Functions: Basic notation

Functions, also called **maps** or **mappings**, are ubiquitous in mathematics. The laymen usually thinks of things like $f(x) = x + 1$ when they hear the word *function*, but the concept is much more general.

Functions convert input arguments to an output

Anything can be thought of as a function as long as it takes a fixed number of **arguments** as its input and converts them to an output. Crucially, the output is not allowed to vary while the input is kept the same.

Example 1 A car wash can be regarded as a function that takes as input a car and returns as its output a clean car (in an ideal world, at least). A dirty Dodge Viper comes out as a clean Dodge Viper, and a clean Audi A4 still comes out as a clean Audi A4. The output is always perfectly predictable from the input.

Example 2 Suppose $f(x)$ can be randomly chosen between $x + 1$ and $2 \times x$. This is not a function because one and the same input can produce different outputs.

Exercise 1 Let f be a function that takes as its input a number n and returns $n + 1$ on a weekday and $n + 2$ on the weekend.

- Is f a function?
- What if f instead takes two arguments: a number n, and the name of the day of the week.

This special property of functions is known as **right uniqueness**. Right uniqueness guarantees that functions are deterministic in the sense that one can predict the output from the input with 100% accuracy.

Caution: The functions used in programming languages are not necessarily functions in the mathematical sense because their output can vary even if the input stays the same.