High-Frequency Trading in Limit Order Markets: Equilibrium Impact and Regulation

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Abstract

We investigate the impact of high-frequency trading (HFT) on market quality and investor welfare using a dynamic general limit order book model. We find that while the presence of HFT always improves market quality under symmetric information, under asymmetric information this is the case only if competition between high-frequency traders is sufficiently strong. While HFT does not negatively impact investor welfare, it reduces the welfare of slow speculators. The flexibility of the model allows investigating the effect of the main recent regulatory initiatives designed to curb HFT on market quality and investor welfare. We consider minimum resting time rules, cancellation fees, transaction taxes, rebate fee structures, and speed bumps. While some of these regulations lead to improvements in a number of market quality measures, this generally does not translate into higher welfare for long-term investors. Rather, the main effect of such regulations is to generate wealth transfers from high-frequency traders to slow speculators. These regulations therefore appear inadequate to enhance investor welfare in the presence of HFTs. Of the different measures, transaction taxes are the least harmful; while they reduce welfare roughly by the amount of the tax, they do not significantly worsen market quality. The common practice by exchanges of granting rebates to limit orders is detrimental to market quality and investor welfare, causing both higher effective spreads and longer execution times.

Keywords: High-frequency trading, Regulation, Market quality

JEL classification: G14, G28, C63, C73, D82

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1 Introduction

The U.S. Securities and Exchange Commission (SEC) describes high-frequency trading (HFT) as "one of the most significant market structure developments in recent years" (SEC, 2010, p. 45). High-frequency traders (HFTs) nowadays account for over half of the volume on many stock, futures and options exchanges. After the Flash crash that occurred in May 2010, a controversy around the impact of HFT on market quality arose in the public discussion, the academic literature, and among regulators and exchanges. Of particular concern are the lack of knowledge about HFTs' strategies, their potentially destabilizing effect on markets in periods of turmoil, and the large volume of quotes not leading to trades which HFTs submit to exchanges, so-called high-frequency spam.[1]

Concerns about the impact of HFT on the functioning of markets are not limited to the U.S. For example, a new European Directive (so-called MiFID 2, 2011) identifies "specific regulatory and supervisory measures necessary in order to adequately deal with the potential threats for the orderly functioning of markets arising from algorithmic and high-frequency trading." HFT and make/take fee structures – the practice common among exchanges of granting fee rebates to qualified market participants providing liquidity – are also among concerns listed in a consultation paper by the Committee of European Securities Regulators (2010). While regulators have been mostly concerned about the impact of HFT on market quality and the welfare of long-term investors, exchanges have been mostly worried about high-frequency spam, as the practice has raised their infrastructure costs without leading to a commensurate increase in trading volume.

The goal of this paper is to assess the impact of HFT on market quality and investor welfare and to investigate the suitability of a number of regulatory measures that have been suggested to improve market quality in the presence of HFT. These measures include a *minimum resting time* for quotes, the imposition of *cancellation fees* when the ratio of quotes to trades exceeds a certain value, *transaction taxes*, so-called *speed bumps* enforcing minimum delays between the time an order is submitted and the time it hits the order book, and the use of *make-take fees* (or *rebate fee structures*) to provide HFTs incentives to make markets.

In spite of the fact that little is known about the impact of these measures on market quality and investor welfare, a number of them have been introduced in several countries either alone or in combination:

1. *Minimum resting time* or similar rules have been introduced in Europe and the U.S. The Eurex options exchange imposes a minimum resting time for quotes as part of market maker obligations.

[1] The term *high-frequency spam* refers to the practice of rapidly canceling and changing quotes without even changing price and with no intention of executing, i.e. anticipatory strategies used to reveal liquidity and trade ahead of it (Hunsader, 2011).

The European Parliament has enacted a requirement for HFTs to maintain their quotes for at least half a second, and Italy has introduced a tax of 0.02% on orders that are cancelled within half a second. In the U.S., minimum resting time requirements have been derived from FINRA rule 5210, which requires quotes to represent bona fide intention to trade. This rule has been explicitly adopted by the NYSE and NYSE Arca in 2011 (see SEC 2011a, 2011b), and the CME enacted a similar regulation in its rule 575 in 2014 (see CFTC, 2014).

2. *Cancellation fees* have been introduced by the Canadian regulatory authority IIROC with the explicit aim of curbing high-frequency spam. The Milan Stock Exchange and the NASDAQ Stock Market introduced an excess order fee for high quote-to-trade ratios in April and July 2012 (see SEC, 2012). Cancellation fees are also imposed by Eurex for order-to-trade ratios exceeding 5, on NASDAQ OMX for ratios exceeding 100, as well as on the German stock exchange. The new MiFID2 European Directive also foresees the introduction of cancellation fees to curb excessive quote-to-trade ratios.

3. *Transaction taxes* are currently imposed in roughly 40 countries globally and mostly serve to finance regulatory activities related to the operation of financial markets. The U.S. imposes a financial transaction tax of 0.0034%. The U.K. collects a stamp duty of 0.5%, but many intermediary transactions are exempted. France introduced a financial transaction tax of 0.2% in August 2012, and Italy a tax of 0.12% in 2013. In January 2013, 11 European countries approved the principle of a financial transaction tax, but postponed its implementation until mid-2016.

4. *Speed bumps* have been introduced by NASDAQ OMX and by a newly founded exchange, IEX, in October 2013. At IEX, the speed bump takes the form of a minimal physical distance between clients' servers and the exchange's main server and is imposed on all orders. NASDAQ OMX applies the speed bump to market orders only, presumably to encourage liquidity provision.

5. *Make-take fees* giving incentives to liquidity providers are used by numerous exchanges in North America (NASDAQ, NYSA Arca, BATS Exchange, NSX, BATS Options Exchange, Boston Options Exchange, International Securities Exchange, TSX) and Europe (Euronext, OMX exchanges, Chi-X). Many other exchanges, including the London Stock Exchange, XETRA and SIX Swiss Exchange grant rebates to designated market makers or other significant providers of liquidity.

We assess the impact of HFT and of proposed regulations on market quality and investor welfare using a unified framework based on the theoretical model of Goettler, Parlour and Rajan (2005, 2009). Goettler et al. develop a theoretical model of a dynamic limit order market with strategic players and asymmetric information and solve for Markov perfect equilibria numerically using an extension of the simulation-based algorithm of Pakes and McGuire (2001) for complete information games. We conduct our investigation by comparing the properties of market equilibria in different settings, specifically settings with HFTs to settings without HFTs, and settings where specific regulatory proposals are implemented to settings where they are not. In addition to measures of market quality and overall welfare, we investigate the impact of HFT and proposed regulations on specific groups of market participants, namely long-term investors and speculators.[2]

We find that the presence of HFTs improves most common measures of market quality. Specifically, markets with HFTs exhibit narrower spreads, improved price discovery and higher depth. These results hold without qualification under symmetric information. When HFTs have an informational advantage over slow traders, however, improvements in market quality arise only if competition between them is sufficiently strong. We also find that HFT does not negatively impact investor welfare. However, it leads to a reduction in the welfare of slow speculators, who are crowded out by HFTs.

We also find that none of the proposed regulations consistently improves the welfare of long-term investors. Admittedly, some of the proposed regulations do lead to an improvement in a number of market quality measures. For example, minimum resting time rules reduce trading costs under asymmetric information, while speed bumps increase book depth and decrease spreads. However, none of the improvements in market quality translates into higher welfare for long-term investors. Rather, the main effect of these regulations is to generate wealth transfers from high-frequency traders to slow speculators. These regulations therefore appear inadequate to enhance investor welfare in the presence of HFTs. Of the different measures, transaction taxes are the least harmful; while they reduce welfare roughly by the amount of the tax (a reduction that can be offset by distributing tax proceeds back to traders), they do not significantly worsen market quality. Importantly, the common practice by exchanges of granting rebates to limit orders is detrimental to market quality and investor welfare, causing both higher effective spreads and longer execution times.

The paper is organized as follows. Section 2 provides a short overview of the existing literature. Section 3 describes the model and the solution methodology. Section 4 details our findings on the impact of HFT on market quality and investor welfare, while Section 5 considers the effect of proposed regulations. Section 6 concludes. Technical aspects of the solution methodology are described in the Appendix.

[2]Consistent with the terminology in Goettler et al. (2005, 2009), we use the term speculator to denote a trader with zero private valuation for an asset; hence the term includes HFTs.

2 Literature Review

High-frequency trading has been a very active area of research in recent years, with the literature focusing on three main questions: the impact of HFT on market quality and welfare, the nature of HFTs' strategies, and the impact of potential regulations on markets.

Theoretical papers examining the welfare implications of HFT and its impact on other market participants have not reached a consensus on whether the effect of HFT is predominantly positive or negative. Jovanovic and Menkveld (2011) find that the presence of HFTs has an ambiguous effect on welfare. Maollemi and Sağlam (2013) show that the presence of HFTs imposes significant costs on market participants with higher latency. Along similar lines, Hoffmann (2014) finds that slow traders are left with a smaller share of overall trading surplus because of their limited ability to avoid their orders being picked-off. On the other hand, Aït-Sahalia and Sağlam (2014) show that competition among HFTs improves the welfare of slow traders and that volatility leads HFTs to reduce their provision of liquidity. Biais, Foucault, and Moinas (2015) establish that investment into fast technology may go beyond its socially optimal level.

Theoretical results on the impact of higher trading speed on market quality are also inconclusive. Using a model with endogenous information acquisition, Baldauf and Mollner (2015) show that an increase in trading speed crowds out information acquisition and causes both a decrease in the bid-ask spread and a deterioration in price efficiency. Menkveld and Zoican (2015) find that the effect of a reduction in a stock exchange's latency on the bid-ask spread is ambiguous; whether market liquidity improves or deteriorates is driven by the ratio of news traders to liquidity traders.

Several empirical papers document that HFTs improve market quality by providing the best bid-ask spreads and contribute significantly to liquidity and price discovery, and that HFT activity is not positively correlated with price and quote volatility (Brogaard, 2010, Hasbrouck and Saar, 2013, Riordan and Storkenmaier, 2012, Conrad, Wahal, and Xiang, 2015, and Hasbrouck, 2015). Hendershott, Jones, and Menkveld (2011) report that HFTs improve liquidity and enhance the informativeness of quotes, but their presence also decreases the depth of the order book and increases the costs of executing large orders. Taking execution costs into consideration, Tong (2015) measures the execution shortfall of institutional investors and finds that HFTs increase transaction costs for them. While it has been found that HFTs did not cause the Flash crash, their presence exacerbated the price movement as they absorbed immediacy ahead of others by withdrawing their orders in the face of higher market volatility (Kirilenko, Kyle, Samadi, and Tuzun, 2014).

Another strand of the literature seeks to better understand the nature of HFTs' strategies. Hagströmer and Nordén (2013) quantify the market-making activity of HFTs on the NASDAQ-OMX Stockholm exchange. They find that HFTs engaged in market making are present about 60% of the time, while other categories of HFTs are on average present only about 5% of the time. Carrion (2013) finds that in the aggregate, HFTs on the NASDAQ make money on average when supplying liquidity and lose money on average when demanding liquidity.[3] Foucault, Hombert, and Roşu (2016) investigate the impact of HFTs engaging in news-based trading on market dynamics. They find that when a HFT has faster access to news, she significantly changes her strategy and her trades account for a much larger fraction of trading volume and forecast short-run price changes. Hasbrouck and Saar (2013) document that HFT leads to a high number of quote cancellations in the millisecond environment. Using a trading game with continuous prices, Baruch and Glosten (2013) find that such a strategy of fleeting orders is supported by equilibrium.

Theoretical predictions regarding the impact of current regulatory initiatives on continuous limit-order markets are scarce.[4] Harris (2013) argues that by causing liquidity-supplying HFTs to lose more often when offering liquidity, minimum resting time rules would ultimately increase investor transaction costs. Aït-Sahalia and Sağlam (2014) find that both minimum resting time rules and cancellation fees induce HFTs to quote more on both sides of the market. However, both measures lead HFTs to provide liquidity countercyclically with volatility, providing high (low) liquidity in low (high) volatility periods. Transaction taxes cause a reduction in HFT quoting and lead to lower transaction volume.

Empirical evidence on the impact of cancellation fees is provided by Malinova, Park, and Riordan (2013), who find that following their introduction on the Toronto Stock Exchange, quoting activity decreased by 30% and spreads increased by 9%. Umlauf (1993) investigates the impact of the introduction of the Swedish transaction tax and documents a large shift in trading volume from Sweden to the U.K. Colliard and Hoffmann (2015) find a large decrease in trading volume following the introduction of the French financial transaction tax in August 2012. Interestingly, the decrease is concentrated in the OTC market. While the main market remained resilient and spreads were not affected, the introduction of the tax led to a decrease in market depth, resiliency, and price efficiency.[5] Malinova and Park (2015) use a change in Toronto Stock Exchange fees to investigate the impact of make-take

[3] Interestingly, this appears to be exactly the opposite of the findings in Brogaard, Hendershott and Riordan (2014) using the same dataset.

[4] An alternative to regulating continuous limit-order markets is to make changes to the trading mechanism as such. A few recent papers consider this approach. Budish, Cramton and Shim (2015) show that changing the design away from continuous limit order books (CLOB) to frequent batch auctions eliminates mechanical arbitrage rents that are built into the CLOB market design, enhance liquidity for investors, and stops the HFT arms race for speed. Du and Zhu (2014) characterize the socially optimal frequency of such batch auctions in a setting with private information. They show that the optimal frequency is low for scheduled information arrivals and high for stochastic information arrivals.

[5] Further results on the effect of the French financial transaction tax on market quality can be found in a number of studies. Becchetti, Ferrari, and Trenta (2014) find a reduction in turnover and intraday volatility, but mixed effects on liquidity. Meyer, Wagener, and Weinhardt (2015) document a reduction in the number of quote and price updates by liquidity suppliers and a decline in top order book depth. Gomber, Haferkorn, and Zimmermann (2015) report a drop in both liquidity demand and supply, an increase in spreads, and a decline in top order book depth.

fees on market quality. They find that rebates improve quoted spreads, decrease adverse selection costs, and increase the aggressiveness of retail traders. They also observe that the effective spread plus total fee retained by the exchange remained constant, confirming the conjecture of Colliard and Foucault (2012) that only total fees (spread plus exchange fees) affect liquidity and trading volume. This last finding contrasts with the predictions of the model of O'Donough (2015), where the total trading cost to investors increases when the taker fee and maker rebate increase, even if the net fee is held fixed. Empirically, O'Donough reports decreasing bid-ask spreads, lower trader participation, higher order aggressiveness and higher probability of execution of limit orders as the taker fee and maker rebate increase.

Most closely related to our analysis is the recent paper by Bernales (2014). He extends the model of Goettler et al. (2009) in a similar way to ours and investigates the impact of HFT and a number of regulatory proposals on market quality. However, our analysis differs from his in several important respects. First, we model HFTs as fast speculators, i.e. as traders that have zero private valuations for the asset. By contrast, Bernales assumes that HFTs have the same distribution of private valuations as slow traders, which is unlikely to be the case in practice. Second, the set of regulatory proposals that we consider is much broader. Bernales only considers cancellation fees and an approximation for latency restrictions. Specifically, regarding the latter, in order to be able to use the original Goettler et al. (2009) model with few modifications, he models latency restrictions using a decrease in the frequency with which traders return to the market. Doing so, however, makes fast traders slower without actually enforcing these restrictions. By contrast, we model latency restrictions exactly in the way that they have been proposed for real-world markets, i.e. by either enforcing minimum resting time rules – a fixed time during which a limit order cannot be modified – or by introducing a speed bump – a delay between the time that a market order is submitted and the time it hits the order book. Modeling latency restrictions in a realistic fashion is important to be able to accurately assess their impact. Indeed, as we show below, these two forms of latency restrictions lead to different outcomes in terms of market quality and welfare. Furthermore, our analysis also considers the impact of rebate fee structures.

3 Model and Solution Methodology

3.1 Model Assumptions

Overview

We consider a continuous-time dynamic trading game in one financial asset similar to that in Goettler et al. (2009). Trading takes place by continuous double auction implemented using a limit order book.

The asset's fundamental value v_t follows a random walk where the timing of price changes is driven by a Poisson process with intensity λ_v; conditional on a price change occurring, up and down moves are equally likely.

Traders arrive to the market randomly according to a Poisson process with intensity λ. They observe both the current state of the order book and the market's transaction history. We investigate both symmetric and asymmetric information settings. Under symmetric information, all traders observe the fundamental value in real time. Under asymmetric information, informed traders observe the fundamental value in real time, while uninformed traders observe it with a lag of Δ units of time and form an expectation about its current value based on their last observed value and the current state of the market.

Upon arrival, each trader selects the best action given the current state of the book, the asset's (observed or estimated) fundamental value, the transaction history and his individual parameters (described in detail below). This action may be to submit a buy or sell market or limit order or not to submit any order.

Traders whose orders are executed leave the market and never return. The others return to the market at random times which are exponentially distributed with intensity λ_r. Upon returning to the market, traders again select the best action, but based on the new state.

Structure of the Limit Order Book

We keep the notation consistent with Goettler et al. (2009). Prices are discrete $\mathscr{P} = \{p^i\}_{i=-\infty}^{+\infty}$ and the distance between any two adjacent prices is identical and called the tick size. With each price $p^i \in \mathscr{P}$ is associated a queue of outstanding limit orders denoted by l_t^i. This quantity is displayed as a positive (negative) integer for buy (sell) limit orders. The limit order book is the collection of the indexed queues $L_t = \{l_t^i\}_{i=-\infty}^{+\infty}$. The best sell order in the book is called the ask price and denoted $A(L) = p^{\min\{i|l_t^i<0\}}$. Analogously, the best buy order is called the bid price and given by $B(L) = p^{\max\{i|l_t^i>0\}}$. In the implementation, we use a grid with a finite number of prices n_p and center these around the fundamental value v_t. We choose the number of ticks n_p sufficiently high that orders never "fall off" the grid. That is, orders are revised by returning traders before becoming too unaggressive for the grid, or get picked off before becoming too aggressive for the grid.

As is standard in limit order markets, the priority of execution of limit orders is based on price and time. Price priority means that buy (sell) orders at higher (lower) price have priority over those at lower (higher) prices. Time priority gives the limit orders in the particular price queue preference depending on the time of arrival. When an order executes against an outstanding limit order, the order which was submitted earlier at that price has priority. A buy (sell) market order is an order which executes against the resting sell (buy) limit orders starting from the best ask (bid). A marketable limit order is the same as a market order with a specified maximum (minimum) price for buying (selling) a specified number of units of the asset.

Trader Types and Behavior

New traders arrive to the market according to a Poisson process with intensity λ. Upon arrival, they may choose to submit a buy or sell market or limit order or not to submit any order. The action they select is optimal given the current state of the limit order book, the history of transactions, the asset's (observed or estimated) fundamental value, and their type. Changes in the fundamental value expose limit orders to two risks: (i) obtaining an undesirable execution when the fundamental value moves in an unfavourable direction, and (ii) not obtaining a desirable execution when the fundamental value moves in a favourable direction.

Traders may submit orders only for one share. Traders whose order is executed leave the market. The others return to the market with intensity λ_r. The reentry (or monitoring) rate λ_r depends on the technology available to the trader – it will be high when monitoring costs are low as in Foucault, Kadan, and Kandel (2013) – and represents a friction which traders have to take into account when making their decisions. A higher monitoring rate allows faster reaction to changing market conditions, giving the trader the ability to execute against a stale limit order or to cancel or resubmit her own order and thus avoid her order being picked-off.

Upon reentry, traders choose their action optimally, depending on the current state of the limit order book, the history of transactions, the asset's (observed or estimated) fundamental value, their type, and the priority and position of their last limit order if they submitted any. They are allowed to retain their existing limit order and they do so if this is the highest expected value action for them. Otherwise, they might cancel the existing order, possibly incurring a cancellation cost and choose a different value-maximizing action.

Traders' type θ is defined by three attributes, $\theta = \{h, \alpha, \rho\}$: whether they are HFT or not ($h \in \{0, 1\}$), their private valuation for the asset, α, and their impatience, ρ. The private valuation α is drawn from a discrete probability distribution in which the probability of a zero private value is positive. Traders with a zero private value are called speculators, those with a nonzero private value are investors. Traders with nonzero private valuation α seek to materialize this potential benefit through trade.

Traders' type is also defined by their impatience parameter ρ, which enters their utility function in a similar fashion as a discount factor. The impatience coefficient represents the trader's disutility of obtaining execution later. Traders are risk neutral and their instantaneous utility at time t is given by:

$$u_t = \begin{cases} \alpha + v_t - p^i & \text{if she executes a buy order at price } p^i \text{ and time } t, \\ p^i - \alpha - v_t & \text{if she executes a sell order at price } p^i \text{ and time } t, \\ 0 & \text{if she does not execute at time } t. \end{cases} \tag{3.1}$$

HFTs are modeled as fast speculators, i.e. their monitoring rate exceeds that of other traders, $\lambda_r^h > \lambda_r$, and their private valuation for the asset is zero.[6] In addition, when investigating asymmetric information settings, we assume that HFTs observe the fundamental value v_t in real time, whereas slow traders observe it with a lag of Δ time units as in Foucault, Hombert, and Roşu (2013). Thus, in the presence of asymmetric information, non-HFTs have to form an expectation about the current fundamental value v_t based on the lagged fundamental value $v_{t-\Delta}$ and the current state of the market in the same way as the uninformed traders in Goettler et al. (2009).

3.2 Time Line and Solution of the Game

Since the game evolves in continuous time and the different events that can occur arrive stochastically, there is no pre-set sequence of events. Rather, when solving the game, the first step is to draw the time of the next event from an exponential distribution with intensity given by the overall event arrival intensity Λ_t, which is the sum of (i) the intensity of arrivals of new traders λ, (ii) the weighted intensity of arrivals of returning traders λ_r and λ_r^h, and (iii) the intensity of changes in the fundamental value λ_v. Formally,

$$\Lambda_t = \lambda + N_t^{h=0}\lambda_r + N_t^{h=1}\lambda_r^h + \lambda_v, \tag{3.2}$$

where $N_t^{h=0}$ denotes the number of slow traders and $N_t^{h=1}$ the number of HFTs that have not left the market at time t.

Once the time of the next event is known, its nature – new arrival, return, or change in the fundamental value – is determined randomly based on the ratio of the intensity of that event to the overall event intensity Λ_t. The game is then played following the workflow summarized in Figure 1. Specifically, when a new trader arrives, the trader performs her optimization and submits her order or no order. When an existing trader returns, the trader performs her optimization and retains or cancels her existing order and, in the latter case, submits a new order or no order. All active orders are stored in the limit order book and if a new order was submitted, it enters in the precise position specified by the trader. If the trader submits a market order, the order executes against a resting limit order;

[6] A categorization of traders based on their intrinsic motivation to trade, presence in the market, information, trading strategies and other characteristics is provided in Harris (2002). Using this categorization, we distinguish two basic categories of traders – those with an intrinsic motivation (investors) and those without (speculators). We model HFTs as fast speculators who may have an information advantage because of faster information processing.

that order is then removed from the book and the priorities of the remaining limit orders are updated. Traders whose orders are executed are removed from the system. Finally, if the event is a change in the fundamental value v_t, the price grid in the limit order book is shifted as described in Section 3.1. Throughout, payoffs, transaction costs and other statistics are computed and stored.

Since traders have privately known α's as well as possibly private information about the fundamental value v_t and time is not a state variable, the solution concept for this game is Markov-perfect Bayesian equilibrium as pointed out in Maskin and Tirole (2001). We focus on symmetric equilibria, where traders of the same type optimally choose the same action in a given state.

The state space consists of the past history of the game, the current limit order book, the trader's type and the position of his past order (if any) in the book. Our state space is richer than that in Goettler et al. (2009) because of the additional parameter h. Let Θ denote the set of feasible agent types. Ideally, the state space considered during decision making would comprise all the variables of the limit order book and the entire transaction history. In order to make the problem tractable, we make certain restrictions. Specifically, we do not consider the entire transaction history but just the last transaction, whether it was buyer- or seller-initiated, and the transaction price. Moreover, we do not consider the full detail of the limit order book, but focus on its most informative variables, namely bid price, ask price, bid size, ask size, depth off bid and depth off ask. We perform tests to determine whether there is a significant difference in actions compared to those arising when using a broader information set, but like Goettler et al. (2009) find that broadening the state space does not improve predictive power. A mathematical description of the state space and the optimization problem faced by traders is provided in the Appendix.

3.3 Parameterization

In setting parameter values, we relate to Goettler et al. (2009) and the empirical literature cited therein in order to ensure the comparability of our results. We set the arrival rate for new traders λ to 0.25. The returning rate of HFTs λ_r^h is set to 4 and that of non-HFTs λ_r to 1. Thus, we assume that HFTs react four times faster than non-HFTs.

The discount rate ρ is set to 0.03 for all traders. The support of the private valuation α in ticks is $\{-4, -2, 0, 2, 4\}$ with probability distribution $F_\alpha = \{0.15, 0.35, 0.65, 0.85, 1.0\}$. This private value distribution roughly corresponds to the empirical findings in Hollifield, Miller, Såndas and Slive (2006) for the Vancouver stock exchange.

The tick size is set to 1/8. The expected time between changes in the fundamental value is set to 10 units of time, i.e. $\lambda_v = 0.1$. When the fundamental value changes, it always does by one tick, with up and down moves equally likely. When considering asymmetric information settings, we set the lag

Δ with which slow traders observe the fundamental value to 2 units of time; we experimented with higher values and found that the results do not change significantly.

The limit orders being tracked in the book may lie between 7 ticks below and 7 ticks above the fundamental value. The fundamental value lies on the tick in the middle of this price range. Limit orders may be submitted at prices between 3 ticks below and 3 ticks above the fundamental value. There are thus seven possible prices at which a limit order can be submitted.

4 The Impact of HFT on Market Quality and Welfare

In this section, we investigate the impact of HFT on market quality and welfare by comparing the properties of equilibrium between a situation without HFTs and situations with HFTs. We assess market quality using the following standard measures:

1. The *bid-ask spread* is defined as the difference between the best ask price and the best bid price. It measures the price for immediacy.

2. The *effective spread* is defined as the absolute value of the difference between the transaction price and the mid-quote at the time of the transaction, and measures effective trading costs.

3. *Market depth* is measured by the number of units of the asset available to buy or sell in the limit order book. We consider both the depth at bid and ask and the depth off-bid and off-ask. Large depth means that the price impact of an order of a given size and thus related transaction costs are small.

4. *Price discovery* measures the percentage distance between the fundamental value and transaction prices. In efficient and transparent markets, the difference between the fundamental value and transaction prices is small.

5. *Microstructure noise volatility* measures the dispersion of transactions around the asset's fundamental value. The higher microstructure noise volatility, the larger execution risk for traders, for example because their limit orders may be picked off or become stale.

To assess the impact of HFT on welfare, we compute each trader's utility using equation (3.1), i.e. as the difference between his valuation for the asset (fundamental value plus private valuation) and the price at which he is able to purchase or sell the asset on the market. For limit orders, utility is discounted by the amount of time it took to execute the limit order. We then compute welfare as the average utility of all traders in the game. Since regulators are likely to be more concerned about the welfare of investors than that of speculators, in addition to total welfare, we also compute welfare separately for investors and speculators.

Before we proceed, it is useful to consider the trade-offs faced by traders in the market and how these might be affected by the presence of HFTs. We say that an order is more aggressive if it is submitted at a price which is closer to the opposite side of the market. The most aggressive order is a market order. The more aggressive an order is, the higher the execution probability prior to the trader returning. This probability, however, also depends on the other orders already in the book and on which new orders are submitted before the trader returns. As mentioned earlier, changes in the fundamental value expose limit orders to two risks: (i) obtaining an undesirable execution when the fundamental value moves in an unfavourable direction, and (ii) not obtaining a desirable execution when the fundamental value moves in a favourable direction. In the first case, the presence of HFTs exacerbates picking-off risk for a slow trader because HFTs are likely to react to the new situation before the slow trader's returning to the market and lift the order. In the second case, the presence of HFTs reduces the probability that a slow trader wishing to update her order given the new situation will be able to pick off a too aggressively priced order on the other side of the market. Furthermore, if she decides to submit a limit rather than a market order when updating her order, that order is likely to lie behind HFTs' orders in the queue, delaying execution.

When conducting our quantitative analysis, we model the situation without HFTs using the base case parameter values described in Section 3.3. Recall that in this parameterization, the shares of traders with private valuations of 0, 2, and 4 ticks in absolute value are 30%, 40%, and 30%, respectively. We assess the impact of HFT in two ways: (1) by gradually replacing slow speculators with HFTs, and (2) starting from a situation in which half of speculators are slow and the other half are HFTs (i.e. a situation in which both groups each make up 15% of the overall population), by gradually adding HFTs until their overall share of the population reaches 25%. Note that while the first approach only affects the distribution of traders, the second increases overall trader arrivals and therefore overall market activity. Arguments could be made in favor of using one or the other approach; as will be shown shortly, however, our findings are robust to the approach chosen.

The impact of HFT on market quality and welfare using the first approach can be seen in Table 1 and Figure 2 for the symmetric information case and Table 3 and Figure 6 for the asymmetric information case. The results using the second approach are reported in Table 2 and Figure 4 for the symmetric information case and Table 4 and Figure 7 for the asymmetric information case. We highlight our main observations regarding market quality and investor welfare and then discuss them in more detail. Throughout, we begin with the results under symmetric information and then contrast them with those under asymmetric information.

Observation 1 (Market Quality)*: The presence of HFTs increases market depth, reduces spreads and transaction costs, and improves price discovery. Under asymmetric information, these effects arise only if the share of HFTs in the market is sufficiently large to generate competition between them.*

Consider first the case where slow speculators are replaced with HFTs. As can be seen in the top panel of Figure 2, an increase in the share of HFTs among speculators significantly improves depth and yields small improvements in the quoted bid-ask spread, the effective spread, and price discovery. The volatility of microstructure noise is also slightly lower (see Table 1). Two channels drive these results. First, HFTs' shorter reaction times allow them to update their limit orders faster, resulting in quotes that track the evolution of the fundamental value more closely and improving price discovery. HFTs' faster reaction times also leads to lower picking-off risk. Accordingly, as is apparent in the bottom panel of Figure 2, the expected payoff of their limit orders is significantly higher than for slow speculators. Second, HFTs are faster at picking-off mispriced orders using market orders. This results in their submitting a larger proportion of market orders than slow speculators, as can be seen in the bottom panel of Figure 2. Although profits from lifting mispriced orders remain constant at one tick (0.125, see the bottom panel of Figure 2, right axis), an increase in the proportion of HFTs increases competition, reducing the probability of lifting mispriced orders. This induces both HFTs and slow speculators to reduce their use of market orders and leads to higher liquidity provision by both groups.[7] With increasing competition, however, pick-off risk increases for both groups of speculators, as is apparent from the decrease in their utility from limit orders in the bottom panel of Figure 2. A closer look at the bottom panel reveals that slow speculators' expected payoff from limit orders in the case without HFTs (the intercept of the solid blue line) exceeds HFTs' expected payoff from limit orders in the case where there are no slow speculators (the value of the dashed blue line on the right). This means that investors demanding liquidity pay a lower price in the case where all speculators are HFTs than in the case without HFTs.

Inspection of traders' strategies in Table 1 reveals that consistent with the findings in Hoffmann (2014), tighter spreads cause slow traders to submit less aggressive orders as competing for the best quotes would increase their risk of being picked-off. Tighter spreads also reduce the volatility of microstructure noise. As their share in the market increases, HFTs provide more liquidity by increasing their share of limit orders (see the bottom panel of Figure 2), which benefits investors in the form of shorter execution times (see the *Transactions* section in Table 1). HFTs' ability to submit high priority orders also results in an increase in depth at the bid and ask and further in the book. These findings are in line with the empirical studies by Hasbrouck and Saar (2013) for U.S. data and Riordan and

[7]Bernales (2014) also finds that HFTs favor market orders when their share of the market is small and limit orders when it is large. However, he does not provide a breakdown of the order types for the different groups of slow traders and only considers an asymmetric information setting.

Storkenmaier (2013) for German data. As can be seen in Figure 3, in the presence of HFTs, the order book is more skewed towards the fundamental value (located in the middle of the price grid).

As can be seen in Table 2 and Figures 4 and 5, under symmetric information, the impact of HFT on market quality is similar if HFTs are added to the market instead of substituting slow speculators.

The impact of HFT on market quality is less clear-cut under asymmetric information. This is best seen in the case where the overall trader population is constant (see Table 3 and Figure 6). Since slow traders observe the fundamental value with a lag, they must base their trading decisions on an estimate of the fundamental value rather than on the true value. This gives HFTs better opportunities to pick-off mispriced orders. Imperfect information about the fundamental value also causes slow traders to make mistakes when submitting market orders to lift orders they believe to be mispriced, resulting in a growing difference between their and HFTs' payoff from market orders (see the bottom panel of Figure 6). The beneficiaries of these mistakes are HFTs, whose average discounted payoff from limit orders increases compared to the symmetric information case (contrast the bottom panels of Figures 2 and 6; the exact values are 0.0348 under symmetric information and 0.0448 under asymmetric information). When the share of HFTs in the market is small, the adverse selection causes price discovery and spreads to worsen. As the share of HFTs in the market increases, however, competition among them leads to an improvement in all market quality measures. When the share of HFTs is sufficiently large, the competition effect dominates the adverse selection effect and market quality is better than in the case without HFTs. In the case where HFTs are added to the market instead of substituting slow speculators (see Table 4 and Figure 7), the share of HFTs is already sizable at 15% initially. As a result, the competition effect dominates and market quality improves as HFTs are added.

It is instructive to contrast our results with those in Bernales (2014). He investigates the asymmetric information case and finds that market quality increases monotonically with the share of HFTs in the market. Our results show that while this statement holds under symmetric information, it does not hold under asymmetric information. Rather, when HFTs have an informational advantage, HFT initially reduces market quality, and increases it once HFTs' share in the market becomes relatively large.[8]

Observation 2 (Welfare): *HFT does not reduce investor welfare but affects the welfare of slow speculators negatively. HFTs' welfare is significantly higher than that of slow speculators. HFTs supply more liquidity than investors, but less than slow speculators.*

Turning to investor welfare and again starting with the case where information is symmetric and slow speculators are replaced with HFTs, we find that both overall welfare and average welfare for all slow

[8]The reason that Bernales (2014) identifies a monotonic impact is that he considers step sizes for the market share of HFTs of 20% and the nonmonotonicity occurs in the range between 0% and 20%.

traders (speculators and investors) are not negatively affected by the presence of HFTs (see Figure 2, middle panel). However, the impact of HFTs on welfare differs across trader categories. Whereas slow speculators' welfare falls markedly, investors experience a slight increase in welfare reflecting the benefits from increased liquidity in the market. Because of improved liquidity and the increase in picking-off risk associated with HFT, investors' use of market orders rises (see the *Trader Strategies* section in Table 1).

Reflecting their speed advantage, HFTs earn higher surpluses than slow speculators. Both aspects of slow speculators' disadvantage compared to HFTs are visible in Table 1: since they are slow at updating their orders when the fundamental value changes, (i) they earn lower profits on limit orders than HFTs, and (ii) they seldom manage to pick off mispriced orders, as can be seen from the lower share of market orders in their transactions compared to HFTs.

Interestingly, the results in Table 1 also reveal that HFTs supply more liquidity than investors, but less than slow speculators. As the share of HFTs increases, slow speculators initially substitute market orders with aggressive limit orders. As the share of HFTs increases further, however, competing for the best quotes with HFTs in an environment with narrower spreads increases slow speculators' pick-off risk, inducing them to submit fewer aggressive limit orders. Slow speculators provide liquidity even more often than HFTs, but their technological disadvantage prevents them from earning substantial payoffs as their orders are frequently picked-off. As one would expect, as the share of HFTs among speculators rises, increased competition between them erodes their trading profits. The decline in profits, however, is even more pronounced for slow speculators.

As can be seen in Table 2 and Figures 4 and 5, the impact of HFT on welfare is similar if HFTs are added to the market instead of substituting slow speculators. Thus, the impact of HFT on both market quality and welfare is robust to the way that it is modeled.

Under asymmetric information (see Table 3 and Figure 6), the impact of HFT on welfare is similar to that under symmetric information: the welfare of investors increases slightly, and that of slow speculators falls. It is worth noting that slow speculators' welfare falls more strongly under asymmetric information than under symmetric information because they face both a speed and an informational disadvantage. Similar patterns can be observed if HFTs are added to the market instead of substituting slow speculators (see Table 4 and Figure 7).

Thus, our results highlight that the welfare impact of HFT on slow traders is heterogeneous. While slow speculators generally experience a welfare reduction from increased competition by HFTs, the welfare of investors increases. Put differently, the negative impact of HFT on slow trader welfare identified by Bernales (2014) does not hold for all subgroups, but only for slow speculators.

5 Impact of Proposed Regulations

In this section, we investigate the effectiveness of a number of regulatory measures that have been suggested to improve market quality in the presence of HFT. We focus on the main regulatory initiatives, namely minimum resting time rules, cancellation fees, transaction taxes, make/take (rebate) fee structures and speed bumps. We compute the equilibria arising when introducing each of these regulations, thus fully accounting for the resulting changes in traders' optimal strategies, and collect the market quality and welfare statistics. We then assess the impact of each regulation by comparing the properties of the resulting equilibrium with those of the unregulated case. Section 5.1 describes the different regulations and how they are implemented in our analysis, and Section 5.2 presents our findings.

5.1 Description of the Regulations and Implementation in the Model

Minimum resting time

Minimum resting time (also known as minimum holding or waiting time and henceforth abbreviated as MRT) is an artificial delay imposed between the time an order is submitted and the time at which it may be changed or cancelled. According to D'Antona (2010), the topic of a minimum resting time for quotes came up during joint meetings on the Flash crash between the SEC and the Commodity Futures Trading Commission that took place in summer 2010. The range of values that were considered in these discussions was from 50 milliseconds to one second. Such a measure would in effect impose on all market participants some of the obligations that are imposed on market makers by some exchanges.[9]

The inability to immediately change orders increases picking-off risk for traders, thus making it more costly to submit limit orders. The implementation of MRT rules in the model is straightforward. A trader that had submitted an order at time s and returns at time $t < s + MRT$, i.e. before the minimum resting time requirement is met, is put into a waiting queue. Further events in the game are then drawn and processed as long as their time is less than $s + MRT$. As soon as an event time after $s + MRT$ is drawn, the trader recorded in the queue is allowed to modify or cancel her order based on the state at time $s + MRT$. We report results for a MRT of one time unit. We have investigated MRT values between 0.25 and five time units and came to similar conclusions.

Cancellation fee

A cancellation fee is a fee that traders must pay whenever they cancel or modify their orders. Cancel-

[9]For example, in addition to submitting quotes on both sides of the market, market makers on the Eurex options exchange have to quote with a minimum holding time of ten seconds. Similar rules are in force on the Hong Kong Exchange and on the ASX options market, where the holding period is 30 seconds.

lation fees make updating orders costly and must be taken into consideration by traders when choosing the aggressiveness of their limit orders. In the implementation, the fee is subtracted from the trader's current utility whenever an order is cancelled or updated. Note that any fees incurred following potential later order cancellations are automatically included in the continuation utility associated with the current state by the solution algorithm and therefore do not need to be tracked explicitly. To make the welfare comparison with the unregulated situation fair, fee proceeds are then distributed equally across all traders. For completeness, we report total welfare both without and with fee proceeds. We report results for a cancellation fee value of 0.015, corresponding to roughly 0.38% of the average asset price.[10] We also experimented with values between 0.001 and 0.5 and found the results to be similar.

Transaction tax

Under a transaction tax, traders have to pay a tax amount equal to a certain proportion of the value of their transactions. In the implementation, the tax is collected at the time of order execution and, consistent with taxes typically imposed in the real world, shared equally between the seller and the buyer. To make the welfare comparison with the unregulated situation fair, tax proceeds are then distributed equally across all traders. For completeness, we report total welfare both without and with tax proceeds. We set the tax rate at 0.1%, a value in the ballpark of rates imposed in several countries. Because the fundamental value follows a random walk and transactions happen at prices around the fundamental value, the transaction tax lowers agents' final payoff by 0.001945. We also investigate varying the tax rate from 0.05% to 0.5% and come to similar conclusions.

Make/take fees

Under a make/take (or rebate) fee structure, whenever a transaction is executed, the consumer of the liquidity has to pay a taker fee and the trader who provided the limit order receives a maker fee (a rebate on transaction fees). As mentioned in the Introduction, such fee structures are used by many exchanges in order to encourage liquidity provision. However, their desirability is controversial.[11] Since such fee structures discourage trading by market orders and encourage submitting limit orders, one would expect them to result in tighter quoted spreads. To implement rebates in the model, we simply subtract the taker fee from the utility of the trader that consumed liquidity and add the maker

[10] According to Hollifield et al. (2006) and Goettler et al. (2009), based on the distribution of private values that we use, the expected stock price is roughly 3.89. Because private valuations are additive in the utility function and orders do not "fall off" the price grid, traders only consider changes in the fundamental value, not its level, which follows a random walk and has an infinite support.

[11] For example, in its Concept Release on Equity Market Structure, the SEC (2010) raises the following concerns: "Are liquidity rebates unfair to long-term investors because they necessarily will be paid primarily to proprietary firms engaging in passive market making strategies? Or do they generally benefit long-term investors by promoting narrower spreads and more immediately accessible liquidity? Do liquidity rebates reward proprietary firms for any particular types of trading that do not benefit long-term investors or market quality?"

fee to the utility of the trader whose limit order got picked off. We set the taker fee at \$0.01 and the maker fee at (minus) \$0.01, which is in line with the actual pricing structures at various exchanges. We set both fees symmetrically in order to make the fee structure budget neutral overall, thus facilitating the comparison with the unregulated case. We investigated the properties of market equilibrium for a range of make/take fee structures, both symmetric and asymmetric, and obtained similar results.

Speed bump

A speed bump is an artificial delay imposed on incoming orders; when submitted, orders are stored for a fixed amount of time before they are routed to the order book. Traders submitting orders have to take into account that by the time their order reaches the book, the book could be different than what they observed when they submitted the order. As mentioned in the Introduction, speed bumps have been imposed on all orders on some exchanges and only on market orders on others. In our analysis, we choose to apply the speed bump to market orders only. Such orders are converted to marketable limit orders upon submission in order to protect traders against adverse quote movements between the time that an order is submitted and the time it hits the order book. However, execution is no longer guaranteed. If the best quote the market order intends to trade on moves in an unfavourable direction, the order is recorded in the intended position in the book and the trader can decide on her optimal action again upon her next reentry. This is consistent with the way that speed bumps have been implemented on Nasdaq. We report results for a speed bump of one time unit. We have experimented with values between 0.25 and five time units and found similar results.

5.2 Results

We conduct our analysis using the parameterization described in Section 3.3. Recall that in this parameterization, the shares of traders with private valuations of 0, 2, and 4 ticks in absolute value are 30%, 40%, and 30%, respectively. We assume that half of speculators are HFTs. As mentioned at the beginning of this section, we assess the impact of each regulation by comparing the properties of the resulting equilibrium with those of the unregulated case. We consider both the symmetric information case, where all traders observe the fundamental value in real time, and a situation with asymmetric information where HFTs observe the fundamental value in real time and all other traders observe it with a lag of two time units. The market quality and welfare indicators under symmetric and asymmetric information are reported in Tables 5 and 6, respectively. We highlight our main observations regarding market quality and investor welfare and then discuss them in more detail.

Observation 3 (Minimum Resting Time)*: MRT does not improve market quality, reduces aggregate welfare, transfers welfare from HFTs to slow speculators, increases the aggressiveness of orders submitted by investors and increases order execution time for all trader groups.*

Overall, MRT has a mixed impact on market quality. It reduces the depth of the book, but leads to a decrease in both quoted and effective spreads in the presence of asymmetric information.[12] At the same time, MRT worsens price discovery and increases the volatility of microstructure noise under both symmetric and asymmetric information. This finding is in line with the results in Kirilenko and Lamacie (2015), who report that an increase in latency leads to higher short-term volatility.

MRT has a negative impact on aggregate welfare. Importantly, the decrease in aggregate welfare is entirely borne by investors and not by speculators. The welfare of HFTs does decrease, but this decrease is exactly offset by an increase in the welfare of slow speculators, which are the only group that is better of under MRT rules. The reason for the welfare transfer from HFTs to slow speculators is that HFTs' inability to modify or cancel their outstanding limit orders sufficiently fast exposes them to increased picking-off risk. HFTs' reduced reaction ability is clearly visible in Tables 5 and 6: the share of limit orders that they cancel falls by an order of magnitude, and the fraction of their transactions from limit orders rises (equivalently, the fraction of transactions that they initiate via market orders falls).

Due to increased picking-off risk, HFTs substitute risky aggressive limit orders with less risky orders at the best quote. By contrast, slow speculators and 4 ticks private value investors, who are now better able to pick-off stale orders, are more likely to submit market orders. Overall, HFTs end up providing more liquidity. However, the fact that liquidity provision becomes more risky for all trader groups results in wider spreads. This lower quality of liquidity reduces liquidity consumption. Accordingly, as can be seen at the bottom of Tables 5 and 6, execution times increase for all groups of traders, negatively affecting welfare.

Observation 4 (Cancellation Fees)*: Cancellation fees decrease market quality and welfare, increase the usage of market orders and limit orders posted at the best quote, and lead to a rise in order execution time for all trader groups.*

Consistent with the empirical findings in Friederich and Payne (2015), cancellation fees decrease depth and worsen price discovery. Under symmetric information, the quoted spread remains the same and the effective spread worsens. Under asymmetric information, the quoted spread falls, but the effective spread remains the same. Thus, even ignoring the fee itself, the introduction of cancellation fees does not reduce transaction costs.

Turning to welfare, cancellation costs lower overall welfare; importantly, distributing fee proceeds to traders does not offset the welfare loss. Welfare losses are registered in all trader groups, but slow

[12] While our results on market depth are in line with those reported by Bernales (2014) who analyzes the asymmetric information case, our results on spreads differ from his. As mentioned in the introduction, while we implement MRT rules exactly in the way that they have been proposed for real-world markets, Bernales (2014) models them by reducing the returning frequency of fast traders. Doing so makes fast traders slower without actually enforcing the MRT restriction.

speculators facing asymmetric information are least affected. As one would expect, order cancellation ratios fall. More strikingly, cancellation fees lead to a sharp decrease in spread-improving (i.e. aggressive) limit orders: realizing that the fees make it more costly to remove orders that have become mispriced to prevent them from being lifted, traders become more prudent in their order submission. Instead of aggressive limit orders, they use market orders or limit orders submitted at the best current quote. Another consequence of cancellation fees is that they induce traders to decide not to submit any order initially and wait until they return to do so, leading to lower overall depth in the book. This causes liquidity to become hidden and execution times to rise significantly for all trader groups. The reason is similar to that in the case of MRT rules: cancellation fees make liquidity provision more costly for all trader groups, leading to lower quality liquidity and reduced liquidity consumption.

Observation 5 (Transaction Taxes): *Transaction taxes reduce the trading gains of all trader groups roughly by the amount of the tax and do not significantly impact market quality, aggregate welfare, and trading strategies.*

Transaction taxes only have a small impact on market quality. Welfare decreases for all groups of traders roughly by the amount of the tax; accordingly, distributing tax proceeds to traders completely offsets the welfare loss. The decrease in welfare is somewhat stronger for slow speculators because the tax represents a larger portion of their welfare without the tax. Under asymmetric information, transaction taxes lead uninformed traders to submit fewer aggressive limit orders because the riskiness of such orders remains the same but their payoff is lower.

It is worth noting that with a finer distribution of private values (or with a higher tax rate), some traders would decide not to trade at all, which would lead to wider spreads and a decrease in market liquidity as was observed by Colliard and Hoffmann (2015) in their event study on the introduction of the French financial transaction tax.

Observation 6 (Rebates): *Rebates reduce market quality and investor welfare, increase the aggressiveness of speculators, and lead to a rise in order execution time for all trader groups.*

Contrary to what one might expect, rebates lead to worse price discovery and reduce depth. Although they improve the quoted spread in the symmetric information case, this does not translate into lower transaction costs as measured by the effective spread. Rather, the effective spread increases under both symmetric and asymmetric information.

Rebates also lead to a reduction in overall welfare. They increase the welfare of slow speculators at the expense of investors, while HFT welfare is almost unaffected. The higher costs of market orders lead traders to avoid such orders and induces longer execution times of limit orders for all groups of traders. Slow speculators and especially HFTs do not compete for the best quotes (the share of aggressive limit orders that they submit is lower than in the base case). Rather, they wait for investor quotes to become mispriced and lift them, as can be seen from the higher proportion of market orders in speculators' strategies and transactions. As HFTs can react to mispriced limit orders faster, the proportion of market orders in their transactions rises more strongly than for slow speculators. Like speculators, investors also submit fewer aggressive limit orders and more limit orders at the best quote than in the base case. However, the proportion of limit orders at the best quote increases more strongly for investors than for speculators, and a sizable part of that increase comes from a reduction in orders submitted below the best quote. The reason that investors increase their use of orders at the best quote is that submitting market orders is costly because of the take fees and submitting aggressive limit orders is risky due to speculators' more frequent lifting of mispriced orders. Accordingly, rebates cause market depth to fall more strongly off the best quote than at the best quote.

Observation 7 (Speed Bumps)*: Speed bumps for market orders increase microstructure noise and worsen price discovery. They reduce the welfare of slow speculators, but do not affect investor welfare. Speed bumps also make order submissions by investors and HFTs more aggressive and increase order execution time for all trader groups.*

Speed bumps generally worsen price discovery, but improve spreads and depth in the book. The welfare of investors is unaffected and that of slow speculators decreases. Speed bumps render the execution of market orders uncertain. Perhaps surprisingly, speed bumps actually lead to an increase in market order submissions in all groups. The reason is that since speed bumps for market orders are implemented as marketable limit order with the limit set at the current best limit order on the other side of the market (consistent with the way that speed bumps have been implemented on Nasdaq), traders might obtain slightly better execution. This induces them to submit blind orders and leads to higher pick-off risk for standing limit orders. This is reflected in lower cancellation ratios for HFTs and a higher proportion of limit orders in their transactions.

The results in Tables 5 and 6 relate to a speed bump with a deterministic delay. Harris (2013) proposed introducing speed bumps with random delays, primarily to deter the technology arms race between HFTs. In order to assess whether such random speed bumps lead to improvements in market quality and welfare, we implemented two versions of his proposal. In the first, the speed bump was drawn from a uniform distribution between zero and one second, while in the second, it was drawn from a uniform distribution with strictly positive support (the cases considered were between 0.1 and 1.1 second and between 0.5 and 1.5 seconds). All these cases led to results that were similar to those for deterministic speed bumps presented in Tables 5 and 6 are are not reported for brevity.

We ran numerous additional cases, varying both the parameter values of the underlying base case (in particular the volatility of the asset's fundamental value and the proportions of investors, slow speculators and HFTs) and the values of the parameters capturing the different regulations. The general conclusion that none of these regulations consistently improves investors' welfare is robust.

6 Conclusion

We investigate the impact of HFT and a number of regulatory initiatives on market quality and investor welfare using a general dynamic limit order book model as in Goettler, Parlour, and Rajan (2009). We solve for the Markov perfect equilibrium numerically using the Q-learning algorithm. We find that the presence of HFTs improves most common measures of market quality, reducing spreads, increasing market depth, and enhancing price discovery. These results hold without qualification under symmetric information. When HFTs have an informational advantage over slow traders, however, improvements in market quality arise only if competition between them is sufficiently strong. We also find that HFT does not negatively impact investor welfare. However, it leads to a reduction in the welfare of slow speculators, who are crowded out by HFTs.

The flexibility of the model allows us to investigate the impact of the main recent regulatory initiatives designed to curb HFT on market quality and investor welfare. We consider minimum resting time rules, cancellation fees, transaction taxes, rebate fee structures and speed bumps. We find that most of these regulatory proposals have a negative impact on market quality and that none consistently improves investor welfare. Admittedly, some of these regulatory proposals lead to improvements in a number of market quality measures; for example, minimum resting time rules reduce trading costs under asymmetric information, while speed bumps increase book depth and decrease spreads. However, these improvements generally do not translate into higher welfare for long-term investors. Rather, the main effect of these regulations is to generate wealth transfers from HFTs to slow speculators. These regulations therefore appear inadequate to enhance investor welfare in the presence of HFTs. Of the different measures, transaction taxes are the least harmful; while they reduce welfare roughly by the amount of the tax (a reduction that can be offset by distributing tax proceeds back to traders), they do not significantly worsen market quality. Importantly, the common practice by exchanges of granting rebates to limit orders is detrimental to market quality and investor welfare, causing both higher effective spreads and longer execution times.

It should be noted that while we have considered asymmetric information between HFTs and other traders in our analysis, we have assumed that markets are not fragmented and ruled out front-running of already submitted orders by HFTs. In order to make the analysis tractable, we have restricted individual traders to trade a single share. A promising avenue for future research is to investigate to what extent our findings generalize to settings where traders may submit large orders, thus requiring them to incorporate the issues of price impact and detection by other traders in their decision-making process.

A Formal Description of the Model

This Appendix provides a detailed formal description of the model. Our description follows that provided in Goettler et al. (2009).

Structure of the Trading Game

We define an action a trader takes as $a = (p, q, x)$, where p is the price at which she submits an order, $q \geq 0$ the priority of her order and x is $+1$ or -1 when the order is buy or sell and 0 if there is no order. The priority of an order is determined by p, x and the current book L. q decreases with increasing priority in the queue. If the order is a market order, the priority is 0. The priority of a limit order is the new order's position in the queue. Formally:

$$q(p,x) = \begin{cases} 0 & \text{if } x = 0 \text{ or} \\ & x = 1,\ p \geq A(L) \text{ or} \\ & x = -1,\ p \leq B(L), \\ |l^p + x| & \text{otherwise.} \end{cases}$$

A trader reentering the market at time t who does not have a pending order in the book faces the same problem as a new trader. A trader having an order in the book additionally has the option of leaving the order unchanged, potentially taking advantage of an improvement in the priority of the order. The optimal action the trader chooses depends on the state in which the trader enters the market.

Let $s(\theta)$ be the state observed on a particular entry to the market by a trader of type θ. The state $s(\cdot)$ includes:

1. the history of the game and its elements as described above, the changes of the fundamental value until time $t - \Delta$ if $h = 0$ or until time t if $h = 1$, where Δ is the lag (if any) with which slow traders observe the fundamental value. The state further contains the limit order book and the status of the previous action $a = (p, q, x)$ if the trader previously submitted a limit order, where p is the price at which the order was submitted, q the current priority at price p and x an indicator of a buy $(+1)$ or sell (-1) order or no order (0). If the trader is entering for the first time, x is set to zero;

2. a variable $z \in \{0, 1\}$ used in the Bellman equation to set the agent's future payoff to zero once she trades. The variable z can also be viewed as the trader's budget or the number of shares she has available to trade. In our model, traders enter the market with $z = 1$.

For our numerical implementation, we have to limit the possibly infinite state space. First, because the fundamental value follows a random walk, it can take unbounded values. For this reason, we only track all prices and quotes relative to the fundamental value. We further collapse the characterization of market conditions into (i) the fundamental value v_t for $h = 1$ and $v_{t-\Delta}$ if $h = 0$; (ii) the price of the last transaction p_t and whether the transaction was buyer- or seller-initiated $b_t = -1$ or $b_t = 1$; and (iii) the limit order book, which is represented by bid and ask prices B_t, A_t, bid and ask sizes l_t^B, l_t^A, cumulative depths at buy and sell $D_t^b = \sum_{i=0}^{N} (l_t^i > 0), D_t^s = \sum_{i=0}^{N} (l_t^i < 0)$. We investigated models with broader market characteristics and found that while convergence occurs more slowly, the results remain practically unchanged.

Traders can choose an action from a feasible action set. In order not to hamper computational tractability, we restrict the number of ticks where traders can place limit orders to k. We choose k such that it does not influence equilibrium strategies. Denote the expectation of the fundamental value given the state s as $\hat{v}(s) = \mathbb{E}(v|s)$. The feasible action set is then formally defined as

$$\mathcal{A}(s) = \{(p, q, x) \mid (i)\, x \in \{-1, 0, 1\},\ (ii)\, q = \hat{q}(p, x),\ (iii) \text{ if } q \neq 0 \Rightarrow p \in [\hat{v}(s) - k, \hat{v}(s) + k] \cap \mathcal{P}\}.$$

A mixed strategy for a trader of type θ is $\sigma_\theta : S_\theta \longrightarrow \prod_{s \in S_\theta} \Delta(\mathcal{A}(s))$, where S_θ is the set of feasible states that a trader of type θ may encounter and $\Delta(\mathcal{A}(s))$ represents possible probability distributions over $\mathcal{A}(s)$.

Every action leads to an expected payoff which is composed of two parts. The first is the payoff from execution prior to reentry and the second the continuation value if the trader reenters prior to the execution of her order. We consider symmetric equilibria and denote by $\sigma = \{\sigma_\theta\}_{\theta \in \Theta}$ the strategies adopted by every other player. Let $\phi(\tau, v; s, \tilde{a}, \sigma)$ be the probability that an action $\tilde{a} = (\tilde{p}, \tilde{q}, \tilde{x})$ taken at time s leads to an execution at time τ when the fundamental value is v and all other players' strategies are σ. We further let $f(v|s, t)$ denote the density function of the fundamental value at time t given state s. Suppose the trader reenters the market at time $w > 0$. Her expected payoff due to execution prior to reentry is

$$\pi(s, \tilde{a}, w, \sigma) = \int_0^w \int_{-\infty}^{\infty} e^{-\rho t} \tilde{x}(\alpha + v_t - \tilde{p}) \phi(\cdot) f(v|s, t) dv dt,$$

where the inner integral is over the possible values of the fundamental value and the outer integral is over the possible times at which execution can occur. Inside the integral is the instantaneous payoff discounted back to the time at which the order was submitted.

The agent chooses her action such that it maximizes her expected payoff (the sum of the payoff due to execution prior to reentry and the continuation value). The reentry time is distributed randomly and exogenously according to $G(\cdot)$. The state s' in which the trader reenters has probability of occurring $v(s'|s, \tilde{a}, w, \sigma)$. We denote the trader's value function in state s by $J(s)$. The trader's optimization problem can be written as a dynamic programming problem, where the Bellman equation is

$$J(s, \sigma) = \max_{\tilde{a} \in \mathcal{A}(s)} \int_0^\infty \left(\pi(s, \tilde{a}, w, \sigma) + e^{-\rho t} \int_{s' \in S_\theta} J(s', \sigma) v(s'|s, \tilde{a}, w, \sigma) ds' \right) dG(w).$$

Since the trader always faces a maximization problem over a well-defined and finite action set, the maximum over all feasible actions exists.

Fixing the strategies of all other traders, a pure strategy y_θ^* for a trader of type θ is a best response if and only if for every $s \in S_\theta$

$$y_\theta^*(s) = \arg \max_{\tilde{a} \in \mathcal{A}(s)} \int_0^\infty \left(\pi(s, \tilde{a}, w, \sigma) + e^{-\rho t} \int_{s' \in S_\theta} J(s', \sigma) v(s'|s, \tilde{a}, w, \sigma) ds' \right) dG(w).$$

A collection of strategies $y^* = \{y_\theta^*\}_{\theta \in \Theta}$ is a Markov perfect equilibrium if and only if for each pair $\theta \in \Theta, y_\theta^*$ is a best response in every feasible state $s \in S_\theta$.

Beliefs and Q-Learning

Equilibrium is obtained by finding common beliefs $Q(a|s)$, where s is the current state and $a = a(s)$ an attainable action at this state. This is done by simulating the market and updating beliefs until they converge. In finding the fixed-point of this game, since an analytic solution is not tractable, we turn to the stochastic algorithm of Pakes and McGuire (2001) implemented for a similar kind of games in Goettler et al. (2005, 2009).

One way to obtain beliefs would be to update them by integrating over all possible sequences of future outcomes that lead to a transaction being executed. By contrast, the algorithm of Pakes and McGuire (2001) tracks each share in the book until it executes in the market simulation. Upon execution, we update the belief $Q(a|s)$ at the state from which the order was submitted by averaging this outcome with the previous outcomes for shares submitted at this state discounted back by the time it took the share to execute. We use an online implementation of this reinforcement or Q-learning algorithm and update the $Q(a|s)$ with the new $Q(a'|s')$ from the new state s' where the trader's action was a', not just relying on the final executions, but incorporating the new belief as soon as a trader reenters and chooses a new action. Figure 8 provides a diagrammatic representation of this procedure.

Not every possible state is visited during the simulation, thus it focuses only on the recurrent subset of the state space.[13] The advantage of this approach is that updates are computed only for the states

[13]A recurrent subset of states has the following properties: (i) regardless of the initial state, the system eventually enters the recurrent class, (ii) once entered, the probability of each state outside the recurrent class is zero, and (iii) each state in the recurrent class is visited infinitely often as t approaches infinity.

actually visited, thus lowering the memory and computation time requirements. The disadvantage is that not all relevant states might have been visited during the simulation. To prevent the algorithm from falling into local equilibria which do not satisfy the perfection condition, a couple of state space exploration techniques are implemented, namely: (i) optimistic initial beliefs – when beliefs of the first explored state-action get corrected, the other un-explored state still possesses higher optimistic initial beliefs and is thus likely to be explored, (ii) trembles – in the learning phase, traders choose suboptimal actions with a small probability, and (iii) resetting the learning speed from time to time in order to speed up learning in different phases of the simulation.

The simulation has two main phases: (i) the learning phase, where the beliefs are updating and the exploration of the state space is promoted, and (ii) the simulation of the equilibrium, during which the beliefs are fixed and the exploration is not artificially promoted. The algorithm switches between phase (i) and phase (ii) depending on the convergence of the beliefs. The convergence during the first phase is called convergence of the first type or learning convergence and if beliefs are found to have converged in the first phase, the second phase with fixed beliefs starts. Convergence of the beliefs in the second phase is called second type convergence or equilibrium convergence. If second type convergence is not satisfied, the algorithm returns to the learning phase. When second type convergence is satisfied, we claim that the fixed point was obtained and the data collected during the simulation phase represent equilibrium values.

Formally, at each time t in each state s encountered in the simulation, each action $\tilde{a}$ has an associated payoff $Q_t(\tilde{a}|s)$. This real number represents the current belief of an agent about the payoff from this action at this state.[14] The beliefs at each time t imply an optimal strategy y_t, which assigns the payoff maximizing action at each state, $a^*(s) \in \arg\max_{\tilde{a}\in\mathcal{A}(s)} Q_t(\tilde{a}|s)$. The value of state s then is $J(s, y_t) = Q_t(a^*(s)|s)$.

The value for a newly encountered state is called an initial belief $Q_0(\tilde{a}|s)$ and is determined in the following way. Consider a buy limit order. The initial belief is determined by the payoff $\alpha + \hat{v} - p$ discounted by the expected time until the arrival of a new trader for whom being a counterparty yields a non-negative payoff. Here, $\hat{v}(s) = \mathbb{E}(v|s)$ is the expectation of the current fundamental value given the trader's type. If the trader is a HFT, she knows the current fundamental value v_t. For the uninformed slow trader, we have to compute the initial expectation of the fundamental value. Let $\delta(s(\theta)) = \mathbb{E}(v|s) - v_{t-\Delta}$. We update this expected difference between the expected current fundamental value and the lagged fundamental value using the following updating rule:

$$\delta_{r+1}(s(\theta)) = \frac{r}{r+1}\delta_r(s(\theta)) + \frac{1}{r+1}(v_t - v_{t-\Delta}).$$

[14] Q-learning was first described as a model for animals' learning in Watkins (1989) and more details and the underlying theory can be found in Bertsekas and Tsitsiklis (1996).

Thus, for an uninformed trader, the estimate of the current fundamental value is given by $\hat{v}(s(\theta)) = v_{t-\Delta} + \delta_r(s(\theta))$. The initial belief for market orders is straightforward. The initial belief of not submitting any order is determined as the average of all non-negative beliefs the trader has available at the current state discounted by the expected return time of the particular trader.

If the previous state was s and the state s' is hit, the continuation value $J(s'|y_{t'})$ is defined as in Figure 8 based on the action a' taken in the new state s':

1. *Market order*: payoff from the market order.
2. *Limit order*: expected value of the limit order represented by the action a', $Q(a'|s')$.
3. *No order*: expected value of taking no action a', $Q(a'|s')$.

Put together, the Q-factors are updated in the following fashion, where $J(s'|y_{t'})$ represents the payoff to the action taken at state s' and time t'.

$$Q_{t'}(a^*|s) = \frac{n}{n+1} Q_t(a^*|s) + \frac{1}{n+1} e^{-\rho(t'-t)} J(s'|y_{t'})$$

Here, $n(\tilde{a}^*, s)$ is a positive integer that is incremented each time $\tilde{a}^*$ is chosen in state s. Periodically during the simulation, we restart n to some small initial value of n_0 for some action and state pairs to obtain quicker convergence.

Similarly, if the previously submitted limit order is executed, the expected payoff at the previous state is updated by $\tilde{x}(\alpha + v_{t'} - \tilde{a}^*)$, where t' is the current time.

A formal proof of convergence to the optimal Q-factors with probability 1 is provided in Section 5.6 of Bertsekas and Tsitsiklis (1996).

Convergence Criteria

We simulate a couple of billion events during which we continuously decrease the artificial exploration of the state space by decreasing the trembling probability in order to achieve a soft landing to the equilibrium values of the Q-factors.[15] We use the same convergence criteria as Goettler et al. (2009). After an initial exploration of the state space, we perform the following computations every 300 million events:

[15]Increasing the trembling probability involves a tradeoff. The benefit is that it allows a faster exploration of the state space. The cost is that it also impacts the strategies of traders that do not tremble by making them consider a certain fraction of "erroneous orders" to be part of general market conditions.

1. If $\frac{|Q^{k_2}_{t_2}(\tilde{a}|s)-Q^{k_1}_{t_1}(\tilde{a}|s)|}{k_2-k_1}$ is small (less than 0.01), then

2. fix beliefs $Q(\cdot)$ and simulate 300 million events,

3. compare fixed beliefs to (a) "one step ahead" $\tilde{J}_1$ and (b) "realized" $\tilde{J}$ empirical payoffs. If

   - the correlation between J^* and both $\tilde{J}_1$ and $\tilde{J}$ exceeds 0.99, and
   - the mean absolute error in beliefs over n is less than 0.01,

   convergence is achieved.

Step (1) corresponds to the learning convergence criterion described above and step (3) corresponds to the equilibrium convergence criteria. If (1) is not satisfied, learning continues, and if (3) is not satisfied, the algorithm returns to the learning phase. Here, k_1 is the number of times that action $\tilde{a}$ has been chosen in state s at the start of the current 300 million events, and k_2 the number of times it has been chosen at the end of the current 300 million new events. Further, t_1 and t_2 represent the corresponding times.

The one step ahead and realized empirical payoffs required for these convergence computations are determined as follows. Note that eventually every trader in this model executes and leaves the market. At the time she executes, she obtains a "realized payoff", computed as follows. Suppose the trader enters at t and executes at t' . If $\tilde{a}$ was her most recent action before execution, her realized payoff is then $\tilde{J}(s, y^*) = e^{-\rho(t'-t)}\tilde{x}(\alpha + v_{t'} - \tilde{p})$. The "one step ahead" payoff is based on the trader's next entry time or execution time, whichever is sooner. Suppose a trader takes an action $\tilde{a}$ at time t, and reenters at $t' > t$ with a new state s'. Her one-step ahead empirical payoff is $\tilde{J}_1(s, y^*) = e^{-\rho(t'-t)}J^*(s', y^*)$. If the trader's order executes prior to reentry, her one-step ahead empirical payoff is $\tilde{J}_1(s, y^*) = e^{-\rho(t'-t)}\tilde{x}(\alpha + v_{t'} - \tilde{p})$.

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Table 1: Impact of HFT on market quality, welfare, and strategies when the overall trader population is constant under symmetric information.

Share of HFTs among Speculators	0%	25%	50%	75%	100%
Market Quality					
Best Bid/Offer size	1.419	1.708	1.976	2.282	2.569
Depth off BBO	4.611	5.024	5.822	6.447	7.558
Quoted spread	1.610	1.611	1.615	1.575	1.575
Effective spread	1.266	1.298	1.176	1.100	1.079
Price discovery $\|p_t - v_t\|/v_t$ %	1.322	1.315	1.289	1.260	1.166
Microstructure noise volatility	0.080	0.080	0.080	0.079	0.077
Welfare					
Average welfare	0.235	0.237	0.239	0.240	0.242
Slow speculator	0.069	0.060	0.050	0.035	-
Investor 2 ticks PV	0.216	0.217	0.219	0.222	0.228
Investor 4 ticks PV	0.434	0.434	0.435	0.436	0.440
HFT	-	0.105	0.096	0.084	0.066
Trader Strategies					
Slow speculator					
Buy market order %	0.153	0.103	0.064	0.046	-
Aggressive buy limit order %	1.282	1.337	1.454	1.200	-
At quote buy limit order %	9.946	11.476	10.779	10.147	-
Below best buy limit order %	38.990	40.100	36.537	37.694	-
Investor 2 ticks private valuation					
Buy market order %	5.570	5.416	5.420	5.736	5.903
Aggressive buy limit order %	10.027	10.450	13.777	16.736	17.940
At quote buy limit order %	45.644	46.547	39.156	31.097	22.696
Below best buy limit order %	34.607	34.243	36.161	42.232	50.017
Investor 4 ticks private valuation					
Buy market order %	18.159	17.558	16.951	16.754	14.030
Aggressive buy limit order %	16.893	16.515	17.898	18.841	18.492
At quote buy limit order %	55.195	56.216	53.357	47.620	45.845
Below best buy limit order %	9.029	8.986	10.880	15.968	21.633
High-frequency trader					
Buy market order %	-	0.073	0.047	0.033	0.022
Aggressive buy limit order %	-	1.026	1.106	0.845	0.779
At quote buy limit order %	-	8.872	11.309	11.421	10.981
Below best buy limit order %	-	40.116	37.588	36.410	39.218
Order cancellation ratio %	-	75.759	72.403	68.491	65.988
Transactions					
Share of Limit Orders					
Slow speculator %	66.157	74.731	81.265	84.376	-
Investor 2 ticks PV %	52.286	52.301	51.537	50.735	46.804
Investor 4 ticks PV %	30.875	32.548	33.865	35.169	39.147
HFT %	-	33.756	47.012	56.990	65.109
Execution Times					
Slow speculator	7.774	6.394	4.939	3.918	-
Investor 2 ticks PV	5.029	4.324	3.649	2.752	1.914
Investor 4 ticks PV	3.789	3.526	3.278	3.174	2.604
HFT	-	3.188	3.746	4.142	3.556

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for five different parameterizations in which the share of HFTs among speculators is progressively increased from 0% to 100%. Overall market activity and the total share of speculators in the trader population are held constant at 30%. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

Table 2: Impact of HFT on market quality, welfare, and strategies when adding HFTs to the trader population under symmetric information.

Share of HFTs in the Market	15%	17.5%	20%	22.5%	25%
Market Quality					
Best Bid/Offer size	1.976	2.162	2.637	3.261	4.281
Depth off BBO	5.822	7.119	8.338	9.909	11.576
Quoted spread	1.615	1.561	1.503	1.469	1.467
Effective spread	1.176	1.078	1.057	1.024	1.029
Price discovery $\lvert p_t - v_t \rvert / v_t$ %	1.289	1.185	1.138	1.101	1.118
Microstructure noise volatility	0.080	0.077	0.076	0.075	0.076
Welfare					
Average welfare	0.239	0.235	0.228	0.220	0.213
Slow speculator	0.050	0.034	0.023	0.011	0.011
Investor 2 ticks PV	0.219	0.228	0.229	0.230	0.228
Investor 4 ticks PV	0.435	0.437	0.440	0.442	0.444
HFT	0.096	0.085	0.074	0.069	0.062
Trader Strategies					
Slow speculator					
Buy market order %	0.064	0.043	0.034	0.025	0.025
Aggressive buy limit order %	1.454	1.291	1.099	0.705	0.707
At quote buy limit order %	10.779	10.174	9.684	9.966	10.013
Below best buy limit order %	36.537	37.885	37.962	40.220	40.380
Investor 2 ticks private valuation					
Buy market order %	5.420	6.203	6.463	6.846	6.859
Aggressive buy limit order %	13.777	18.525	17.020	14.812	11.870
At quote buy limit order %	39.156	23.535	23.974	23.650	30.606
Below best buy limit order %	36.161	44.639	45.313	47.052	44.746
Investor 4 ticks private valuation					
Buy market order %	16.951	16.522	14.384	15.300	15.359
Aggressive buy limit order %	17.898	18.662	17.164	16.664	19.036
At quote buy limit order %	53.357	50.203	46.913	46.831	39.505
Below best buy limit order %	10.880	14.361	17.811	20.744	19.995
High-frequency trader					
Buy market order %	0.047	0.035	0.029	0.023	0.021
Aggressive buy limit order %	1.106	1.144	0.855	0.449	0.425
At quote buy limit order %	11.309	9.568	10.471	11.742	12.881
Below best buy limit order %	37.588	38.910	38.285	37.883	35.807
Order cancellation ratio %	72.403	69.300	60.705	52.054	36.435
Transactions					
Share of Limit Orders					
Slow speculator %	81.265	85.839	87.097	90.778	90.212
Investor 2 ticks PV %	51.537	45.541	42.803	40.411	38.702
Investor 4 ticks PV %	33.865	37.331	38.214	37.085	35.739
HFT %	47.012	50.984	53.750	56.382	59.944
Execution Times					
Slow speculator	4.939	3.902	3.632	3.904	4.701
Investor 2 ticks PV	3.649	2.041	2.018	2.046	2.674
Investor 4 ticks PV	3.278	3.142	3.037	2.830	2.473
HFT	3.746	2.624	3.009	4.077	4.216

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for five different parameterizations in which the share of HFTs in the overall trader population is progressively increased from 15% to 25% by gradually adding HFTs. Overall market activity increases in this scenario. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

Table 3: Impact of HFT on market quality, welfare, and strategies when the overall trader population is constant under asymmetric information.

Share of HFTs among Speculators	0%	25%	50%	75%	100%
Market Quality					
Best Bid/Offer size	1.334	1.454	1.582	1.837	2.441
Depth off BBO	3.523	3.530	3.822	4.522	6.765
Quoted spread	1.697	1.878	1.780	1.683	1.619
Effective spread	1.413	1.492	1.452	1.415	1.163
Price discovery $\|p_t - v_t\|/v_t$ %	1.356	1.363	1.376	1.382	1.287
Microstructure noise volatility	0.082	0.082	0.083	0.084	0.080
Welfare					
Average welfare	0.235	0.237	0.239	0.240	0.241
Slow speculator	0.069	0.062	0.053	0.034	-
Investor 2 ticks PV	0.215	0.215	0.217	0.218	0.225
Investor 4 ticks PV	0.432	0.433	0.433	0.434	0.437
HFT	-	0.108	0.098	0.092	0.073
Trader Strategies					
Slow speculator					
Buy market order %	0.199	0.163	0.132	0.121	-
Aggressive buy limit order %	2.045	3.312	2.237	2.488	-
At quote buy limit order %	11.066	11.377	11.032	8.121	-
Below best buy limit order %	38.403	36.298	37.108	37.172	-
Investor 2 ticks private valuation					
Buy market order %	5.499	5.188	5.031	5.381	5.721
Aggressive buy limit order %	10.030	10.369	12.311	13.558	18.796
At quote buy limit order %	45.305	45.089	43.027	39.022	23.746
Below best buy limit order %	32.551	33.789	32.868	33.342	43.805
Investor 4 ticks private valuation					
Buy market order %	19.777	16.810	17.571	17.435	15.686
Aggressive buy limit order %	19.543	17.508	21.585	21.419	20.295
At quote buy limit order %	47.963	55.758	44.745	42.703	42.498
Below best buy limit order %	6.766	8.381	13.174	16.518	20.794
High-frequency trader					
Buy market order %	-	0.100	0.069	0.050	0.024
Aggressive buy limit order %	-	2.110	1.782	1.604	0.724
At quote buy limit order %	-	9.801	14.471	12.895	11.884
Below best buy limit order %	-	37.688	34.955	33.916	37.387
Order cancellation ratio %	-	81.879	74.740	72.260	69.670
Transactions					
Share of Limit Orders					
Slow speculator %	66.003	71.591	74.231	70.361	-
Investor 2 ticks PV %	52.942	53.329	53.753	53.525	47.813
Investor 4 ticks PV %	32.748	34.740	32.635	34.021	37.755
HFT %	-	28.979	50.596	58.298	65.110
Execution Times					
Slow speculator	6.546	5.053	3.968	2.461	-
Investor 2 ticks PV	5.100	4.917	4.141	3.400	1.732
Investor 4 ticks PV	3.747	3.448	2.935	2.544	2.409
HFT	-	2.631	3.448	4.227	4.400

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for five different parameterizations in which the share of HFTs among speculators is progressively increased from 0% to 100%. Overall market activity and the total share of speculators in the trader population are held constant at 30%. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

Table 4: Impact of HFT on market quality, welfare, and strategies when adding HFTs to the trader population under asymmetric information.

Share of HFTs in the Market	15%	17.5%	20%	22.5%	25%
Market Quality					
Best Bid/Offer size	1.594	1.835	1.946	2.281	2.495
Depth off BBO	3.732	5.003	5.684	6.665	7.150
Quoted spread	1.844	1.779	1.756	1.699	1.625
Effective spread	1.530	1.439	1.356	1.332	1.261
Price discovery $\lvert p_t - v_t \rvert / v_t$ %	1.376	1.310	1.266	1.239	1.194
Microstructure noise volatility	0.084	0.081	0.082	0.081	0.080
Welfare					
Average welfare	0.250	0.242	0.235	0.228	0.221
Slow speculator	0.051	0.033	0.013	-0.001	-0.013
Investor 2 ticks PV	0.217	0.223	0.228	0.229	0.229
Investor 4 ticks PV	0.434	0.436	0.437	0.442	0.447
HFT	0.101	0.094	0.088	0.080	0.073
Trader Strategies					
Slow speculator					
Buy market order %	0.154	0.120	0.125	0.115	0.134
Aggressive buy limit order %	3.525	2.551	2.409	1.724	1.529
At quote buy limit order %	10.045	9.060	8.889	9.578	10.478
Below best buy limit order %	34.896	38.626	38.139	38.588	39.642
Investor 2 ticks private valuation					
Buy market order %	5.420	5.763	6.044	6.217	6.653
Aggressive buy limit order %	12.440	16.459	18.500	17.078	16.379
At quote buy limit order %	41.317	30.980	22.593	23.842	23.337
Below best buy limit order %	33.382	37.852	41.395	44.093	46.961
Investor 4 ticks private valuation					
Buy market order %	18.543	16.263	14.509	13.232	11.566
Aggressive buy limit order %	22.554	21.351	20.549	20.428	18.278
At quote buy limit order %	40.609	46.282	48.886	39.764	33.156
Below best buy limit order %	15.667	15.667	14.399	18.110	24.482
High-frequency trader					
Buy market order %	0.078	0.056	0.044	0.039	0.031
Aggressive buy limit order %	2.484	1.987	1.824	1.200	1.044
At quote buy limit order %	11.290	11.721	10.156	11.070	11.718
Below best buy limit order %	34.209	36.410	37.674	38.794	37.973
Order cancellation ratio %	77.457	75.674	71.732	66.213	62.043
Transactions					
Share of Limit Orders					
Slow speculator %	73.135	73.036	71.930	71.937	70.757
Investor 2 ticks PV %	54.129	49.471	45.203	43.629	41.855
Investor 4 ticks PV %	34.551	38.871	42.138	41.716	40.451
HFT %	46.788	50.580	54.623	57.075	60.599
Execution Times					
Slow speculator	3.818	2.598	2.057	2.040	2.007
Investor 2 ticks PV	3.980	2.567	1.758	1.601	1.650
Investor 4 ticks PV	2.978	2.467	2.575	2.204	1.996
HFT	3.170	2.946	2.608	2.923	2.969

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for five different parameterizations in which the share of HFTs in the overall trader population is progressively increased from 15% to 25% by gradually adding HFTs. Overall market activity increases in this scenario. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

Table 5: Impact of regulations on market quality, welfare, and strategies under symmetric information.

Setting	Base case	Minimum resting time 1.0s	Cancellation fee $0.015	Transaction tax 0.1%	Rebates $0.01/-$0.01	Speed bump 1.0s
Market Quality						
Best Bid/Offer size	1.976	1.964	1.460	2.042	1.868	4.592
Depth off BBO	5.822	5.292	2.466	5.998	4.684	12.862
Quoted spread	1.615	1.609	1.616	1.592	1.557	1.265
Effective spread	1.176	1.303	1.428	1.172	1.275	1.065
Price discovery $\|p_t - v_t\|/v_t$ %	1.289	1.358	1.360	1.282	1.341	1.337
Microstructure noise volatility	0.080	0.082	0.082	0.080	0.081	0.083
Welfare						
Average welfare	0.239	0.234	0.219	0.237	0.234	0.233
Average welfare + proceeds	0.239	0.234	0.222	0.239	0.234	0.233
Slow speculator	0.050	0.057	0.042	0.047	0.056	0.034
Investor 2 ticks PV	0.219	0.214	0.206	0.218	0.213	0.222
Investor 4 ticks PV	0.435	0.431	0.426	0.433	0.427	0.434
HFT	0.096	0.089	0.083	0.094	0.095	0.089
Trader Strategies						
Slow speculator						
Buy market order %	0.064	0.102	0.128	0.063	0.101	0.241
Aggressive buy limit order %	1.454	0.705	0.648	1.450	1.172	0.179
At quote buy limit order %	10.779	11.699	8.197	11.137	12.739	10.388
Investor 2 ticks private valuation						
Buy market order %	5.420	5.803	5.136	5.471	5.642	7.194
Aggressive buy limit order %	13.777	10.080	6.201	14.618	9.511	7.068
At quote buy limit order %	39.156	49.321	52.062	35.261	48.603	55.778
Investor 4 ticks private valuation						
Buy market order %	16.951	20.645	17.439	17.320	17.786	30.567
Aggressive buy limit order %	17.898	17.415	12.743	19.707	15.932	14.481
At quote buy limit order %	53.357	50.763	65.568	48.191	56.785	41.407
High-frequency trader						
Buy market order %	0.047	0.073	0.087	0.047	0.066	0.082
Aggressive buy limit order %	1.106	0.294	0.236	1.181	0.975	0.127
At quote buy limit order %	11.309	16.212	16.050	10.920	12.197	13.444
Order cancellation ratio %	72.403	7.892	1.127	73.185	74.833	29.026
Transactions						
Share of Limit Orders						
Slow speculator %	81.265	75.936	73.510	81.856	78.869	87.530
Investor 2 ticks PV %	51.537	54.144	53.195	51.461	53.860	51.243
Investor 4 ticks PV %	33.865	29.475	35.975	34.024	33.671	17.593
HFT %	47.012	54.069	45.960	46.120	43.242	74.078
Execution Times						
Slow speculator	4.939	7.672	21.359	4.936	6.648	9.533
Investor 2 ticks PV	3.649	4.962	6.869	3.387	5.286	4.769
Investor 4 ticks PV	3.278	3.854	4.841	3.302	4.448	3.397
HFT	3.746	9.211	14.867	3.452	4.650	10.614

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for six different parameterizations. The base case is a situation where 30% of traders are speculators, half of which are HFTs, and HFTs do not have an information advantage over other traders. The five other cases introduce a minimum resting time, cancellation fees, transaction taxes, rebate fees and speed bumps of a size reported in the respective column headers. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

Table 6: Impact of regulations on market quality, welfare, and strategies under asymmetric information.

Setting	Base case	Minimum resting time 1.0s	Cancellation fee \$0.015	Transaction tax 0.1%	Rebates \$0.01/-\$0.01	Speed bump 1.0s
Market Quality						
Best Bid/Offer size	1.596	1.570	1.545	1.633	1.547	2.205
Depth off BBO	3.865	3.687	2.601	3.953	3.435	4.895
Quoted spread	1.844	1.670	1.692	1.807	2.005	1.339
Effective spread	1.530	1.463	1.542	1.453	1.939	1.332
Price discovery $\|p_t - v_t\|/v_t$ %	1.376	1.440	1.419	1.381	1.522	1.395
Microstructure noise volatility	0.084	0.085	0.084	0.083	0.087	0.086
Welfare						
Average welfare	0.239	0.233	0.221	0.237	0.236	0.237
Average welfare + proceeds	0.239	0.233	0.224	0.239	0.236	0.237
Slow speculator	0.051	0.058	0.049	0.048	0.064	0.039
Investor 2 ticks PV	0.217	0.213	0.205	0.215	0.212	0.218
Investor 4 ticks PV	0.434	0.430	0.424	0.431	0.425	0.431
HFT	0.101	0.092	0.083	0.099	0.103	0.102
Trader Strategies						
Slow speculator						
Buy market order %	0.154	0.170	0.135	0.143	0.223	0.397
Aggressive buy limit order %	3.525	1.272	0.608	3.358	3.548	0.913
At quote buy limit order %	10.045	11.798	11.315	10.412	10.537	12.191
Investor 2 ticks private valuation						
Buy market order %	5.420	5.325	4.870	5.272	4.924	6.180
Aggressive buy limit order %	12.440	10.077	6.651	11.862	9.917	7.265
At quote buy limit order %	41.317	50.117	56.761	42.962	48.141	53.765
Investor 4 ticks private valuation						
Buy market order %	18.543	19.115	17.107	17.638	15.757	37.369
Aggressive buy limit order %	22.554	18.086	13.076	21.552	17.933	10.934
At quote buy limit order %	40.609	52.817	64.357	45.604	55.863	32.865
High-frequency trader						
Buy market order %	0.078	0.097	0.083	0.071	0.111	0.160
Aggressive buy limit order %	2.484	0.434	0.208	2.553	2.360	0.460
At quote buy limit order %	11.290	18.273	16.747	11.253	12.974	14.485
Order cancellation ratio %	77.457	7.975	1.195	79.108	77.173	64.577
Transactions						
Share of Limit Orders						
Slow speculator %	73.135	70.136	69.571	73.218	62.305	79.815
Investor 2 ticks PV %	54.129	55.252	54.852	53.519	56.627	56.939
Investor 4 ticks PV %	34.551	31.122	35.422	35.261	42.767	13.417
HFT %	46.788	53.369	46.644	46.803	34.623	74.599
Execution Times						
Slow speculator	3.968	12.585	21.244	4.936	4.055	10.144
Investor 2 ticks PV	4.141	6.716	9.108	3.387	8.908	3.919
Investor 4 ticks PV	2.978	3.376	4.393	3.211	3.015	3.488
HFT	3.170	8.806	14.522	3.075	3.786	6.228

This table presents the market quality indicators, welfare, trader strategies and transactions from model simulations performed for six different parameterizations. The base case is a situation where 30% of traders are speculators, half of which are HFTs. The five other cases introduce a minimum resting time, cancellation fees, transaction taxes, rebate fees and speed bumps of a size reported in the respective column headers. Throughout, HFTs have an information advantage over other traders, which only observe the fundamental value with a lag of two time units. To better compare strategies across trader groups, we report strategies for positive private valuation investors. Strategies for negative private value traders are symmetric.

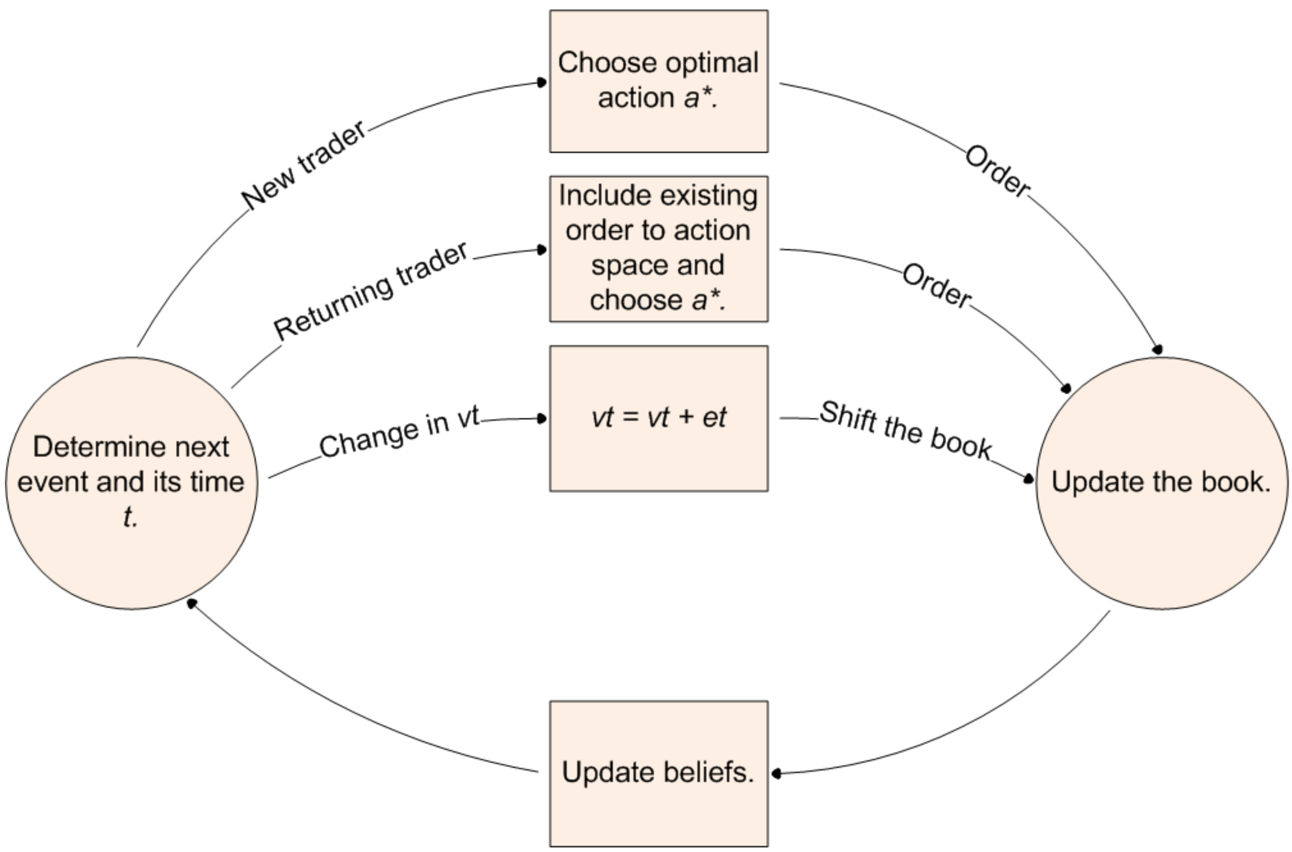

Figure 1: Workflow of the game.

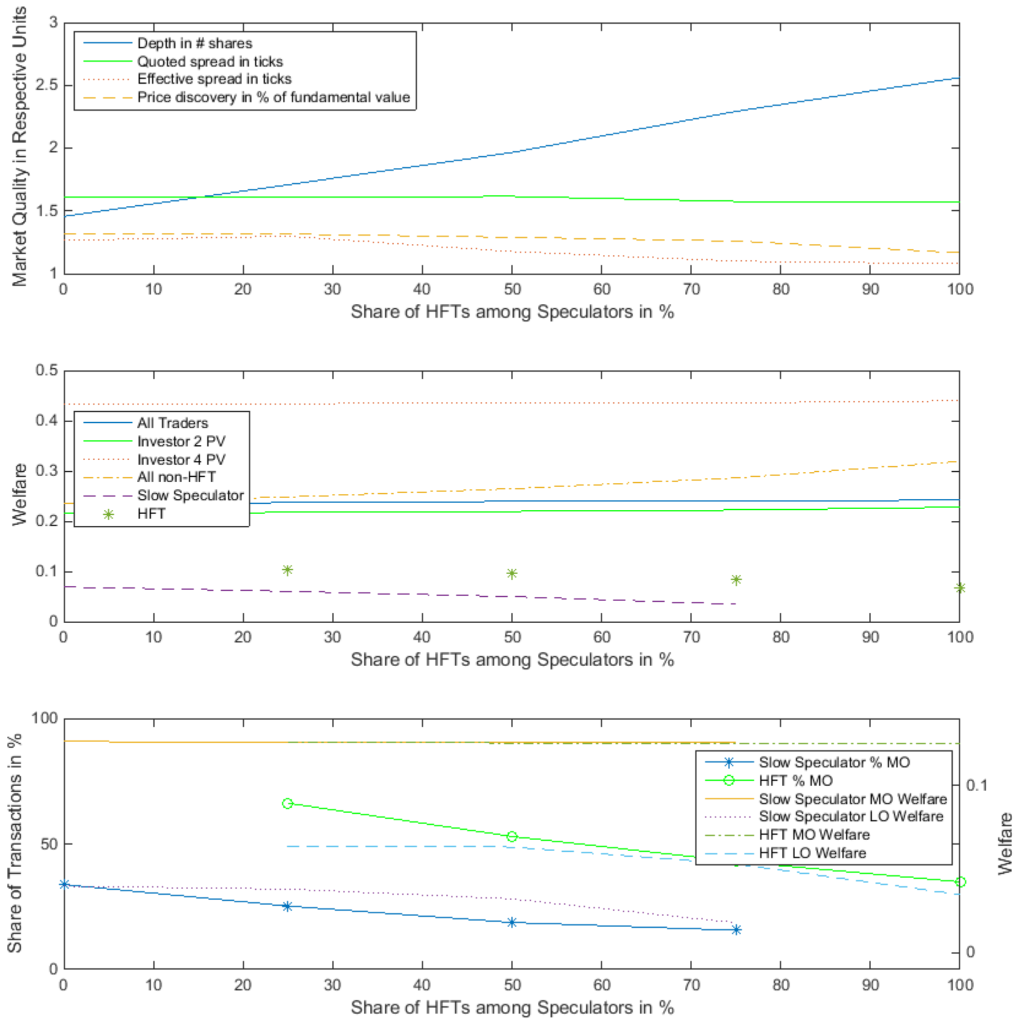

Figure 2: Impact of HFT on market quality, welfare, and strategies under symmetric information when the overall trader population is constant.

This figure presents the market quality indicators, welfare, and trader strategies from model simulations performed for five different parameterizations in which the share of HFTs among speculators is progressively increased from 0% to 100%. Overall market activity and the total share of speculators in the trader population are held constant at 30%.
The top panel reports the different market quality measures as a function of the share of speculators in the market that are HFTs. The middle panel reports welfare for the different groups and the average welfare of all traders in the market. The bottom panel reports the share of market orders in slow speculators' and HFTs' transactions (left axis) and their welfare from market orders and limit orders (right axis).

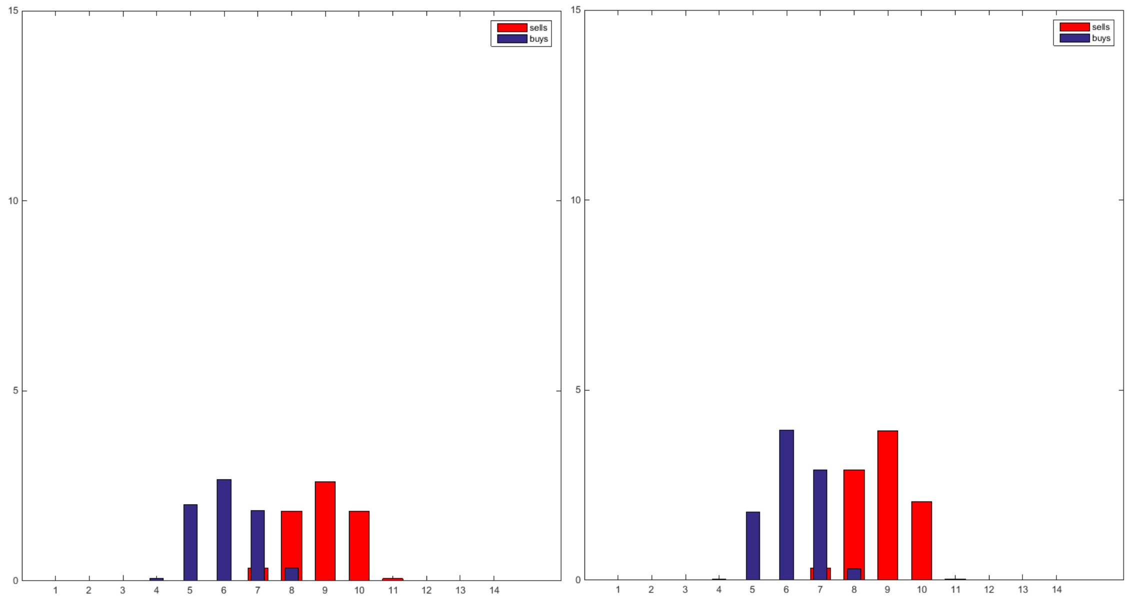

Figure 3: Average composition of the limit order book from model simulations performed for two different parameterizations in which the share of HFTs among speculators is increased from 25% (left) to 75% (right). Overall market activity and the total share of speculators in the trader population are held constant at 30%.

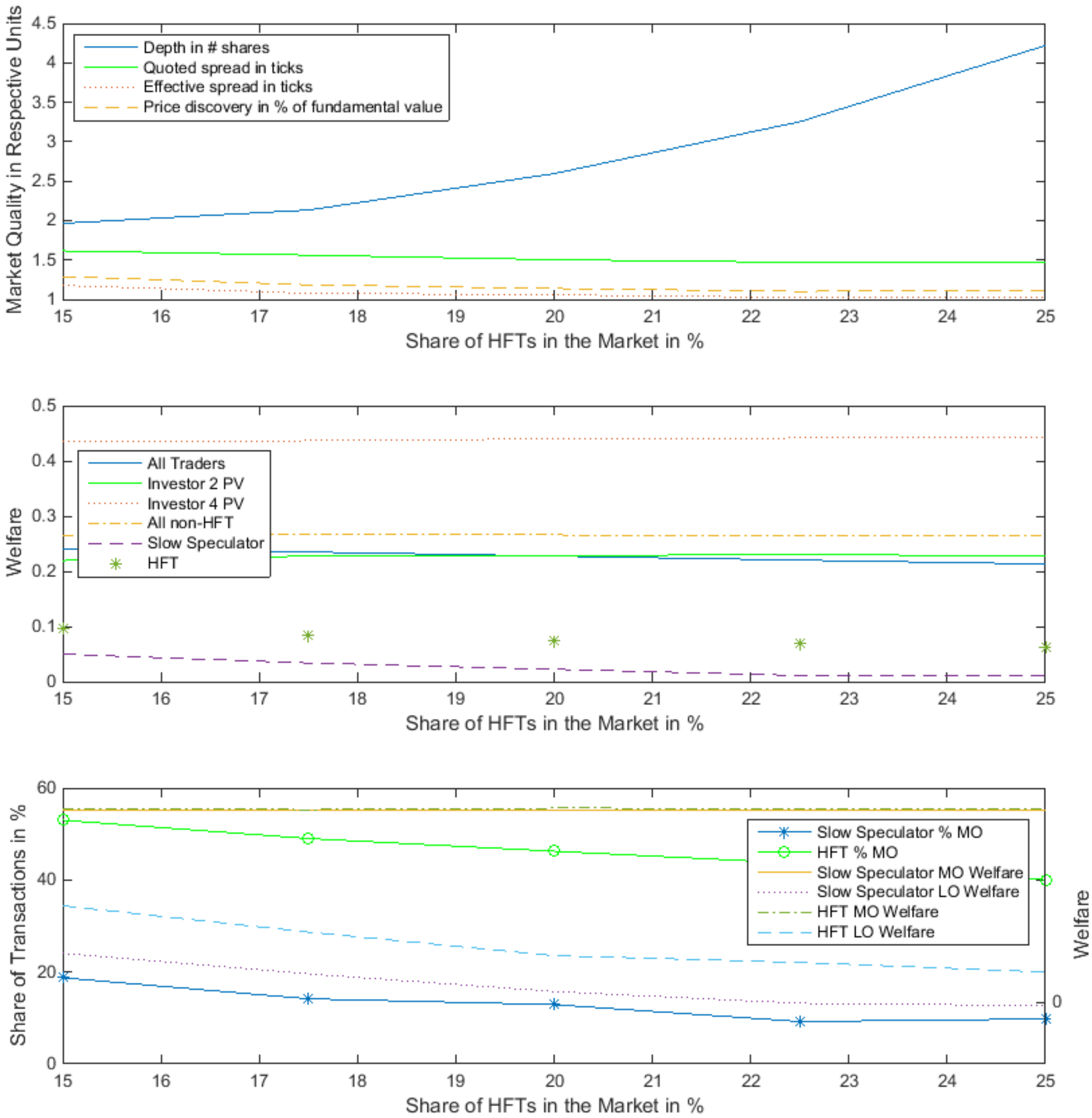

Figure 4: Impact of HFT on market quality, welfare, and strategies under symmetric information when adding HFTs to the trader population.

This figure presents the market quality indicators, welfare, and trader strategies from model simulations performed for five different parameterizations in which the share of HFTs in the overall trader population is progressively increased from 15% to 25% by gradually adding HFTs. Overall market activity increases in this scenario.

The top panel reports the different market quality measures as a function of the share of traders in the market that are HFTs. The middle panel reports welfare for the different groups and the average welfare of all traders in the market. The bottom panel reports the share of market orders in slow speculators' and HFTs' transactions (left axis) and their welfare from market orders and limit orders (right axis).

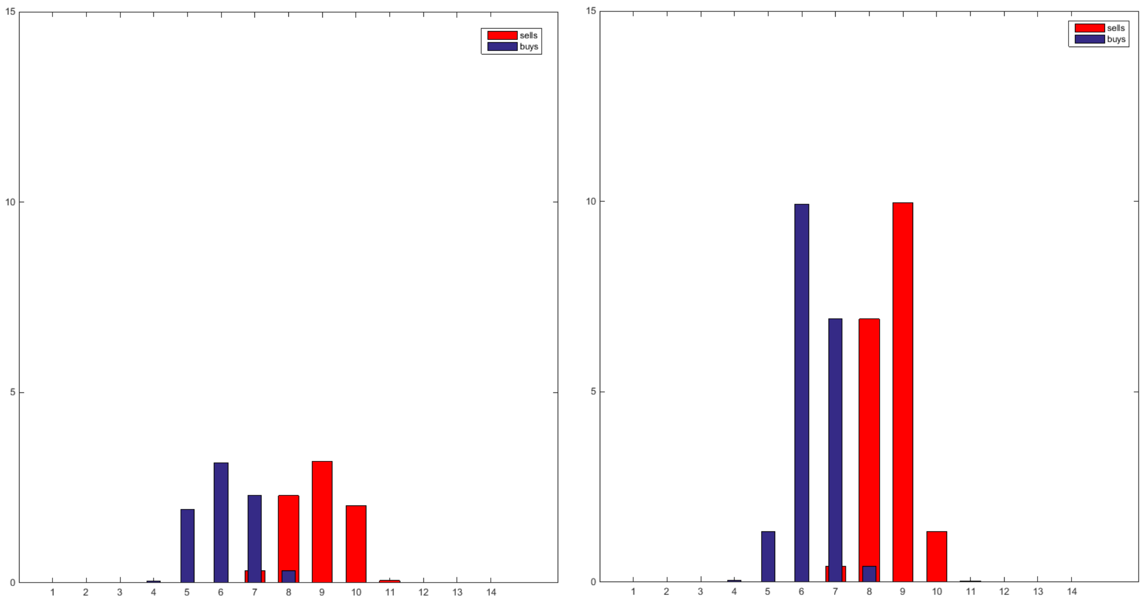

Figure 5: Average composition of the limit order book from model simulations performed for two different parameterizations in which the share of HFTs in the overall trader population is increased from 15% (left) to 25% (right) by adding HFTs. Overall market activity increases in this scenario.

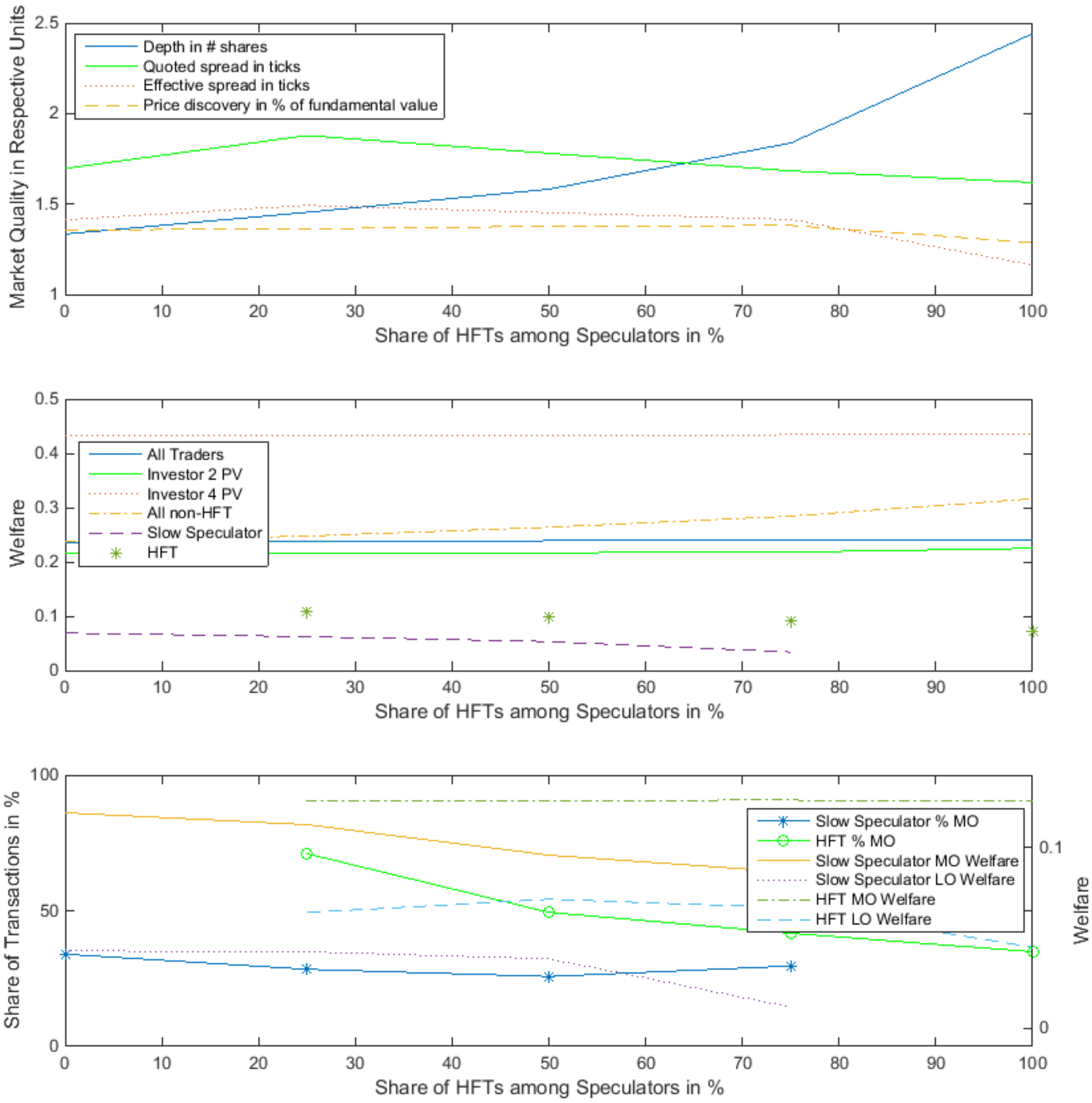

Figure 6: Impact of HFT on market quality, welfare, and strategies under asymmetric information when the overall trader population is constant.

This figure presents the market quality indicators, welfare, and trader strategies from model simulations performed for five different parameterizations in which the share of HFTs among speculators is progressively increased from 0% to 100%. Overall market activity and the total share of speculators in the trader population are held constant at 30%.
The top panel reports the different market quality measures as a function of the share of speculators in the market that are HFTs. The middle panel reports welfare for the different groups and the average welfare of all traders in the market. The bottom panel reports the share of market orders in slow speculators' and HFTs' transactions (left axis) and their welfare from market orders and limit orders (right axis).

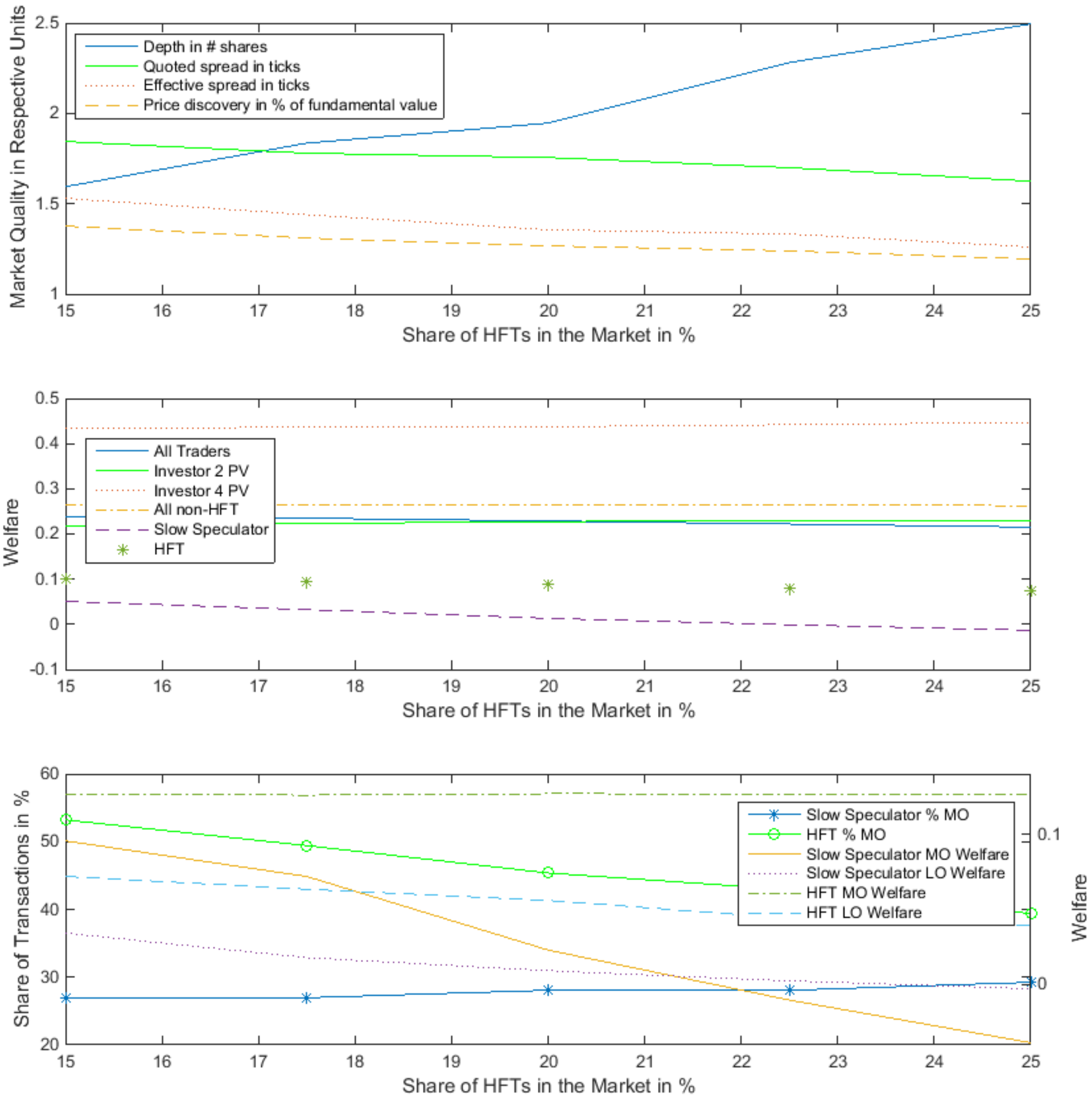

Figure 7: Impact of HFT on market quality, welfare, and strategies under asymmetric information when adding HFTs to the trader population.

This figure presents the market quality indicators, welfare, and trader strategies from model simulations performed for five different parameterizations in which the share of HFTs in the overall trader population is progressively increased from 15% to 25% by gradually adding HFTs. Overall market activity increases in this scenario.

The top panel reports the different market quality measures as a function of the share of traders in the market that are HFTs. The middle panel reports welfare for the different groups and the average welfare of all traders in the market. The bottom panel reports the share of market orders in slow speculators' and HFTs' transactions (left axis) and their welfare from market orders and limit orders (right axis).

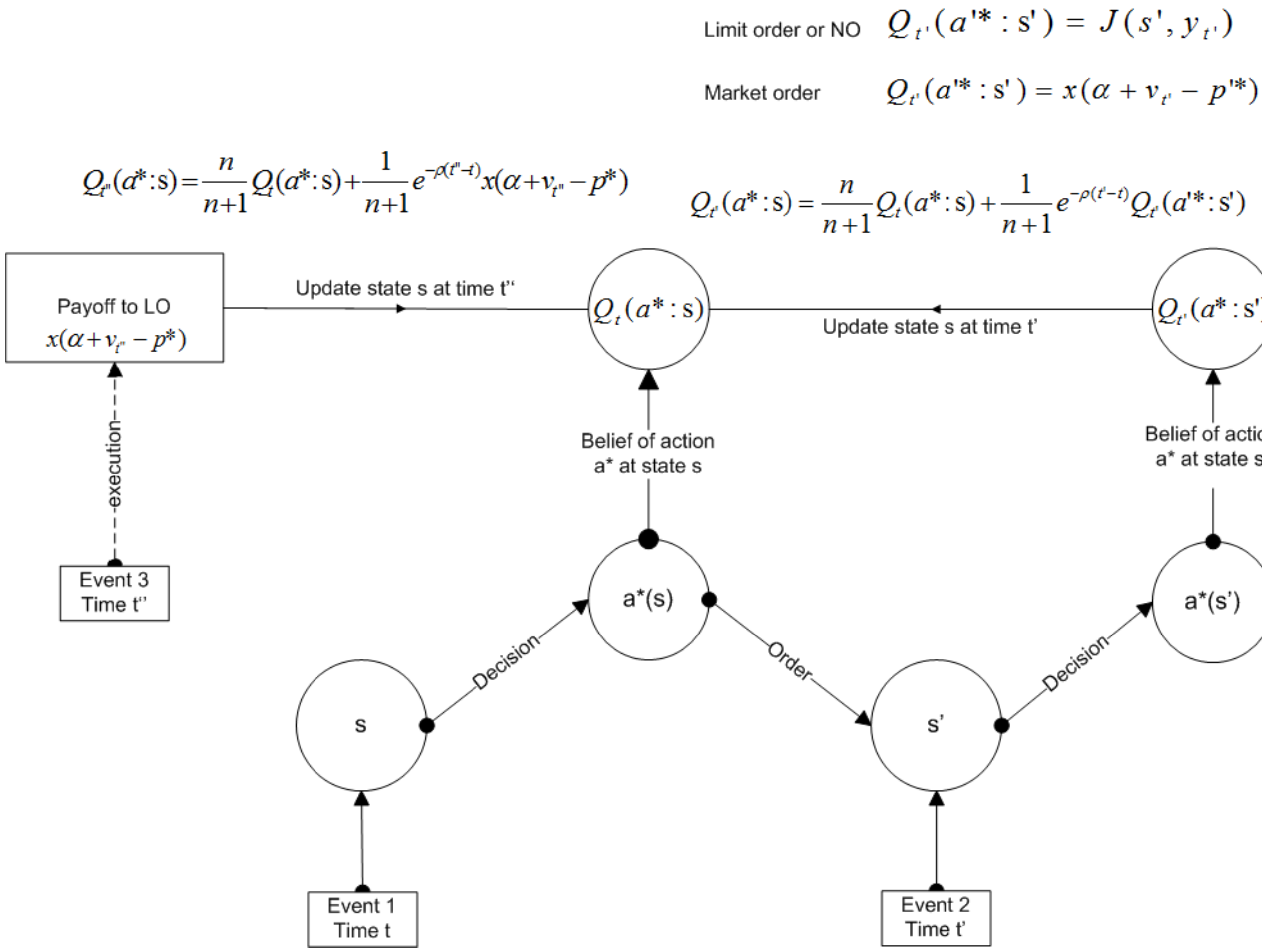

Figure 8: Updates of beliefs about payoffs for action a taken at state s using Q-learning.