

`G.strong_product(H)`
`G.tensor_product(H)`
`G.categorical_product(H)`
Same as the tensor product.
`G.disjunctive_product(H)`
`G.lexicographic_product(H)`
`G.cartesian_product(H)`

Boolean Queries

`G.is_tree()`
`G.is_forest()`
`G.is_gallai_tree()`
`G.is_interval()`
`G.is_regular()`
`G.is_chordal()`
`G.is_eulerian()`
`G.is_hamiltonian()`
`G.is_interval()`
`G.is_independent_set([vertices])`
`G.is_overfull()`
`G.is_regular(k)`
Can test for being k-regular, by default k=None.

Common Invariants

`G.diameter()`
`G.average_distance()`
`G.edge_disjoint_spanning_trees(k)`
`G.girth()`
`G.size()`
`G.order()`
`G.radius()`

Graph Coloring

`G.chromatic_polynomial()`
`G.chromatic_number(algorithm="DLX")`
You can change DLX (dancing links) to CP (chromatic polynomial coefficients) or MILP (mixed integer linear program)
`G.coloring(algorithm="DLX")`
You can change DLX to MILP
`G.is_perfect(certificate=False)`

Planarity

`G.is_planar()`
`G.is_circular_planar()`
`G.is_drawn_free_of_edge_crossings()`
`G.layout_planar(test=True, set_embedding=True`
`G.set_planar_positions()`

Search and Shortest Path

`list(G.depth_first_search([vertices], distance=4)`
`list(G.breadth_first_search([vertices])`
`dist,pred = graph.shortest_path_all_pairs(by_weig`
Choice of algorithms: BFS or Floyd-Warshall-Python
`G.shortest_path_length(v_1,v_2, by_weight=True`
`G.shortest_path_lengths(v_1)`
`G.shortest_path(v_1,v_2)`

Spanning Trees

`G.steiner_tree(g.vertices()[:10])`

`G.spanning_trees_count()`
`G.edge_disjoint_spanning_trees(2, root vertex)`
`G.min_spanning_tree(weight_function=somefunction, algorithm='Kruskal',starting_vertex=3)`
Kruskal can be change to Prim_fringe, Prim_edge, or NetworkX

Linear Algebra

Matrices

`G.kirchhoff_matrix()`
`G.laplacian_matrix()`
Same as the kirchoff matrix
`G.weighted_adjacency_matrix()`
`G.adjacency_matrix()`
`G.incidence_matrix()`

Operations

`G.characteristic_polynomial()`
`G.cycle_basis()`
`G.spectrum()`
`G.eigenspaces(laplacian=True)`
`G.eigenvectors(laplacian=True)`

Automorphism and Isomorphism Related

`G.automorphism_group()`
`G.is_isomorphic(H)`
`G.is_vertex_transitive()`
`G.canonical_label()`
`G.minor(graph of minor to find)`

Generic Clustering

`G.cluster_transitivity()`
`G.cluster_triangles()`
`G.clustering_average()`
`G..clustering_coeff(nbunch=[0,1,2],weights=True)`

Clique Analysis

`G.is_clique([vertices])`
`G.cliques_vertex_clique_number(vertices=[(0, 1), (1, 2)],algorithm="networkx")`
networkx can be replaced with cliquer.
`G.cliques_number_of()`
`G.cliques_maximum()`
`G.cliques_maximal()`
`G.cliques_get_max_clique_graph()`
`G.cliques_get_clique_bipartite()`
`G.cliques_containing_vertex()`
`G.clique_number(algorithm="cliquer")`
cliquer can be replaced with networkx.
`G.clique_maximum()`
`G.clique_complex()`

Component Algorithms

`G.is_connected()`
`G.connected_component_containing_vertex(vertex)`
`G.connected_components_number()`
`G.connected_components_subgraphs()`
`G.strong_orientation()`
`G.strongly_connected_components()`
`G.strongly_connected_components_digraph()`
`G.strongly_connected_components_subgraphs()`
`G.strongly_connected_component_containing_vertex(vertex)`
`G.is_strongly_connected()`

NP Problems

`G.vertex_cover(algorithm='Cliquer')`
The algorithm can be changed to MILP (mixed integer linear program. Note that MILP requires packages GLPK or CBC.
`G.hamiltonian_cycle()`
`G.traveling_salesman_problem()`

Sage Dynamics Ref Card v3.0
(for Sage 8.1)

Rings and Fields

`ZZ`	integer ring	`Zmod(m)`	$\mathbb{Z}/m\mathbb{Z}$
`QQ`	rational field	`QQbar`	alg. clos. of QQ
`RR`	real field	`CC`	complex field
`Qp(p)`	p-adic field	`Zp(p)`	p-adic integers
`QQ[]`	polynomials	`QQ[[]]`	power series
`GF(p)`	prime field	`GF(p^n,'v')`	finite field

`CyclotomicField(n)`	$\mathbb{Q}(\zeta_n)$
`FractionField(ring)`	field of fractions
`QuadraticField(d)`	$\mathbb{Q}(\sqrt{d})$
`NumberField(poly,'var',[emb])`	number field
`K.absolute_field()`	—
`K.degree()`	$[K:\mathbb{Q}]$
`K.extension(poly)`	fld ext
`QQ.range_by_height(bd)`	iterator
`K.elements_of_bounded_height(bd, [params])`	
`number_field_elements_from_algebraics(pts)`	

Spaces and Schemes

`A.<vars>=AffineSpace(ring,dim)`	$\mathbb{A}^n$
`P.<vars>=ProjectiveSpace(ring,dim)`	$\mathbb{P}^n$
`PP.<vars>=ProductProjectiveSpaces(ring,dims)`	$\mathbb{P}^n \times \cdots \times \mathbb{P}^m$
`WehlerK3Surface(polys)`	—
`S.affine_patch(i,[A])`	—
`S.base_ring()`	base ring S
`S.change_ring()`	change base ring
`S.coordinate_ring()`	coor. ring of S
`S.defining_ideal()`	—
`S.defining_polynomials()`	—
`S.dimension()`	rel. dim of S
`S.gens()`	vars of coord. ring
`S.point_transformation_matrix([pts,pts])`	find PGL element
`S.projective_embedding([i,P])`	—
`S.projective_closure([i,P])`	—
`S.rational_points([bd,fld])`	—
`S.subscheme(polys)`	subscheme of S
`S.vars()`	vars of coord. ring
`S.variable_names()`	vars as strings
`S.weil_restriction()`	restric. of const.

Dynamical System Initialization

`DynamicalSystem(polys, [domain])`	projective if no domain
`DynamicalSystem_affine(polys, [domain])`	
`DynamicalSystem_projective(polys, [domain])`	
`f.as_dynamical_system()`	End $\to$ DS

Periodic Behavior

`f.dynatomic_polynomial([m,n])`	—
`Q.is_preperiodic(f)`	—
`Q.multiplier(f,n)`	$(f^n)'(Q)$
`Q.orbit_structure(f)`	[tail,period]
`f.periodic_points(n,[params])`	—
`f.rational_periodic_points([params])`	—
`f.rational_periodic_graph([params])`	—
`f.rational_preperiodic_points([params])`	—
`f.rational_preperiodic_graph([params])`	—
`f.possible_periods([params])`	via good red.

Heights and Measures

`Q.canonical_height(f,[params])`	$\hat{h}_f(Q)$
`f.critical_height()`	$\sum_{c\in \mathrm{Crit}} \hat{h}_f(c)$
`Q.global_height([prec])`	$h(Q)$
`f.global_height([prec])`	—
`Q.green_function(v,[prec])`	at v
`f.height_difference_bound()`	$\lvert h(Q)-\hat{h}_f(Q)\rvert$
`f.local_height_arch(i,[prec])`	at ∞

Critical Points

`f.critical_points()`	—
`f.critical_subscheme()`	—
`f.critical_point_portrait()`	—
`f.critical_height()`	$\sum_{c\in \mathrm{Crit}} \hat{h}_f(c)$
`f.is_postcritically_finite()`	—
`f.wronskian_ideal()`	crit locus

Cyclic Structures

`f.all_rational_preimages(points)`	—
`f.cyclegraph()` $\boxed{\mathbb{F}_q}$	digraph
`Q.orbit_structure(f)` $\boxed{\mathbb{F}_q}$	[tail,per]
`Q.rational_preimages(f)`	—
`Q.rational_connected_component(f)`	—

Rational Functions

`f.dynamical_degree()`	—
`f.degree_sequence()`	deg. of iterates
`f.indeterminacy_locus()`	—
`f.indeterminancy_points()`	if fin. many

Functions

`f[i]`	ith coord
`f.automorphism_group()`	$\{\phi : f^{\phi} = f\}$
`f.autmorphism_group()`	$\mathrm{Hom}(f,f)$
`f.base_ring()`	—
`f.change_ring()`	—
`P.chebyshev_polynomial(k,kind)`	
`f.codomain()`	—
`f.conjugate(φ)`	$\phi^{-1}\circ f \circ \phi$
`f.conjugating_set(g)`	$\mathrm{Hom}(f,g)$
`f.defining_polynomials()`	—
`f.degree()`	—
`f.dehomogenize(k)`	—
`f.domain()`	—
`f.homogenize(k)`	
`f.is_morphism()`	—
`f.normalize_coordinates()`	remove gcd
`f.nth_iterate(Q,n)`	$f^n(Q)$
`f.nth_iterate_map(n)`	f^n
`P.Lattes(E,m)`	create Lattès map
`f.orbit(Q,[m,n])`	$\{f^m(Q),\dots,f^n(Q)\}$
`f.primes_of_bad_reduction()`	—
`f.resultant()`	—
`f.scale_by(t)`	$t\cdot f$
`f.specialization()`	subs value of param

Points

`Q[i]`	ith coord
`Q.change_ring()`	—
`Q.clear_denominator()`	—
`Q.codomain()`	ambient space
`Q.dehomogenize(i)`	—
`Q.domain()`	base ring
`Q.homogenize(i)`	—
`Q.normalize_coordinates()`	remove gcd
`Q.nth_iterate(f,n)`	$f^n(Q)$
`Q.orbit(f,(m,n))`	$[f^m(Q),\dots,f^n(Q)]$
`Q.scale_by(t)`	$t\cdot Q$

Iteration

`f.nth_iterate(Q,n)`	$f^n(Q)$
`f.nth_iterate_map(n)`	f^n
`f.orbit(Q,[m,n])`	$[f^m(Q),\ldots,f^n(Q)]$
`f.rational_preimages(Q,k)`	$f^{-k}(Q)$

Moduli Spaces

`f.is_polynomial()`	has tot. ram. fixed pt.
`f.is_PGL_minimal()`	
`f.is_conjugate(g)`	$g \stackrel{?}{=} f^\phi$
`f.normal_form()`	$x^n + a_{n-2}x^{n-2} + \cdots + a_0$
`f.minimal_model()`	min resultant f^ϕ
`f.multiplier_spectra(n,[params])`	$\{\lambda_f(Q) : Q \in \mathrm{Per}_n\}$
`f.sigma_invariants(n,[params])`	$\{\sigma_i(\lambda_f(Q)) : Q \in \mathrm{Per}_n\}$

Finite Fields

`f.cyclegraph()`	iteration digraph
`Q.orbit_structure(f)`	[tail,period]

Mandelbrot and Julia Sets

`external_ray(v)`	list or single angle
`mandelbrot_plot([params])`	for z^2+c
`julia_plot([params])`	for z^2+c

Miscellaneous / Help

`_`	last output
`%time`	execution time
`timeit('cmd',number=#)`	time multiple iterations
`s.<tab>`	show all cmds on s
`s.cmd?`	info about cmd on s
`set_verbose(None)`	disable warnings
`load(''path to file'')`	load code file
`copy(obj)`	—
`latex(obj)`	—
`all(list of bool)`	—
`any(list of bool)`	—
`sum(list)`	—
`max(list)`	—
`isinstance(f, type)`	check for type
`preparser(bool)`	on/off notebk preparsing

Matrices

`matrix(K,n,m,list)`	create matrix
`matrix(K,list of lists)`	create matrix
`M.charpoly()`	—
`M.determinant()`	—
`M.height()`	global height
`M.inverse()`	—
`M.LLL([args])`	LLL reduced lattice
`M.minors(k)`	dets of $k\times k$ minors
`M.rank()`	—

Polynomial Rings

`R.<a,b>=PolynomialRing(K,2)`	poly ring over K
`R.<a>=PolynomialRing(K)`	univar poly ring
`R.<a>=PolynomialRing(K,1)`	multivar poly ring
`R.gen(k)`	kth variable
`R.gens()`	all variables
`R.hom(im_gens,S)`	Hom(R,S)
`R.ideal(polys)`	—
`I.dimension()`	krull dim of R/I
`I.elimination_ideal(vars)`	—
`I.gens()`	—
`I.groebner_basis()`	—
`I.is_prime()`	—
`I.is_maximal()`	—
`I.is_principal()`	—
`I.is_one()`	—
`I.primary_decomposition()`	—
`I.radical()`	—
`I.ring()`	R
`I.variety()`	rat pts of dim 0
`I.vector_space_dimension()`	of R/I
`F.monomial_coefficient(mon)`	base ring element
`F.polynomial(x)`	make univariate
`F.subs(dict)`	substitution
`F(tuple)`	substitution
`F.coefficient(mon)`	poly element
`F.coefficients()`	—
`list(F)`	list of (coeff,mon)
`F[list]`	coeff of mon with exp *list*
`F.dict()`	dict of mon:coef via exp
`F.lift(I)`	coeff of gens of I to get F

Algebraic Geometry

`S.Chow_form()`	associated Chow form
`S.coordinate_ring()`	—
`S.defining_ideal()`	—
`S.defining_polynomials()`	—
`S.degree()`	from lc of hil poly
`S.dimension()`	relative dimension
`S.intersection(T)`	—
`S.intersection_multiplicity(T,Q)`	Serre's Tor
`S.irreducible_components()`	—
`S.is_smooth()`	—
`S.Jacobian()`	Jacobian ideal
`S.projective_closure([ℙ])`	—
`S.rational_points([bd])`	—
`S.subscheme(ideal)`	—
`S.veronese_embedding(d)`	—
`S.weil_restriction()`	—
`S*T`	$S\times T$
`S**n`	$S\times\cdots\times S$
`I.radical()`	radical ideal

`PP.components()`	—
`PP.dimension_components()`	list of dims
`PP.segre_embedding([codomain])`	—

`C.arithmetic_genus()`	—
`C.genus()`	—
`C.is_complete_intersection()`	—
`C.is_ordinary_singularity(Q)`	—
`C.is_transverse(D,Q)`	—
`C.tangents(Q)`	

 Thanks to ICERM for hosting us while version 1.0 was written.