

Functions, Linear Forms

LATEX

November 20, 2023

Functions

1. Let R be the relation defined in N , as $R = \{(x, y) : 2x + 3y = 15, x, y \in N\}$, then $R = \{\text{____}, \text{____}\}$.
2. If the function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 2, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is ____.
3. Show that the relation R in the set \mathbb{R} of all real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric.
4. Find the value of $\tan^{-1} \left[2 \cos \left(2 \sin^{-1} \left(\frac{1}{2} \right) \right) \right]$
5. Let a function $f : \mathbb{R} - \left\{ \frac{-4}{3} \right\} \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{4x}{3x+4}$. To show that f is one-one function. Hence, find the inverse of the function $f : \mathbb{R} - \left\{ \frac{-4}{3} \right\} \rightarrow \text{Range of } f$.
6. If $f : R \rightarrow R$ be given by $f(x) = (3 - x^3)^{1/3}$, then find $(f \circ f)(x)$.
7. Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \text{ such } x \text{ and } y \text{ have at least one letter in common}\}$. Show that this relation R is reflexive and symmetric, but not transitive.
8. Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4} \right)$