

## Summary

This unit discussed monotonicity as an abstract yet very important aspect of natural language. Monotonicity is a mathematical property of functions and breaks down into two distinct notions.

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**Definition 1.** Let  $f : S \rightarrow T$  be a function from  $S$  to  $T$ . Assume that  $\leq_S$  is an ordering of elements of  $S$ , whereas  $\leq_T$  is an ordering of the elements of  $T$ . Then  $f$  is **monotonically increasing** (or **isotonic**) iff  $x \leq_S y$  implies  $f(x) \leq_T f(y)$ .

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**Definition 2.** Let  $f$ ,  $S$ ,  $T$ ,  $\leq_S$ , and  $\leq_T$  be as before. Then  $f$  is **monotonically decreasing** (or **antitonic**) iff  $x \leq_S y$  implies  $f(x) \geq_T f(y)$ .

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Monotonicity is only of interest if the domain and co-domain of a function aren't just flat sets but have additional structure that puts the elements into an order. A monotonically increasing function preserves order, a monotonically decreasing function reverses it. The structure may be a poset, a join semilattice, a meet semilattice, or a lattice.

Various phenomena can be analyzed as involving monotonicity:

- the No-Crossing Branches constraint in Autosegmental Phonology,
- morphological syncretism (adjectival gradation, person pronouns, case), and
- NPI licensing based on left/right downward entailment, and
- the crosslinguistic typology of the Person Case Constraint, and
- the Adjunct island Constraint, and
- the learning of SL grammars, and
- many more that we didn't have time to discuss.

Monotonicity thus seems to permeate all aspects of language, from phonology and morphology to syntax, semantics, and even learning. It is a formal universal that can be combined with substantive universals (e.g. person hierarchies, adjunct algebras) to reduce overgeneration. It adds a new facet to our formal understanding. While a purely computational approach often fails to distinguish the linguistically natural from the unnatural, the combination of monotonicity and substantive universals can often capture the relevant distinction.