

## Computer Number Systems

All computers – from large mainframes to hand-held micros – ultimately can do one thing: detect whether an electrical signal is “on” or “off”. Computer programs in BASIC and Pascal are converted by various pieces of systems software into sequences of bits (*Binary digITs*) which correspond to sequences of on/off (equivalently TRUE/FALSE or 1/0) signals. Proficiency in the binary number system is essential to understanding how a computer works.

Since binary numbers representing moderate values quickly become rather lengthy, bases eight (octal) and sixteen (hexadecimal) are frequently used as short-hand. Octal numbers group binary numbers in bunches of 3 digits and convert the triplet to a single digit between 0 and 7, inclusive. For example,  $1001010110_2 = 001\ 001\ 010\ 110_2 = 1126_8$ . Hexadecimal numbers group binary numbers by fours, and convert the quadruplet to a single digit in the range 0, 1, 2 ..., 9, A, B, C, D, E, F. For example,  $10110110100101_2 = 0010\ 1101\ 1010\ 0101_2 = 2DA5_{16}$ .

### References

Many pre-Algebra textbooks cover bases other than 10. From the computer science point of view, most books covering Assembly Language also cover binary, octal and hex number systems. The texts cited for the Boolean Algebra category cover computer number systems.

### Sample Problems

<p>Solve for <math>X</math>.</p> $X_{16} = 3676_8$	<p>One method of solution is to convert <math>3676_8</math> into base 10, and then convert that number into base 16 to yield the value of <math>X</math>.</p> <p>An easier solution, less prone to arithmetic mistakes, is to convert from octal (base 8) to hexadecimal (base 16) through the binary (base 2) representation of the number:</p> $\begin{aligned} 3676_8 &= 011\ 110\ 111\ 110_8 \\ &= 0111\ 1011\ 1110_8 \\ &= 7BE_{16} \end{aligned}$
<p>Solve for <math>X</math>.</p> $X_{16} = \text{FEED}_{16} - 6ACE_{16}$	<p>The rightmost digit becomes <math>F</math>, because <math>1D-E=F</math>. Next, <math>D-C=1</math> (the <math>E</math> becomes a <math>D</math> because we had to borrow from it to do the units' subtraction), <math>E-A=4</math>, and then <math>F-6=9</math>. Combining these results of each column, we get a final answer of <math>941F_{16}</math>.</p>

In the ACSL computer, each “word” of memory contains 20 bits representing 3 pieces of information. The most significant 6 bits represent Field A; the next 11 bits, Field B; and the last 3 bits represent Field C. For example, the 20 bits comprising the “word”  $18149_{16}$  has fields with values of  $6_{16}$ ,  $29_{16}$  and  $1_{16}$ . What is Field B in  $E1B7D_{16}$ ? (Express your answer as a base 16 number.)

$$\begin{aligned} E1B7D &= 1110\ 0001\ 1011\ 0111\ 1101 \\ &= 1110\ 00\ 01\ 1011\ 0111\ 1\ 101 \\ \text{Field B} &= 01\ 1011\ 0111\ 1 \\ &= 011\ 0110\ 1111 \\ &= 3\ 6\ F_{16} \end{aligned}$$