Deriving spectra location on detector from slit location on DMD

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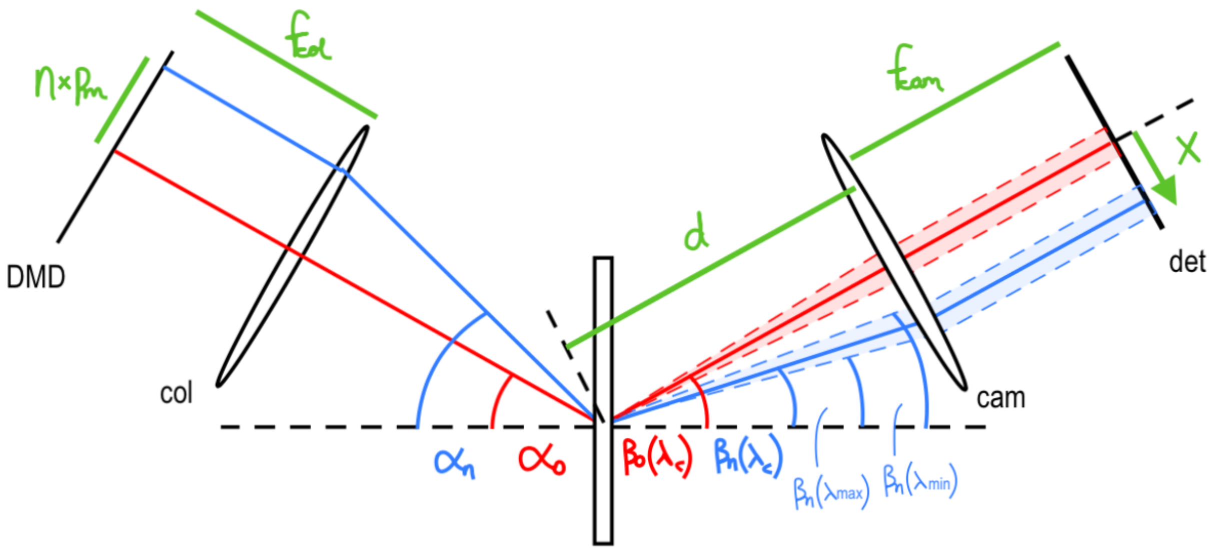

Figure 1: Schematic illustrating most of the significant variables in this analysis

Under the Bragg condition for the central wavelength λ_c, we can assume that the chief ray from the central slit (n=0) has an entrance angle α_0 equal to its exit angle $\beta_0(\lambda_c)$. This angle can be calculated using the grating equation, as shown in equation 1.

$$G\lambda_c = \sin\alpha_0 + \sin\beta_0(\lambda_c) = 2\sin\alpha_0$$
$$\alpha_0 = \arcsin\left(\frac{G\lambda_c}{2}\right) \tag{1}$$

$$= arcsin\left(\frac{600}{mm} \times \frac{.00055mm}{2}\right)$$
$$\simeq 0.1658rad \simeq \boxed{9.497\text{deg}}$$

With some trigonometry referencing figure 1, the entrance angle for slits n micromirrors from the center can be calculated using equation 2.

$$\alpha_n = \alpha_0 + \arctan(\frac{np_M}{f_{col}}) \tag{2}$$

Using the grating equation, the corresponding exit angle β_n can be written as a function of wavelength $lambda$, as shown in equation 3.

$$\begin{aligned} G\lambda &= \sin\alpha_n + \sin\beta_n(\lambda) \\ \beta_n(\lambda) &= \arcsin(G\lambda - \sin(\alpha_n)) \end{aligned} \tag{3}$$

To calculate entrance and exit angles for the slit on the outermost edge of the DMD, we need to set $n = n_{edge}$. For a 2x2 micromirror slit, the value of n_{edge} is defined by equation 4 below. See figure 2 for a visualization of why this is the case.

$$\begin{aligned} n_{edge} &= \frac{N_{MX}}{2} - 1 \\ &= \frac{750}{2} - 1 = 374 \end{aligned} \tag{4}$$

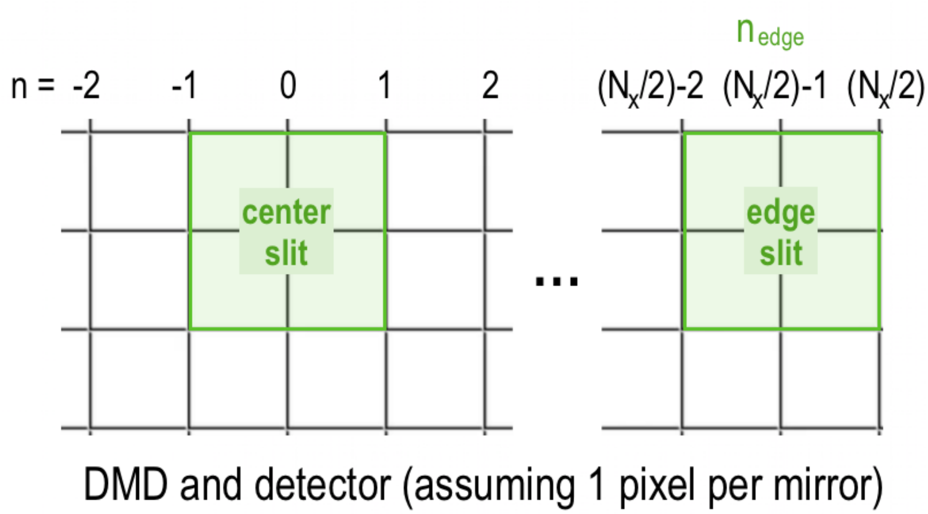

Figure 2: Diagram illustrating the allocation of slits on the micromirror/detector

Now, using equations 2 and 3, we can calculate the entrance and exit angles of the edge slit.

$$\begin{aligned} \alpha_{n_{edge}} &= \alpha_0 + \arctan(\frac{n_{edge}p_M}{f_{col}}) = 0.1658rad + \arctan(\frac{374(.0137mm)}{85mm}) \\ &\simeq 0.2260rad \simeq 12.95\text{deg} \end{aligned}$$

$$\begin{aligned} \beta_{n_{edge}}(\lambda_c) &= \arcsin(G\lambda_c - \sin(\alpha_{n_{edge}})) = \arcsin(\frac{600}{mm}.00055mm - \sin(.2260)) \\ &\simeq 0.1062rad \simeq 6.082\text{deg} \end{aligned}$$

The next step is to translate exit angles β_n into corresponding distances from the center of the detector along the spectral direction X_n. This requires first calculating the distance d between the camera and the central point of dispersion. To find d, we can make use of the known values $X_{n_{edge}}$ and $\beta_{n_{edge}}$, as shown in equations 5 and 6. (In retrospect, d is equal to the focal length of the camera f_{cam})

$$\begin{aligned} X_{n_{edge}}(\lambda_c) &= N_{p1} p_D n_{edge} \\ &= (1)(0.0065mm)(374) = \text{2.431mm} \end{aligned} \tag{5}$$

$$\begin{aligned} d &= \frac{X_{n_{edge}}(\lambda_c)}{\tan(\beta_0(\lambda_c) - \beta_{n_{edge}}(\lambda_c))} \\ &= \frac{2.431mm}{\tan(.1062rad)} = \text{40.74mm} \end{aligned} \tag{6}$$

With all these pieces in place, we can finally calculate the distance from the center of the detector X as a function of n and λ, as shown in equation 7. This can then be converted to corresponding pixel number $P_x(n, \lambda)$ using equation 8.

$$\begin{aligned} X_n(\lambda) &= d \tan(\beta_0(\lambda_c) - \beta_n(\lambda)) \\ &= d \tan(\arcsin(G\lambda_c - \sin(\alpha_0)) - \arcsin(G\lambda - \sin(\alpha_n))) \end{aligned} \tag{7}$$

$$P_x(n, \lambda) = \frac{N_{DX}}{2} + \frac{X_n(\lambda)}{p_D} \tag{8}$$

As an example, we can use this equation to calculate the spectrum location for the edgemost slit.

$$\begin{aligned} P_x(n_{edge}, \lambda_{max}) &= \frac{N_{DX}}{2} + \frac{X_{n_{edge}}(\lambda_{max})}{p_D} \\ &= \frac{N_{DX}}{2} + \frac{d \tan(\arcsin(G\lambda_c - \sin(\alpha_0)) - \arcsin(G\lambda_{max} - \sin(\alpha_{n_{edge}})))}{p_D} \\ &= 1000pix + \frac{(40.74mm)\tan(\arcsin(\frac{600}{mm}(.00055mm) - \sin(.1658rad)) - \arcsin(\frac{600}{mm}(.0007mm) - \sin(.2260rad))}{.0065mm} \\ &\simeq \text{pixel802} \end{aligned}$$

$$\begin{aligned} P_x(n_{edge}, \lambda_{min}) &= \frac{N_{DX}}{2} + \frac{X_{n_{edge}}(\lambda_{min})}{p_D} \\ &= \frac{N_{DX}}{2} + \frac{d \tan(\arcsin(G\lambda_c - \sin(\alpha_0)) - \arcsin(G\lambda_{min} - \sin(\alpha_{n_{edge}})))}{p_D} \\ &= 1000pix + \frac{(40.74mm)\tan(\arcsin(\frac{600}{mm}(.00055mm) - \sin(.1658rad)) - \arcsin(\frac{600}{mm}(.0004mm) - \sin(.2260rad))}{.0065mm} \\ &\simeq \text{pixel1945} \end{aligned}$$

Therefore, the outermost slit on the DMD will cast a spectrum on the detector spanning pixels 802 to 1945 in the spectral direction