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ASSIGNMENT I

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1 solve the following recurrance relations

a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

$x(1) = 0$

Substitute $n = 2$: $x(2) = x(2-1) + 5$

$= x(1) + 5$

$= 0 + 5$

$x = 5$

$n = 3$: $x(3) = x(3-1) + 5$

$= x(2) + 5$

$= 5 + 5$

$x(3) = 10$

$n = 4$ $x(4) = x(4-1) + 5$

$= x(3) + 5$

$= 10 + 5$

$x(4) = 15$

for this recurrance relations, each term is 5 more than previous term.

so, $x(n) = 5n$ for $n > 1$

b) $x(n) = 3x(n-1)$ for $n = 1$, $x(1) = 4$

$x(1) = 4$

$n = 2$ $x(2) = 3x(2-1)$

$= 3x(1)$

$= 3(4)$ $x(2) = 12$

$n=3$ $= 3x(3-1)$

$= 3x(2)$

$= 3(12)$

$x(3) = 36$

$n=4$ $x(4) = 3x(4-1)$

$= 3x(3)$

$= 3(36)$

$x(4) = 108$

for this recurrance relation, each term is 3 times the previous term

So, $x(n) = 4 * 3^{n-1}$ for $n>1$

c) $x(n) = x(n/2) + n$ for $n>1$ $x(1) = 1$

tolve for $n=2k$

$x(1) = 1$

$n=2$ $x(2) = x(2/2) + 2$

$= x(1) + 2$

$n = 2^1 = 2$ $x(2) = 3$

$n=4$ $x(4) = x(4/2) + 4$

$n = 2^2 = 4$ $= x(2) + 4$

$= 3+4$

$x(4) = 7$

$n=8$ $x(8) = x(8/2) + 8$

$n = 2^3 = 8$ $= x(4) + 8$

$= 7+8$

$x(8) = 15$

for this recursion relation $2^{n}-1 = 2^{k}-1$

for $n = 2^{t}$, $x(2k) = 2^{2k}-1$

d) $x(n) = x(n/3)$ ~~+~~ for $n > 1$ $x(1) = 1$ $n = 3k$

$x(1) = 1$

$n = 3$ $x(3) = x(3/3)+1$

$= x(1)+1$

$= 1+1$

$x(3) = 2$

$n = 9$ $x(9) = x(9/3)+1$

$= x(3)+1$

$= 2+1$

$x(9) = 3$

$n = 27$ $x(27) = x(27/3)+1$

$= x(9)+1$

$= 3+1$

$x(27) = 4$

for this recursance relation

$x(n) = \log_3 n$

$x(3) = \log_3 3^{k}$

2 i) Evaluate following occurances.

i) $T(n) = T(n/2)+1$

By using Substitution Method.

$T(n) = T(n/2)+1 \rightarrow (1)$

$T(n/2) = T(n/2^2)+1 \quad -(2)$

Sub (2) in (1)

$T(n) = T(n/2^2)+2 \quad -(3)$

$T(n) = T(n/2^3)+3 \quad -(4)$

$T(n) = T(n/2^k)+k \quad -(5)$

Assume $\frac{n}{2^k} = 1, \; n = 2^k$.

$n = \log n$

$T(n) = T(1) + \log n$

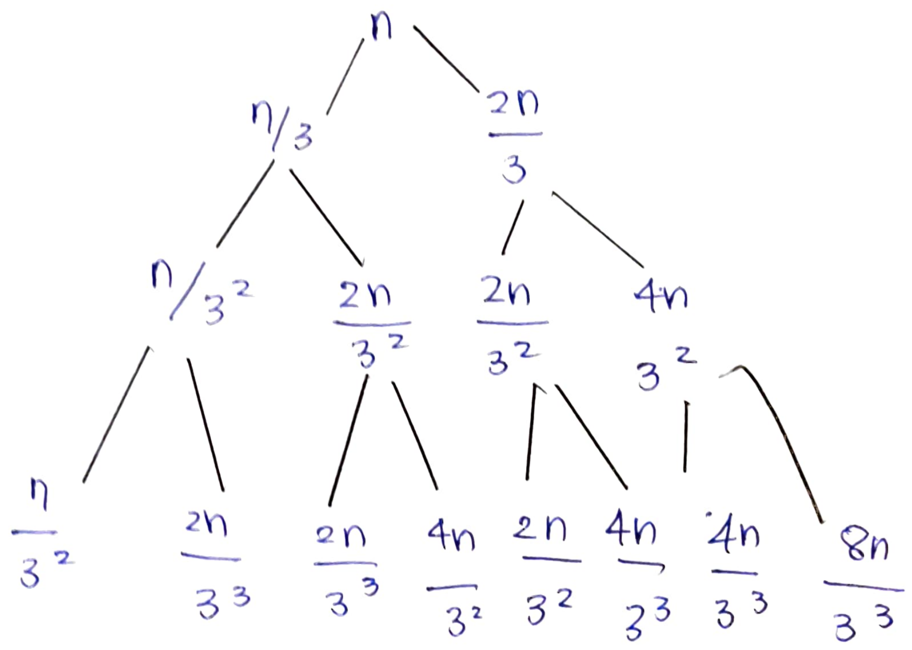
$T(n) = 1 + \log n$.

$\Rightarrow O(\log n)$

ii) $T(n) = T(N/3) + T(2^n/3) + C_n$

$T(n/3) = T(n/3^2) + \cancel{T}\left(\frac{2n}{3^2}\right) + \frac{c \cdot n}{3}$

$T(2n/3) = T(2n/3^2) + T(4n/3^2) + \frac{c \cdot 2n}{3}$



$n/3^2 + 2n/3^2 + 2n/3^2 + \frac{4n}{3^2} = 9n/3^2$

$n/3k = 1$, $k = \log_3 n$ ($\log_3 n \rightarrow$ with bas 3)

$= c \cdot n \log_3 n$

$= O(n \log n)$

3 a) What does the algorithm compute?

The algorithm finds the minimum value in the array, efficiently breaking down the problem into smaller sub-problems

$n = 1 \rightarrow$ there is only one element.

b) Setup a recurrance relation for algorithm basic operation count and solve it

$T(n) = T(n-1)+1$ when $n>1$

$T(1) = 0$ (no comparission)

$T(n) = T(1) + T(n-1)$

$= 0 + (n-1)$

$= n-1$

The complexity $= O(n)$

4) Analyze the order of growth.

i) $F(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega g(n)$ notation.

	F(n)	g(n)
	$2n^2+5$	$7n$
$n=1$	7	7
$n=2$	13	14
$n=3$	23	21

$n \geq 3$ $\quad F(n) \geq g(n) \cdot c$

F(n) is always greater than or equal to g(n) where $n \geq 3$

$F(n) = \Omega(g(n))$