ASSIGNMENT 5 — OPTIMAL FEEDBACK CONTROL

For all questions below, provide all programming code and plots in the report. Please refer to slides for LQG equations.

Initial Conditions and Constants

- Simulation time from 0.0 to 0.5s (51 time steps) using a step size (h) of $0.01s$.
- $m(4.0), b(1.0), k(0.25)$
- $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $R_k = \begin{bmatrix} 0.0000001 \end{bmatrix}$
- $Q_k = \begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, & \text{if } k \neq N \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & \text{if } k = N \end{cases}$
- $v_0 = v_k = \mathcal{N}\left(\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.005 & 0.0 \\ 0.0 & 0.005 \end{bmatrix}\right)$
- $w_0 = w_k = \mathcal{N}\left(\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}\right)$
- $W = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}$
- $V = \begin{bmatrix} 0.005 & 0.0 \\ 0.0 & 0.005 \end{bmatrix}$
- $P_0^{prior} = W$
- $x_0 = \begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix}$
- $y_0 = x_0 + w_0$
- $\hat{x}_0^{post} = y_0$

LQR

1. Convert the following continuous, 2^{nd} order differential equation into 2 coupled ODEs:

   a. $m\ddot{x} = -b\dot{x} - kx + F$, such that $u = F$. 1 mark.

   b. express your answer found in **(a)** into matrix form. 1 mark.

   c. convert the above continuous system found in **(b)** into a discrete system (i.e., A_d, B_d). 1 mark.

2. Calculate the optimal feedback gains (F_k) based on the following (backwards) recursive equations:

   . Initial Condition: $P_{k+1} = Q_N$

   . $F_k = (R_k + B_d^T P_{k+1} B_d)^{-1}(B_d^T P_{k+1} A_d)$

   . $P_k = A_d^T P_{k+1} A_d - (A_d^T P_{k+1} B_d)\left(R_k + B_d^T P_{k+1} B_d\right)^{-1}(B_d^T P_{k+1} A_d) + Q_k$

   a. Plot the optimal feedback gains (F_k). 3 marks.

3. Run an LQR controller **without** noise based on the following equations:

   . Initial conditions: x_0

   . $u_k = -F_k x_k$

   . $x_{k+1} = A_d x_k + B_d u_k$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 3 marks.

4. Run an LQR controller **with** noise based on the following equations:

   . Initial conditions: x_0

   . $u_k = -F_k x_k$

   . $x_{k+1} = A_d x_k + B_d u_k + v_k$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 1 mark.

LQG

5. Compute the Kalman Gain based on the following equations (**Graduate Only**):

   . Initial Condition: $P_0^{prior} = W$

   . $S_k = CP_k^{prior}C^T + W$

   . $K_k = P_k^{prior}C^T S_k^{-1}$

   . $P_k^{post} = (I - K_k C)\, P_k^{prior}$

   . $P_{k+1}^{prior} = A_d P_k^{post} A_d^T + V$

   a. No plotting here, just show code (marks included in question 6, below)

6. Run an LQG controller **without** noise based on the following equations (**Graduate Only**):

   . Initial Conditions: x_0 ; w_0 ; $y_0 = x_0 + w_0$; $\hat{x}_0^{post} = y_0$

   . $u_k = -F_k \hat{x}_k^{post}$

   . $x_{k+1} = A_d x_k + B_d u_k$

   . $y_{k+1} = Cx_{k+1}$

   . $\hat{x}_{k+1}^{prior} = A_d \hat{x}_k^{post} + B_d u_k$

   . $\tilde{y}_{k+1} = y_{k+1} - C\hat{x}_{k+1}^{prior}$

   . $\hat{x}_{k+1}^{post} = \hat{x}_{k+1}^{prior} + K_{k+1}\tilde{y}_{k+1}$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 6 marks.

7. Run an LQG controller **with** noise based on the following equations (**Graduate Only**):

. Initial Conditions: x_0 ; w_0 ; $y_0 = x_0 + w_0$; $\hat{x}_0^{post} = y_0$

. $u_k = -F_k \hat{x}_k^{post}$

. $x_{k+1} = A_d x_k + B_d u_k + v_k$

. $y_{k+1} = C x_{k+1} + w_{k+1}$

. $\hat{x}_{k+1}^{prior} = A_d \hat{x}_k^{post} + B_d u_k$

. $\tilde{y}_{k+1} = y_{k+1} - C \hat{x}_{k+1}^{prior}$

. $\hat{x}_{k+1}^{post} = \hat{x}_{k+1}^{prior} + K_{k+1} \tilde{y}_{k+1}$

a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 1 mark.