Range

For many functions it won't be the case that the every value in their domain is actually a possible output of a function. Given a function $f : D \to C$, we use the term **range** to refer to the set of elements of C that are an output for at least one input in D.

Example 1 Consider the function $f : \mathbb{N} \to \mathbb{N}$ with $x \mapsto 2x$. Not every natural number is a possible output of this function:

1. $f(0) = 0$
2. $f(1) = 2$
3. $f(2) = 4$
4. $f(3) = 6$
5. and so on

The range thus does not contain all members of $\mathbb{N}$. Instead, it consists of all even natural numbers, and nothing else.

Example 2 Now suppose that we have $f : \mathbb{R} \to \mathbb{N}$ with $x \mapsto 2x$. For every natural number n, $\frac{n}{/}2$ is a real number and thus an element of $\mathbb{R}$. Hence it must be the case that for every natural number n there is at least one element e in the domain of f such that $f(e) = n$. So this is an example where a function's range is identical to its co-domain.

Exercise 1 For each one of the following functions, describe its range and say whether it is the same as the function's co-domain. Justify your answer. As in many other exercises, getting the correct answer is less important than giving a good argument for you answer.

1. $f : \mathbb{N} \to \mathbb{N}$, $x \mapsto x + 1$
2. $f : \mathbb{N} \to \mathbb{N}$, $x \mapsto x - 1$
3. len $: \Sigma^* \to \mathbb{N}$ with $s \mapsto |s|$ (remember that $|s|$ denotes the length of string s)
4. the child-of kinship relation among humans, limited to women (for instance, child(Sue) = Mary iff Sue is a child of Mary)
5. a benchmark that sorts graphics card models by their speed for neutral network training