Overview.

The simulations start with a single polarization of the circular waveguide TE_{11} mode propagated out through a corrugated horn.

There are two experimental configurations modeled by these simulations. One is a differencing configuration in which orthogonal polarizations of the TE_{11} mode are coupled to a pair of detectors such as waveguide OMTs or antennas in the space opposite the sky side of the waveguide. The difference of these signals is a measurement of the Stokes parameter Q in the coordinate system defined by the orientation of the detectors. This differs from an exact measurement of Q on the sky due only to systematic errors introduced by the telescope optics.

The other configuration involves a half-wave plate (HWP). In this configuration, the polarization is rotated by 90° somewhere between the horn and the secondary mirror. **Check that this occurs in the horn farfield, if this matters. Also, check HWP rotation sign.**

Conventions.

The GRASP convention is that the fields and currents carry time dependence $\mathrm{e}^{\mathrm{j}\omega t}$. I'll use $\mathrm{e}^{\mathrm{i}\omega t}$ throughout. This is the opposite of the most common convention in physics texts, where the phase of a plane wave is usually given by $\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)$. Both modern physics and engineering use the same convention for circular polarization, which is the opposite of the optics convention.

The modern physics convention matches the definition of spin for other particles: positive helicity (right circular polarized) light carries angular momentum with a positive projection along the positive axis of propagation, and for negative helicity (left circular polarized) light the projection of angular momentum is opposite the direction of propagation. This is clearly superior to the optics convention, which switches left and right. Fortunately, GRASP uses the modern physics convention, as I will here. To minimize confusion I'll use only the terms positive and negative helicity.

At a fixed point in space, the electric and magnetic fields of positive helicity light rotate in the right-handed sense with respect to the direction of propagation.

Plotting and orientation.

GRASP can report linear polarization according to the Ludwig-3 definition, which defines two unit vectors tangent to the sphere:

$$\begin{aligned}
\hat{\mathbf{e}}_{\mathrm{co}} &\equiv \hat{\boldsymbol{\theta}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi \\
\hat{\mathbf{e}}_{\mathrm{cx}} &\equiv \hat{\boldsymbol{\theta}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi.
\end{aligned}$$

GRASP also refers to $\hat{\mathbf{e}}_{co}$ as $\mathbf{u}$ and to $\hat{\mathbf{e}}_{cx}$ as $\mathbf{v}$. Near the positive z-axis, with $\theta \approx 0$, $\hat{\mathbf{e}}_{\mathrm{co}} \approx \hat{\mathbf{x}}$ and $\hat{\mathbf{e}}_{\mathrm{cx}} \approx \hat{\mathbf{y}}$. When plotting the electric fields in a grid, GRASP by default chooses right-handed axes labeled U and V, where in the grid coordinate system U corresponds to x and V corresponds to y. This is equivalent to looking at the radiation pattern from the storage grid back toward the radiating feed.

Using these unit vectors, the circular polarization basis vectors are

$$\begin{aligned}
\hat{\mathbf{e}}_{+} = \hat{\mathbf{e}}_{rhc} &= 2^{-1/2}(\hat{\mathbf{e}}_{co} - \mathrm{i}\hat{\mathbf{e}}_{cx}) \\
\hat{\mathbf{e}}_{-} = \hat{\mathbf{e}}_{lhc} &= 2^{-1/2}(\hat{\mathbf{e}}_{co} + \mathrm{i}\hat{\mathbf{e}}_{cx}).
\end{aligned}$$

Because the GRASP time dependence is $\mathrm{e}^{\mathrm{i}\omega t}$, the positive and negative helicity unit vectors are exchanged from those in Jackson, which uses $\mathrm{e}^{-\mathrm{i}\omega t}$.

Power.

When power normalization is turned on, the radiation pattern from each feed is normalized to 4π W, meaning that

$$P_{\text{total}} = \iint_{4\pi} |\mathbf{E}(\theta,\phi)|^2 \mathrm{d}\Omega = 4\pi\,\mathrm{W},$$

where $\bar{E}$ is the far field and the integration is over the entire sphere. A simulation of the far field of an optical system generally produces a storage grid that covers only part of the sphere, often a very small part. GRASP calculates the field at each of the grid points, and separates it into two projections along the given basis vectors tangent to the sphere.

In the far field, the radiated power per solid angle is independent of distance from the source. The electric fields calculated by GRASP have units $\mathrm{W}^{1/2}$, but in this case it seems that the units should be $(\mathrm{W}\ /\ \mathrm{sr})^{1/2}$ so that the integral of $|\mathbf{E}|^2$ over the sphere has units of watts. Assuming that the field is well-sampled, it should be approximately constant over the area of a pixel. If the grid indices are α, β, then

$$P_{\text{grid}} = \sum_{\alpha,\beta} |\mathbf{E}_{\alpha,\beta}|^2 \Delta x_\alpha\, \Delta y_\beta,$$

where x and y represent the coordinates, possibly non-Cartesian, in which the map is computed and the quantity $\Delta x_\alpha\, \Delta y_\beta$ is the area of a pixel in these coordinates. Unless the map is calculated near a pole of the coordinate system, in the flat sky limit all pixels will have the same area $\Delta x \Delta y$. The fractional spillover is thus $1 - P_{\text{grid}}/4\pi\,\mathrm{W}$, which is the fraction of power that radiates to areas of the sphere other than the storage grid or is absorbed in an optical component.

In some orthonormal coordinate system, a Jones matrix $\mathbf{J}$ transforms an electric field $\mathbf{E} = \binom{E_u}{E_v}$ into a different electric field

$$\mathbf{E}' = \mathbf{J}\mathbf{E},$$

so the elements of $\mathbf{J}$ are dimensionless. To characterize an optical instrument we form $n_x \times n_y \times 2 \times 2$ Jones matrix maps $\mathbf{J}(x, y)$ that have the following meaning: the two components of the initial field $\mathbf{E}$ represent the field amplitudes of orthogonal waveguide modes, and an element (x, y) of the Jones matrix gives the resulting far field

$$\mathbf{E}'(x, y) = \mathbf{J}(x, y)\mathbf{E}.$$

By the previous argument, the units of this type of Jones matrix are $\mathrm{sr}^{-1/2}$. A simulation of the far field radiation pattern of a single polarization of a feed mode produces co-pol and cross-pol maps in a particular coordinate system. If the feed pattern for a given polarization is normalized to radiate a total of 4π W over the whole sphere, then this is the total power entering the optical system.

Brad's simulations.

Grid centers.

The focal plane positions of the feeds are given in millimeters in the files `v6/dat_files/HF_xxx.dat`, where `xxx` is the frequency. There are some differences between between the coordinate systems defined in this file and the .tor files used in the simulations: in particular, the file defines coordinate systems `Beam_Coor_Sys_xxx` that depend on `Coor_Sys_Cut_1`, while the .tor files define these coordinate systems relative to `Coor_Sys_Subrefl_1`.

Brad's `make_sky_beam_centers.pro` computes the plate scale $s = 1/f_{\text{eff}}$, where f_{eff} is the effective focal length of the instrument. For the low-frequency instrument at 97 GHz the effective focal length is 2961.05 mm, and for the high-frequency instrument at 150 GHz and 225 GHz it is 2700 mm. **Figure out how to compute f_{eff}.** The plate scale has dimensions $\mathrm{radians}\,\mathrm{length}^{-1}$.

For a feed centered at position (x_n, y_n), with $(0,0)$ at the center of symmetry of the focal plane, the radial distance is $r_n = \sqrt{x_n^2 + y_n^2}$. The zenith angle from boresight is $\theta_n = r_n s$, and the azimuth angle relative to the x-coordinate is $\phi_n = \arctan(y_n/x_n)$, wrapped to $[0, 2\pi)$. Note that this is the correct angle for a right-handed coordinate system.

The values saved in `sky_beam_centers.pro` are converted approximately to angles in degrees using

$$x_n = \tfrac{180^\circ}{\pi}\theta_n \cos\phi_n,$$

and

$$y_n = \tfrac{180^\circ}{\pi}\theta_n \sin\phi_n.$$

These values are converted by `sky_beam_centers_to_dict.py` to dimensionless unit vector components (u, v), where u corresponds to the axis from which ϕ is measured in the right-handed direction.

Time reversal and orientation.

A simulation shows the antenna pattern of a detector as it radiates while an actual instrument absorbs radiation from the sky, so the true detector beam is the time-reversed version of the antenna pattern.