HOMEWORK — BAYES' THEOREM

Answer all the Questions below.

1. Point Probability 1:

   a. As in the lecture example, lets say we are recording from a monkey and there is some probability that the electrode will spike when the monkey's hand is open. Initial conditions: $p(open) = 0.5$, $p(spike|open) = 0.6$, $p(spike|closed) = 0.35$. You observe 10 spikes in row. List and plot probability that the hand is open for each observed spike

   b. Change your initial prior to 0.01. List and plot probability that the hand is open for each observed spike. What happens and why?

   c. Change your initial prior to 0.0. List and plot probability that the hand is open for each observed spike. What happens and why?

   d. Once again, lets assume $p(open) = 0.5$, $p(spike|open) = 0.6$, $p(spike|closed) = 0.35$. This time, estimate the probability $p(open|no\ spike)$ given you observe ten trials in a row with no spikes. Tip, $p(no\ spike|open)$ is the complement of $p(spike|open)$, and $p(no\ spike|closed)$ is the complement of $p(spike|closed)$.

   e. Again assuming $p(open) = 0.5$, $p(spike|open) = 0.6$, $p(spike|closed) = 0.35$. Lets say that over 10 trials sometimes you observe spikes and other times you do not observe spikes according to: [1,0,0,1,0,1,1,0,1,1], such that $1 = spike$, $0 = no\ spike$. List and plot the probability that the hand is open for each observation

2. Point Probability 2 (**Graduate Only**):

   a. Lets pretend we are recording from a neuron in a mouse, but a label has fallen off one of the electrodes and we don't know which type of neuron the electrode is recording. Luckily, we have some information: a) We know that a Type 1 Neuron will spike at a probability of 90.4% [$p(spike|Type\ 1) = .904$] when the mice moves its right paw, but Type 1 neurons only represent 20% of the neurons we typically record from [$p(Type\ 1) = 0.2$]. b) A Type 2 Neuron will spike at a probability of 75.8% when the mice moves its right paw, and represents 30% of the neurons we typically record. c) Finally, a Type 3 Neuron will spike at a probability of 52.7% when the mice moves its right paw, and represents 50% of the neurons we typically record. The mouse moves its right paw and we observe a spike. What is the probability that it is a Type 1 Neuron [$p(Type1|spike)$]? Tip: for the marginal probability you will need to also consider the remaining possible types.

   b. What is the probability that it is a Type 2 neuron?

   c. What is the probability that it is a Type 3 neuron?

d. Lets say the mouse moves its right paw two more times and each time we observe a spike. List the probability that the electrode is recording Type 1, Type 2, or Type 3 neurons for each of the three movements. Interpret these findings.

3. Continuous Probability 1:

   a. Similar to lecture, lets assume we are lifting a box a single trial ($n = 1$). Looking at the box, you originally assumed the box weighs about 5N but are uncertain about your visual estimate (i.e., there is some amount of σ_0). When you lift the box once, you feel that it weighs about 7.0N with some level of uncertainty ($\sigma_1 = 1.0$). We want to find the posterior, $p(x|\mu, \sigma^2)$, were x is probability of different possible box weights. Use hyperparameters to calculate the posterior. Assume: prior = $\mathcal{N}(5.0, 100^2)$, likelihood = $\mathcal{N}(7.0, 1.0^2)$. Plot the prior, likelihood, and posterior.
   b. This time, lets assume a different prior. Again, use hyperparameters to calculate the posterior. Assume: prior = $\mathcal{N}(5, 1.5^2)$, likelihood = $\mathcal{N}(7.0, 1.0^2)$. Plot the prior, likelihood, and posterior. How does this result differ from the question above and why?
   c. Lets iterate through four trials, **one at a time (n = 1 for each lift)**. Lets use the initial prior = $\mathcal{N}(5.0, 1.5^2)$. Lets assume the box is always the same weight (7N) and we have the same level of uncertainty of its weight on each lift ($\sigma = 1.0$). On each new iteration, update the prior with the newly found posterior. Plot the prior, likelihood, and posterior for each lift. What is the most likely weight of the box after each lift?
   d. Use the same initial prior and likelihood as above (prior = $\mathcal{N}(5.0, 1.5^2)$; likelihood = $\mathcal{N}(7.0, 1.0^2)$), but do not iterate through each trial. Instead, use the hyperparameters to calculate the the posterior after 4 lifts all at once (i.e., $n = 4$). Plot the prior, likelihood, and posterior.

4. Continuous Probability 2 (**Graduate Only**):

   a. Repeat question **3c** above (same initial prior and likelihood), but do it by numerically solving the posterior by discretizing the prior and likelihood along the x-axis. Use 101 data points for each Normal curve and keep the range of the x-axis from 0 to 10. Plot the prior, likelihood, and posterior. What are the advantages of numerical estimates. What are the weaknesses?
   b. Pretend you are running an experiment where a participant is facing several speakers and several lights that are placed in a semicircle ($\pm 90°$) around them. As the experimenter, you can manipulate the average location and noise of auditory and visual feedback by controlling which lights and speakers will turn on. On a particular trial, you control a few lights in a general area to light up, but it is hard to pinpoint exactly where ($\mu_v = -10.0°$, $\sigma_v = 5°$). Simultaneously, you set a few speakers in a general area to produce a tone ($\mu_a = 25.0°$, $\sigma_a = 15°$). The participant has to point towards a single location that best captures the source of the light and sound. Assuming no prior knowledge of the source location, where should they point?