## Order book approach to price impact

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Buying and selling stocks causes price changes, which are described by the price impact function. To explain the shape of this function, we study the Island ECN orderbook. In addition to transaction data, the orderbook contains information about potential supply and demand for a stock. The virtual price impact calculated from this information is four times stronger than the actual one and explains it only partially. However, we find a strong anticorrelation between price changes and order flow, which strongly reduces the virtual price impact and provides for a quantitative explanation of the empirical price impact function.

In a perfectly efficient market, stock prices change due to the arrival of new information about the underlying company. From a mechanistic point of view, stock prices change if there is an imbalance between buy and sell orders for a stock. These ideas can be linked by assuming that someone who trades a large number of stocks might have private information about the underlying company, and that an imbalance between supply and demand transmits this information to the market. In this sense, order imbalance and stock price changes should be connected causally, i.e. prices go up if demand exceeds supply and go down if supply exceeds demand. The analysis of huge financial data sets [1] allows a detailed study of the price impact function [2–12], which quantifies the relation between order imbalance and price changes.

Potential supply and demand for a stock is stored in the limit order book. If a trader is willing to sell a certain volume (number of shares) of a stock at a given or higher price, she places a limit sell order. For buying at a given or lower price, a limit buy order is placed. An impatient trader who wants to buy immediately places a market buy order, which is matched with the limit sell orders offering the stock for the lowest price, the ask price  $S_{\rm ask}$  for that stock. Similarly, a market sell order is matched with the limit buy orders offering the highest price, the bid price  $S_{\rm bid}$ .

In previous studies [1–10, 12] (with the exception of [11]), the price impact of trades was calculated by determining whether a given trade was buyer or seller initiated [13]. Here, we analyze order book data which unambiguously allow to identify the character of a transaction. We first calculate the price impact of market orders, which are aggregated in time intervals of length  $\Delta t = 5 \text{min}$ , and compare it to the virtual or instantaneous price impact, which would be caused by a market order matched with limit orders from the order book. The virtual price impact is found to be four times stronger than the actual one. To explain this surprising discrepancy, we study time dependent correlations between order flow and returns and find that limit orders are anticorrelated with returns in contrast to the positive correlations between returns and market orders. We suggest that limit orders placed in response to returns provide for a quantitative link between virtual and actual price impact.

We analyzed data from the Island ECN, NASDAQ's largest electronic communication network, which comprises about 20 percent of all trades. We chose the 10 most frequently traded stocks for the year 2002 [14]. The volume of market buy orders is counted as positive and the volume of market sell orders as negative, and the sum of all signed market orders placed in the time interval  $[t, t + \Delta t]$  with  $\Delta t = 5 \text{min}$  is denoted by Q(t) [15]. Stock price changes are measured by the return G(t) in the same time interval as

$$G(t) = \ln S_M(t + \Delta t) - \ln S_M(t), \tag{1}$$

where the midquote price  $S_M(t) = \frac{1}{2}(S_{\text{bid}}(t) + S_{\text{ask}}(t))$  is the arithmetic mean of bid and ask price. To make different stocks comparable, we normalize the return time series G by their standard deviation  $\sigma_G$  and the volume time series Q by  $\sigma_Q = \langle |Q - \langle Q \rangle| \rangle$  as their second moment is not well defined due to a slow decay of the probability distribution.

**Price impact of market orders:** We define the price impact of market orders as the conditional expectation value

$$I_{\text{market}}(Q) = \langle G_{\Delta t}(t) \rangle_Q$$
 (2)

for overlapping time intervals of market order flow and returns. The functional form of  $I_{\text{market}}(Q)$  is shown in Fig. 1. We find that  $I_{\text{market}}(Q)$  is a concave function of volume [2], which can be well fitted by a power law  $G = 0.48 \ Q^{0.76}$  with  $R^2 = 0.997$ . We note that the exponent 0.76 as compared to 0.5 found in [5, 9, 16] is due to the fact that we compute returns for midquote prices as compared to returns for transaction prices. The concave shape of the function is very surprising: This type of price impact would theoretically be an incentive to make large trades as they would be less costly. In contrast, a convex price impact would encourage a trader to brake up a large trade into several smaller ones, which is what actually happens. Having this in mind, we want to understand the mechanism responsible for this concave shape and analyze the trading information contained in the limit order book.

Order book and virtual price impact: At each instant in time and for each stock i, the limit order book

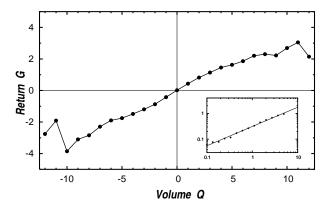


FIG. 1: The price impact function  $I_{\mathrm{market}}(Q)$  for market orders is a monotonously increasing and concave function of the signed market order volume. A logarithmic plot (inset) shows that the function can be fitted by a power law.

can be described by a density function  $\rho_i(\gamma, t)$  for the number of limit orders where

$$\gamma = \begin{cases} (\ln(S_{\text{limit}}) - \ln(S_{\text{bid}})) & \text{limit buy order} \\ (\ln(S_{\text{limit}}) - \ln(S_{\text{ask}})) & \text{limit sell order} \end{cases} . (3)$$

We reconstructed the time dependent density functions for all ten stocks from information about placement, cancellation, and execution of limit orders contained in the Island ECN data base, thereby processing about 60GB of data.

First, we study the average order book  $\rho_{\text{book}}(\gamma) = \langle \rho_i(\gamma,t) \rangle$ , where  $\langle ... \rangle$  denotes an average over both time and different stocks. It is characterized by a flat maximum at  $\gamma \approx 1$  and a slow decay for large  $\gamma$  (Fig. 2a). Its overall shape agrees with the results of [17–19]. We note that we have approximated  $\rho_{\text{book}}(G)$  on a grid with spacing  $0.3 \sigma_G$ .

Consider a trader who wants to buy a volume Q of stocks and has only offers from the order book available. Beginning at the ask price, she executes as many limit orders as necessary to match her market order, and changes the ask price by an amount of G. Traded volume Q and return G are related by

$$Q_{\text{book}} = \int_{0}^{G} \rho_{\text{book}}(\gamma) \, d\gamma \quad . \tag{4}$$

The virtual price impact  $I_{\text{book}}(Q)$  is obtained by inverting this relation [20]. We assume that the bid-ask spread remains constant in the process and that the midquote price changes by the same amount as the ask price. The virtual price impact is four times stronger than the price impact of actual market orders (see Fig. 2b), a volume of  $5\sigma_Q$  causes a virtual price change of  $8\sigma_G$  but only an actual price change of  $2\sigma_G$ . In addition,  $I_{\text{book}}(Q)$  is a convex function that can be fitted well by a power law  $G=1.22~Q_{\text{book}}^{1.19}$  with an  $R^2=0.998$  and not a concave function as  $I_{\text{market}}(Q)$ . The average order book and thus the virtual price impact can be

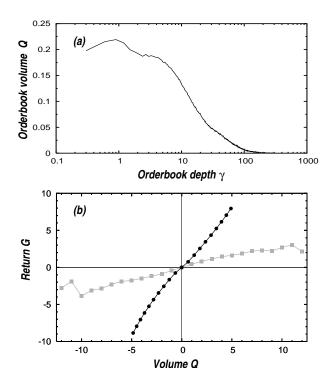


FIG. 2: (a) The average order book is characterized by a maximum at  $\gamma=1$  and a slow decay up to  $\gamma=100$ . The negative side (buy orders) of the order book is similar to the positive one. (b) The virtual price impact function  $I_{\text{book}}(Q)$  (circles) calculated from the average limit order book is a convex function of order volume and much steeper than the price impact of market orders (squares).

decsribed by "zero intelligence models" [19, 21], in which orders are placed randomly. As the virtual price impact is not a good approximation for the actual one, it seems that an additional mechanism describing "intelligent" or collective behavior is needed to explain it.

## Correlations between order flow and returns:

Which effect is responsible for the pronounced difference between virtual and actual price impact? In the following, we will argue that a strong anticorrelation between returns and limit orders reduces the virtual price impact and provides for the link between virtual and actual price impact. In order to understand how order flow and price changes are related, we study the correlation functions

$$c_{\alpha}(\tau) = \frac{\langle Q_{\alpha}(t+\tau)G(t)\rangle - \langle Q_{\alpha}(t)\rangle\langle G(t)\rangle}{\sigma_{Q_{\alpha}}\sigma_{G}}$$
 (5)

between the volume of market orders ( $\alpha = \text{market}$ ) or limit orders ( $\alpha = \text{limit}$ ) and returns. The order volume is measured in intervals  $[t,t+\delta t]$  with width  $\delta t = 50s$ , and the returns are recorded for five minute intervals. For  $\alpha = \text{market}$ ,  $Q_{\text{market}}(t)$  is the volume of signed market orders, and for  $\alpha = \text{limit}$ 

$$Q_{\text{limit}}(t) = \int_{-\infty}^{\infty} \text{sign}(\gamma) \left( Q_{\delta t}^{\text{add}}(\gamma) - Q_{\delta t}^{\text{canc}}(\gamma) \right) d\gamma \qquad (6)$$

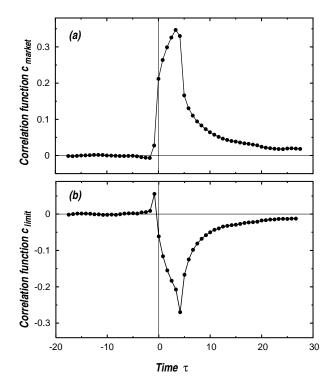


FIG. 3: Correlation functions between return and signed order flow (buy minus sell orders). (a) Market orders and returns show strong positive equal time correlations decaying slowly to zero. (b) Limit orders preceding returns have weak positive correlations with them, while equal time correlations are strongly negative.

is the net volume of limit sell orders minus the net volume of limit buy orders. In Eq. 6,  $Q_{\delta t}^{\rm add}(\gamma)$  is the volume of limit orders added to the book at a depth  $\gamma$ , and  $Q_{\delta t}^{\rm canc}(\gamma)$  is the volume of orders canceled from the book.

The correlation functions are plotted in Fig. 3. We find that  $c_{\rm market}(\tau)$  is zero for  $\tau < -50s$  as required for an efficient market where returns cannot be predicted over extended periods of time. For times  $\tau \geq -50s$ , we find positive correlations which are strongest when the time intervals for orders and returns overlap. For  $\tau > 250s$  (non overlapping time intervals), we observe a slow decay of the correlation function which is probably caused by the strong autocorrelations of the market order flow [2, 11, 22].

The correlation function between limit orders and returns vanishes for negative times  $\tau < -50s$  and has a small positive value  $c_{\text{limit}}(-50s) = 0.04$ . Surprisingly, for zero and positive time differences there is a significant anticorrelation between limit orders and returns, which is strongest for  $\tau = 250s$  (overlapping time intervals) and decays slowly to zero for large positive times. We interpret this anticorrelation as an indication that rising prices cause an increased number of sell limit orders and vice versa for falling prices. Price changes seem to be counteracted by an orchestrated flow of limit orders.

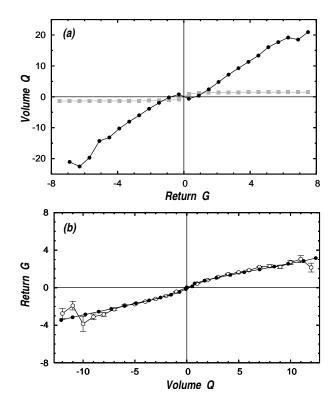


FIG. 4: (a) The average flow of limit orders integrated up to an order book depth G (squares) changes rapidly at small returns and stays constant then. The additional volume of limit orders  $Q_{\rm corr}$  in response to a return G (circles) increases linearly for large G. (b) Empirical price impact  $I_{\rm market}(Q)$  of market orders (open circles) compared to the theoretical price impact  $I_{\rm theory}(Q)$  (full circles), which takes into account orders from the order book, the average flow of limit orders and the additional flow  $Q_{\rm corr}$ .

Limit order flow and feedback mechanism: The anticorrelation between returns and the flow of limit orders suggests that dynamical effects are responsible for the difference between virtual and actual price impact. First, one should take into account the average order flow. The average density of this order flow is described by

$$\rho_{\text{flow}}(\gamma) = \langle Q_{\Delta t}^{\text{add}}(\gamma) - Q_{\Delta t}^{\text{canc}}(\gamma) \rangle \tag{7}$$

with  $\Delta t=5$ min. Near the ask price, the net volume of incoming limit orders is five times larger than the volume stored in the average order book. More than one  $\sigma_G$  away from the bid and ask price, the order flow decreases rapidly. Integration of the order flow density up to a given return G contributes the additional volume  $Q_{\text{flow}}(G)=\int_0^G \rho_{\text{flow}}(\gamma)\ d\gamma$ , which is displayed in Fig. 4a. It grows fast for small returns and saturates for larger returns.

Furthermore, there is an additional volume of incoming limit orders generated by the returns G due to the anticorrelation between returns and limit orders. The density of these additional orders is described by the conditional expectation value

$$\rho_{\mathrm{c}}(\gamma,G) \!=\! \left\langle Q_{t_0}^{\mathrm{add}}(\gamma) \!-\! Q_{t_0}^{\mathrm{canc}}(\gamma) \right\rangle_{G} \!-\! \left\langle Q_{t_0}^{\mathrm{add}}(\gamma) \!-\! Q_{t_0}^{\mathrm{canc}}(\gamma) \right\rangle. \tag{8}$$

Here,  $Q_{t_0}^{\mathrm{add}}(\gamma)$  is the number of limit orders added to the book at a depth  $\gamma$  within the time interval  $[t, t + t_0]$ . We find that  $\rho_{\mathrm{c}}(\gamma, G)$  approximately saturates for  $t_0 \geq 30 \mathrm{min}$ .

We consider a situation with "stationary price changes" by assuming that  $G(t) \equiv G$  is constant in time. Then, the choice  $t_0 = 30 \text{min}$  makes sure that also the additional limit order volume due to returns in past time intervals is taken into account. The correlation volume corresponding to a return G is

$$Q_{\rm corr}(G) = \int_0^G \rho_{\rm c}(\gamma, G) d\gamma \ . \tag{9}$$

 $Q_{\text{corr}}(G)$  is slightly negative for small G and increases then almost linearly for larger G (see Fig.4a).

The total volume Q(G) corresponding to a return G is the sum

$$Q(G) = Q_{\text{book}}(G) + Q_{\text{flow}}(G) + Q_{\text{corr}}(G)$$
 (10)

of the volume  $Q_{\text{book}}(G)$  of orders stored in the limit order book up to a depth G, the volume  $Q_{\text{flow}}(G)$  arriving within a five minute interval on average, and the correlation volume  $Q_{\text{corr}}(G)$ . The theoretical price impact

function  $I_{\text{theory}}(Q)$  calculated by inverting this relation is shown in Fig. 4b.

The agreement between  $I_{\text{theory}}(Q)$  and  $I_{\text{market}}(Q)$  is excellent, up to  $G = 10\sigma_G$  there are no deviations within the error bars of  $I_{\text{market}}(Q)$ . The additional liquidity due the influx of limit orders correlated with past returns has a very strong influence on the price impact of market orders. It strongly reduces the virtual price impact and is responsible for the empirically observed concave shape of the price impact function. We note that a reduction of "bare" price impact by liquidity providers was recently postulated in [12] in order to reconcile the strong autocorrelations of market orders with the uncorrelated random walk of returns, and that [22] explains the uncorrelated nature of returns by liquidity fluctuations.

In summary, we find that the virtual price impact function as calculated from the average order book is convex and increases much faster than the concave price impact function for market orders. This difference can be explained by taking into account dynamical properties of the order book, i.e. the average net order flow and the strong anticorrelation between returns and limit order flow. This anticorrelation leads to an additional influx of limit orders as a reaction to price changes, which reduces the price impact of market orders. Including these dynamical effects, we quantitatively model the price impact of market orders.

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