

Algebra assignment

November 25, 2023

Questions

1. If one zero of the polynomial $p(x) = (a^2 + 4)X^2 + 20X + 4a$ is reciprocal of the other, find the value of a .
2. Find the roots of the quadratic equation

$$X^2 + X - (a + 1)(a + 2) = 0 \quad (1)$$

3. Solve for x :

$$10X - \frac{1}{X} = 3, X \neq 0 \quad (2)$$

4. In $\triangle ABC$, $\angle B = 90^\circ$ and $\tan A = \frac{1}{\sqrt{3}}$. Then find the value of $\sin A \cos C + \cos A \sin C$
5. If $X = a \sin \theta + b \cos \theta$ and $y = a \cos \theta - b \sin \theta$, then find the value of $(X^2 + Y^2)$.
6. Answer any **four** of the following questions:
 - (a) The sum and the product of the zeroes of a quadratic polynomial are -1 and -12 respectively. The polynomial is
 - i. $X^2 - X - 12$
 - ii. $X^2 + X - 12$
 - iii. $X^2 - X + 12$
 - iv. $X^2 + X + 12$
 - (b) The zeroes of the quadratic polynomial $x^2 + 20x + 91$ are
 - i. both positive.
 - ii. both equal.
 - iii. both negative.
 - iv. one positive and one negative.
 - (c) If the zeroes of the polynomial $5x^2 - 26x + k$ are reciprocal of each other, then the value of k is

- i. 5
 - ii. -5
 - iii. $\frac{1}{5}$
 - iv. $-\frac{1}{5}$
- (d) If α, β are the zeroes of the polynomial $x^2 - 5x - 14$, then the value of $\alpha\beta - \alpha - \beta$ is
- i. -9
 - ii. 19
 - iii. 9
 - iv. -19
- (e) What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of the resulting polynomial ?
- i. 5
 - ii. 4
 - iii. 2
 - iv. 1
7. If $2 \sin 2A = \sqrt{3}$, then find the value of A.
8. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$, then show that $\tan \theta = \frac{1}{\sqrt{3}}$, $0^\circ < \theta < 90^\circ$
9. Find the quadratic polynomial whose zeroes are $(\sqrt{5} - 4)$ and $(\sqrt{5} + 4)$.
10. If the sum of *LCM* and *HCF* of two numbers is 1260 and the *LCM* is 900 more than their *HCF*, find their *LCM*.
11. Find the values of m and n for which x=2 and x=3 are the roots of the quadratic equation $3x^2 - 2mx + 2n = 0$.
12. Divide 19 into two parts such that sum of their squares is 193.
13. The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building.
14. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed on the top of a 20m high building are 45° and 60° respectively. Find the height of the tower.
15. As observed from the top of 75m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
16. It takes 12 hours to fill a swimming pool using two pipes together. If the larger pipe is used for 4 hours and smaller pipe is used for 9 hours, only half of the pool is filled. How long will it take for each pipe alone to fill the pool?