

## Sets: The basics

Sets are the fundamental building block of modern mathematics. Intuitively, a set is a collection of objects, but with two important twists:

1. Sets are unordered.
2. Sets contain no duplicates.

**Example 1** Suppose you want to keep a record of which words occur in a text. You aren't interested in how often a given word occurred, just whether it occurs at all. Nor do you care in which order the words occurred in the text. So you are actually interested in the *set* of words that occur in the text.

Each property is explained in detail below, but let's first put some helpful notation in place.

### List notation

Sets are often written as lists with curly braces around them. So  $\{a, b, c, d\}$  denotes the set containing  $a, b, c, d$ . Here  $a, b, c, d$  are some arbitrary objects. This is known as **list notation**. More complex sets are defined with **set-builder notation**, which will be covered in a later unit.

**Example 2** Consider the string *If John slept, then Mary left*. Its set of words (ignoring sentence-initial capitalization) is  $\{\text{if, John, left, Mary, slept, then}\}$ .

**Exercise 1** Write the following as a set:

- the first names of your three favorite actors/actresses,
- the colors of the rainbow,
- all prime numbers between 1 and 10 (remember, 1 is not a prime number!)

### Elements and set membership

The objects contained in a set are called its **elements** or **members**. One writes  $e \in S$  to indicate that  $e$  is an element of  $S$ . The opposite is denoted  $e \notin S$ :  $e$  is not an element of  $S$ . The symbol  $\in$  thus indicates **set membership**.

**Example 3** Let  $W$  be the set of words in the string *If John slept, then Mary left*. Then it holds that  $\text{left} \in W$  and  $\text{right} \notin W$ . But it is not the case that  $\text{then} \notin W$  or  $\text{awake} \in W$ .

Sometimes  $\ni$  is used as the mirror image of  $\in$ . For example,  $a \in S$  could also be written as  $S \ni a$ .

**Example 4** Continuing the previous example, it is true that  $\text{left} \in W \ni \text{then}$ . That is to say, both  $\text{left} \in W$  and  $\text{then} \in W$  are true.

**Exercise 2** Put  $\in$ ,  $\exists$ ,  $\notin$ ,  $\nexists$  in the gaps below as appropriate:

- $5\_ \{1, 2, 4, 5, 8\}$
- $6\_ \{1, 2, 4, 5, 8\}$
- $\{5\}\_ \{1, 2, 4, 5, 8\}$
- $5\_ \{1, 2, 4, 5, 8\}\_ 6$

### Lack of order

Even though we may write sets in a linear fashion as lists, they have no internal order. The set  $\{a, b\}$  could also be written as  $\{b, a\}$ . So we have  $\{a, b\} = \{b, a\}$ , and  $\{a, b, c\} = \{a, c, b\} = \{b, a, c\} = \{b, c, a\} = \{c, a, b\} = \{c, b, a\}$ .

**Example 5** Consider the strings *If John slept, then Mary left* and *If Mary left, then John slept*. While they are clearly distinct sentences, their sets of words are identical.

**Exercise 3** For each one of the following, fill the gap with  $=$  or  $\neq$  as appropriate:

- $\{a, b\}\_ \{a, b\}$
- $\{b, a\}\_ \{a, b\}$
- $\{b, a, c, d\}\_ \{e, a, b, d\}$

### Lack of duplicates/Idempotency

Sets are **idempotent**, which means that duplicates are ignored. So  $\{a, b\} = \{a, a, b\} = \{a, b, b, a, b, a, b, a, a\}$ . It also holds that  $\{a\} = \{a, a\} = \{a, a, a\}$ , and so on.

**Example 6** Linguists distinguish between **word types** and **word tokens**. The sentence *dogs love dogs* contain two tokens of the type *dogs*, and one token of the type *love*. The sentences *dogs love* and *dogs love dogs* are different with respect to word tokens, but identical with respect to word types. So if you care about word types rather than word tokens, you're dealing with a set because the only thing that matters is which words the text contains, not how many tokens of each word.

**Example 7** Consider the sentence *If police police police, then police police police*. Its set of words (ignoring capitalization) is  $\{\text{if, police, then}\}$ .

**Exercise 4** For each one of the following, fill the gap with  $=$  or  $\neq$  as appropriate:

- $\{a, b\}\_ \{a, a, b, b\}$
- $\{b, a\}\_ \{a, b, a\}$
- $\{c, b, a, a, d, c\}\_ \{a, a, b, d, c, c, c\}$
- $\{a\}\_ \{a, a, a, a, a, a, c, a, a, a, a, a, a\}$

**Exercise 5** The sentence *If police police police, then police police police* actually uses two different word types. It just just so happens that both are pronounced and spelled *police*. But one is the noun *police*, the other one the verb *police*. So we might want to annotate the string as follows: *If*

*police[N] police[V] police[N], then police[N] police[V] police[N].* Assume that words are annotated with their part of speech in this fashion. Then what would be the corresponding set of words?

### Recap

- Sets are collections of arbitrary objects.
- Sets are unordered and idempotent (= duplicates are ignored).
- Sets can be defined with list notation, e.g.  $\{a, b\}$ .
- The objects contained in a set are called its *elements* or *members*.
- The symbols  $\in$  and  $\notin$  are used to indicate membership and non-membership, respectively.
- Occasionally,  $\ni$  is used as the mirror image of  $\in$ .