3

The Julian Calendar

Atque hic erat anni Romani status cum C. Cæsar ei manum admovit: qui ex lunari non malo in pessimum a Numa aut alio rupice et rustico depravatus, vitio intercalationis veteres fines suos tamen tueri non potuit. Vt non semel miratus sim, orbis terrarum dominam gentem, quæ generi humano leges dabat, sibi unam legem anni ordinati statuere non potuisse, ut post hominum memoriam nulla gens in terris ineptiore anni forma usa sit. [*Such was the condition of the Roman calendar when Julius Cæsar went about his work on it. Numa or some other rustic clod took a lunar calendar that was not too bad and made it appalling. Thanks to his faulty system of intercalation it could not stay in its original bounds. I have been amazed more than once that the people who ruled the entire world and gave laws to the entire human race could not make one law for itself for an orderly calendar. As a result, no nation in human memory has used a worse calendar than theirs.*]

Joseph Justus Scaliger: *De Emendatione Temporum* (1583)[1]

3.1 Structure and Implementation

The calculations for the Julian calendar, which we described in introducing the Gregorian calendar in Chapter 2, are nearly identical to those for the Gregorian calendar, but we must change the leap-year rule to

$$\textbf{julian-leap-year?}\,(\textit{j-year}) \stackrel{\text{def}}{=} (\textit{j-year} \bmod 4) = \left\{ \begin{array}{ll} 0 & \textbf{if}\ \textit{j-year} > 0 \\ 3 & \textbf{otherwise} \end{array} \right\} \tag{3.1}$$

The upper part is formula (1.82); the lower part is formula (1.83) with $\Delta = 1$ because there is no year 0 on the Julian calendar. Note that the Julian leap-year rule was applied inconsistently for a period of years prior to 8 C.E. (see [6, pp. 156–158]).

The months of the Julian calendar are the same as those of the Gregorian calendar (see page 55).

Converting from a Julian date to an R.D. date requires a calculation similar to that in the Gregorian case but with two minor adjustments: we no longer need consider century-year leap days, and we must define the epoch of the Julian calendar

[1] *Lectores ne credant huius libri auctores his sententiis subscribere.*

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in terms of our fixed dating. For the epoch, we know that R.D. 1 is January 3, 1 C.E. (Julian), and thus the first day of the Julian calendar, January 1, 1 C.E. (Julian) must be December 30, 0 (Gregorian), that is, R.D. −1:

$$\textbf{julian-epoch} \stackrel{\text{def}}{=} \textbf{fixed-from-gregorian}\left(\boxed{0 \mid \textbf{december} \mid 30}\right) \tag{3.2}$$

Now we can write

$$\textbf{fixed-from-julian}\left(\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}\right) \stackrel{\text{def}}{=} \tag{3.3}$$

$$\textbf{julian-epoch} - 1 + 365 \times (y - 1) + \left\lfloor \frac{y-1}{4} \right\rfloor + \left\lfloor \tfrac{1}{12} \times (367 \times \mathit{month} - 362) \right\rfloor + \begin{cases} 0 & \textbf{if } \mathit{month} \leqslant 2 \\ -1 & \textbf{if julian-leap-year?}\,(\mathit{year}) \\ -2 & \textbf{otherwise} \end{cases} + \mathit{day}$$

where

$$y = \begin{cases} \mathit{year} + 1 & \textbf{if } \mathit{year} < 0 \\ \mathit{year} & \textbf{otherwise} \end{cases}$$

This function is similar in structure to that of **fixed-from-gregorian**. We start at **julian-epoch**−1, the R.D. number of the last day before the epoch; to this, we add the number of nonleap days (positive for positive years, negative otherwise) between the last day before the epoch and the last day of the year preceding the given year, the corresponding (positive or negative) number of leap days, the number of days in prior months of the given year, and the number of days in the given month up to and including the given day. For nonpositive years, we adjust the year to accommodate the lack of year 0.

For the inverse function, we handle the missing year 0 by subtracting 1 from the year as determined by formula (1.90) for dates before the epoch:

$$\textbf{julian-from-fixed}\,(\mathit{date}) \stackrel{\text{def}}{=} \boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}} \tag{3.4}$$

where

$$\mathit{approx} = \left\lfloor \frac{1}{1461} \times (4 \times (\mathit{date} - \textbf{julian-epoch}) + 1464) \right\rfloor$$

$$\mathit{year} = \begin{cases} \mathit{approx} - 1 & \textbf{if } \mathit{approx} \leqslant 0 \\ \mathit{approx} & \textbf{otherwise} \end{cases}$$

$$\mathit{prior\text{-}days} = \mathit{date} - \textbf{fixed-from-julian}\left(\boxed{\mathit{year} \mid \textbf{january} \mid 1}\right)$$

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$$correction = \begin{cases} 0 & \textbf{if } \mathit{date} < \textbf{fixed-from-julian}\left(\boxed{\mathit{year} \mid \textbf{march} \mid 1}\right) \\ 1 & \textbf{if julian-leap-year?}\,(\mathit{year}) \\ 2 & \textbf{otherwise} \end{cases}$$

$$month = \left\lfloor \frac{1}{367} \times (12 \times (\mathit{prior\text{-}days} + \mathit{correction}) + 373) \right\rfloor$$

$$day = \mathit{date} - \textbf{fixed-from-julian}\left(\boxed{\mathit{year} \mid \mathit{month} \mid 1}\right) + 1$$

We can construct alternative functions in the style of **alt-fixed-from-gregorian** and **alt-gregorian-from-fixed** from Section 2.3 for the functions **fixed-from-julian** and **julian-from-fixed**.

3.2 Roman Nomenclature

Brutus: Is not tomorrow, boy, the ides of March?
Lucius: I know not, sir.
Brutus: Look in the calendar and bring me word.

Shakespeare: *Julius Cæsar*, Act II, scene i (1623)

In ancient Rome it was customary to refer to days of the month by counting down to certain key events in the month: the *kalends*, the *nones*, and the *ides*. This custom, in popular use well past the middle ages, is evidently quite ancient, coming from a time in which the month was still synchronized with the lunar cycle: the kalends were the new moon, the nones the first quarter moon, and the ides the full moon. (Indeed, the word *calendar* is derived from *kalendæ*, meaning "account book," for loans were due on the first of the month.) We define three special constants,

$$\textbf{kalends} \stackrel{\text{def}}{=} 1 \tag{3.5}$$

$$\textbf{nones} \stackrel{\text{def}}{=} 2 \tag{3.6}$$

$$\textbf{ides} \stackrel{\text{def}}{=} 3 \tag{3.7}$$

to identify these events.

The kalends are always the first of the month. The ides are near the middle of the month—the thirteenth of the month, except in March, May, July, and October when they fall on the fifteenth; hence

$$\textbf{ides-of-month}\,(\mathit{month}) \stackrel{\text{def}}{=} \tag{3.8}$$

$$\begin{cases} 15 & \textbf{if } \mathit{month} \in \{\textbf{march}, \textbf{may}, \textbf{july}, \textbf{october}\} \\ 13 & \textbf{otherwise} \end{cases}$$

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The nones are always 8 days before the ides:

$$\textbf{nones-of-month}\,(month) \stackrel{\text{def}}{=} \textbf{ides-of-month}\,(month) - 8 \tag{3.9}$$

Dates that fall on the kalends, the nones, or the ides are referred to as such. Thus, March 15 is called "the ides of March," for example, whereas January 1 and 5 are, respectively, the kalends and nones of January. Dates that fall on the day before one of these special days are called *pridie* ("day before" in Latin); for example, July 6, the day before the nones of July, is *pridie Non. Jul.* in Latin. All dates other than the kalends, nones, or ides, or days immediately preceding them are described by the number of days (inclusive) until the next upcoming event: The Roman name for October 30 is *ante diem III Kal. Nov.*, meaning 3 days (inclusive) before the kalends of November; the idiomatic English usage would describe this as "2 days before the first of November," but the Roman custom uses the inclusive count.

In a leap year February has an extra day, and modern authorities understand the Roman custom as intercalating that day after February 24, before February 25 (see [6] for another possibility; see [1, pp. 92–94, 678–680] for a discussion of the placement of the leap day on the Julian calendar). Because February 24 was *ante diem VI Kal. Mar.*, the extra day was called *ante diem bis VI Kal. Mar.* or "the second sixth day before the kalends of March." The phrase *bis VI* was read *bis sextum* which gave rise to the English words *bissextus* for leap day and *bissextile* as an adjective to describe a leap year [3, p. 795]. Despite the official Roman calendar, unofficial and medieval usage made the day after February 23 the leap day. The necessary changes to our functions **fixed-from-roman** and **roman-from-fixed** are simple, should one want to follow that variant rule.

Table 3.1 gives abbreviated names for all the days according to the Roman system. Full spellings of all the names are given for each in [1]; details of the Latin grammar of those names can also be found there [1, pp. 672–673].

We represent the Roman method of referring to a day of the month by a list containing the year number, the month, the next event, a count (inclusive) of days until that event, and a **true**/**false** leap-day indicator:

year	*month*	*event*	*count*	*leap*

Although the Roman method of referring to days of the month is sometimes used in the context of Gregorian calendar dates, such references are archaic and it is more sensible to tie the Roman nomenclature to the Julian calendar, as we do here. Determining the Roman name for a Gregorian date is easily done by making the appropriate substitutions of **gregorian-from-fixed** and **gregorian-leap-year?** for the corresponding Julian functions.

Determining the fixed date corresponding to a given Roman form involves subtracting the count from the date of the event in the specified month and year, while adjusting for the leap day if the event is the kalends of March in a leap year:

Table 3.1 Roman nomenclature for days of the month on the Julian calendar. The abbreviation "a.d." stands for the Latin *ante diem*. In dates after the ides of a month, "Kal." means the kalends of the coming month; "Non." and "Id." mean the nones and ides, respectively, of the current month. Adapted from [2] and [5].

Day	January August December	February (ordinary)	February (leap)	March May July October	April June September November
1	**Kalends**	**Kalends**	**Kalends**	**Kalends**	**Kalends**
2	a.d. iv Non.	a.d. iv Non.	a.d. iv Non.	a.d. vi Non.	a.d. iv Non.
3	a.d. iii Non.	a.d. iii Non.	a.d. iii Non.	a.d. v Non.	a.d. iii Non.
4	pridie Non.	pridie Non.	pridie Non.	a.d. iv Non.	pridie Non.
5	**Nones**	**Nones**	**Nones**	a.d. iii Non.	**Nones**
6	a.d. viii Id.	a.d. viii Id.	a.d. viii Id.	pridie Non.	a.d. viii Id.
7	a.d. vii Id.	a.d. vii Id.	a.d. vii Id.	**Nones**	a.d. vii Id.
8	a.d. vi Id.	a.d. vi Id.	a.d. vi Id.	a.d. viii Id.	a.d. vi Id.
9	a.d. v Id.	a.d. v Id.	a.d. v Id.	a.d. vii Id.	a.d. v Id.
10	a.d. iv Id.	a.d. iv Id.	a.d. iv Id.	a.d. vi Id.	a.d. iv Id.
11	a.d. iii Id.	a.d. iii Id.	a.d. iii Id.	a.d. v Id.	a.d. iii Id.
12	pridie Id.	pridie Id.	pridie Id.	a.d. iv Id.	pridie Id.
13	**Ides**	**Ides**	**Ides**	a.d. iii Id.	**Ides**
14	a.d. xix Kal.	a.d. xvi Kal.	a.d. xvi Kal.	pridie Id.	a.d. xviii Kal.
15	a.d. xviii Kal.	a.d. xv Kal.	a.d. xv Kal.	**Ides**	a.d. xvii Kal.
16	a.d. xvii Kal.	a.d. xiv Kal.	a.d. xiv Kal.	a.d. xvii Kal.	a.d. xvi Kal.
17	a.d. xvi Kal.	a.d. xiii Kal.	a.d. xiii Kal.	a.d. xvi Kal.	a.d. xv Kal.
18	a.d. xv Kal.	a.d. xii Kal.	a.d. xii Kal.	a.d. xv Kal.	a.d. xiv Kal.
19	a.d. xiv Kal.	a.d. xi Kal.	a.d. xi Kal.	a.d. xiv Kal.	a.d. xiii Kal.
20	a.d. xiii Kal.	a.d. x Kal.	a.d. x Kal.	a.d. xiii Kal.	a.d. xii Kal.
21	a.d. xii Kal.	a.d. ix Kal.	a.d. ix Kal.	a.d. xii Kal.	a.d. xi Kal.
22	a.d. xi Kal.	a.d. viii Kal.	a.d. viii Kal.	a.d. xi Kal.	a.d. x Kal.
23	a.d. x Kal.	a.d. vii Kal.	a.d. vii Kal.	a.d. x Kal.	a.d. ix Kal.
24	a.d. ix Kal.	a.d. vi Kal.	a.d. vi Kal.	a.d. ix Kal.	a.d. viii Kal.
25	a.d. viii Kal.	a.d. v Kal.	a.d. bis vi Kal.	a.d. viii Kal.	a.d. vii Kal.
26	a.d. vii Kal.	a.d. iv Kal.	a.d. v Kal.	a.d. vii Kal.	a.d. vi Kal.
27	a.d. vi Kal.	a.d. iii Kal.	a.d. iv Kal.	a.d. vi Kal.	a.d. v Kal.
28	a.d. v Kal.	pridie Kal.	a.d. iii Kal.	a.d. v Kal.	a.d. iv Kal.
29	a.d. iv Kal.		pridie Kal.	a.d. iv Kal.	a.d. iii Kal.
30	a.d. iii Kal.			a.d. iii Kal.	pridie Kal.
31	pridie Kal.			pridie Kal.	

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$$\textbf{fixed-from-roman}\left(\begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \mathit{month} & \mathit{event} & \mathit{count} & \mathit{leap} \\ \hline\end{array}\right) \stackrel{\text{def}}{=} \tag{3.10}$$

$$\left\{\begin{array}{ll} \textbf{fixed-from-julian}\left(\begin{array}{|c|c|c|}\hline \mathit{year} & \mathit{month} & 1 \\ \hline\end{array}\right) & \textbf{if } \mathit{event} = \textbf{kalends} \\ \textbf{fixed-from-julian} & \\ \quad\left(\begin{array}{|c|c|l|}\hline \mathit{year} & \mathit{month} & \textbf{nones-of-month} \\ & & (\mathit{month}) \\ \hline\end{array}\right) & \textbf{if } \mathit{event} = \textbf{nones} \\ \textbf{fixed-from-julian} & \\ \quad\left(\begin{array}{|c|c|l|}\hline \mathit{year} & \mathit{month} & \textbf{ides-of-month} \\ & & (\mathit{month}) \\ \hline\end{array}\right) & \textbf{if } \mathit{event} = \textbf{ides} \end{array}\right\}$$
$$- \mathit{count}$$
$$+ \left\{\begin{array}{ll} 0 & \textbf{if julian-leap-year?}\,(\mathit{year}) \text{ and } \mathit{month} = \textbf{march} \text{ and} \\ & \quad \mathit{event} = \textbf{kalends} \text{ and } 16 \geqslant \mathit{count} \geqslant 6 \\ 1 & \textbf{otherwise} \end{array}\right\}$$
$$+ \left\{\begin{array}{ll} 1 & \textbf{if } \mathit{leap} \\ 0 & \textbf{otherwise} \end{array}\right\}$$

Converting a fixed date to the Roman form thus requires converting that fixed date to a Julian year-month-day and then determining the next event. If the month is February of a leap year, the special cases must be handled separately:

$$\textbf{roman-from-fixed}\,(\mathit{date}) \stackrel{\text{def}}{=} \tag{3.11}$$

$$\left\{\begin{array}{l} \begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \mathit{month} & \textbf{kalends} & 1 & \text{false} \\ \hline\end{array} \\ \quad \textbf{if } \mathit{day} = 1 \\ \begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \mathit{month} & \textbf{nones} & \textbf{nones-of-month}\,(\mathit{month}) - \mathit{day} + 1 & \text{false} \\ \hline\end{array} \\ \quad \textbf{if } \mathit{day} \leqslant \textbf{nones-of-month}\,(\mathit{month}) \\ \begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \mathit{month} & \textbf{ides} & \textbf{ides-of-month}\,(\mathit{month}) - \mathit{day} + 1 & \text{false} \\ \hline\end{array} \\ \quad \textbf{if } \mathit{day} \leqslant \textbf{ides-of-month}\,(\mathit{month}) \\ \begin{array}{|c|c|c|c|c|}\hline \mathit{year}' & \mathit{month}' & \textbf{kalends} & \mathit{kalends}_1 - \mathit{date} + 1 & \text{false} \\ \hline\end{array} \\ \quad \textbf{if } \mathit{month} \neq \textbf{february} \text{ or not } \textbf{julian-leap-year?}\,(\mathit{year}) \\ \begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \textbf{march} & \textbf{kalends} & 30 - \mathit{day} & \text{false} \\ \hline\end{array} \\ \quad \textbf{if } \mathit{day} < 25 \\ \begin{array}{|c|c|c|c|c|}\hline \mathit{year} & \textbf{march} & \textbf{kalends} & 31 - \mathit{day} & \mathit{day} = 25 \\ \hline\end{array} \\ \quad \textbf{otherwise} \end{array}\right.$$

where

$$\begin{aligned} \mathit{j\text{-}date} &= \textbf{julian-from-fixed}\,(\mathit{date}) \\ \mathit{month} &= \mathit{j\text{-}date}_{\textbf{month}} \\ \mathit{day} &= \mathit{j\text{-}date}_{\textbf{day}} \\ \mathit{year} &= \mathit{j\text{-}date}_{\textbf{year}} \end{aligned}$$

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$$
\begin{aligned}
\mathit{month}' &= (\mathit{month} + 1) \bmod [1 \mathinner{..} 12] \\
\mathit{year}' &= \begin{cases} \mathit{year} & \textbf{if } \mathit{month}' \neq 1 \\ \mathit{year} + 1 & \textbf{if } \mathit{year} \neq -1 \\ 1 & \textbf{otherwise} \end{cases} \\
\mathit{kalends}_1 &= \textbf{fixed-from-roman}\left(\boxed{\mathit{year}'} \boxed{\mathit{month}'} \boxed{\textbf{kalends}} \boxed{1} \boxed{\text{false}} \right)
\end{aligned}
$$

Note that when the upcoming event is the kalends, it is the kalends of the *next* month, not the present month; thus, dates following the ides of a month carry the name of the next month, and after the ides of December dates carry the following year number.

3.3 Roman Years

Cæsar set out the problem before the best philosophers and mathematicians and, from the methods available, he concocted his own correction that was more precise.

Plutarch: *Life of Cæsar* (75 C.E.)

Roman years were specified A.U.C., *Ab Urbe Condita*, from the founding of the city (of Rome). There is some uncertainty about the precise traditional year of the founding of Rome, so we make it a symbolic value:

$$\textbf{year-rome-founded} \stackrel{\text{def}}{=} 753 \text{ B.C.E.} \tag{3.12}$$

We want to convert between A.U.C. years and Julian years. Because years were not counted from zero on the Julian calendar, we assume that they should not be counted from zero A.U.C.; recalling that B.C.E. years are represented internally as negative integers we write

$$\textbf{julian-year-from-auc}\,(\mathit{year}) \stackrel{\text{def}}{=} \tag{3.13}$$

$$\begin{cases} \mathit{year} + \textbf{year-rome-founded} - 1 & \textbf{if } 1 \leqslant \mathit{year} \leqslant -\textbf{year-rome-founded} \\ \mathit{year} + \textbf{year-rome-founded} & \textbf{otherwise} \end{cases}$$

and

$$\textbf{auc-year-from-julian}\,(\mathit{year}) \stackrel{\text{def}}{=} \tag{3.14}$$

$$\begin{cases} \mathit{year} - \textbf{year-rome-founded} + 1 & \textbf{if year-rome-founded} \leqslant \mathit{year} \leqslant -1 \\ \mathit{year} - \textbf{year-rome-founded} & \textbf{otherwise} \end{cases}$$

3.4 Olympiads

Therefore those who prophesied in the time of Darius Hystaspes, about the second year of his reign—Haggai, and Zechariah, and the angel of the twelve, who prophesied about the first year of the forty-eighth Olympiad—are demonstrated to be older than Pythagoras, who is said to have lived in the sixty-second Olympiad, and than Thales, the oldest of the wise men of the Greeks, who lived about the fiftieth Olympiad.

Clement of Alexandria: *The Stromata* (c. 200 C.E.)

Another common historical method of denoting years was in terms of the Olympiad, the four-year cycle of Olympic games, said to have been introduced by either the Greek historian Timaeus or by Eratosthenes. The recently reconstructed Antikythera mechanism included this reckoning, along with eclipse predictions and displays of the phases of the moon and positions of the visible planets.[2] The games were held every fourth year, so each Olympiad comprises four years; we represent an Olympiad by a pair of integers,

$$\boxed{\textit{cycle}}\boxed{\textit{year}}$$

The first recorded games were in 776 B.C.E., so this was year 1 of cycle 1:

$$\textbf{olympiad-start} \stackrel{\text{def}}{=} 776 \text{ B.C.E.} \tag{3.15}$$

To convert a Julian year into its Olympiad equivalent and vice versa, we need to count quadrenniel periods:

$$\textbf{julian-year-from-olympiad}\,(\textit{o-date}) \stackrel{\text{def}}{=} \begin{cases} \textit{years} & \textbf{if } \textit{years} < 0 \\ \textit{years} + 1 & \textbf{otherwise} \end{cases} \tag{3.16}$$

where

$$\begin{aligned} \textit{cycle} &= \textit{o-date}_{\textbf{cycle}} \\ \textit{year} &= \textit{o-date}_{\textbf{year}} \\ \textit{years} &= \textbf{olympiad-start} + 4 \times (\textit{cycle} - 1) + \textit{year} - 1 \end{aligned}$$

In the other direction

$$\textbf{olympiad-from-julian-year}\,(\textit{j-year}) \stackrel{\text{def}}{=} \tag{3.17}$$

$$\boxed{\left\lfloor \frac{\textit{years}}{4} \right\rfloor + 1}\boxed{(\textit{years} \bmod 4) + 1}$$

where

$$\textit{years} = \textit{j-year} - \textbf{olympiad-start} - \left\{ \begin{array}{ll} 0 & \textbf{if } \textit{j-year} < 0 \\ 1 & \textbf{otherwise} \end{array} \right\}$$

[2] For more about this marvelous device, see "The Antikythera Mechanism: A Computer Science Perspective," D. Spinellis, *IEEE Computer* vol. 41, no. 5, pp. 22–27, May 2008. Also, see "Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism," T. Freeth, A. Jones, J. M. Steele, and Y. Bitsakis, *Nature*, vol. 454, pp. 614–617, July 2008. A popular treatment, with many interesting photographs, is given in J. Marchant's *Decoding the Heavens*, William Heinemann, London, 2008.

3.5 Seasons

Civilized nations in general now agree to begin reckoning the new year from the first of January. Yet it may seem strange to call that a new *season, when everything is most inactive and lifeless; when animals are benumbed by the cold, and vegetables are all dead or withered. For this reason, some have thought to begin the year in Spring.*

Thomas Gosden: *The Calendar of Nature: Designed for the Instruction and Entertainment of Young Persons* (1822)

As the Julian year of 365.25 days is longer than the Gregorian year of 365.2425 days, the times and dates of the Julian seasons shift over the years with respect to the Gregorian calendar. Let *season* be any value in the range $[0° \,..\, 360°)$; in particular, let the four seasons be defined by the following values:

$$\textbf{spring} \stackrel{\text{def}}{=} 0° \tag{3.18}$$

$$\textbf{summer} \stackrel{\text{def}}{=} 90° \tag{3.19}$$

$$\textbf{autumn} \stackrel{\text{def}}{=} 180° \tag{3.20}$$

$$\textbf{winter} \stackrel{\text{def}}{=} 270° \tag{3.21}$$

(These are the celestial longitudes of the sun at the start of those seasons; see Section 14.4.)

To compute the occurrences of the Julian *season* in the Gregorian year *g-year*, we first define a generic function for calculating occurrences of seasons of a year of any length *L*. We must allow for the possibility of multiple occurrences (if the calendar year is short) or none (if the year is long and the season falls near January 1 on the Gregorian calendar). We use **positions-in-range** (1.40):

$$\textbf{cycle-in-gregorian}\,(\textit{season}, \textit{g-year}, L, \textit{start}) \stackrel{\text{def}}{=} \tag{3.22}$$

$$\textbf{positions-in-range}\,(\textit{pos}, L, \Delta, \textit{year})$$

where

$$\begin{aligned} \textit{year} &= \textbf{gregorian-year-range}\,(\textit{g-year}) \\ \textit{pos} &= \frac{\textit{season}}{360°} \times L \\ \Delta &= \textit{pos} - (\textit{start} \bmod L) \end{aligned}$$

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Then, for Julian seasons, we have

$$\textbf{julian-season-in-gregorian}\,(season, g\text{-}year) \stackrel{\text{def}}{=} \tag{3.23}$$

$$\textbf{cycle-in-gregorian}\left(season, g\text{-}year, Y, \textbf{fixed-from-julian}\left(\boxed{1 \text{ B.C.E.} \mid \textbf{march} \mid 23} \right) + offset \right)$$

where

$$Y = 365 + 6^{\text{h}}$$

$$offset = \frac{season}{360^\circ} \times Y$$

This is based on the assumption of a spring equinox on March 23 in 1 B.C.E., as used in computations of Easter. See Chapter 9.

3.6 Holidays

It is related that once a Roman asked a question to Rabbi Yoḥanan ben Zakkai: We have festivals and you have festivals; we have the Kalends, Saturnalia, and Kratesis, and you have Passover, Shavuot, and Sukkot; which is the day whereon we and you rejoice alike? Rabbi Yoḥanan ben Zakkai replied: "It is the day when rain falls."

Deuteronomy Rabbah, VII, 7

Until 1923 the date of the Eastern Orthodox Christmas depended on the Julian calendar. At that time, the Ecumenical Patriarch, Meletios IV, convened a congress at which it was decided to use the Gregorian date instead.[3] By 1968 all but the churches of Jerusalem, Russia, and Serbia had adopted the new date, December 25 on the Gregorian calendar. There remain, however, *Palaioemerologitai* groups, especially in Greece, who continue to use the old calendar. Virtually all Orthodox churches continue to celebrate Easter according to the Julian calendar (see Chapter 9).

The occurrence of the old Eastern Orthodox Christmas in a given Gregorian year is somewhat involved. With the current alignment of the Julian and Gregorian calendars, and because the Julian year is always at least as long as the corresponding Gregorian year, Eastern Orthodox Christmas occurs at most once in a given Gregorian year—in modern times it occurs near the beginning. However, far in the past or the future, there are Gregorian years in which it does not occur at all (1100, for example); as the two calendars get further out of alignment (it will take some 50000 years for them to be a full year out of alignment), Eastern Orthodox Christmas will migrate throughout the Gregorian year.

We can write a general function that gives a list of the corresponding R.D. dates of occurrence, within a specified Gregorian year, of a given month and day on the Julian calendar:

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[3] The Congress of the Orthodox Oriental Churches actually adopted a "revised" Gregorian leap-year rule; see footnote on page 57.

https://doi.org/10.1017/9781107415058.006 Published online by Cambridge University Press

$$\textbf{julian-in-gregorian}\,(j\text{-}month, j\text{-}day, g\text{-}year) \stackrel{\text{def}}{=} \tag{3.24}$$

$$\{date_0, date_1\} \cap \textbf{gregorian-year-range}\,(g\text{-}year)$$

where

$$\begin{aligned}
jan_1 &= \textbf{gregorian-new-year}\,(g\text{-}year)\\
y &= (\textbf{julian-from-fixed}\,(jan_1))_{\textbf{year}}\\
y' &= \begin{cases} 1 & \textbf{if } y = -1\\ y+1 & \textbf{otherwise}\end{cases}\\
date_0 &= \textbf{fixed-from-julian}\left(\boxed{y \mid j\text{-}month \mid j\text{-}day}\right)\\
date_1 &= \textbf{fixed-from-julian}\left(\boxed{y' \mid j\text{-}month \mid j\text{-}day}\right)
\end{aligned}$$

Tens of thousands of years from the present, the alignment of the Gregorian and Julian calendars will be such that some Julian dates occur twice in a Gregorian year—the first example of this is in Gregorian year 41104 when Julian date February 28 occurs twice; the function **julian-in-gregorian** correctly returns a list of two R.D. dates in such cases.

For example, we can use this function to determine a list of R.D. dates of December 25 (Julian) for a given year of the Gregorian calendar:

$$\textbf{eastern-orthodox-christmas}\,(g\text{-}year) \stackrel{\text{def}}{=} \tag{3.25}$$

$$\textbf{julian-in-gregorian}\,(\textbf{december}, 25, g\text{-}year)$$

Other fixed Orthodox holidays are the Nativity of the Virgin Mary (September 8), the Elevation of the Life-Giving Cross (September 14), the Presentation of the Virgin Mary in the Temple (November 21), Theophany (January 6), the Presentation of Christ in the Temple (February 2), the Annunciation (March 25), the Transfiguration (August 6), and the Repose of the Virgin Mary (August 15). Orthodox periods of fasting include the Fast of the Repose of the Virgin Mary (August 1–14) and the 40-day Christmas Fast (November 15–December 24).

Orthodox movable holidays and fasts are explained in Chapter 9.

The Armenian church celebrates Christmas on January 6 (Julian) in Jerusalem, and on January 6 (Gregorian) elsewhere.

References

[1] B. Blackburn and L. Holford-Strevens, *The Oxford Companion to the Year*, Oxford University Press, Oxford, 1999.

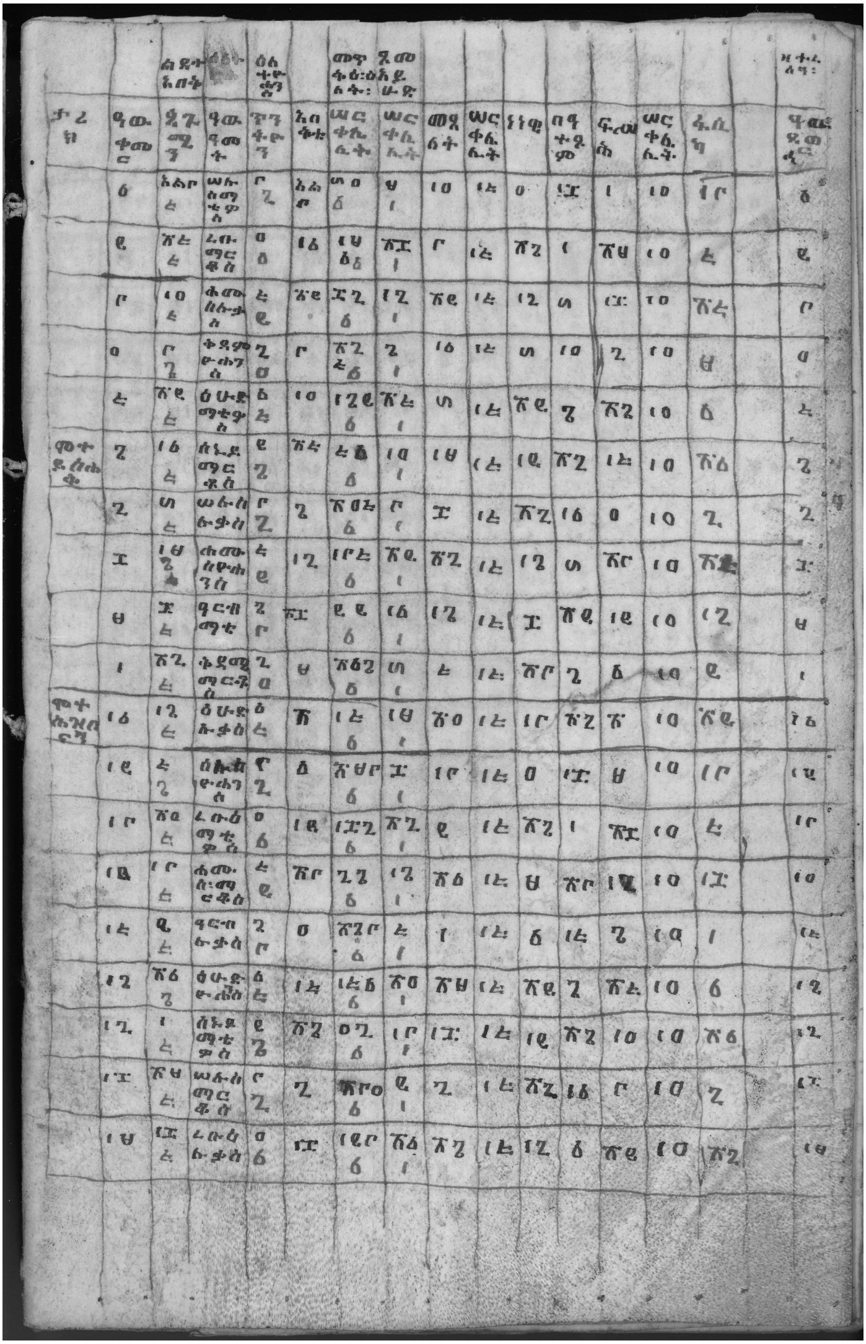
[2] L. E. Doggett, "Calendars," *Explanatory Supplement to the Astronomical Almanac*, P. K. Seidelmann, ed., University Science Books, Mill Valley, CA, pp. 575–608, 1992.

[3] J. K. Fotheringham, "The Calendar," *The Nautical Almanac and Astronomical Ephemeris*, His Majesty's Stationery Office, London, 1931−1934; revised 1935−1938; abridged 1939−1941.

[4] A. K. Michels, *The Calendar of the Roman Republic*, Princeton University Press, Princeton, NJ, 1967.

[5] E. G. Richards, *Mapping Time: The Calendar and its History*, Oxford University Press, Oxford, 1998.

[6] A. E. Samuel, *Greek and Roman Chronology*: *Calendars and Years in Classical Antiquity*, C. H. Beck'sche Verlagsbuchhandlung, Munich, 1972.



Ethiopic computus (*Baḥrā Ḥassab*) in Ge‘ez (1904 E.E.). The third column from the right gives the day of the Ethiopic month (Magābit or Miyāzyā) of Nicæan Easter for years 1417–1435 E.E. (= 1425–1443 C.E.). Other columns give the number of epagomenæ (third column from the left), the epact (sixth column from the left), the day of the Ethiopic month of various fasts, and (nonstandard) dates for Jewish holidays. See [4] for detailed information on such tables. (Courtesy of the Library of Congress, Washington, D.C.; MS 23, Thomas Leiper Kane Manuscript Collection.)