Optimization and Quantization of Multibeam Beamforming Vector for Joint Communication and Radio Sensing

Project Seminar Emerging Topics in MIMO Communication Networks

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Outline

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Overview

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Introduction
Joint Communication And Sensing (JCAS)

- Demand on the systems with communication and sensing capabilities
- Shared hardware, reduced cost, higher spectral and energy efficiencies
- Mutual information sharing
- Challenge: different beamformer (BF) requirements
  - Communication – stable accurately pointed beam
  - Sensing – time-varying scanning beams

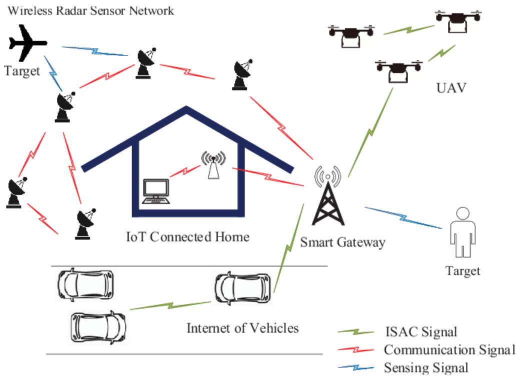
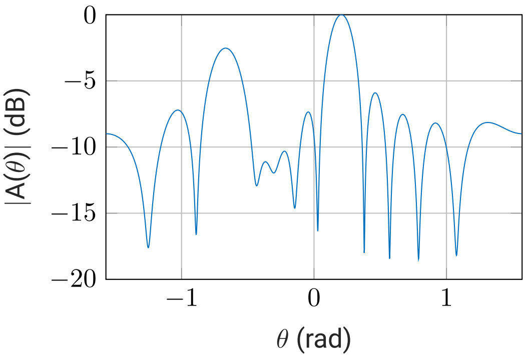


Figure: An example of a wireless network [LH21]

Introduction

Our approach

- Multibeam [Zha+18]
  - Beam with two or more main lobes
  - One lobe for communication, another for sensing
- Generated by single analog antenna array
- Can meet various requirements, such as power level, side-lobe level, beamwidth
- Communication and sensing beams can be combined to improve SNR at Rx



Overview

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System Model

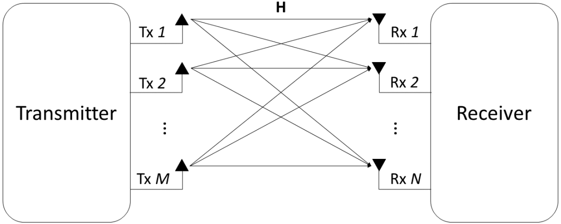
- Uniform linear array (ULA) with M elements

$$\mathbf{a}(\theta) = \left[1, e^{j\pi \sin(\theta)}, \ldots, e^{j\pi(M-1)\sin(\theta)}\right]^{\mathsf{T}}$$

- L multipath paths with AoDs $\theta_{\mathrm{t},l}$ and AoAs $\theta_{\mathrm{r},l}$

$$\mathbf{H} = \sum_{l=1}^{L} b_l \delta(t - \tau_l) e^{j2\pi f_{\mathrm{D},l} t} \mathbf{a}(\theta_{\mathrm{t},l}) \mathbf{a}^{\mathsf{T}}(\theta_{\mathrm{r},l})$$

where $b_l \in \mathbb{C}$ is the amplitude of l-th path, τ_l is the propagation delay, and $f_{\mathrm{D},l}$ is the associated Doppler frequency



System Model

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- The receive signal

$$y(t) = \mathbf{w}_\mathrm{r}^\mathsf{T}\mathbf{H}\mathbf{w}_\mathrm{t}s(t-\tau_l) + \mathbf{w}_\mathrm{r}^\mathsf{T}\mathbf{z}(t) = \sum_{l=1}^{L} b_l e^{j2\pi f_{\mathrm{D},l}t}(\mathbf{w}_\mathrm{r}^\mathsf{T}\mathbf{a}(\theta_{\mathrm{r},l}))(\mathbf{a}^\mathsf{T}(\theta_{\mathrm{t},l})\mathbf{w}_\mathrm{t})s(t-\tau_l) + \mathbf{w}_\mathrm{r}^\mathsf{T}\mathbf{z}(t)$$

  where $\mathbf{z}(t)$ is the additive white Gaussian noise vector with zero mean and variance σ_n^2
- Assuming the mean power of the transmit signal $s(t)$ is σ_s^2, we can write the receive signal-to-noise ratio (SNR) as

$$\gamma = \frac{|\mathbf{w}_\mathrm{r}^\mathsf{T}\mathbf{H}\mathbf{w}_\mathrm{t}|^2}{||\mathbf{w}_\mathrm{r}||^2} \cdot \frac{\sigma_\mathrm{s}^2}{\sigma_\mathrm{n}^2}$$

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Beam Generation

- Specify the desired array response
  $\mathbf{v} = [v_1, \ldots, v_M]^\mathsf{T}$

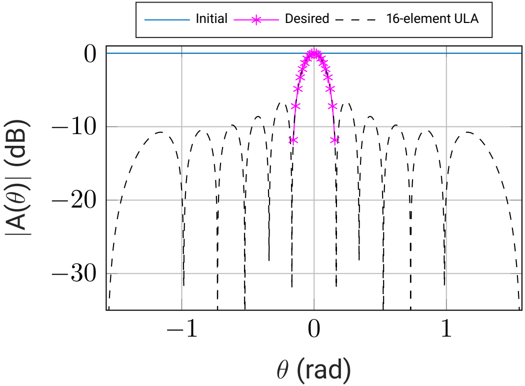
$$\mathbf{v} = \mathbf{D}_v \mathbf{p}_v$$

  where $\mathbf{D}_v \in \mathbb{R}^{M \times M}$ and $\mathbf{p}_v \in \mathbb{R}^{M \times 1}$ are the desired pattern magnitude and phase respectively

- The optimization problem can be formulated as

$$\mathbf{p}_{v,\text{opt}} = \arg\min_{\mathbf{p}_v} ||(\mathbf{A}\mathbf{A}^\dagger - \mathbf{I})\mathbf{D}_v \mathbf{p}_v||_2^2$$

  $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_K]^\mathsf{T}$, K - analyzed directions.



NTS

Beam Generation and Simple Combination

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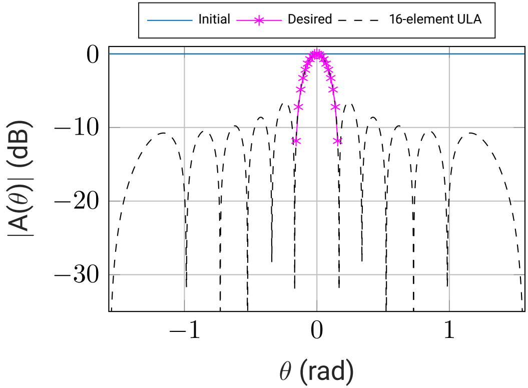
- Two-step Iterative Least Squares (ILS) algorithm [SF05] will provide a sub-optimal solution for $\mathbf{w}_{\text{t,c}}$ and $\mathbf{w}_{\text{t,s}}$
- $\mathbf{w}_{\text{t}} = \sqrt{\rho}\mathbf{w}_{\text{t,c}} + \sqrt{1-\rho}\mathbf{w}_{\text{t,s}}$, with energy parameter $0 \leq \rho \leq 1$



Optimized Combination

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- Simple Combination does not provide an optimal combination of communication beam and sensing beam.
- The new expression for combining two subbeams is given as

$$\mathbf{w}_\mathrm{t} = \sqrt{\rho}\mathbf{w}_\mathrm{t,c} + \sqrt{1-\rho}e^{j\varphi}\mathbf{w}_\mathrm{t,s}$$

  where $e^{j\varphi}$ is a phase shifting term.
- The optimization of the phase-shifting term is considered for two cases:
  - When the full channel matrix $\mathbf{H}$ is known at the Rx,
  - When only the AoD θ_t of the dominating path is known at the Rx.

Optimized Combination
Known Channel Matrix

- The optimal φ, φ_{opt}, can be obtained through maximizing the receiver SNR.
- The optimization problem is represented as

$$\varphi_{\mathrm{opt}} = \arg\max_{\varphi} \frac{|\mathbf{w}_{\mathrm{r}}^{\mathrm{T}} \mathbf{H} \mathbf{w}_{\mathrm{t}}(\varphi)|^2}{||\mathbf{w}_{\mathrm{r}}||^2 ||\mathbf{w}_{\mathrm{t}}(\varphi)||^2}.$$

- The maximal ratio combining (MRC) is used at the receiver side. Thus, the receive BF vector becomes $\mathbf{w}_{\mathrm{r}} = (\mathbf{H}\mathbf{w}_{\mathrm{t}}(\varphi))^*$.
- The problem is rewritten as

$$\varphi_{\mathrm{opt}} = \arg\max_{\varphi} \frac{\mathbf{w}_{\mathrm{t}}(\varphi)^{\mathrm{H}} \mathbf{H}^{\mathrm{H}} \mathbf{H} \mathbf{w}_{\mathrm{t}}(\varphi)}{||\mathbf{w}_{\mathrm{t}}(\varphi)||^2}.$$

- The objective function is 2π periodic.

Optimized Combination
Known dominating AoD

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- In practice, it is hard to obtain full knowledge of the channel matrix.
- The optimization problem is represented as

$$\tilde{\varphi}_{\mathrm{opt}} = \arg\max_{\varphi} \frac{||\mathbf{a}^{\mathsf{T}}(\theta_{\mathrm{t}})\tilde{\mathbf{w}}_{\mathrm{t}}(\hat{\varphi})||^2}{||\tilde{\mathbf{w}}_{\mathrm{t}}(\hat{\varphi})||^2}.$$

  where θ_{t} is the AoD of the dominating path.
- The objective function is also 2π periodic.

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Quantization of Multibeam Beamforming Vector

One Phase Shifter

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- Most of the BF vector $\mathbf{w}_\mathrm{t}$ cannot be realized in a practical analog array.
- It must be transformed into predefined discrete phase values.
- Each element of $\mathbf{w}_\mathrm{t}$ is of the form $w_i = |w_i|e^{j\psi_i}$, $i = 1, ..., M$, where ψ_i is the phase of w_i.
- The phase of w_i is matched into the quantized value

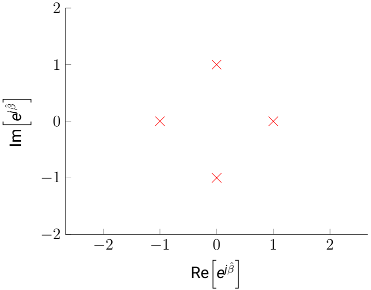
$$\hat{\beta}^{(i)} = \arg\min_{\hat{\beta}\in\mathcal{B}} |\mathrm{mod}_{2\pi}(\psi_i - \hat{\beta})|,$$

  where $\hat{\beta} \in \mathcal{B} = \left\{0, \triangle_\beta, 2\triangle_\beta, ..., (2^b - 1)\triangle_\beta\right\}$ with quantization step $\triangle_\beta = 2\pi/2^b$, and b is the number of quantization bits.
- Problem: the mismatch of the amplitudes

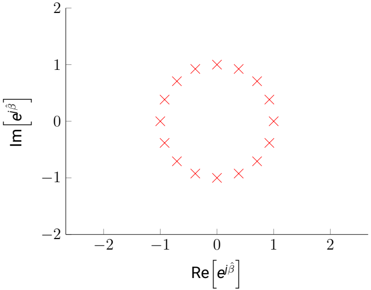
Quantization of Multibeam Beamforming Vector

Constellations of Single Phase shifter

- All points have different phase, but same magnitude.



(a) b = 2



(b) b = 4

Two Phase Shifters

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- Two phase Shifters can solve the issue of the amplitude mismatch.
- Three methods for creating the phase shifting values:
  - separate quantization of individual phase shifters,
  - joint quantization using combined quantization codebooks,
  - quantization with optimized scalling factor.

Two Phase Shifters
Separate Quantization of Individual Phase Shifters

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- The elements w_i of BF vector $\mathbf{w}$ can be rewritten as

$$w_i = |w_i|e^{j\psi_i} = e^{j\beta_1^{(i)}} + e^{j\beta_2^{(i)}},$$

- The quantized phase shifts are then determined separately

$$\hat{\beta}_1^{(i)} = \arg\min_{\hat{\beta}_1 \in \mathcal{B}_1} |\text{mod}_{2\pi}(\beta_1^{(i)} - \hat{\beta}_1)|,$$
$$\hat{\beta}_2^{(i)} = \arg\min_{\hat{\beta}_2 \in \mathcal{B}_2} |\text{mod}_{2\pi}(\beta_2^{(i)} - \hat{\beta}_2)|.$$

where $\hat{\beta}_1 \in \mathcal{B}_1 = \{0, \triangle_{\beta_1}, 2\triangle_{\beta_1}, ..., (2^{b_1} - 1)\triangle_{\beta_1}\}$ and $\hat{\beta}_2 \in \mathcal{B}_2 = \{0, \triangle_{\beta_2}, 2\triangle_{\beta_2}, ..., (2^{b_2} - 1)\triangle_{\beta_2}\}$ are the sets of the quantized phase values.

Two Phase Shifters

Joint Quantization Using Combined Quantization Codebooks

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- The codebook $\mathcal{C}$ with codes $\hat{c}_k$, which are generated by

$$\hat{c}_k = e^{j\hat{\beta}_1} + e^{j\hat{\beta}_2},$$

where $\hat{c}_k$ is the k-th element of $\mathcal{C}$.

- Two separate codebooks are defined as

$$\hat{\beta}_1 \in \mathcal{B}_1 = \left\{0, \triangle_{\beta_1}, 2\triangle_{\beta_1}, ..., (2^{b_1} - 1)\triangle_{\beta_1}\right\},$$

$$\hat{\beta}_2 \in \mathcal{B}_2 = \left\{\phi, \phi + \triangle_{\beta_2}, ..., \phi + (2^{b_2} - 1)\triangle_{\beta_2}\right\},$$

where $\phi \in [0, \triangle_{\beta_2}/2]$ is a constant.

Two Phase Shifters
Joint Quantization Using Combined Quantization Codebooks

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- The constellation points $\hat{c}_k$ are normalized so that $\mathrm{E}[|\hat{c}_k|^2] = 1/M$, e.g., the normalization factor for $\phi = 0$ and $\phi = \triangle_{\beta_2}/2$ are

$$h_1 = \sqrt{\frac{M}{2^{b-1}} \sum_{k=1}^{2^{b-1}} \hat{c}_k^2} = \sqrt{2 + 2^{2-b}}\sqrt{M},$$

$$h_2 = \sqrt{\frac{M}{2^b} \sum_{k=1}^{2^b} \hat{c}_k^2} = \sqrt{2M},$$

respectively.

- The BF weight w_i can then be obtained by

$$\hat{w}_i = \arg\min_{\hat{c}_k \in \mathcal{C}} |w_i - \hat{c}_k|^2.$$

Two Phase Shifters

Quantization with Optimized Scaling Factor

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- In a joint quantization scheme, the normalization factors depend on the number of bits b and the dimension of the array M.
- Instantaneous optimality for quantizing particular BF vectors guaranteed by IGSS-Q based on the improved golden section search (IGSS) algorithm [Höpfinger ’1976]
- The IGSS-Q algorithm aims to find the optimal scaling factor υ_{opt} by iteratively solving

$$\upsilon_{\text{opt}} = \arg\min_{\upsilon} ||\upsilon \mathbf{w}_t - \hat{\mathbf{q}}(\upsilon)||_2^2,$$

where $\hat{\mathbf{q}}(\upsilon)$ is quantized BF vector.

IGSS-Q

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- Initial two different points x_1 and x_2 ($x_2 > x_1$).
- The searching interval is defined as

$$d = x_2 - x_1.$$

- Define two interior points a_1 and a_2 ($a_2 > a_1$ and $a_1, a_2 \in [x_1, x_2]$).
- Compute the errors at the four points in each iteration.
- The error is computed by using

$$e(v) = \sum_{i=1}^{M} |v w_i - \hat{q}_i|^2.$$

where $\hat{q}_i$ can be obtained by

$$\hat{q}_i = \arg\min_{\hat{c}_k \in \mathcal{C}} |v w_i - \hat{c}_k|^2.$$

IGSS-Q

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- Compare the errors of four points ($x_1 < a_1 < a_2 < x_2$).
- Update the searching interval:
  - If the smallest error $e_{min} \in [e(x_1), e(a_1)]$, then

    $$x_2 = a_2, a_2 = a_1, a_1 = \alpha(a_2 - x_1)$$

    where $\alpha \in (0, 1)$.
  - Otherwise, i.e., $e_{min} \in [e(a_2), e(x_2)]$

    $$x_1 = a_1, a_1 = a_2, a_2 = \alpha(a_2 - x_1)$$

- The searching interval becomes narrow after each interation.

IGSS-Q

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- When the interval d is smaller than a threshold, take either the upper bound x_2 or the lower bound x_1 as υ_{opt}.
- Each elements $\hat{q}_i$ of vector $\hat{\mathbf{q}}$ can obtained by

$$\hat{q}_i = \arg\min_{\hat{c}_k \in \mathcal{C}} |\upsilon_{\text{opt}} w_i - \hat{c}_k|^2.$$

- The final quantized BF vector $\hat{\mathbf{w}}_{\text{t}}$ is given as

$$\hat{\mathbf{w}}_{\text{t}} = \frac{\hat{\mathbf{q}}}{||\hat{\mathbf{q}}||}.$$

Overview

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Simulation Settings

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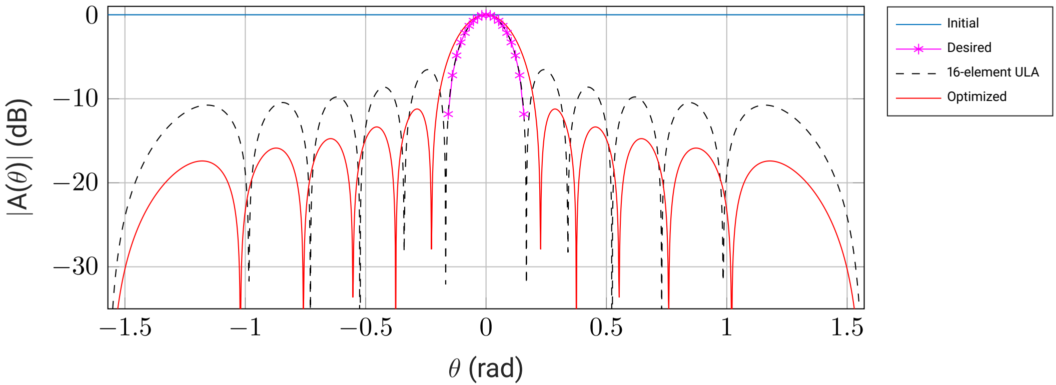
- Analog uniform linear array with 16 omni-directional antennas for both communication and sensing subbeams.
- There is an LOS path (0°) for communication. All the other multipath components are uniformly distributed within an angular range of 14° centered at the LOS direction.
- The mean power ratio between the LOS and NOLS signals is 10 dB.

Beam Generation

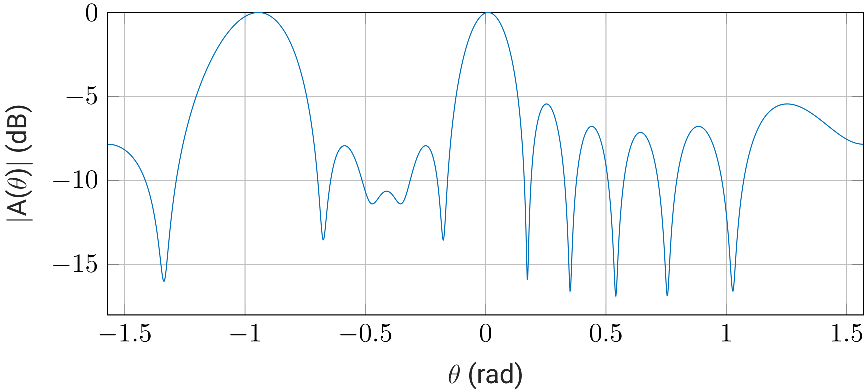
Pattern optimization using two-step ILS



- The optimized beam has lower side-lobe-level as the conventional ULA pattern

Simple Combination

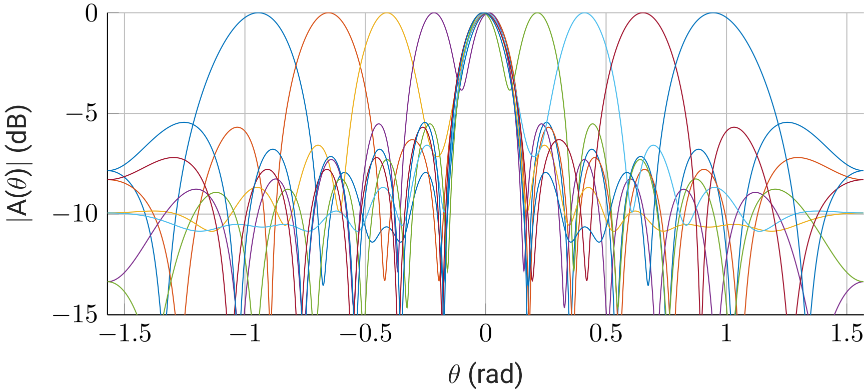
An optimized multibeam



- A multibeam obtained via Simple Combination with $\rho = 0.5$

Simple Combination

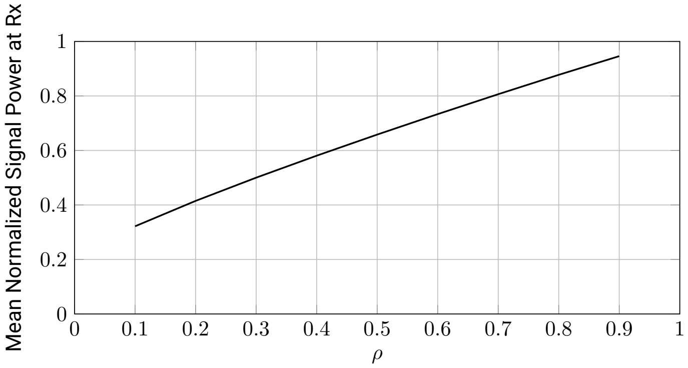
A set of optimized multibeams



- A set of multibeams can be computed offline

Simple Combination

Performance w.r.t. ρ



- We expect that Optimized Combination will outperform Simple Combination

Optimized Combination

Simple Combination vs. Optimized Combination

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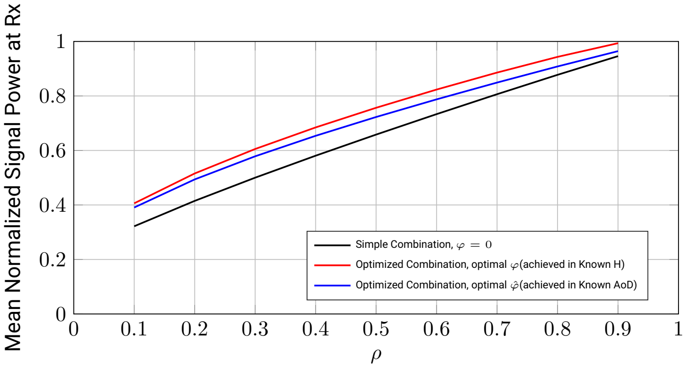

- Optimized Combination outperforms Simple Combination.

Optimized Combination

Signal Powers at the Receiver and at the Dominating AoD

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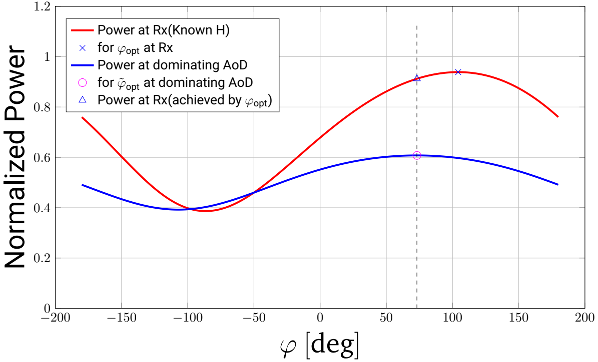
- The communication subbeam is set pointing to the dominating AoD($0°$), while the sensing subbeam is fixed at $10.8°$.



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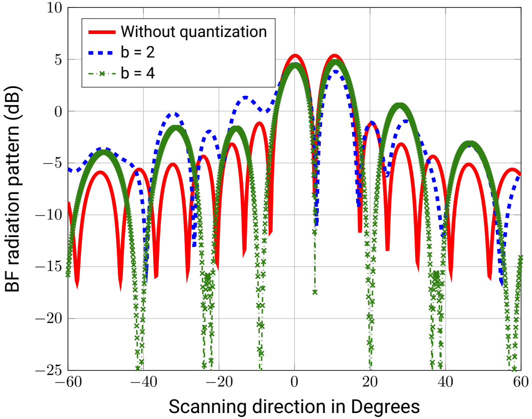
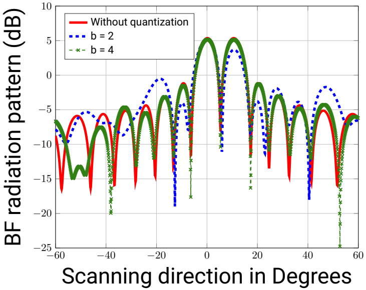
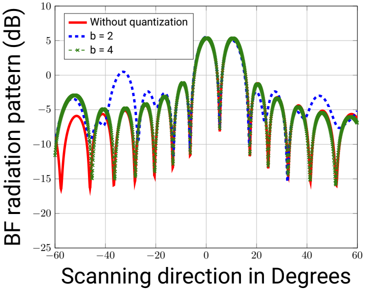
Quantization
Single phase shifter



- Codebook $\mathcal{C}_1$ with $\phi = 0$ and Codebook $\mathcal{C}_2$ with $\phi = \triangle_{\beta_2}/2$

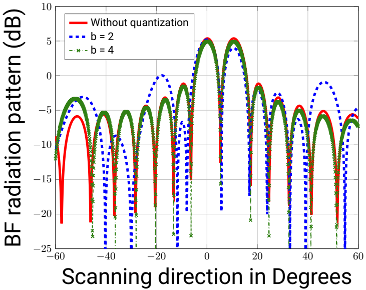


(a) Codebook $\mathcal{C}_1$

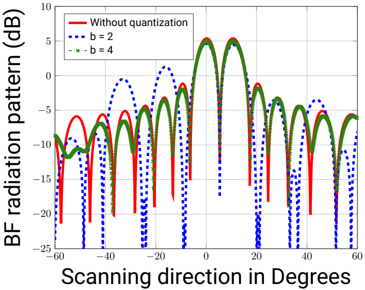


(b) Codebook $\mathcal{C}_2$

Quantization
Quantization with Optimized Scaling Factor



(a) Codebook $\mathcal{C}_1$



(b) Codebook $\mathcal{C}_2$

Overview

Conclusion

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- In this project, we reproduced Two-Step ILS algorithm to generate the beams and studied Simple and Optimized Combination methods to combine them.
- The beams combination methods provide great flexibility for varying BF directions and enable the constructive combination of two subbeams.
- We investigated the performances of single phase shifter and the two phase shifters.
- **Our own contribution** is the performance comparison of Simple and Optimized Combination methods.
- Remaining questions:
  - How will the optimization algorithms perform in the arrays different from ULA?
  - How can we obtain the optimal communication and sensing beams?
  - Is there a ϕ which can be used in creating an optimal codebook for quantization?

References

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[SF05] Z. Shi and Z. Feng. "A new array pattern synthesis algorithm using the two-step least-squares method". In: IEEE Signal Processing Letters 12.3 (2005), pp. 250–253.

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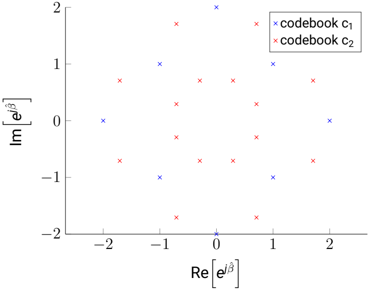
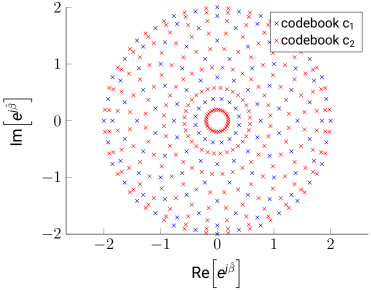
Thank you for your attention!

Constellations of Codebook 1 and 2



(a) b = 2



(b) b = 4

Two-Step ILS Algorithm

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Step 1 Given $\mathbf{w}_0$, $n = 0$

Step 2
- $n = n + 1$
- $\mathbf{p}_{v,n} = \mathbf{w}_{n-1}^{\mathsf{H}} \cdot \mathbf{v} \cdot \mathbf{D}_v^{-1}$
- Project all elements of $\mathbf{p}_v$ to the closest values on the unit circle to produce a new vector $\mathbf{p}_{v0,n}$
- $\mathbf{w}_n = (\mathbf{v} \cdot \mathbf{v}^{\mathsf{H}})^{-1} \cdot \mathbf{v} \cdot \mathbf{D}_v \cdot \mathbf{p}_{v0,n}$