—:UNIT – 1:—

Stress:- When some external forces applied on a body, then the body offers some internal resistance to .

- The magnitude of the internal resistance force is numerically equals to the applied forces.
- This internal resisting force per unit area is known as stress (σ)

A body is subjected to no. of forces Fig.1

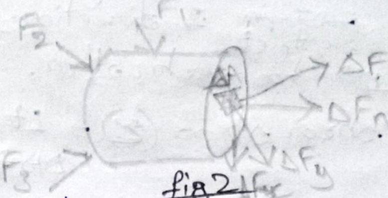


fig 2

Defining stress at a point

- we have to understand the concept of stress, consider a body subjected to no. of forces as shown in fig1.
- An imaginary cut section (Perfection) plane 11, on the left portion is drawn seperatly as shown in fig2.
- Let us consider elementary force ΔF, which is acting on the elemental area ΔA

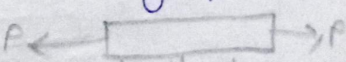
from the definition, stress $$\boxed{(\sigma) = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}}$$

normal stress $$\sigma_n = \lim_{\Delta A \to 0} \frac{\Delta F_n}{A} = \frac{dF_n}{dA}$$

tensile stress | compressive stress

tensile stress:- when a body subjected to 2 equal & opposite pulls from both sides then it causes extending of length of the bar.

- Such a stress which causes the increase in the length of the bar, is known as tensile stress.



tensile stress

Compressive stress:-

When a body subjected to 2 equal forces pushing it from both sides then it causes an decrease in the length of the bar.

- Such a stress which causes decrease in the length of the bar is known as compressive stress.

- The tensile & compressive stress are also known as the direct stresses.

Strain: — It is ~~remained~~ as ratio of change in length to original length.

~~to~~ generally, strain $(\epsilon) = \dfrac{\text{change in length}}{\text{original length}}$

Shear stress (τ): — The forces ΔF may be resolved into infinite no. of components along the surface of the area ΔA. However if we ~~restrain~~ restrict 2 directions (along x & y directions) which are $\perp^{er}$ to each other.

i.e, shear stress $(\tau) = \lim_{\Delta A \to 0} \dfrac{\Delta F_x}{\Delta A} = \dfrac{dF_x}{dA}$

$$(\tau) = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} = \frac{dF_y}{dA}$$

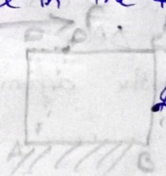
when dF_x, dF_y are known as shear forces, dA is known as shear Area.

Shear strain (ϕ): — Shear strain is produced ~~reduce~~ on the action of shear stresses & it is measured by the change in the ~~two~~ angle.

$$\tan\phi = \frac{CC'}{CB}$$

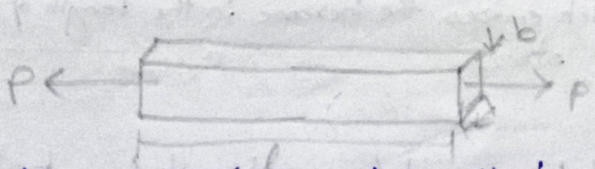
$$\tan\phi = \frac{\delta L}{L}$$

$$\boxed{\phi = \frac{\delta L}{L}}$$



for small angle, $\tan\phi = \phi$

consider a rectangle block ABCD, In which the botton portion AB is fixed. A tangential force F is applied along the surface on the top layer DC as shown in fig. Due to the application of the force the block ABCD is ~~reastorted~~ deformed to ABC'D' due some angle ϕ.



consider a rectangular bar of ~~length, area~~ l, b, t subjected to the equal forces on both sides as shown in fig. ~~to the~~.
due to the application of force there is a deformation takes place along the longitudinal & lateral directions.

$$\epsilon_L = \frac{\delta L}{L}$$

$$\epsilon_b = \frac{\delta b}{b}$$

$$\epsilon_t = \frac{\delta t}{t}$$

longitudinal strain $\epsilon_L = \frac{\delta L}{L}$

lateral strain, $= \epsilon_b$ & ϵ_t

$\frac{\delta b}{b}$ & $\frac{\delta t}{t}$

Poisson's ration (μ) (or) $\frac{1}{m}$ = $\frac{\text{lateral strain}}{\text{longitudinal strain}}$ = $\frac{\left(\frac{\delta b}{b}\right) \text{(or)} \left(\frac{\delta t}{t}\right)}{\left(\frac{\delta L}{L}\right)}$

Volumetric strain (ϵ_v) = $\frac{\text{change in volume}}{\text{original volume}} = \frac{\delta v}{v}$

Young's Modulus (or) Modulus of Elasticity :- $(Y) = \frac{\text{tensile stress}}{\text{tensile strain}}$ (or) $\frac{\text{compressive stress}}{\text{compressive strain}}$

i.e $\boxed{Y = \frac{\sigma}{\epsilon}}$ $\Rightarrow Y$ (or) $E = \frac{\left(\frac{P}{A}\right)}{\left(\frac{\delta L}{L}\right)}$ $\qquad \epsilon = Y\sigma$

$\Rightarrow E = \frac{PL}{A(\delta L)}$

$\therefore$ deformation, $\boxed{\delta L = \frac{PL}{AE}}$

$1 N/m^2 = 1 Pa$
$\sigma = N/mm^2$
ϵ = No units
$Y = \frac{\sigma}{\epsilon} = N/mm^2$
$\delta L = mm$
$C, G, N = N/mm^2$
$K =$

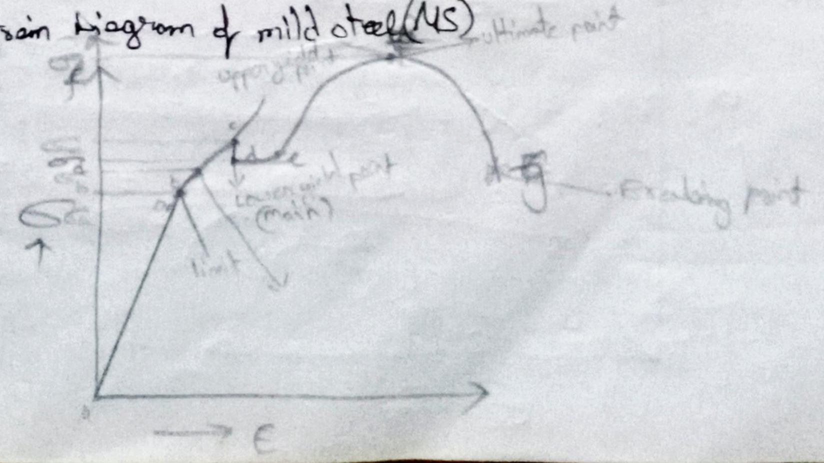
shear modulus
Modulus of Rigidity C (or) G (or) N = $\frac{\text{shear stress}}{\text{shear strain}}$

C (or) G (or) N $= \frac{\tau}{\phi}$ (or) $\frac{\tau}{\gamma}$

Bulk Modulus $(K) = \frac{\text{uniform stress/direct stress}}{\text{Volumetric strain}}$

$K = \frac{\sigma}{\left(\frac{\delta v}{v}\right)}$

stress-strain diagram of mild steel (MS)



⊙ Consider a mild steel bar subjected to a tensile load. The ~~curve~~ curve starts from o & there is no initial stress or strain in the specimen

- upto point A a Hook's law is obeyed (stress ∝ strain) & therefore oa represents as a st. line. And the point a is known as limit of proportionality. & corresponding stress is known as limit of proportionality stress (σ_a).

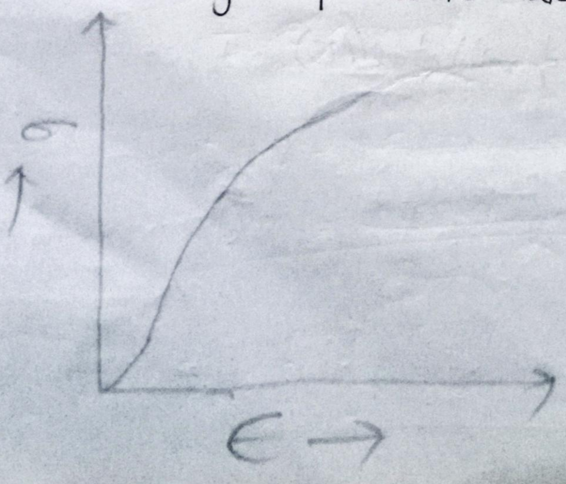
- The portion of the diagram b/w a & b is not a st. line. But upto point b, the bar is in elastic in nature i.e, on the removal of the load the bar regain its original shape. & the corresponding stress at point b is known as Elastic limit stress (σ_b).

- After the point b, the bar is a plastic deformation until the point c which is known as upper yield point then after that from point c to d, there is a decrease in the stress & the point d is known as lower yield point. At this point without any application of load, there is a increase in the length of the specimen. that point is known as yield point corresponding stress is known as yield stress.

- from the point e the strength of the material increases & reaches the ultimate point (f) corresponding load is known as unltimate load & corresponding stress is known as ultimate stress (σ_u). At this point onward neck formation takes place in the specimen, thereby Area decreases gradually & finally broken into 2 pieces. At the point g

- The point g is known as breaking point & the corresponding stress is known as breaking stress

stress - strain Diagram for Brittle materials



Ex:- Cast Iron (C.I)
Glass, wood

- In Brittle materials, there is no appriable rate of deformation i.e, there is no yield point & there is no neck formation.
- ultimate point & breaking points are same for brittle materials

① A circular rod of diameter 20mm & 500mm long is subjected to tensile force of 45kN. The modulus of elasticity for the steel is $200 kN/mm^2$. Find stress, strain, & elongation in the bar.

Ⓐ Given, $P = 4\cancel{8}^{5} \times 10^3 N$; $E = 200\times10^3 N/mm^2$; $L = 500mm$; $D = 20mm$

(i) stress $= \frac{P}{A} = \frac{4\cancel{8}^{5}\times10^3}{\frac{3.14\times 20^2}{4}} = 143.24 N/mm^2$

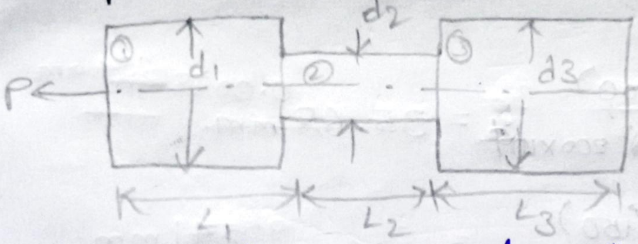
~~(ii) strain = ε =~~

(iii) $\delta L = \frac{PL}{AE} = \frac{45\times10^3\times500}{\frac{\pi}{4}(20)^2\times200\times10^3} = 0.358mm$

(ii) strain $= \frac{\delta L}{L} = \frac{0.358}{500} = \cancel{0.00}\,7\times10^{-4}$

Stress in Bars of varying sections:
1. Bars of Different sections
2. Bars of uniformly tappering sections (circular)
3. Bars of composite sections.

① Bars of Different sections



consider a bar of varying circular sections as shown in above fig and subjected to an axial load of 'P' throughout,

Area of different sections are given by

$A_1 = \frac{\pi}{4}d_1^2$; $A_2 = \frac{\pi}{4}d_2^2$; $A_3 = \frac{\pi}{4}d_3^2$

Let $\sigma_1, \sigma_2, \sigma_3$ are the stresses included in sections

$||^{ly}$ $\epsilon_1, \epsilon_2, \epsilon_3$ are the strains in different sections

$\sigma_1 = \frac{P}{A_1}$; $\sigma_2 = \frac{P}{A_2}$; $\sigma_3 = \frac{P}{A_3}$

$\epsilon_1 = \frac{\delta L_1}{L_1}$; $\epsilon_2 = \frac{\delta L_2}{L_2}$; $\epsilon_3 = \frac{\delta L_3}{L_3}$

change in lengths,

$\delta L_1 = \frac{PL_1}{A_1E}$; $\delta L_2 = \frac{PL_2}{A_2E}$; $\delta L_3 = \frac{PL_3}{A_3E}$

Total elongation in the bar = $\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$$\boxed{\delta L = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}}$$

law of superposition

0.0.2456

(1) A round bar subjected an axial load of 100kN, what must be the diameter 'd', if stress their is to be 100 MN/m². Find the total elongation in the bar if E = 200 GPa

$1\,N/m^2 = 1\,Pa$

$1\,MPa = 1\times10^6\,N/m^2$

$1\,MPa = 1\times10^6\,N \times 10^{-6}\,N/mm^2$

$\boxed{1\,MPa = 1\,N/mm^2}$

$\boxed{1\,GPa = 10^3\,N/mm^2}$

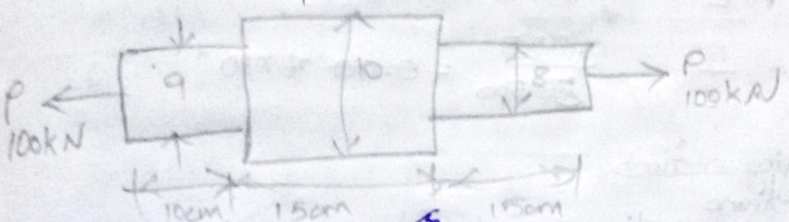
$\boxed{G = 10^9}$

~~Sol:- Given, $P = 1\times10^5\,N$, $\sigma = 1\times10^8\,N/m^2$~~

~~$E = 2\times10^{11}\,N/m^2$~~

~~W.K.T, $\sigma = \frac{P}{A}$~~

~~$10^8 = \frac{10^5}{\frac{\pi}{4}d^2}$~~



Sol:- Given, $\sigma = 100\,N/m^2 \div 100\,N/mm^2$

$E = 200\,GPa = 200\times10^3\,N/mm^2$

$P = 10^5\,N$

$$\delta L_1 = \frac{PL_1}{A_1E} = \frac{10^5(100)}{\frac{\pi}{4}(90)^2\,200\times10^3} = \cancel{35.68\,mm}\ 0.0078634\,mm$$

$$\delta L_2 = \frac{PL_2}{A_2E} = \frac{10^5(150)}{\frac{\pi}{4}(100)^2\times200\times10^3} = 0.0095541\,mm$$

$$\delta L_3 = \frac{PL_3}{A_3E} = \frac{10^5(150)}{\frac{\pi}{4}(80)^2\times200\times10^3} = 0.014928\,mm$$

$\therefore \delta L = \delta L_1 + \delta L_2 + \delta L_3 = \cancel{35.68 + 0.0078 + 0.0042 + 0.0149}$ $0.0078634 + 0.0095541 + 0.014928$

$= \cancel{0.0269\,mm} = 0.0323455\,mm$

$$\delta L = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

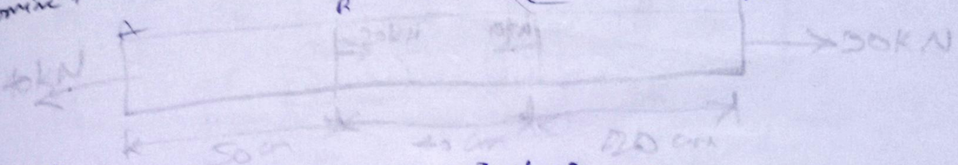
$$\Rightarrow \delta l = \frac{100\times10^3}{200\times10^3}\left[\frac{100}{\frac{\pi}{4}(75.68)^2}+\frac{150}{\frac{\pi}{4}(100)^2}+\frac{150}{\frac{\pi}{4}(80)^2}\right]$$

$$\boxed{\Rightarrow \delta l = 0.0323\ \text{mm}}$$

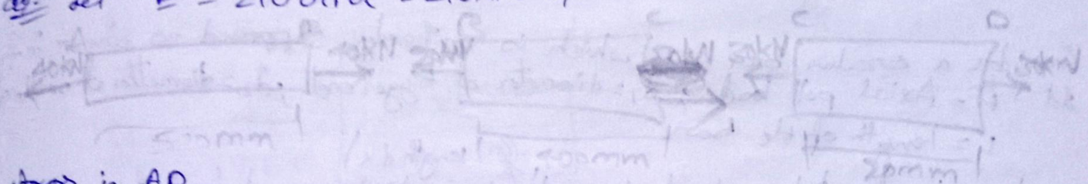
$$\Rightarrow \delta l = \frac{100\times10^3}{200\times10^3}\left[\frac{100}{\frac{\pi}{4}(90)^2}+\frac{150}{\frac{\pi}{4}(100)^2}+\frac{150}{\frac{\pi}{4}(80)^2}\right]$$

$$\boxed{\Rightarrow \delta l = 0.0323\ mm}$$

(2) A steel bar of 25mm diameter is loaded as shown in fig. Determine the stresses in each part & total elongation in the bar.



Sol: Let $E = 210\ GPa = 210\times10^3\ N/mm^2$



stress in AB –

$$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{40{,}000}{\frac{\pi}{4}(25)^2} = 81.48\ N/mm^2$$

stress in BC,

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{20{,}000}{\frac{\pi}{4}(25)^2} = 40.76\ N/mm^2$$

stress in CD,

$$\sigma_{CD} = \frac{P_{CD}}{A} = \frac{30{,}000}{\frac{\pi}{4}(25)^2} = 61.11\ N/mm^2$$

elongation in AB, total elongation,

$$\delta l_{AB} = \frac{P_{AB}L_{AB}}{A}$$

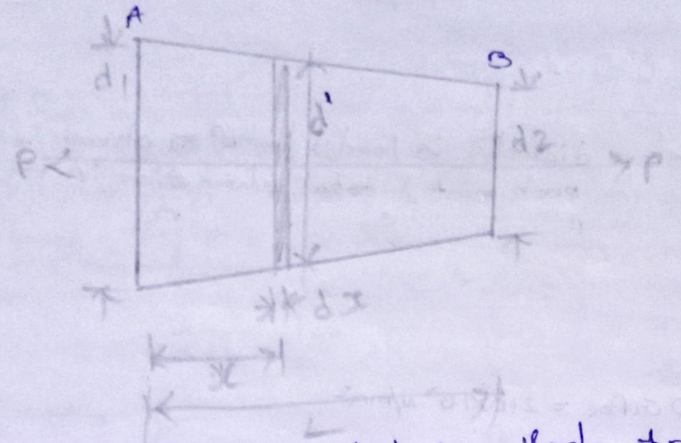
$$\delta L = \delta L_{AB} + \delta L_{BC} + \delta L_{CD}$$

$$\delta L = \frac{1}{AE}\left[P_1L_1 + P_2L_2 + P_3L_3\right]$$

$$\delta l = \frac{1}{\frac{\pi}{4}(25)^2\times10^3\times210}\left[40{,}000\times500 + 20000\times400 + 30000\times200\right]$$

$$\boxed{\delta l = 0.32mm}$$

stress in uniformly tappering section bars:-



Consider a circular bar AB which is uniformly tappered as shown in fig.
Let P = Axial pull load ; d_1 = diameter at bigger end ; d_2 = diametter at smaller end.
L = length of the bar
Consider, an elemental part of bar d' at a distance of 'x' from bigger end (or) length dx)
Now, Diameter of elemental strip (d') at a distance of x from the bigger end is given by

$$d' = d_1 - \left(\frac{d_1-d_2}{L}\right)x$$

(∵ from sles) $\frac{d_1-d_2}{L_1} = \frac{d_1-d'}{x} \Rightarrow \left(\frac{d_1-d_2}{L_1}\right)x = x_1 - d' \Rightarrow d' = d_1 - \left(\frac{d_1-d_2}{L}x\right)$

Area at the section, $A' = \frac{\pi}{4}(d')^2$

$$\boxed{A' = \frac{\pi}{4}(d_1 - kx)^2} \quad \left(\because k = \frac{d_1-d_2}{L}\right)$$

stress in the elemental section (σ') $= \frac{P}{A'}$

$$\sigma' = \frac{P}{\frac{\pi}{4}(d_1 + kx)^2}$$

$$\boxed{\sigma' = \frac{4P}{\pi(d_1-kx)^2}} \text{ — (1)}$$

strain in the section, $\epsilon' = \frac{\sigma'}{E}$

$$\boxed{\epsilon' = \frac{4P}{\pi E(d_1-kx)^2}} \text{ — (3)}$$

But strain at the section length dx is given by,

$$\epsilon' = \frac{\delta dx}{dx} \text{ — (4)}$$

from (3) & (4)

$$\frac{\delta dx}{dx} = \frac{4P}{\pi E(d_1-kx)^2}$$

$$\delta dx = \left(\frac{4P}{\pi E(d_1-kx)^2}\right)dx$$

Total elongation in the bar,

$$(\delta L) = \int_0^L \frac{4P\,dx}{\pi E (d_1 - kx)^2}$$

$$\delta L = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1-kx)^2}$$

$$\delta L = \frac{4P}{\pi E}\left[\frac{(d_1-kx)^{-1}}{(-1)(-k)}\right]_0^L$$

$$\delta L = \frac{4P}{\pi E}\left[\frac{(d_1-kL)^{-1}}{k} - \frac{(d_1)^{-1}}{k}\right]$$

$$\delta L = \frac{4P}{\pi E k}\left[\frac{1}{d_1-kL} - \frac{1}{d_1}\right]$$

$$\cancel{\delta L = \frac{4P}{\pi E k}\left[\frac{kL}{d_1(d_1-kL)}\right]}$$

$$\cancel{\delta L = \frac{4PL}{\pi E}\left[1\right]}$$

$$\delta L = \frac{4P}{\pi E k}\left[\frac{1}{d_1 - \cancel{\frac{(d_1-d_2)}{L}L}} - \frac{1}{d_1}\right]$$

$$\delta L = \frac{4PL}{\pi E(d_1-d_2)}\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

$$\boxed{\delta L = \frac{4PL}{\pi E d_1 d_2}}$$

if, $d_1 = d_2 = d$

then, $\delta L = \frac{4PL}{\pi E d^2}$

$$\delta L = \frac{PL}{\frac{\pi}{4}(d^2)E}$$

$$\boxed{\delta L = \frac{PL}{AE}}$$

A circular bar of 2m long uniformly tappering from 30mm to 20mm diametter, calculate elongation of a bar a load of 50kN, take $E = 140$ GPa (Al)

Sol:- Given, $d_1 = 30$mm, $d_2 = 20$mm, $E = 140 \times 10^3$ N/mm^2, $P = 50 \times 10^3$ N

$L = 2 \times 10^3$ mm.

w.k.T, total elongation,

$$\delta l = \frac{4PL}{\pi E d_1 d_2} = \frac{4 \times 50 \times 10^3 \times 2 \times 10^3}{3.14 \times 140 \times 10^3 \times 30 \times 20} = 1.515 \text{mm}$$

A 2m long steel bar as a uniform diameter of 40mm for a length of 1m form 1 end. for next 0.5m length the diameter decreases uniformly to 'd'. for remaining 0.5m length it has uniform diameter of 'd' mm. when a load of 150 KN is applied. It is observed that extention of 2.4mm in the bar. Calculate 'd'.

Sol:- Given, $L = 2 \times 10^3$mm, $P = 150 \times 10^3$ N, $E = 2 \times 10^5$ N/mm^2, $d_1 = 40$mm, $d_2 = d$

Elongation at AB, $\delta L_1 = \frac{PL}{AE} = \frac{150 \times 10^3 \times 10^3 \times 4}{\pi (40)^2 \times 2 \times 10^5} = 0.5971$ mm.

Elongation at BC, $\delta L_2 = \frac{4PL}{\pi E d_1 d_2} = \frac{4 \times 150 \times 10^3 \times 500}{\pi \, 2 \times 10^5 \times 40 \times d} = \frac{11.93}{d}$

Total elongation, $\delta L = \frac{PL_1}{AE} + \frac{4PL_2}{\pi E d_1 d} + \frac{PL_2}{A_2 E}$

$\delta L = \frac{P}{E}$.

24

$\therefore \delta l = 0.5971 + \frac{11.93}{d} + \frac{477.46}{d^2}$

$24d^2 = 0.5971d^2 + 11.93d + 477.46$

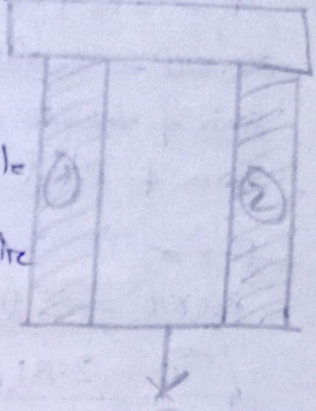
$23.4029d^2 - 11.93d + 477.46 = 0$

$d = 19.9$mm (or) $d = -13.28$ mm

$\boxed{\therefore d = 19.9\text{mm}}$

Stress in bars of composite section

Composite Bars - A bars which is made up of 2 (or) more different materials joined together.



- the bars are joined together in such a manner that the system expands (or) contracts as 1 unit equally when it is subjected to a tensile (or) compressive load resp.
- the following assumptions are made in composite bars

① extension (or) contraction must be equal,
$\boxed{\text{i.e., strains are equal.}}$

② Total external load in bar is equal to, sum of loads carried by individual bars.

Let P = total external load; L = length of the bar; P_1 & P_2 are the external load applied in bar ① & ② resp; E_1 = young's modulus of material of bar ①; A_1 = C.S.A of bar ①; E_2 = young's modulus of material of bar ②, A_2 = C.S.A of bar ②.

① from 1st condition, $\boxed{\epsilon_1 = \epsilon_2}$

$$\Rightarrow \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \boxed{\sigma_1 = \left(\frac{E_1}{E_2}\right)\sigma_2}$$

② from second condition,

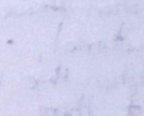
$$\boxed{P = P_1 + P_2}$$

$\Rightarrow \boxed{P = \sigma_1 A_1 + \sigma_2 A_2}$ is the equation

Note:-

① The ratio of $\left(\frac{E_1}{E_2}\right)$ is known as Modulus Ratio.

② If the lengths of the bars are different, then the elongations, (δL), carried out & can be calculated separately & equated

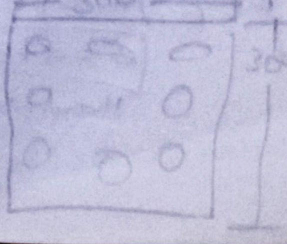


$\epsilon_1 = \epsilon_2$ ✗

$\frac{\delta L_1}{L_1} = \frac{\delta L_2}{L_2}$ ✓

- A reinforced concrete column is 300x300 mm. The column is provided with 8 reinforced steel bar of 20mm diameter, the column carries a load of 360 kN. Find the stresses in concrete & steel bar.

Sol:- Given, $E_s = 2.1\times10^5\ N/mm^2$; $E_c = 0.14\times10^5\ N/mm^2$; $\sigma_c = ?$; $\sigma_s = ?$



① W.K.T, $\epsilon_c = \epsilon_s$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

$$\frac{\sigma_c}{0.14\times10^5} = \frac{\sigma_s}{2.1\times10^5} \Rightarrow \boxed{\sigma_s = 15\sigma_c}$$

(2) $P = P_c + P_s$

$P = \sigma_c A_c + \sigma_s A_s$

~~$360 \times 10^3 = \sigma_s (9 \times 10^4) + 15\sigma_c (9 \times 10^4)$~~

Area of concrete ~~concrete~~ $= 300 \times 300 = 9 \times 10^4\ mm^2$

Area of steel bar $= 8 \left(\frac{\pi}{4}\right)(20)^2 = 2514\ mm^2$

~~$P = \sigma_s$~~

$360 \times 10^3 = \sigma_s (9 \times 10^4) + 15\sigma_c (2514)$

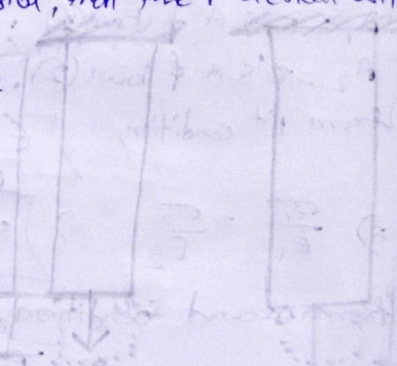
$\boxed{\sigma_c = 2.81\ N/mm^2}$

$\boxed{\sigma_s = 42.2\ N/mm^2}$

17/10/2023

Thermal stresses:-

- when a thermal force is applied to material, then the material will expand, ~~as same a~~
- when we apply the the load on the same material,
- when the elongation is same to both of them,
- then the rise in temp. is equals to the load applied on it.



$\Rightarrow$ W.K.T $\Delta = \frac{PL}{AE}$ (mech.) & $\Delta = \alpha TL$ (thermal)

$\Rightarrow \boxed{\alpha TL = \frac{PL}{AE}}$

$\boxed{P = t}$ rise in temp.

thermal resistance force:-

- when we resist, the elongation due to thermal force, then there will be a compressive resistance force which will compress it.

$\boxed{\therefore \Delta = \alpha TL - \frac{PL}{AE}}$

steel — $12 \times 10^{-6}/°C$

copper — $17.5 \times 10^{-6}/°C$

stainless steel — $18 \times 10^{-6}/°C$

Brass, Bronze — $19 \times 10^{-6}/°C$

Aluminium — $23 \times 10^{-6}/°C$

$\delta L = \alpha tL$

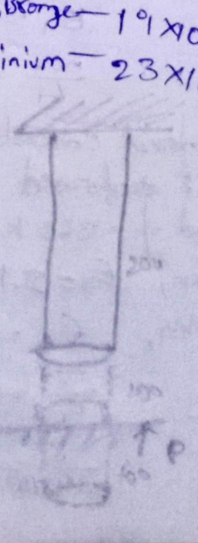
$\frac{\delta L}{L} = \alpha t$

thermal strain $\boxed{\epsilon = \alpha t}$

$E = \frac{\sigma}{\epsilon} \Rightarrow \sigma = E\epsilon$

$\boxed{\sigma = \alpha t E}$

thermal stress



Sai $\boxed{\Delta = \alpha T L = \frac{PL}{AE}}$

$\Delta = 2\times10^{-4}\times 100\times 10$

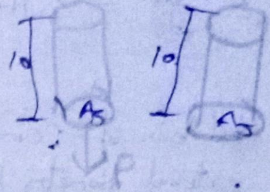
$\boxed{\Delta = 120}$

Jeeva, $\Delta = \alpha_J T_J L_J$

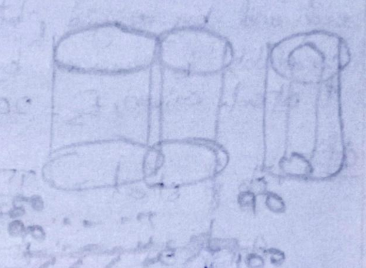
$\boxed{\Delta = 100}$

$E_S = 200$ $E_J = 560$

$\alpha_S = 23\times10^{-6}/°c$ $\alpha_J \to 12\times10^{-6}/°c$



- If two materials are compounded, then they will share each other acc. to their strength (α).



Sai $\Rightarrow \Delta = 80 + \frac{PL}{AE}$ (t)

Jeeva $\Delta = 100 - \frac{PL}{AE}$ (c)

$\Rightarrow \alpha T L = \frac{PL}{AE}$

$\boxed{P = \alpha T A E}$

① A steel rile of 12 m long & is lied at a temp of 18°c, the max. temp. expected is 40°c (i) estimate the min. gap b/w two rails to be left so that the temp. stresses don't develop. (ii) calc. temp. stresses developed in the rails (a) if no expansion joint is provided.
(b) If a 1.5 mm gap is provided for expansion
(iii) If the stress developed is 20 N/mm² what is the gap provided b/w the rails - take $E = 2\times10^5$ N/mm²
$\alpha = 12\times10^{-6}/°C$

Sol:- Given

i) $\Delta = \alpha t L = (12\times10^{-6})(22)(12\times10^3) = 3.168$ mm

(ii) ⓐ $\delta = \alpha t L = (12\times10^{-6})(22)(12\times10^3) = 3.168$ mm.

w.k.T $\alpha t \cancel{L} = \frac{P\cancel{L}}{AE}$

$\Rightarrow \frac{P}{A} = \alpha t E$

$\boxed{\sigma_t = 52.8 \text{ N/mm}^2}$

ⓑ $\Delta = \alpha t L = 3.16$ mm

Given, $\Delta = 1.5$ mm

then, $1.5 = 3.16 - \frac{PL}{AE}$

$\frac{PL}{AE} = 1.66$

$\boxed{\frac{P}{A} = 27.67}$

(c) $\delta = \alpha t L = $

$\delta L = 3.16 - \frac{PL}{AE} = 3.16 - \frac{\cancel{20}\times 12000}{\cancel{2}\times 10^{\cancel{5}} 10} = 3.16 - 1.2 = 1.96$

(1) A steel bar of 20mm diameter passes centrally to a copper tube of 50 mm external diameter & 40mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The temp of assembly is raised by 50°C, Calc. the stresses developed in cu & steel. Given, $E_s = 200\ GN/m^2$, $E_{cu} = 100\ GN/m^2$; $\alpha_s = 12\times10^{-6}/°C$

$\alpha_{cu} = 18\times10^{-6}/°C$

Sol:-

A_r of steel $= \frac{\pi}{4} 20^2 = 314.15\ mm^2$

A_r of cu $= \frac{\pi}{4}(50^2 - 40^2) = 706.85\ mm^2$

$t = 50°C$

(1) As ~~α for cu is more~~ the $\alpha_{cu} > \alpha_{st}$, the expansion in cu is more than to steel bar.

$\cancel{\delta L_s = \alpha_s T L + \frac{PL}{AE}}$

$\cancel{\delta L_s =}\ P_{st} = P_{cu}$

$\sigma_s A_s = \sigma_{cu} A_{cu}$

$\sigma_s = \frac{706.85}{314.15}\sigma_c$

$$\boxed{\sigma_s = 2.25\sigma_c}$$

direct,

$\alpha_s t L + \frac{PL}{AE} = \sigma_c t L - \frac{PL}{AE}$

$(12\times10^{-6}\times 50) + \left(\frac{2.25\sigma_c}{200\times10^3}\right) = (18\times10^{-6}\times 50) - \left(\frac{\sigma_c}{100\times10^3}\right)$

$\frac{2.25\sigma_c}{2\times10^5} + \frac{\sigma_c}{10^5} = (18\times10^{-6}\times 50) - (12\times10^{-6}\times 50)$

$\frac{2.25\sigma_c + 2\sigma_c}{2\times10^5} = 3\times10^{-4}$

$2.25\sigma_c + 2\sigma_c = 60$

$4.25\sigma_c = 60$

$\boxed{\sigma_c = 14.11\ N/mm^2}$ $\sigma_c = 240\ N/mm^2$

$\boxed{\sigma_s = 31.76\ N/mm^2}$ $\sigma_s = 540\ N/mm^2$

Relation b/w Elastic constants:-

Youngs Modulus, $E = \frac{\sigma}{e}$ N/mm²

Rigidity Modulus, $G = \frac{\tau}{\phi}$ N/mm²

Bulk Modulus, $K = \frac{\sigma}{e_v}$ N/mm²

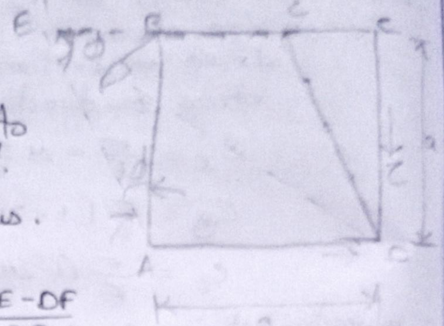
Poission Ratio,

$= \frac{\text{Lateral strain}}{\text{longitudinal (or) linear strain}}$

① Relation b/w E & G

- Consider a sample element a, b, c, d of side 'a', subjected to pure shear 'τ' as shown in fig., Deformed shape due to 'τ' drop perpendicular BF to Diagonal DE.

→ Let φ shear strain & G be rigidity modulus.



strain in Diagonal is,

$$\text{Diagonal BD}, (e) = \frac{DE - BD}{BD} = \frac{DE - DF}{DF}$$

$$= \frac{EF}{DB}$$

$$= \frac{EF}{AB\sqrt{2}}$$

∴ angle & deformation is very small we can assume $\angle DEF = 45^\circ$ & hence $EF = BE\cos 45^\circ$

$$\text{strain in Diagonal BD} = \frac{EF}{BD} = \frac{BE\cos 45^\circ}{AB\sqrt{2}} = \frac{G\tan\phi\cos 45^\circ}{a\sqrt{2}}$$

$$= G \times \phi \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

($\tan\phi = \phi$ ∵ $\tan\phi$ is very small)

$$= \frac{\tau}{2G}$$

($\because \phi = \frac{\tau}{G}$)

- we know that above pure shear gives to axial tensile stress in the ~~...~~ Diagonal Direction of 'BD' & axial compression at right angle to it. these two stress causes tensile stress along BD.

- tensile stress along the Diagonal BD,

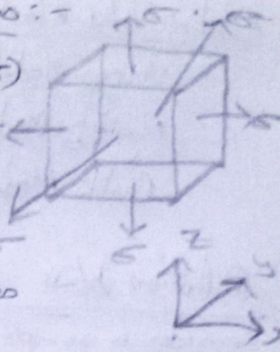
$$= \frac{\tau}{E} + \mu\frac{\tau}{E}$$

$$= \frac{\tau}{E}(1+\mu)$$

$$\Rightarrow \frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

Relation b/w Modulus of Elasticity & Bulk Modulus:-



- consider a cubic element subjected to stress(σ) in a 3 Mutually ⊥er Direction x, y, z as shown in above fig.
- Now stress 'σ' in 'x' direction causes tensile strain, stress in x-direction $= \frac{\sigma}{E}$
- while the stress 'σ' is 'y' & 'z' direction causes compression strain.

$$\text{stress y-direction} = \mu \frac{\sigma}{E}$$

$$\text{stress z-direction} = \mu \frac{\sigma}{E}$$

$$e_x = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$e_y = \frac{\sigma}{E}(1-2\mu)$$

$$e_z = \frac{\sigma}{E}(1-2\mu)$$

- volumetric strain $\Rightarrow e_v = e_x + e_y + e_z = \frac{3\sigma}{E}(1-2\mu)$

$$K = \frac{\sigma}{e_v}$$

$$\Rightarrow K = \frac{\cancel{\sigma} \times E}{3\cancel{\sigma}(1-2\mu)}$$

$$\Rightarrow E = 3K(1-2\mu)$$

$$E = 3K(1-2\mu)$$

$$E = 2G(1+\mu)$$

$$\Rightarrow 3K(1-2\mu) = 2G(1+\mu)$$

$$3K - 6\mu K = 2G + 2G\mu$$

$$3K - 2G = \mu(2G + 6K)$$

$$\Rightarrow$$

$$\Rightarrow \boxed{\frac{9}{E} = \frac{3}{G} + \frac{1}{K}}$$

∴ Relation b/w Young's Modulus, Rigid Modulus, Bulk Modulus is,

$$\boxed{\frac{9}{E} = \frac{3}{G} + \frac{1}{K}}$$

1) A steel tube of 30 mm External Diameter & 20 mm internal diameter ~~at each~~ encloses a copper rod of 15mm to which it is rigid joined at each end. If the temp 10°C. There is no longitudinal stress. Calc. the stresses in rod, when the temp. raised to 200°C. $E_s = 2.1 \times 10^5$ N/mm²; $E_c = 1 \times 10^5$ N/mm²; $\alpha_s = 11 \times 10^{-6}/°C$

$\alpha_c = 18 \times 10^{-6}/°C$

Sol:- $A_c = \frac{\pi}{4}(15)^2 = 176.71$ mm²

$A_s = \frac{\pi}{4}(30^2 - 20^2) = 392.69$ mm²

Rise in temp T = 190°C

~~Direct~~, at equilibrium conditions,

$$P_c = P_s$$

$$\sigma_c A_c = \sigma_s A_s$$

$$\sigma_c (176.71) = \sigma_s (392.69)$$

$$\boxed{\sigma_c = 2.22 \sigma_s}$$

Direct,

$$\Rightarrow \alpha_s T \cancel{L_s} + \frac{\sigma_s \cancel{L_s}}{E_s} = \alpha_c T \cancel{L_c} - \frac{\sigma_c \cancel{L_c}}{E_c}$$

$$11 \times 10^{-6} \times 190 + \frac{\sigma_s}{2.1 \times 10^5} = 18 \times 10^{-6} \times 190 - \frac{2.22 \sigma_s}{1 \times 10^5}$$

$$\sigma_s + (2.22 \times 2.1)\sigma_s = 279.3$$

$$5.662 \sigma_s = 279.3$$

$$\boxed{\sigma_s = 49.32 \text{ N/mm}^2}$$

$$\boxed{\sigma_c = 109.51 \text{ N/mm}^2}$$

2) A bar of 30mm Diameter subjected a pull of 60kN. The measured extension on guage length of 200mm is 0.1mm change in diameter is 0.004mm. Calculate 'E' & Poission ratio (μ) & Bulk Modulus.

Sol:- Given, $P = 60 \times 10^3$ N ; $d = 30$mm ; $L = 200$mm ; $\delta L = 0.1$mm

$\delta d = 0.004$mm

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2$$

W.K.T,

$$E = \frac{PL}{A(\delta L)} = \frac{6 \times 10^4 \times 200 \times 10}{706.85 \times 1} = 1.69 \times 10^5 \text{ N/mm}^2$$

$$\& \ \mu = \frac{\text{Lateral}}{\text{longitudinal}} = \frac{\left(\frac{\delta d}{d}\right)}{\left(\frac{\delta L}{L}\right)} = \frac{0.004 \times 200}{30 \times 0.1} = 0.26$$

$$\& \ E = 3K(1 - 2\mu)$$

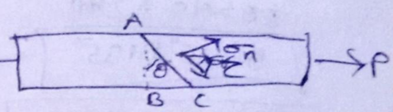
$$K = \frac{1.69 \times 10^5}{3(1 - 2(0.26))} = 0.58 \times 10^5 \text{ N/mm}^2$$

Principal stresses & strains:-

- Introduction
- σ_n, τ for the uniaxial load
- σ_n, τ for the Biaxial load
- σ_n, τ for the Biaxial & along shear
- $\sigma_x, \sigma_n \rightarrow$ principal stresses
- Problems – Analytical, Method
- Mohr's circle – Graphical

Method of finding stresses on a oblique plane under uni-axial loading:-

- Let us consider a bar of crosectional area 'A' shown in fig. which is carrying a load 'P' applied along the axis of the bar.



- This load 'P' is applied on a plane AB, which is $\perp^{r}$ to the plane AB
- Let an oblique plane AC is considered which makes an angle 'θ' to the Vertical AB.
- The stress induced on a plane AB is,

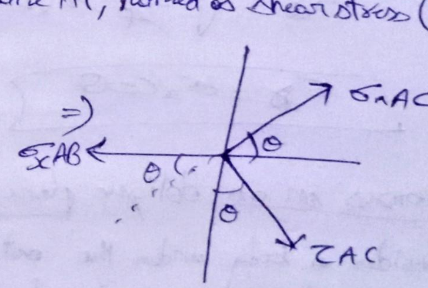
$$\boxed{\sigma_x = \text{Direct stress} = \frac{P}{A}}$$

$\therefore P = \frac{P}{AB}$

(force diagram)

- The stresses induced on a oblique plane AC are of 2 types
  i) $\perp^{r}$ to the direction of an oblique plane AC, named as normal stress (σ_n)
  ii) along the direction of an oblique plane AC, named as shear stress (τ).

$\Rightarrow$



Resolve the forces along the direction of $\sigma_n AC$

$\Rightarrow \sigma_n (AC) = \sigma_x AB \cos\theta$

$\Rightarrow \sigma_n = \sigma_x \left(\frac{AB}{AC}\right) \cos\theta \qquad (\because \frac{AB}{AC} = \cos\theta)$

$$\boxed{\sigma_n = \sigma_x \cos^2\theta} \quad - (2)$$

$$\boxed{\sigma_n = \sigma_x \left(\frac{1+\cos 2\theta}{2}\right)} \quad - (3)$$

$\therefore \sigma_n$ should be max. (or) min dependers upon the value of $\cos\theta$.

If $\theta = 0^\circ \Rightarrow \boxed{\sigma_n = \sigma_x}$

If $2\theta = 180^\circ \Rightarrow \boxed{\sigma_n = 0}$
(or)
$\theta = 90$

- for uniaxial loading σ_n is max. & min at 0° & 90° resp.

Resolving the forces along the τAC :-

$$\tau AC = \sigma_x \overline{AB} \sin\theta$$

$$\tau = \sigma_x \frac{AB}{AC} \sin\theta$$

$$\tau = \sigma_x (\cos\theta) \sin\theta$$

$$\boxed{\tau = \frac{\sigma_x}{2} \sin 2\theta}$$

$\therefore \tau$ is max. when $\sin 2\theta = \pm 1$

$2\theta = 90^\circ$ & 270°

$$\boxed{\theta = 45^\circ \; \& \; 135^\circ}$$

$$\boxed{\tau_{max} = \pm \frac{\sigma_x}{2}}$$

$\therefore \tau$ is min. when $\sin 2\theta = 0$

$$\boxed{\theta = 0 \; \& \; 90^\circ}$$

$$\boxed{\tau_{min} = 0}$$

Resultant stress $\Rightarrow \sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$$\sigma_r = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 (1+\cos 2\theta)^2 + \left(\frac{\sigma_x}{2}\right)^2 \sin^2 2\theta}$$

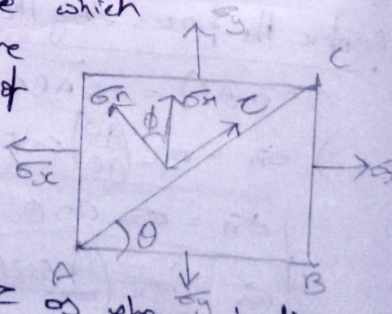
$$\sigma_r = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 (1+\cos^2 2\theta + 2\cos 2\theta + \sin^2 2\theta)}$$

$$\sigma_r = \sqrt{\frac{\sigma_x^2}{\cancel{4}} \cancel{2}(\cancel{2}\cos^2\theta)}$$

$$\boxed{\sigma_r = \sigma_x \cos\theta}$$

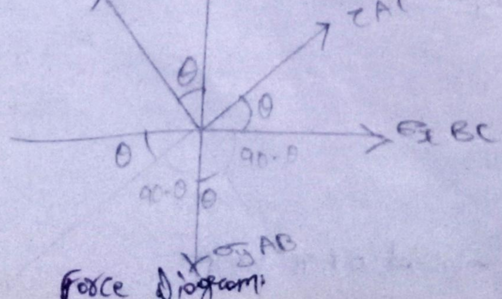
Stresses on an oblique plane under Bi-axial loading :-

- Consider a body under the action of Bi-axial stresses σ_x & σ_y as shown in fig. Let AC be the oblique plane which makes an angle 'θ' with horizontal (with the plane AB) which is $\parallel^{el}$ to the line of action of σ_x stress. Let σ_n & τ be the normal stress & shear stress resp. on the oblique plane AC.

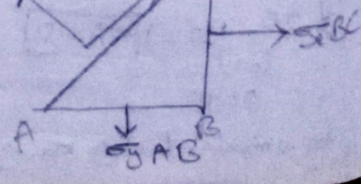


The forces can be resolved along σ_n & τ as shown in fig.

$\because$ thickness is unity



Force Diagram.



Resolve the forces along σ_n & τ :-

$$\frac{AB}{AC} = \cos\theta$$

$$\frac{BC}{AC} = \sin\theta$$

(1) $\sigma_n AC \cancel{\sigma_x BC} = \sigma_x BC \sin\theta + \sigma_y AB \cos\theta$

$$\sigma_n = \sigma_x \frac{BC}{AC}\sin\theta + \sigma_y \frac{AB}{AC}\cos\theta$$

$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta = \sigma_x\frac{(1 - \cos 2\theta)}{2} + \sigma_y\frac{(1+\cos 2\theta)}{2}$$

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_y - \sigma_x}{2}\right)\cos 2\theta}$$

resolve the forces along τ :-

(2). $\cancel{\tau AC = \sigma_y AB \cos\theta + \sigma_x BC}$

$$\tau AC + \sigma_x BC \cos\theta = \sigma_y AB \sin\theta$$

$$\tau = \sigma_y \frac{AB}{AC}\sin\theta - \sigma_x \frac{BC}{AC}\cos\theta$$

$$\tau = \sigma_y \sin\theta\cos\theta - \sigma_x \sin\theta\cos\theta$$

$$\boxed{\tau = \left(\frac{\sigma_y - \sigma_x}{2}\right)\sin 2\theta}$$

τ is max. at $\theta = 45°$ $\Rightarrow$ $\boxed{\tau_{max} = \frac{\sigma_y - \sigma_x}{2}}$ $\Rightarrow$ $\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2}}$

τ is min at $\theta = 0$ $\Rightarrow$ $\tau_{min} =$

Resultant stress $\Rightarrow$ $\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$$= \sqrt{\sigma}$$

if resultant stress σ_r makes an angle ϕ with σ_n stress

$$\boxed{\tan\phi = \frac{\tau}{\sigma_n}}$$

where ϕ is called as angle of obliquity

$$\Rightarrow \tan\phi = \frac{(\sigma_y - \sigma_x)\sin\theta\cos\theta}{\sigma_x \sin^2\theta + \sigma_y\cos^2\theta}$$

$$\boxed{\tan\phi = \frac{\sigma_y - \sigma_x}{\sigma_x \tan\theta + \sigma_y \cot\theta}}$$

Determine greatest angle of obliquity;
to

$$\frac{d}{d\theta}(\tan\phi)=0$$

at denominator, $\sigma_x \sin^2\theta - \sigma_y \cos^2\theta = 0$
after simplification,

$$\boxed{\tan\theta = \sqrt{\frac{\sigma_y}{\sigma_x}}}$$ ($\because$ max. obliquity exists at this angle)

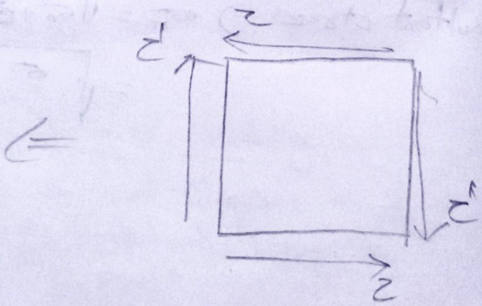
Max. obliquity,

$$(\phi_{max}) = \tan^{-1}\left[\frac{\sigma_y - \sigma_x}{\sigma_x\left(\sqrt{\frac{\sigma_y}{\sigma_x}}\right) + \sigma_y\left(\sqrt{\frac{\sigma_x}{\sigma_y}}\right)}\right]$$

$$\boxed{(\phi_{max}) = \tan^{-1}\left[\frac{\sigma_y - \sigma_x}{2\sqrt{\sigma_x\sigma_y}}\right]}$$

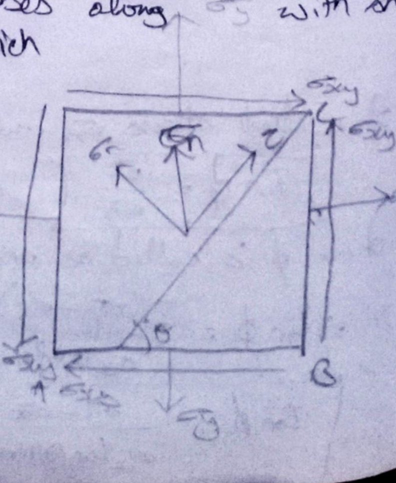
<u>complementary shear stress:-</u>
The existance of shear stress on any 2 sides of an element induces a complementary shear stresses on the other 2 sides of an element to maintain an equilibrium conditions.

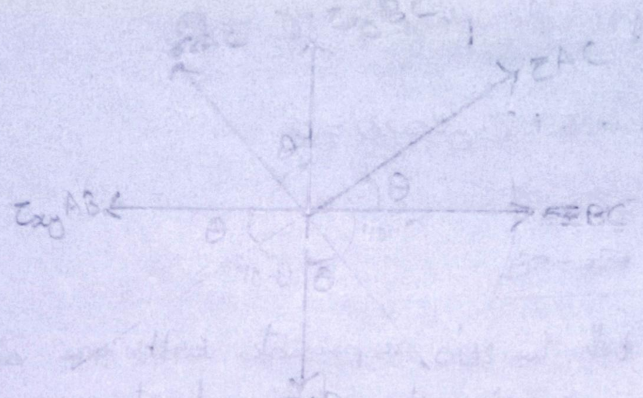
$$\boxed{\tau = \tau'}$$

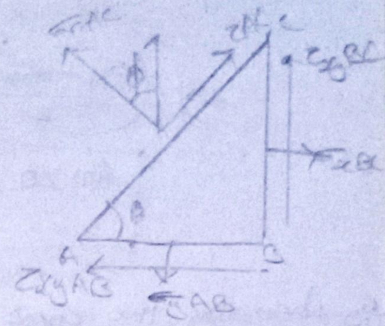


<u>A Body is subjected to biaxial stresses along with shear stress.</u>
Consider a body subjected to biaxial stresses along with shear stress. consider a oblique plane AC which makes an angle θ with plane AB which is ||el to line of action of σ_x stress.

Let σ_n & τ be the normal & shear stress resp. acting on a oblique plane AC.







$\cos\theta = \frac{AB}{AC}$

$\sin\theta = \frac{BC}{AC}$

Resolve the forces in σ_n direction:-

$$\sigma_n AC + \cancel{\sigma}\, \tau_{xy} BC\cos\theta + \tau_{xy} AB\sin\theta \cancel{+ \sigma_y AB(\cos\theta)}$$

$$= \sigma_y AB\cos\theta + \sigma_x BC\sin\theta$$

$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - \tau_{xy} \sin\theta\cos\theta - \tau_{xy}\sin\theta\cos\theta$$

$$\sigma_n = \sigma_x\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1+\cos 2\theta}{2}\right) - \tau_{xy}\sin 2\theta$$

$$\boxed{\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_y-\sigma_x}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta}$$

Resolve the forces in τ direction:-

$$\tau AC + \sigma_x BC\cos\theta + \tau_{xy} BC\sin\theta = \tau_{xy} AB\cos\theta + \sigma_y AB\sin\theta$$

$$\tau = \sigma_y \sin\theta\cos\theta - \sigma_x\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\boxed{\tau = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

- The direction & magnitudes of σ_n & τ will depend upon the relative magnitudes of σ_x, σ_y & τ_{xy}.
- If the direction of τ_{xy} is reversed (left side upwards), then put $\tau_{xy} = -\tau_{xy}$ in final equation.

For σ_n to max (or) min value;

$$\frac{d}{d\theta}(\sigma_n) = 0$$

$$\left(\frac{\sigma_x-\sigma_y}{\cancel{2}}\right)\sin 2\theta - 2\tau_{xy}\cos 2\theta = 0$$

$$\boxed{\tan 2\theta = \frac{+2\tau_{xy}}{(\sigma_x-\sigma_y)}}$$ ⇒ this gives orientation of

from the eqn, For a principle plane, $\tau = 0$

$$\Rightarrow \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}}$$

By observing the eqns both $\tan 2\theta$'s, represents both are similar in nature i.e, to max & min ~~stress~~ of normal stress will occurs on a planes where $\tau = 0$ (shear stress $= 0$)

- A planes having a shear stress $= 0$ ~~or only norm~~ & only normal stress exists, such planes are called as principle planes & corresponding normal stresses is named as principle stress.
- In case of 2-D system, the principle stresses are named as "Major principle stress" (σ_1) & other is "minimum principle stress" σ_2
- In case of 3-D, σ_1, σ_2, σ_3 are there

$$\underset{\text{major}}{\sigma_1},\ \underset{\text{intermediate}}{\sigma_2},\ \underset{\text{minimum}}{\sigma_3}$$

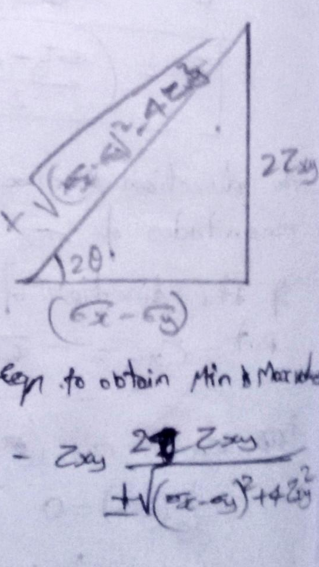
The max. & min values of σ_n can be determined as

$$\because \tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x-\sigma_y)}$$

from the $\triangle$le $\therefore$

$$\sin 2\theta = \frac{2\tau_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\cos 2\theta = \frac{\sigma_x-\sigma_y}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$



substitute the 2 values of $\sin 2\theta$ & $\cos 2\theta$ in σ_n eqn to obtain Min & Max values

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_y-\sigma_x}{2}\right)\cdot\frac{\sigma_x-\sigma_y}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} - \tau_{xy}\frac{2\tau_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} - \frac{\frac{1}{2}(\sigma_x-\sigma_y)^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} - \frac{2\tau_{xy}^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\left[\frac{(\sigma_x-\sigma_y)^2 + 4\tau^2_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau^2_{xy}}}\right]$$

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} \pm \frac{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2_{xy}}}{2}$$

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}$$

i.e, σ_n max $\Rightarrow \sigma_1$, major principle stress $= \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}$

i.e, σ_n min $\Rightarrow \sigma_2$, minimum principle stress $= \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}$

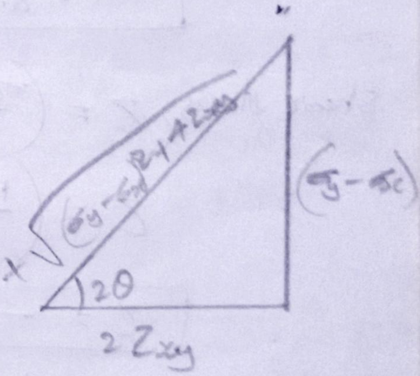
For shear stress to be max,

$$\frac{d}{d\theta}(\tau) = 0$$

$$(\sigma_y-\sigma_x)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\boxed{\tan 2\theta = \cancel{\frac{2\tau_{xy}}{\sigma_y-\sigma_x}} \; \frac{\sigma_y-\sigma_x}{2\tau_{xy}}}$$

from the Δ^{le}, $\sin 2\theta = \dfrac{\sigma_y-\sigma_x}{\pm\sqrt{(\sigma_y-\sigma_x)^2+4\tau^2_{xy}}}$



$$\cos 2\theta = \frac{2\tau_{xy}}{\pm\sqrt{(\sigma_y-\sigma_x)^2+4\tau^2_{xy}}}$$

Substituting both values of $\sin 2\theta$ & $\cos 2\theta$ in (τ) eqn,

$$\tau = \frac{\sigma_y-\sigma_x}{2}\left(\frac{\sigma_y-\sigma_x}{\pm\sqrt{(\sigma_y-\sigma_x)^2+4\tau^2_{xy}}}\right) + \tau_{xy}\frac{2\tau_{xy}}{\pm\sqrt{(\sigma_y-\sigma_x)^2+4\tau^2_{xy}}}$$

$$\tau = \frac{1}{2}\left(\frac{(\sigma_y-\sigma_x)^2+4\tau_{xy}}{\pm\sqrt{(\sigma_y-\sigma_x)^2+4\tau^2_{xy}}}\right)$$

$$\boxed{\tau_{max} = \pm\frac{1}{2}\left(\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2_{xy}}\right)}$$

$$\boxed{(\tau_{max}) = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2_{xy}}}$$

$$\therefore \boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$$

① At a point in a material, there are 2 normal tensile stresses of magnitudes 20 Mpa & 10 Mpa acting ⊥ to each other. there is also a positive shear stress of 5 Mpa acting at a point. Det. the normal, shear ~~&~~ on a plane, whose normal is inclined at 60° to 20 Mpa stress.

Sol:- Given, $\sigma_x = 20\ N/mm^2$; $\sigma_y = 10\ N/mm^2$

$\tau = 5\ N/mm^2$; $\theta = \cancel{60}\ 30°$

w.k.T,

Normal stress, $\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_y-\sigma_x}{2}\right)\cos 2\theta - \tau \sin 2\theta$

$$\sigma_n = \left(\frac{20+10}{2}\right) + \left(\frac{10-20}{2}\right)\cos 60° - 5 \sin 60°$$

$$\boxed{\sigma_n = 8.1698\ N/mm^2}$$

Shear stress, $\tau = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$$\tau = \left(\frac{10-20}{2}\right)\sin 60 + 5 \cos 60$$

$$\boxed{\tau = -1.83\ N/mm^2}$$

∴ The -ve sign indicates the direction of τ in opp. direction that we have taken.

2

2nd block

② A tensile stress of 110 N/mm² & ~~40~~ 7 N/mm² at right angles to each other. combining with shear stress of 63 N/mm²,
(i) find the direction & orientation of principle planes.
(ii) Find the magnitude of principle stresses.
(iii) Max. shear stress.
(iv) Max. shear stress planes.

Sol:- Given, $\sigma_x = 110\ N/mm^2$; $\sigma_y = \cancel{47}\ N/mm^2$; $\tau = 63\ N/mm^2$.

i) orientation of principle planes,

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\boxed{\theta = 31.7^\circ}$$

$\theta_1 = 31.7^\circ$ & $\theta_2 = 31.7+90 = 121.7^\circ$

ii) $\boxed{\sigma_1 = 148.93\ N/mm^2}$ > 110

$\boxed{\sigma_2 = 8.06\ N/mm^2}$ < 47

iii) $\boxed{\tau_{max} = 70.42\ N/mm^2}$ > 63

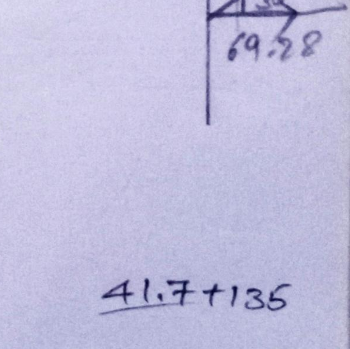
iv) Max. shear stress planes are, orientation of

$\theta = \theta_1 + 45$; $\theta_2 = \theta_1 + 135$

$\theta = 13.7 + 45$

$\boxed{\theta_1 = 76.7}$ $\boxed{\theta_2 = 166.7}$

③ The intensity of resultant stress of a plane AB is as shown in fig- at a point in a material under the stress is 80 Mpa & which is inclined 30° to the normal to the plane the normal component of the stress on the another plane BC at right angles to the plane AB is 60 N/mm². Det.
i) the resultant stress on BC
ii) Principle stresses and their orientations
iii) Max. shear stress & their orientation.



Sol:- Given, $\sigma_y = 60\ N/mm^2$; $\sigma_x = 69\ N/mm^2$

$\tau = 40\ N/mm^2$;

$\sigma_1 = 104.75\ N/mm^2$

$\sigma_2 = 24.24\ N/mm^2$

$\tan 2\theta =$

$\boxed{\theta = 41.7^\circ}$ 41.7+45 41.7+135

$$\boxed{\tau_{(max)} = 40.25}$$

Mohr's circle:- Method

graphical → $\sigma_1, \sigma_2, \sigma_n, \sigma_t, \sigma_R$

we can find, τ_{max}, oblique angle & shear angle.

Procedure:-

step-1:- Line diagram

step-2:- Draw a quadrant system.

step-3:- Declare x-axis (normal stress) - σ_x & σ_y, y-axis - τ

step-4:- locate σ_x, σ_y, & τ_{xy} values on both axis.

step-5:- locate points, x, y.

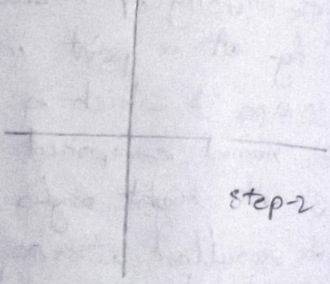
Distance b/w

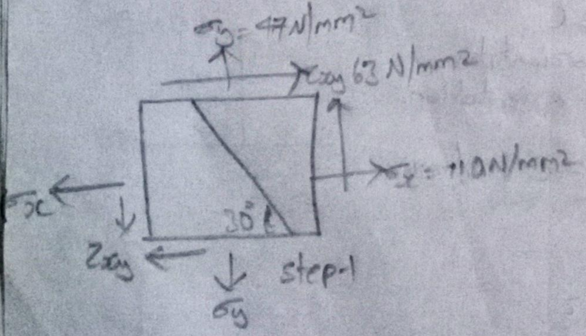
O-D — σ_2 (min)

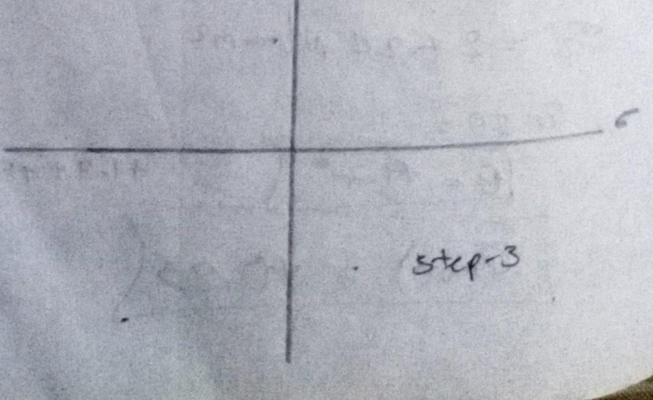
O-E — σ_1 (max)

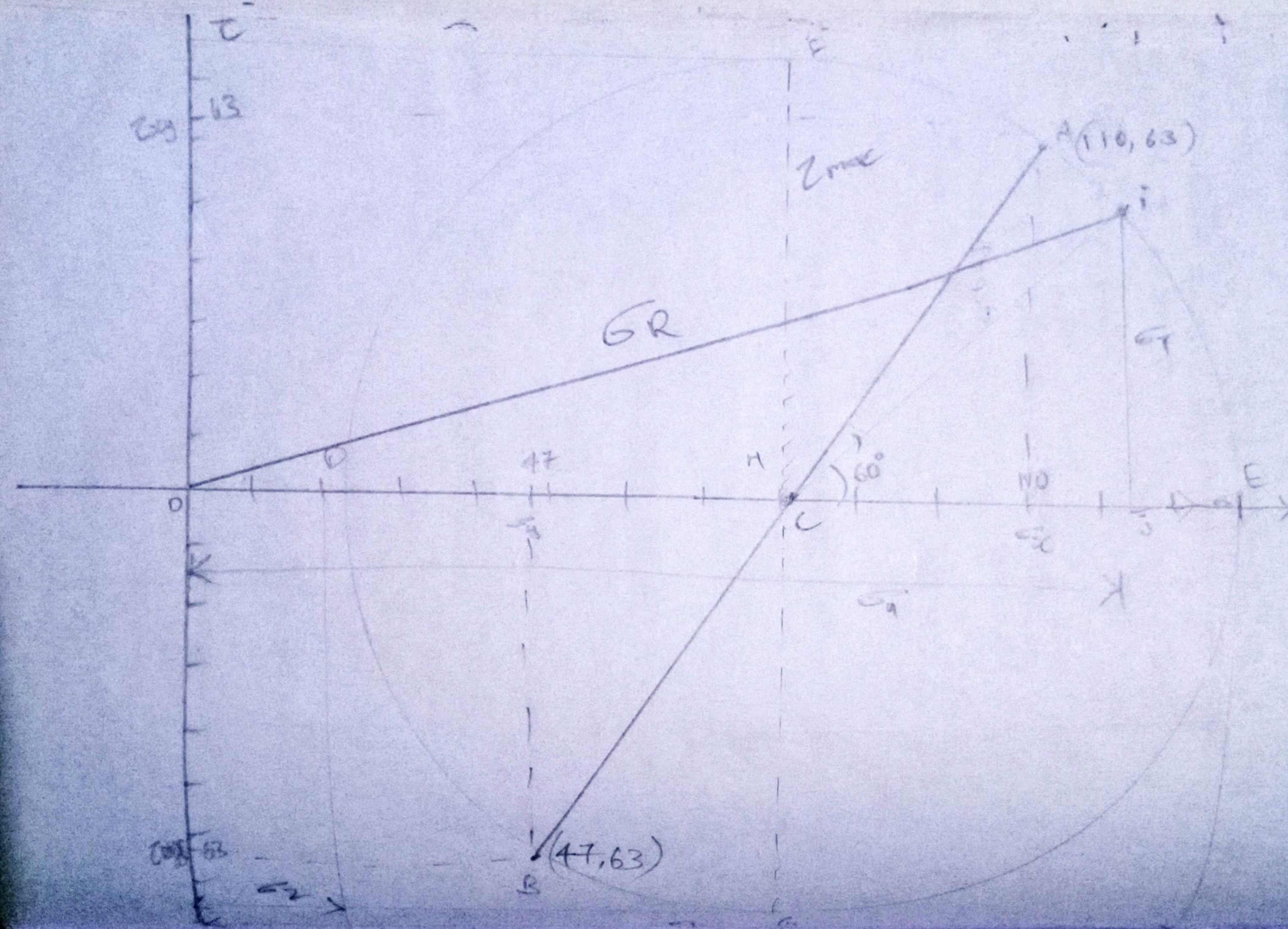
radius of Mohr's cle = $\frac{\sigma_1 - \sigma_2}{2}$

τ_{max} = M.








τ
τxy
63
Zmax
A (110, 63)
GR
60°
47
110
O
C
H
E
σx
σy
σT
σ1
σ2
(47, 63)
B

step-5:- Draw the st. line to Joint A & B

step-6:- Draw the ole with radius BC (or) AC / ~~(or)~~ with diameter AB. Draw a ole with centre C

step-7:- locate starting point of the ole from origin 'O', end point of the ole from origin 'E'.

step-8:-

step-9:- ~~To~~ for drawing σ_t from normal axis, oblique angle double, $2\theta = 2 \times 30^\circ = 60^\circ$

step-10:- Draw 60° from 'C' to bisect ole at point & the point is i