DMD-MOS Parameter Calculations

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Inputs

INPUTS	Symbol	Value	Unit	Notice
Telescope	D	406.40	mm	Diameter of Telescope
	F/#_T	18.00		Output F/#
	S_resolu	3.00	"	Seeing Resolution
DMD	p_M	13.70	μm	Pitch of micro mirror
	NM_X	0.75	K	Number of micro mirrors in X
	NM_Y	1.00	K	Number of micro mirrors in Y
Spectrograph	G	600.00	1/mm	Groove density of grating
Detector	p_D	6.00	μm	Pitch of pixel
	ND_X	2.00	K	Number of pixels in X
	ND_Y	2.00	K	Number of pixels in Y
Requirements	FOV_X	19.20	'	Field of View in X
	FOV_Y	25.60	'	Field of View in Y
	Wave_cen	0.55	μm	Centre wavelength
	Wave_min	0.40	μm	Minium wavelength
	Wave_max	0.70	μm	Maximum wavelength
	R	1000.00		Spectral Resolution
	S_M	1.50	"	Scale on DMD
	S_D	0.75	"	Scale on Detector in spectral channel
	EE80_M	3.00	"	Image performance on DMD
	EE80_D	3.00	"	Image performance on detector
	S_effi	30.00	%	System efficiency

Telescope outputs

To calculate the focal length of the telescope f_T, the relationship between F ratio, focal length and lens diameter was used.

$$F = \frac{f}{D}$$

$$f_T = F_T \times D_T = 18 \times 406.40mm = 7315.20mm$$

To calculate the effective image size S on the focal plane of the telescope, the relationship between angular field FOV_θ of view and image size was used.

$$FOV_\theta = 2 \times \arctan \frac{S}{2f}$$

This equation was rearranged, and applied to both the X and Y components of FOV_θ and telescope image size S_T. For accurate results, conversion from arcminutes to radians was performed.

$$S_{T_X} = 2f_T \times \tan \frac{FOV_{\theta_X}}{2} = 2(7315.20mm) \times \tan \frac{19.20' \frac{\pi}{60 \times 180}}{2} = 40.86mm$$

$$S_{T_Y} = 2f_T \times \tan \frac{FOV_{\theta_Y}}{2} = 2(7315.20mm) \times \tan \frac{25.60' \frac{\pi}{60 \times 180}}{2} = 54.47mm$$

Relay optics outputs

Magnification of the relay optics M_R can be calculated by taking the ratio of the DMD image size $S_{DMD}$to the image size on the focal plane of the telescope S_T. For simplicity, the x-component height will be used as size.

$$M_R = \frac{S_{DMD_X}}{S_{T_X}}$$

The image size of the DMD can be calculated by multiplying the mirror pitch p_M by the number of mirrors N_M along the x direction. This is then applied to get the relay optics magnification M_R.

$$S_{DMD_X} = p_M \times N_{M_X} = (.0137mm)(750) = 10.275mm$$

$$M_R = \frac{S_{DMD_X}}{S_{T_X}} = \frac{10.275mm}{40.86mm} = 0.25$$

Magnification can also be expressed as a ratio of F numbers.

$$M = \frac{F_{out}}{F_{in}}$$

This can be rearranged and applied to calculate the F number of the relay optics F_R from its magnification and telescope F number.

$$F_R = M_R \times F_T = (0.25)(18) = 4.5$$

Spectrograph outputs

Since input to the collimator is the direct output of the relay optics, the F number of the two should be equal.

$$F_{col} = F_R = 4.5$$

To calculate the number of pixels per one micromirror N_{p1} (along the x-direction), the ratio DMD plate scale S_M and detector plate scale S_D can be taken.

$$N_{p1} = \frac{S_M}{S_D} = \frac{1.50''}{0.75''} = 2$$

According to Nyquist sampling theorem, the number of micromirrors per slit N_{m1} must be equal to 2. It follows that the number of pixels per slit N_{ps} is a product of N_{p1} and N_{m1}.

$$N_{ps} = N_{p1} \times N_{m1} = 2 \times 2 = 4 pixels$$

Multiplying the pixel to micromirror ratio by the ratio of pixel pitch p_D to micromirror pitch p_M yields the spectrograph magnification.

$$M_S = \frac{p_D}{p_M} \times N_{p1} = \frac{.006mm}{.0137mm} \times 2 = 0.88$$

Now, using the spectrograph magnification with the previously calculated F number of the collimator, the F number of the camera can be calculated.

$$F_{cam} = M_S \times F_{col} = (0.88)(4.5) = 3.94$$

To solve for diffraction angle β, the grating equation can be rearranged for angular dispersion assuming a constant incident angle. This will be a function of grating density G and wavelength λ.

$$\frac{d\beta}{d\lambda} = \frac{G}{\cos\beta}$$

Noticing this is a separable differential equation, we can rearrange into respective integrals of β and λ, making use of the proper wavelength range.

$$\int \cos\beta \, d\beta = G \int_{\lambda_{min}}^{\lambda_{max}} d\lambda = 600/mm \int_{.0004mm}^{.0007mm} d\lambda$$

$$\sin\beta = (600/mm)(.0003mm) = 0.18$$
$$\beta = \arcsin 0.18 = 0.181 rad$$

The number of spectral band passes contained along the central spectrum $Spec_N$ can be found by dividing the wavelength range by the limit of resolution $\delta\lambda$.

$$Spec_N = \frac{\lambda_{max} - \lambda_{min}}{\Delta\lambda}$$

The limit for the central wavelength can be found by rearranging the formula for spectral resolution R, which can then be applied to solve for $Spec_N$

$$R = \frac{\lambda}{\Delta\lambda}$$

$$\Delta\lambda = \frac{\lambda_{cen}}{R} = \frac{550nm}{1000} = .55nm$$

$$Spec_N = \frac{\lambda_{max} - \lambda_{min}}{\Delta\lambda} = \frac{700nm - 400nm}{.55nm} = 545$$

To calculate the number of pixels the central spectrum spans in the x direction ND_{X1}, multiply the number of pixels per micromirror by the number of band passes per spectrum we just found. Similarly, the number of pixels along the spatial direction ND_{Y1} is a simple reflection of the pixel to micromirror ratio.

$$ND_{X1} = N_{p1} \times Spec_N = 2(545) = 1090 pixels$$

$$ND_{Y1} = N_{p1} = 2 pixels$$

The next step is to calculate is the focal length of the camera f_{cam}, which is related to the angle of dispersion through the reciprocal linear dispersion.

$$\frac{d\lambda}{dx} = \frac{d\lambda}{f_{cam} d\beta} = \frac{\cos\beta}{f_{cam} G}$$

Again, this is a separable differential equation which can be rearranged to solve for the linear size of the spectrum along the detector. However, we already know that this value is equal to the number of bandpasses $Spec_N$ multiplied by pixel pitch p_D.

$$\int_{\lambda_{min}}^{\lambda_{max}} d\lambda = \frac{\cos\beta}{f_{cam} G} \int dx$$

$$\frac{f_{cam} G}{\cos\beta}[\lambda_{max} - \lambda_{min}] = x = Spec_N \times p_D$$

By rearranging the solution of the above integral, we can find the required focal length of the camera.

$$f_{cam} = (Spec_N \times p_D) \frac{\cos\beta}{G[\lambda_{max} - \lambda_{min}]}$$

$$= (1090mm \times .006mm) \frac{\cos 0.181 rad}{600 \frac{1}{mm} (.0003mm)}$$

$$= 35.7mm$$

Finally, using the spectrograph magnification and camera focal length, we can calculate the focal length of the collimator f_{col}

$$M_S = \frac{f_{cam}}{f_{col}}$$

$$f_{col} = \frac{f_{cam}}{M_S} = \frac{35.7mm}{0.88}$$

$$= 40.76mm$$

Outputs summary

OUTPUTS				
Telescope	f_T	7315.20	mm	Focal length of telescope
	S_TX	40.86	mm	Effective image size on focal plane of telescope in X
	S_TY	54.47	mm	Effective image size on focal plane of telescope in Y
Relay Optics	M_R	0.25		Magnification of relay optics
	F/#_R	4.50		Focal ration of relay optics on DMD
Spectrograph	F/#_Coll	4.50		Focal ration of coolimator
	f_Coll	40.76	mm	Focal length of collimator
	N_p1	2.00		Number of pixels for one micro mirror
	N_m1	2.00		Number of micro mirrors for one slit
	N_ps	4.00		Number of pixels for one slit
	M_S	0.88		Magnification of spectrograph
	F/#_Cam	3.94		Focal ration of camera
	f_Cam	35.70	mm	Focal length of camera
	beta	0.18	rad	Diffractive angle of grating
	Spec_N	545.00		Number of spectral band passes
	ND_X1	1090.00		Total number of pixel in X (Spectral)
	ND_Y1	2.00		Total number of pixel in Y (Spatical)