Cooling Coffee in a Busy Laboratory

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Abstract

The cooling of coffee is carried out in a laboratory with results indicating that neither Newton's or Lorentz's cooling law is obeyed. It is suggested that the cause of this is a lack of information about heat lost due to radiative effects in these laws. Results are also found to suggest that additives such as cold milk increase the overall cooling time by 3% compared to black coffee.

Introduction

A cup of coffee left on a table will steadily cool down to room temperature as time passes. The time it takes to cool is dependent on its starting temperature, as well as the environmental conditions. There are laws relating these pieces of information that can be used to predict the temperature of the coffee at some time later, assuming the environmental conditions coincide with those required by the given law. This work serves to determine which law best describes the cooling of coffee in a busy laboratory environment.

Theory

Two laws may be considered to affect the rate of cooling corresponding to two different types of convection. The Process of forced convection [1] occurs when a cooling body is subject to fluid (such as air) circulation by pump or fan and corresponds to Newton's law of cooling given by

$$\frac{dT}{dt} = -\frac{1}{\tau}\Delta T \ . \tag{1}$$

This differential equation says that the rate of cooling is proportional to the difference in temperature of the body and its environment ΔT, where τ is the characteristic time constant. Natural convection is the process by which fluid motion occurs due to thermal expansion only and corresponds similarly to Lorentz's cooling law given by

$$\frac{dT}{dt} = -\frac{1}{\tau}\Delta T^{\frac{5}{4}} \, . \tag{2}$$

Experiments performed by Rees and Viney found that the cooling of coffee in "still air" obeyed Newton's law [2]. Although not considered to be, this is an unexpected result given the theoretical implications of Newtonian cooling. The reproducibility of this result may be tested by repeating a selection of the tests carried out in their original research. Furthermore, by considering τ in both Eq. (1) and Eq. (2), the law providing the best fit of the data may be determined if either one provides a constant value within the experimental error.

Solving by integration and rearranging Eq. (1) and Eq. (2) for τ respectively yields

$$\tau = \frac{t_0 - t}{\ln\left(\frac{\Delta T}{\Delta T_0}\right)} \tag{3}$$

and

$$\tau = \frac{t_0 - t}{4\left(\Delta T_0^{-\frac{1}{4}} - \Delta T^{-\frac{1}{4}}\right)}. \tag{4}$$

Given a starting time t_0 and temperature excess ΔT_0, a value of τ may be calculated at each consecutive data point providing a much more rigorous test of each law's validity.

Experimental Setup

The setup consisted simply of a black mug and a digital thermometer constructed on a breadboard. The digital thermometer consisted of a Wheatstone Bridge circuit, with a forward-biased diode as the temperature sensor. Advantages of this type of circuit are its precision and the ability to zero it when the bridge, represented by the region AB in Figure 1, is balanced at 0 V. Several adjustments were made to the helical potentiometers R_2 and R_4 so that the digital multimeter read 0 mV at 0 °C and so that a change of 1°C corresponded to a change of 1 mV across the bridge over a range of temperatures from 0 – 100 °C.

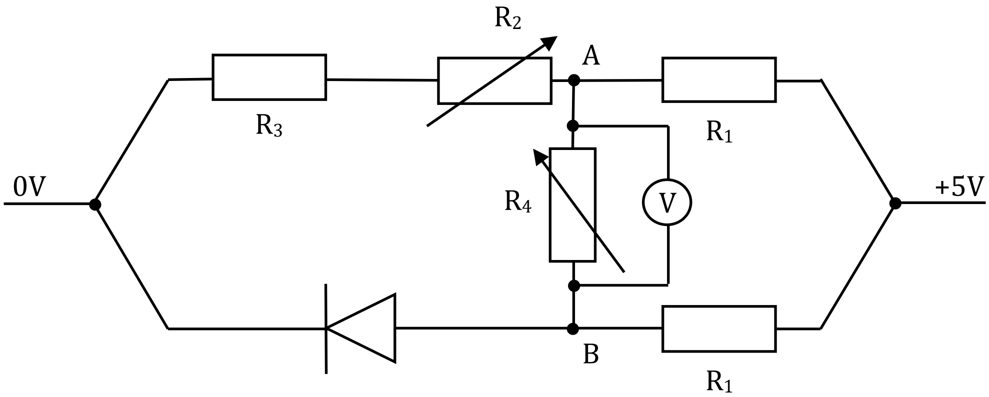

Figure 1: Wheatstone Bridge circuit used to make the digital thermometer. The circuit contains three fixed resistors and two helical potentiometers with resistances R_1 = 4.7 kΩ, R_2 = 200 Ω , R_3 = 620 Ω, R_4 = 10 kΩ, as well as a forward-biased diode used as the temperature probe and a digital multimeter to measure voltage in mV.

Calibration of the digital thermometer was carried out against an alcohol-in-glass thermometer which was graduated in a 1 °C scale and could be read to a precision of 1/3 °C . The analogue and digital thermometers were compared over a range of temperatures as shown in Figure 2. The error in the digital thermometer was taken to be the mean of the absolute values of the differences between the two thermometer readings, equalling to ±0.3 mV.

Alcohol-in-glass Thermometer (± 1/3 °C)	Digital Thermometer (mV)	|Difference|
78	78.3	0.3
70	70.8	0.8
60	60.3	0.3
50	50.0	0
40	39.9	0.1
30	29.8	0.2

$n = 6$ $\quad \sum|\text{Difference}| = 1.7$ $\quad \frac{\sum|\text{Difference}|}{n} = 0.28$ $\quad$ Error = ±0.3 mV

Figure 2: Results of comparing an alcohol-in-glass thermometer to the digital thermometer. The error in the digital thermometer was taken as the average difference in measurements between the two. n represents the number of temperatures compared.

The digital thermometer was then automated by connecting it to a pre-made amplifier, measured at room temperature to increase the output voltage by a factor of 49.8, as well as an Arduino UNO using some pre-written code to record the voltage across the bridge every 30 seconds and return the value with time to a computer running Arduino software.

Experimental Observations

Four tests were carried out varying only in the contents of the mug, with a level teaspoon of instant coffee granules used in coffee-related tests. In all tests (200 ± 2) ml of boiling water was measured at the level of the meniscus of a measuring cylinder and added directly to the mug. When adding cold milk or water, (20 ± 0.5) ml was added by syringe to the mug of coffee, stirred, and the same quantity was then removed by syringe to keep a constant volume for each test.

The room temperature was measured using the digital thermometer after each test resulting in an average of (20.2 ± 0.2) °C, with the uncertainty determined by the addition in quadrature of the error in each measurement (determined by the calibration) divided by four. A systematic error was found due to the Arduino amplification resulting in temperatures being recorded 3 °C higher than the circuit alone measured and was removed by reducing all temperature measurements by this amount.

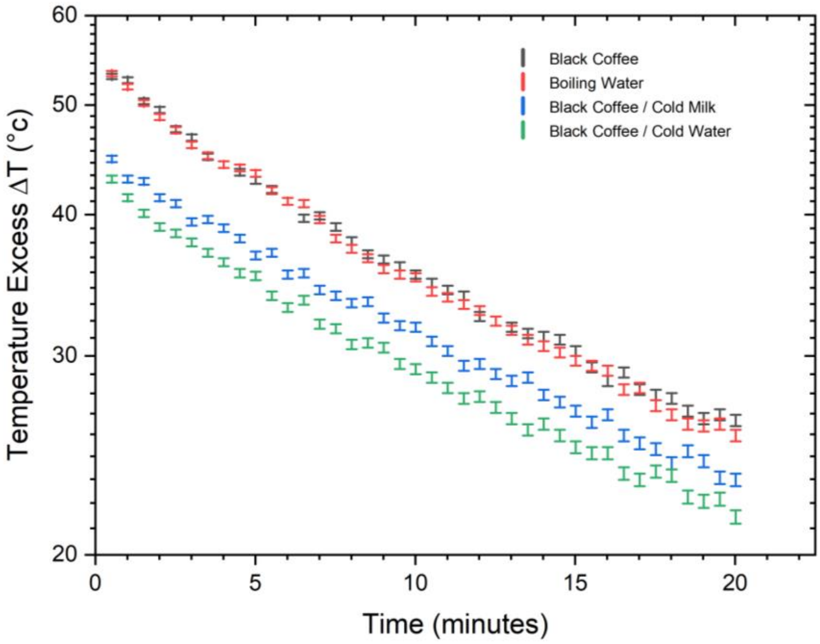

Figure 3: Log-linear plot of Temperature excess against time for four different fluids, each with a non-linear relationship showing that Newton's cooling law is not obeyed.

The results from Figure 3 show similar rates of cooling for all 4 tests with the difference in starting temperatures being due to cooling that took place whilst making up the mixtures. The non-linearity shows that Newton's law does not provide the best fit for the data. Using Python 3.7 and each test's data file, a loop was made to calculate τ given by Eq. (3) and Eq. (4) at each data point. This produced a range of results, as shown in Figure 4 and is further evidence that Newton's law does not provide the correct fit since τ is represented by the gradient of Figure 3. The values of τ given by Lorentz's law show a range 2-3 times greater than that of Newton's, resulting in an even worse fit. Closer inspection of the cooling times given by Newton's law reveals an accelerated rate of cooling in the first minute followed by a steadily decreasing rate.

The effect of adding cold milk can be seen overall to increase the largest cooling time by 3%, and cold water by 6%, compared to black coffee.

Test	Newton's Law, τ (min)	Lorentz's Law, τ (min)
Boiling Water	17.7– 28.8	48.0 – 72.5
Black Coffee	19.7 – 30.1	52.2 – 75.8
Black Coffee / Cold Milk	13.0 – 31.0	34.1 – 80.9
Black Coffee / Cold Water	14.1 – 32.0	36.6 – 72.2

Figure 4: Results of calculating the time constant τ for Newton's and Lorentz's cooling laws. The lack of a constant value shows neither law is obeyed explicitly but that Newton's provides a better fit of the two because of the smaller range.

Discussion

Neither Newton's, nor Lorentz's cooling laws provided the best fit for this data. It is believed that the main reason for this is because the effects of heat lost to radiation are not considered in either law. O'Sullivan [3] has shown that the Newton-Stefan cooling law, which considers both conductive-convective and radiative effects, provides a consistently good fit for cooling in various conditions and that Newton's law is only obeyed when a body is subjected to considerable air flow. The conditions under which this experiment took place most likely favoured Newton's law because of the busy environment, with air conditioning systems and doors being in constant use providing significant drafts.

Comparing to the results of Rees and Viney, it was found that whilst the there is a disagreement between the validity of Newton's law, it was observed that coffee with milk cooled 3% slower than black coffee which fits with their theoretical prediction given in Section IV (A) but differs with their experimental result of 17% slower. Direct comparison of time constants would not provide useful information since the value is so dependent on the environmental conditions.

It would be possible to confirm the presence of significant heat loss to radiation and whether the addition of milk increases cooling time by more than 3% by repeating these tests in a more controlled environment, and by comparing results produced by Newton's cooling law and the Newton-Stefan cooling law.

Conclusion

This experiment has found that Newtonian cooling may predict temperature changes of coffee over a short time to a reasonable accuracy, and certainly more so than Lorentzian cooling if there is a considerable draft in the room. For the most accurate results, it is suggested that the radiative effects are also considered. It has also been shown that the addition of cold milk to black coffee has only a small effect of increasing cooling times by 3%.

References

[1] Hugh D. Young & Roger A. Freedman, Sears and Zemansky's University Physics with Modern Physics 14th Edition, Pearson (2012) Pg. 568

[2] W. G. Rees, C. Viney, On cooling tea and coffee, Am. J. Phys. 56 (5), May 1988

[3] Colm T. O'Sullivan, Newton's law of cooling – A critical assessment, Am. J. Phys.58 (10), October 1990