Prerequisites

- general (big operators)
- sets (cardinality)
- multisets (operations)

Cardinality of multisets

Given a multiset S_M, we write $S_M(a)$ for the count of a in S_M.

Example 1 Let $A_M := \{a : 3, b : 2, c : 1\}$ and $B_M := \{a : 1, b : 1, c : 1, d : 1\}$. Then $A_M(a) = 3$ and $B_M(a) = 1$, whereas $A_M(c) = B_M(c) = 1$.

The **cardinality** of a multiset with finitely many elements is the sum of all its counts. As a formula, this looks as follows:

$$|S_M| := \sum_{s \in S_M} S_M(s)$$

This formula uses the symbol $\sum$ as an abbreviation for addition. The subscript on $\sum$ tells us that s should be substituted in all possible ways by elements of S_M. For multiset $S_M := \{s_1 : c_1, s_2 : c_2, \dots, s_n : c_n\}$ the formula expands to $S_M(s_1) + S_M(s_2) + \cdots + S_M(s_n)$, which in turn reduces to $c_1 + c_2 + \cdots + c_n$.

Example 2 Consider once more the multisets $A_M := \{a : 3, b : 2, c : 1\}$ and $B_M := \{a : 1, b : 1, c : 2, d : 1\}$, for which the following hold:

$$\begin{aligned}
|A_M| &= \sum_{s \in A_M} A_M(s) \\
&= A_M(a) + A_M(b) + A_M(c) \\
&= 3 + 2 + 1 \\
&= 6 \\
|B_M| &= \sum_{s \in B_M} B_M(s) \\
&= B_M(a) + B_M(b) + B_M(c) + B_M(d) \\
&= 1 + 1 + 2 + 1 \\
&= 5 \\
|A_M \cup B_M| &= 8 \\
|A_M \cap B_M| &= 3 \\
|A_M - B_M| &= 3 \\
|B_M - A_M| &= 2
\end{aligned}$$

Exercise 1 Calculate the cardinality of the following multisets:

- $\{a : 17\}$
- $\{\text{John} : 5, \text{Mary} : 5, \text{Bill} : 10\}$

- $\{a : 0, b : 0, c : 0\}$