8

The Hebrew Calendar

Do not take these [*visibility*] *calculations lightly . . . for they are deep and difficult and constitute the "secret of intercalation" that was* [*only*] *known to the great sages . . . On the other hand, this computation that is calculated nowadays . . . even school children can master in three or four days.*

Maimonides: *Mishneh Torah, Book of Seasons* (1178)

The Hebrew calendar, promulgated by the patriarch Hillel II in the mid-fourth century[1] and attributed by Sa'adia Gaon to Mosaic revelation, is more complicated than the other calendars we have considered so far. Its complexity is inherent in the requirement that calendar months be strictly lunar whereas Passover must always occur in the spring. Because the seasons depend on the solar year, the Hebrew calendar must harmonize simultaneously with both lunar and solar events, as do all lunisolar calendars, including the Hindu and Chinese calendars described in Chapters 10, 19, and 20. The earliest extant description of the Hebrew calendar is by the famous al-Khowārizmī [13], after whom the words *algebra* and *algorithm* were coined. The most comprehensive early work is by Savasorda of the eleventh century [19]. Much information about the Hebrew calendar in the early modern period can be found in [6].

The earlier, observation-based, Hebrew calendar is described in Section 18.4.

As in the Islamic calendar, days begin at sunset, the week begins on Sunday, and the days for the most part are numbered, not named, as follows:

Sunday	yom rishon (first day)	יום ראשון
Monday	yom sheni (second day)	יום שני
Tuesday	yom shelishi (third day)	יום שלישי
Wednesday	yom revi'i (fourth day)	יום רביעי
Thursday	yom ḥamishi (fifth day)	יום חמישי
Friday	yom shishi (sixth day)	יום ששי
Saturday	yom shabbat (sabbath day)	יום שבת

[1] Bornstein [3] and others dispute the assertion of the tenth-century Hai Gaon that the current calendar was formulated in 359 C.E. (year 670 of the Seleucid Era); see also [17, p. 118]. Stern [23] argues that the Hebrew calendar calculations were not standardized or fixed until at least the ninth century.

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8.1 Structure and History

Iudaicus computus, omnium qui hodie extant antiquissimus, articiosissimus, et elegantissimus. [*Of all methods of intercalation which exist today the Jewish calculation is the oldest, the most skillful, and the most elegant.*]

Joseph Justus Scaliger: *De Emendatione Temporum* (1593)[2]

The Hebrew year consists of 12 months in a common year and 13 in a leap ("gravid" or "embolismic") year:

(1) Nisan	30 days	ניסן
(2) Iyyar	29 days	אייר
(3) Sivan	30 days	סיון
(4) Tammuz	29 days	תמוז
(5) Av	30 days	אב
(6) Elul	29 days	אלול
(7) Tishri	30 days	תשרי
(8) Marḥeshvan	29 or 30 days	חשון or מרחשון
(9) Kislev	29 or 30 days	כסלו
(10) Tevet	29 days	טבת
(11) Shevat	30 days	שבט
{(12) Adar I	30 days	{אדר ראשון
(12) {(13)} Adar {II}	29 days	אדר {שני}

The leap-year structure is given in braces—in a leap year there is an interpolated twelfth month of 30 days called Adar I to distinguish it from the final month, Adar II. The lengths of the eighth and ninth months vary from year to year according to criteria that will be explained below. Our ordering of the Hebrew months follows biblical convention (Leviticus 23:5) in which (what is now called) Nisan is the first month. This numbering causes the Hebrew New Year (Rosh ha-Shanah) to begin on the first of Tishri, which by our ordering is the seventh month—but this too agrees with biblical usage (Leviticus 23:24).

Adding up the lengths of the months, we see that a normal year has 353−355 days, whereas a leap year has 383−385 days. These are the same year lengths as would be possible with an astronomical lunisolar calendar; see Section 18.4.

It will be convenient to have the following constants defined for the Hebrew months:

$$\textbf{nisan} \stackrel{\text{def}}{=} 1 \tag{8.1}$$

$$\textbf{iyyar} \stackrel{\text{def}}{=} 2 \tag{8.2}$$

$$\textbf{sivan} \stackrel{\text{def}}{=} 3 \tag{8.3}$$

$$\textbf{tammuz} \stackrel{\text{def}}{=} 4 \tag{8.4}$$

$$\textbf{av} \stackrel{\text{def}}{=} 5 \tag{8.5}$$

[2] המובאות אינן מיצגות בהכרח את דעות המחברים.

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$$\textbf{elul} \stackrel{\text{def}}{=} 6 \tag{8.6}$$

$$\textbf{tishri} \stackrel{\text{def}}{=} 7 \tag{8.7}$$

$$\textbf{marheshvan} \stackrel{\text{def}}{=} 8 \tag{8.8}$$

$$\textbf{kislev} \stackrel{\text{def}}{=} 9 \tag{8.9}$$

$$\textbf{tevet} \stackrel{\text{def}}{=} 10 \tag{8.10}$$

$$\textbf{shevat} \stackrel{\text{def}}{=} 11 \tag{8.11}$$

$$\textbf{adar} \stackrel{\text{def}}{=} 12 \tag{8.12}$$

$$\textbf{adarii} \stackrel{\text{def}}{=} 13 \tag{8.13}$$

In the Hebrew calendar, leap years occur in years 3, 6, 8, 11, 14, 17, and 19 of the 19-year Metonic cycle. This sequence can be computed concisely by noting that Hebrew year y is a leap year if and only if $(7y + 1) \bmod 19$ is less than 7—another instance of formula (1.83)[3] with $c = 19$, $l = 7$, and $\Delta = 11$. Thus, we determine whether a year is a Hebrew leap year by

$$\textbf{hebrew-leap-year?}\,(\textit{h-year}) \stackrel{\text{def}}{=} ((7 \times \textit{h-year} + 1) \bmod 19) < 7 \tag{8.14}$$

and the number of months in a Hebrew year by

$$\textbf{last-month-of-hebrew-year}\,(\textit{h-year}) \stackrel{\text{def}}{=} \tag{8.15}$$

$$\begin{cases} \textbf{adarii} & \textbf{if hebrew-leap-year?}\,(\textit{h-year}) \\ \textbf{adar} & \textbf{otherwise} \end{cases}$$

The biblically mandated sabbatical years (Exodus 23:10–11) are—by current reckoning—those whose Hebrew year number is a multiple of 7:

$$\textbf{hebrew-sabbatical-year?}\,(\textit{h-year}) \stackrel{\text{def}}{=} (\textit{h-year} \bmod 7) = 0 \tag{8.16}$$

Sabbatical years no longer bear calendrical significance.

The number of days in a Hebrew month is a more complex issue. The twelfth month, Adar or Adar I, has 29 days in a common year and 30 days in a leap year, but the numbers of days in the eighth month (Marḥeshvan) and ninth month (Kislev) depend on the overall length of the year, which in turn depends on factors discussed later in this section.

[3] An equivalent formula appears in Slonimski [20, p. 21]; see [21] for a fascinating article about this polymath.

The beginning of the Hebrew New Year is determined by the occurrence of the mean new moon (conjunction) of the seventh month (Tishri), subject to possible postponements of 1 or 2 days. The new moon of Tishri A.M.[4] 1, the first day of the first year for the Hebrew calendar, is fixed at Sunday night at 11:11:20 p.m. Because Hebrew days begin at sunset, whereas our fixed dates begin at midnight, we define the epoch of the Hebrew calendar (that is, Tishri 1, A.M. 1) to be Monday, September 7, −3760 (Gregorian) or October 7, 3761 B.C.E. (Julian).

The Hebrew day is traditionally divided into 24 hours, and the hour is divided into 1080 *parts* (*ḥalaqim*), and thus a day has 25920 parts of $3\frac{1}{3}$ seconds duration each. These divisions are of Babylonian origin. The new moon of Tishri A.M. 1, which occurred 5 hours and 204 parts after sunset (6 p.m.) on Sunday night, is called *molad* ***beharad***, because the numerical value of the letter ***beth*** is 2, signifying the second day of the week; ***heh*** is 5 (hours); ***resh*** = 200 parts; ***daleth*** = 4 parts. Other epochs and leap-year distributions appear in classical and medieval literature. In particular, the initial conjunction of the epoch starting 1 year later, called ***weyad*** (signifying 6 days, 14 hours), occurred on Friday at exactly 8 a.m. on the morning when Adam and Eve were created according to the traditional chronology.[5]

The length of a mean lunar period in the traditional representation is 29 days, 12 hours, and 793 parts, or $29\frac{13753}{25920} \approx 29.530594$ days. This is a classical value for the lunar (synodic) month, attributed to Cidenas in about 383 B.C.E. and was an integral part of what is called "System B" of Babylonian astronomy [16]; it was used by Ptolemy in his *Almagest*.[6] With $354^{\mathrm{d}}8^{\mathrm{h}}48^{\mathrm{m}}40^{\mathrm{s}}$ for an ordinary year and $383^{\mathrm{d}}21^{\mathrm{h}}32^{\mathrm{m}}43\frac{1}{3}^{\mathrm{s}}$ for a leap year, this value gives an average Hebrew year length of about 365.2468 days. The start of each New Year, Rosh ha-Shanah (Tishri 1), coincides with the calculated day of the mean conjunction (new moon) of Tishri—12 months after the previous New Year conjunction in ordinary years, and 13 in leap years—unless one of 4 delays is mandated:

1. If the time of mean conjunction is at midday or after, then the New Year is delayed.[7]

[4] *Anno Mundi*; in the (traditional) year of the world (since creation).

[5] The ambiguities in the Hebrew epoch have led to some confusion in medieval, as well as modern, times. For example, M. Kantor's *The Jewish Time Line Encyclopedia*, Jason Aronson, Northvale, NJ (1989), erroneously gives 69 C.E., rather than 70 C.E., as the date on which Titus captured Jerusalem. Similarly, Sephardic Jews, every Tishah be-Av (see page 130), announce in the synagogue the wrong number of elapsed years since the fall of Jerusalem.

[6] The astronomer and mathematician Abraham bar Ḥiyya Savasorda (eleventh century) suggested that the reason for the choice of 1080 parts per hour is that it is the smallest number that allows this particular value of the length of a month to be expressed with an integral number of parts (in other words, 793/1080 is irreducible).

[7] According to Maimonides [15, 6:2] (cf. al-Bīrūnī [2, p. 149]), seasonal time is used in which "daylight hours" and "nighttime hours" have different lengths, which vary according to the seasons (see Section 14.8). Postponement occurs if the conjunction is 18 variable-length hours or more after sunset. Others use fixed-length hours, but the computation is unaffected, because true noon is 18 temporal (seasonal) hours after true sunset just as mean local noon is 18 civil hours after mean local sunset (6 p.m. local mean time). Savasorda and others say that equal-length hours, not variable-length hours, are intended in the Hebrew calendar calculations; dates would be unaffected.

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2. In no event may the New Year (Rosh ha-Shanah) be on a Sunday, Wednesday, or Friday. (This rule is called *lo iddo rosh*.)[8] If the conjunction is on Saturday, Tuesday, or Thursday afternoon, then this rule combines with the previous rule and results in a 2-day delay.

3. In some cases (about once in 30 years) an additional delaying factor may need to be employed to keep the length of a year within the allowable ranges. It is the irregular effect of the second delay that makes this necessary: if the conjunction is before noon on a Tuesday of a common year, and the conjunction of the following year is at noon on Saturday or later (possibly after sunset), then the previous rules would delay Rosh ha-Shanah until Monday (a Saturday afternoon conjunction is put off by the first rule and Rosh ha-Shanah on Sunday is precluded by the second rule). This would require a unacceptable year length of 356 days, and thus instead the *current* Rosh ha-Shanah is delayed (skipping Wednesday) until Thursday, giving a 354-day year. For the following year's conjunction to fall on a Saturday afternoon, the current year's must have occurred after 3:11:20 a.m. The prior year cannot become too long because of this delay, for its New Year conjunction must have been on Friday (in a common year) or Wednesday (in a leap year) and would have been delayed a day by the second rule.

4. In rare cases (about once in 186 years), Rosh ha-Shanah on a Monday after a leap year can pose a similar problem by causing the year just ending to be too short—when the *prior* New Year conjunction was after midday on Tuesday and was, therefore, delayed until Thursday. If the conjunction were after midday Tuesday the previous year, then in the current year it would be after 9:32:43$\frac{1}{3}$ a.m. on Monday. In this case, Rosh ha-Shanah is postponed from Monday to Tuesday, extending the leap year just ending from 382 days to 383. The current year cannot become too short because of this delay; it is shortened from 355 days to 354, the following Rosh ha-Shanah being delayed until Saturday.

The precise rules for delays were the subject of a short-lived dispute (921–923 C.E.) between Palestinian and Babylonian Jewish authorities (the best source in English for details of the controversy is [23, pp. 264–275]). In 923 C.E. the calculated conjunction fell just after midday, but the Palestinian authority, Aaron ben Meir, insisted that the first delaying rule applied only when the conjunction was at 12:35:40 p.m. (that is, noon plus 624 parts) or later, presumably because they (the Palestinians) did their calculations from Nisan instead of Tishri, and rounded the time of the epochal new moon differently. Because of the retroactive effect of the third delay, this had already affected the dates in 921 (see the sample calculation beginning on page 124). In the end, the Babylonian gaon, Sa'adia ben

[8] Excluding Wednesday and Friday serves the ritual purpose of preventing Yom Kippur (Tishri 10) from falling on Friday or Sunday; excluding Sunday prevents Hoshana Rabba (Tishri 21) from falling on Saturday. Maimonides [15, 7:8] ascribes this correction in the calendar of approximately half a day, on the average, to the need to better match the mean date of appearance of the new moon of the month of Tishri; al-Bīrūnī [2] attributes it to astrological considerations. The real purpose of the delay is a moot point.

Joseph al-Fayyūmi, prevailed, and the rules have since been fixed as given above. (Some scant details can be found in [17, vol. III, p. 119] and [10, col. 539–540]; [3] gives a full discussion of the controversy; see also [12].) Interestingly, according to Maimonides [15, 5:13], the final authority in calendrical matters is vested in the residents of the Holy Land, and their decision—even if erroneous—should be followed worldwide:

> Our own calculations are solely for the purpose of making the matter available to public knowledge. Since we know that residents of the Land of Israel use the same method of calculation, we perform the same operations in order to find out and ascertain what day it is that has been determined by the people of Israel.

One fairly common misconception regarding the Hebrew calendar is that the correspondence with the Gregorian calendar repeats every 19 years. This, however, is usually not the case because of the irregular Gregorian leap-year rule and the irregular applicability of the delays. Nor does the Hebrew calendar repeat its pattern every 247 years. In the seventeenth century, Hezekiah ben David da Silva of Jerusalem complained about published tables for the Hebrew calendar:[9]

> I have seen disaster and scandal [on the part] of some intercalators who are of the [erroneous] opinion that the character [of years] repeats every thirteen cycles [$13 \times 19 = 247$ years]. For the sake of God, do not rely and do not lean on them. "Far be it from thee to do after this manner," which will—perish the thought—cause the holy and awesome fast to be nullified, leaven to be eaten on Passover, and the holidays to be desecrated. Therefore, you the reader, "Hearken now unto my voice, I will give thee counsel, and God be with thee." Be cautious and careful lest you forget ... what I am writing regarding this matter, since it is done according to exact arithmetic, "divided well," and is precise on all counts ... from the 278th cycle [1521 C.E.] until the end of time. "Anyone who separates from it, it is as if he separates [himself] from life [itself]."

By the "character" of a year da Silva means the day of the week on which New Year falls and the length of the year. In fact, the Hebrew calendar repeats only after 689472 years (as was pointed out by the celebrated Persian Muslim writer, al-Bīrūnī [2, p. 154] in 1000 C.E.): The 19-year cycle contains exactly

$$\begin{aligned}&991 \text{ weeks, 2 days, 16 hours, and 595 parts}\\ &\qquad = 991 \text{ weeks and } 69715 \text{ parts}\end{aligned}$$

A week has 181440 parts, so it takes

$$\begin{aligned}\frac{\operatorname{lcm}(69715, 181440) \text{ parts}}{69715 \text{ parts/cycle}} &= \frac{2529817920 \text{ parts}}{69715 \text{ parts/cycle}}\\ &= 36288 \text{ cycles}\\ &= 689472 \text{ years}\end{aligned}$$

[9] *Peri Ḥadash, Oraḥ Ḥayyim*, 428. For a recent study of the early origin of this imagined cycle, see "The Origins of the 247-year Calendar Cycle," N. Vidro, *Aleph: Historical Studies in Science and Judaism*, vol. 17, pp. 95–137, 2017.

for the excess parts to accumulate into an integer number of weeks, and for the calendar to return to the same pattern of delays. Thus, the exact correspondence of Hebrew dates (which has a mean year length of $365\frac{24311}{98496}$ days) and dates on the Gregorian calendar (which has a 400-year cycle) repeats only after

$$\begin{aligned}\operatorname{lcm}\left(689472 \times 365\tfrac{24311}{98496}, 400 \times 365\tfrac{97}{400}\right)\\ &= 5255890855047 \text{ days}\\ &= 14390140400 \text{ Gregorian years}\\ &= 14389970112 \text{ Hebrew years}\end{aligned}$$

Similar astronomically long periods are needed for other pairs of calendars to match up exactly.

8.2 Implementation

You have already seen … how much computation is involved, how many additions and subtractions are still necessary, despite our having exerted ourselves greatly to invent approximations that do not require complicated calculations. For the path of the moon is convoluted. Hence wise men have said: the sun knows its way, the moon does not …

Maimonides: *Mishneh Torah, Book of Seasons* (1178)

The epoch of the Hebrew calendar is R.D. −1373427:

$$\textbf{hebrew-epoch} \stackrel{\text{def}}{=} \tag{8.17}$$

$$\textbf{fixed-from-julian}\left(\boxed{3761 \text{ B.C.E.} \mid \textbf{october} \mid 7}\right)$$

We can calculate the time elapsed on the Hebrew calendar from the Hebrew epoch until the new moon of Tishri for Hebrew year y by computing

$$m \times \left(29^{\text{d}}12^{\text{h}}44^{\text{m}}3\tfrac{1}{3}^{\text{s}}\right) - (48^{\text{m}}40^{\text{s}}) \tag{8.18}$$

where m is the number of months before year y, because the first mean conjunction was $48^{\text{m}}40^{\text{s}}$(= 876 parts) before midnight on the epoch, or 5 hours 204 parts after nominal sunset (see page 116). To compute the total number of months, leap and regular, we just apply formula (1.86) with $c = 19$, $l = 7$, and $\Delta = 11$:[10]

$$\lfloor(7y - 6)/19\rfloor + 12(y - 1) = \lfloor(235y - 234)/19\rfloor$$

More generally, the fixed moment of the mean conjunction, called the *molad* (plural, *moladot*), of any month of the Hebrew calendar is computed by

$$\textbf{molad}\,(h\text{-}year, h\text{-}month) \stackrel{\text{def}}{=} \tag{8.19}$$

$$\textbf{hebrew-epoch} - \frac{876}{25920} + months\text{-}elapsed \times \left(29 + 12^{\text{h}} + \frac{793}{25920}\right)$$

[10] An analogous formula for the number of nonleap years was used by Gauss [9].

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where

$$y = \begin{cases} \textit{h-year} + 1 & \textbf{if } \textit{h-month} < \textbf{tishri} \\ \textit{h-year} & \textbf{otherwise} \end{cases}$$

$$\textit{months-elapsed} = \textit{h-month} - \textbf{tishri} + \left\lfloor \frac{1}{19} \times (235 \times y - 234) \right\rfloor$$

readjusting for the year starting with Tishri. The degree to which **molad** approximates the astronomical new moon can be seen in Figure 8.1, which shows a scatter plot of the error (in hours) for Nisan for Gregorian years −1000 to 5000 (= 2760 – −8760 A.M.). Indeed, any arithmetic calendar that uses a mean value for the lunar month, such as the Old Hindu lunisolar calendar (Section 10.3), must show similar deviations, since the true length of the month varies greatly (see Section 14.6).

To implement the first of the four delays (putting off the New Year if the calculated conjunction is in the afternoon), all we need to do is add 12 hours to the time of the epochal conjunction and let the day be the integer part (the floor) of the value obtained. This is analogous to equation (1.92), except that we are counting the days in months of average length $29\frac{13753}{25920}$ days rather than in years. The initial conjunction is $11^{\mathrm{h}}11^{\mathrm{m}}20^{\mathrm{s}}$—that is, 12084 parts—into the determining period, which began at noon on the day before the epoch.

To test for Sunday, Wednesday, and Friday, as required by the second delay, we can use $(3d \bmod 7) < 3$, as in equation (1.82) with $c = 7$ and $l = 3$, to determine

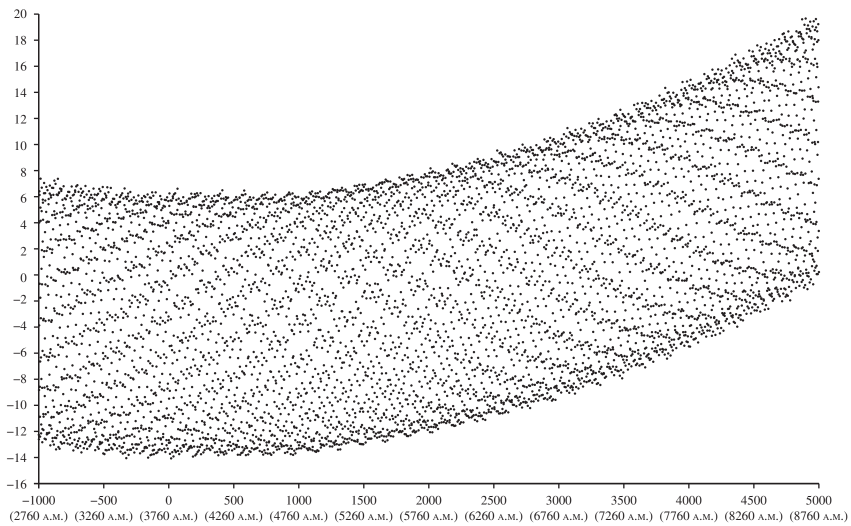

Figure 8.1 Molad of Nisan minus the actual moment of the new moon, Jerusalem local time, in hours, for Gregorian years −1000 to 5000 (= 2760−8760 A.M.). (Suggested by I. L. Bromberg.)

whether d is one of the three evenly spaced excluded days. These two delays are incorporated in the following function:

$$\textbf{hebrew-calendar-elapsed-days}\,(h\text{-}year) \stackrel{\text{def}}{=} \tag{8.20}$$

$$\begin{cases} days+1 & \textbf{if } ((3\times(days+1)) \bmod 7) < 3 \\ days & \textbf{otherwise} \end{cases}$$

where

$$\begin{aligned} months\text{-}elapsed &= \left\lfloor \frac{1}{19} \times (235 \times h\text{-}year - 234) \right\rfloor \\ parts\text{-}elapsed &= 12084 + 13753 \times months\text{-}elapsed \\ days &= 29 \times months\text{-}elapsed + \left\lfloor \frac{parts\text{-}elapsed}{25920} \right\rfloor \end{aligned}$$

Because the count of elapsed days begins with Sunday evening (which is already the second day of the week from the point of view of the Hebrew calendar), we use $days + 1$ for the number of days since the Sunday before the first molad. Whole days and fractional days (parts) are computed separately, so that 32 bits suffice for dates in the foreseeable future; however, this calculation comes close to the 32-bit limit. To avoid such large numbers one can compute days, hours, and parts separately:

$$\begin{aligned} parts\text{-}elapsed &= 204 + 793 \times (months\text{-}elapsed \bmod 1080) \\ hours\text{-}elapsed &= 11 + 12 \times months\text{-}elapsed \\ &\quad + 793 \times \left\lfloor \frac{months\text{-}elapsed}{1080} \right\rfloor + \left\lfloor \frac{parts\text{-}elapsed}{1080} \right\rfloor \\ days &= 29 \times months\text{-}elapsed + \left\lfloor \frac{hours\text{-}elapsed}{24} \right\rfloor \end{aligned}$$

When one can work directly with rational numbers, one may just let

$$days = \left\lfloor \textbf{molad}(\textbf{tishri}, h\text{-}year) - \textbf{hebrew-epoch} + 12^{\text{h}} \right\rfloor$$

using the **molad** function.

The two remaining delays depend on the lengths of the prior and current years that would result from the putative New Year dates suggested by the previous function. If the current year were 356 days then it would be too long, and we would delay its start by 2 days. If the prior year were 382 days long then we delay its end by 1 day. Rather than check the day of the week, the time of conjunction, and the leap-year status of the prior and current year, as in the traditional formulation of these delays, we just check for unacceptable year lengths:

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$$\textbf{hebrew-year-length-correction}\,(h\text{-}year) \stackrel{\text{def}}{=} \tag{8.21}$$

$$\begin{cases} 2 & \textbf{if } ny_2 - ny_1 = 356 \\ 1 & \textbf{if } ny_1 - ny_0 = 382 \\ 0 & \textbf{otherwise} \end{cases}$$

where

$$\begin{aligned} ny_0 &= \textbf{hebrew-calendar-elapsed-days}\,(h\text{-}year - 1) \\ ny_1 &= \textbf{hebrew-calendar-elapsed-days}\,(h\text{-}year) \\ ny_2 &= \textbf{hebrew-calendar-elapsed-days}\,(h\text{-}year + 1) \end{aligned}$$

Adding the value of this function to the number of elapsed days determines the day on which the year begins. To get the R.D. date of the New Year, we have to add the (negative) epoch:

$$\textbf{hebrew-new-year}\,(h\text{-}year) \stackrel{\text{def}}{=} \tag{8.22}$$

$$\begin{aligned} &\textbf{hebrew-epoch} + \textbf{hebrew-calendar-elapsed-days}\,(h\text{-}year) \\ &\quad + \textbf{hebrew-year-length-correction}\,(h\text{-}year) \end{aligned}$$

As already mentioned, the length of the year determines the lengths of the two varying months, Marḥeshvan and Kislev. Marḥeshvan is long (30 days) if the year has 355 or 385 days; Kislev is short (29 days) if the year has 353 or 383 days. The length of the year, in turn, is determined by the dates of the Hebrew New Years (Tishri 1) preceding and following the year in question:

$$\textbf{last-day-of-hebrew-month}\,(h\text{-}year, h\text{-}month) \stackrel{\text{def}}{=} \tag{8.23}$$

$$\begin{cases} 29 & \textbf{if } h\text{-}month \in \{\textbf{iyyar}, \textbf{tammuz}, \textbf{elul}, \textbf{tevet}, \textbf{adarii}\} \text{ or} \\ & \quad \{h\text{-}month = \textbf{adar} \text{ and not } \textbf{hebrew-leap-year?}\,(h\text{-}year)\} \text{ or} \\ & \quad \left\{ \begin{array}{l} h\text{-}month = \textbf{marheshvan} \text{ and} \\ \text{not } \textbf{long-marheshvan?}\,(h\text{-}year) \end{array} \right\} \text{ or} \\ & \quad \{h\text{-}month = \textbf{kislev} \text{ and } \textbf{short-kislev?}\,(h\text{-}year)\} \\ 30 & \textbf{otherwise} \end{cases}$$

Here,

$$\textbf{long-marheshvan?}\,(h\text{-}year) \stackrel{\text{def}}{=} \tag{8.24}$$

$$\textbf{days-in-hebrew-year}\,(h\text{-}year) \in \{355, 385\}$$

Also,

$$\textbf{short-kislev?}\,(h\text{-}year) \stackrel{\text{def}}{=} \tag{8.25}$$

$$\textbf{days-in-hebrew-year}\,(h\text{-}year) \in \{353, 383\}$$

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and

$$\textbf{days-in-hebrew-year}\,(h\text{-}year) \stackrel{\text{def}}{=} \qquad (8.26)$$

$$\textbf{hebrew-new-year}\,(h\text{-}year+1) - \textbf{hebrew-new-year}\,(h\text{-}year)$$

With the foregoing machinery, we are now ready to convert from any Hebrew date to an R.D. date:

$$\textbf{fixed-from-hebrew}\left(\begin{array}{|c|c|c|}\hline year & month & day \\ \hline\end{array}\right) \stackrel{\text{def}}{=} \qquad (8.27)$$

$$\textbf{hebrew-new-year}\,(year) + day - 1$$
$$+ \left\{\begin{array}{ll} \left(\sum_{m \geqslant \textbf{tishri}}^{p(m)} \textbf{last-day-of-hebrew-month}\,(year, m)\right) & \textbf{if } month < \textbf{tishri} \\ + \left(\sum_{m \geqslant \textbf{nisan}}^{m < month} \textbf{last-day-of-hebrew-month}\,(year, m)\right) & \\ \sum_{m \geqslant \textbf{tishri}}^{m < month} \textbf{last-day-of-hebrew-month}\,(year, m) & \textbf{otherwise} \end{array}\right\}$$

where

$$p\,(m) = m \leqslant \textbf{last-month-of-hebrew-year}\,(year)$$

To the fixed date of the start of the given year we add the number of elapsed days in the given month and the length of each elapsed month. We distinguish between months before and after Tishri, which is the seventh month, though the New Year begins with its new moon. For dates in the second half of the year (months 1 through 6) we need to include the lengths of all months from Tishri until **last-month-of-hebrew-year** (month 12 or 13).

Conversion to Hebrew dates is done as follows:

$$\textbf{hebrew-from-fixed}\,(date) \stackrel{\text{def}}{=} \begin{array}{|c|c|c|}\hline year & month & day \\ \hline\end{array} \qquad (8.28)$$

where

$$approx = \left\lfloor \frac{98496}{35975351} \times (date - \textbf{hebrew-epoch}) \right\rfloor + 1$$

$$year = \underset{y \geqslant approx-1}{\textbf{MAX}} \left\{ \textbf{hebrew-new-year}\,(y) \leqslant date \right\}$$

$$start = \left\{\begin{array}{ll} \textbf{tishri} & \textbf{if } date < \textbf{fixed-from-hebrew}\left(\begin{array}{|c|c|c|}\hline year & \textbf{nisan} & 1 \\ \hline\end{array}\right) \\ \textbf{nisan} & \textbf{otherwise} \end{array}\right.$$

$$month = \underset{m \geqslant start}{\textbf{MIN}} \left\{ date \leqslant \textbf{fixed-from-hebrew}\left(\begin{array}{|c|c|c|}\hline year & m & \begin{array}{l}\textbf{last-day-of-}\\ \textbf{hebrew-month}\\ (year, m)\end{array} \\ \hline\end{array}\right)\right\}$$

$$day = date - \textbf{fixed-from-hebrew}\left(\begin{array}{|c|c|c|}\hline year & month & 1 \\ \hline\end{array}\right) + 1$$

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We first approximate the Hebrew year by dividing the number of elapsed days by the average year length, 35975351/98496 days. (A simpler value—even 365.25—can be used instead.) The irregularity of the year lengths means that the estimate *approx* can be off by 1 in either direction. Thus we search for the right year, adding 1 to *approx* − 1 for each year *y* whose New Year is not after *date*. To determine the Hebrew month, we search forward from Nisan or Tishri until we reach the first month that ends on or after *date*.

Consider, as an example, the calculation of the date of Passover in 922 C.E.—that is, Nisan 15, A.M. 4682 (see page 117 for the historical significance of this year). The mean conjunction of the preceding Tishri fell on Wednesday, September 5, 921 C.E. (Julian), R.D. 336276, at 5:51:46$\frac{2}{3}$ a.m. The mean conjunction of the following Tishri fell on Tuesday, September 29, 922 C.E. (Julian), at 3:24:30 a.m. At the latter time, $57909 = (235 \times 4683 - 234)/19$ months of mean length $29\frac{13753}{25920}$ had elapsed since the primeval conjunction, to which we add 12084/25920 to count from noon on the Sunday before the epoch. By the traditional reckoning, that is Tuesday, 9 hours and 441 parts since sunset the preceding evening. Hebrew year 4683 was year 9 of the 247th 19-year cycle, which is not a leap year, making 4683 an instance of the third delay. Because this conjunction was later than 9 hours and 204 parts, the conjunction of the following year, 4684, fell on Saturday afternoon, just 237 parts (13.167 minutes) after midday, for which time the first two delays apply. Specifically, equation (8.20) yields

hebrew-calendar-elapsed-days(4682) = 1709704
hebrew-calendar-elapsed-days(4683) = 1710087
hebrew-calendar-elapsed-days(4684) = 1710443

With the first two delays, but without the third delay, year 4683 would be of $1710443 - 1710087 = 356$ days in duration, an unacceptable length. Thus, the first of Tishri 4683 is put off 2 days to Thursday, September 26, R.D. 336662. The start of year 4682 is delayed until Thursday, making 4682 a "long" leap year with a total of 385 days. Tishri (month 7) and Shevat (month 11) are always 30 days long, Tevet (month 10) is 29 days, Marḥeshvan (month 8) and Kislev (month 9) both have 30 days in a long year, and in a leap year Adar I (month 12) has 30 days and Adar II (month 13) has 29. Adding these ($5 \times 30 + 2 \times 29 = 208$), plus the 14 days of Nisan (month 1), to the R.D. date of Rosh ha-Shanah of 4682, we arrive at R.D. $336277 + 208 + 14 = 336499$ as the starting date of Passover.[11] That date is Tuesday, April 16, 922 C.E. (Julian) and April 21, 922 (Gregorian). Were the first delay not applied in 4684, there would have been no need for the third delay in 4683. Were it not for the third delay, Hebrew year 4682 would have been "short," and Passover in 922—as well as all other dates between Tevet 1 in late 921 and

[11] Dates during the second half of the Hebrew year (from Nisan through Elul) depend *only* on the date of the following Rosh ha-Shanah, because the intervening months are all of fixed length, and thus for hand calculations it is easier to count backwards from the following Rosh ha-Shanah, subtracting 30 days for Sivan and Av, 29 days for Iyyar, Tammuz, and Elul, and 16 for the remainder of Nisan, rather than always starting with the preceding Rosh ha-Shanah, as in our algorithm.

Elul 29 in the summer of 922—would have occurred 2 days earlier. Dates in Kislev would have been 1 day earlier.

8.3 Inverting the Molad[12]

If you see such calculations in other tables, which differ from what I say—as I have seen that what I have calculated in my tables does not agree with them—ignore their reckonings and do not consent to their calculations, but rely on what I have counted for you, no less no more.

Issachar ben Mordecai Susan: *Tikkun Yissakhar* (1564)

Suppose we are told at what time of day and on which day of the week the molad of some Hebrew month occurs; can we determine the date (month and year) of that molad? Surprisingly, the answer is yes, if we assume that the date is within a range of about 14000 years.

Recall that the interval from molad to molad is $29\frac{1}{2}$ days and 793 parts of an hour. There are 1080 parts per hour, so there are $w = 7 \times 24 \times 1080 = 181440$ parts in a week; there are four weeks plus $r = 36 \times 1080 + 793 = 39673$ parts in a molad, so each successive molad advances in the week by r parts. Because r and w are relatively prime, a molad will recur on the same day of the week and at the same time of day as another molad only after w months, about 14670 years—this means that since the epoch of the Hebrew calendar and for more than 8000 years into the future the day/time combination of the molad uniquely determines the Hebrew month and year.

Imagine time as a sequence of Hebrew calendar parts numbered $0, 1, 2, \ldots,$ each labeled with a pair of numbers $\langle a, b\rangle$, where a is the part number within the molad and b is the part number within the week. Thus the first component repeats after $4\times w+r = 765433$ parts and the second repeats after 181440 parts. Because 765433 and 181440 are relatively prime, there are $765433 \times 181440 = 138880163520$ labels; we want to determine n, $0 \leqslant n < 138880163520$, from the pair $\langle a, b\rangle$ such that $a = n \bmod 765433$ and $b = n \bmod 181440$. This is precisely the matter discussed in Section 1.13, where equation (1.70) gives us the answer by setting the cycle lengths $c = 765433$ and $d = 181440$ and the values $a = 0$ (a specifies that it is the start of the molad), $\Gamma = 0$ (the offset Γ specifies that part 0 is the start of a molad), $\Delta = (18 + 5) \times 1080 + 204 = 25044$ [the offset Δ specifies that the cycle began 5 hours, 204 parts after sunset on weekday 0 (Sunday)], and b to the given position in the week of the molad we seek to determine. We need the multiplicative inverse of c modulo d, which by equation (1.72) is

$$k = c^{\varphi(d)-1} \bmod d = 74377$$

The ordinal position of the specified molad in the sequence of 138880163520 labels is hence

$$n = (56930610241b - 1425770202875604) \bmod 138880163520$$

Because the greatest common divisor of the three integers is 765433, this becomes

$$n/765433 = (74377b - 34548) \bmod 181440$$

[12] This problem was suggested by Sacha Stern. Tabular methods have been around at least since the work of Isaac Israeli [11] in the early fourteenth century.

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Of course, since n is in parts and 765433 is the number of parts per molad, $n/765433$ is the number of moladot; that is, $74377b - 34548$ is the residue class of the desired molad (one occurring at part b in the week), modulo the cycle of 181440 moladot after which the moladot repeat their positions.

Working with the rationals, we can express everything in terms of days, rather than parts. Let $m = b/25920$, where 25920 is the number of parts in a 24-hour day. We divide the previous equation through by 25920, giving

$$r = (74377m - 2879/2160) \bmod 7$$

where $r \times 765433 = n/25920$ counts the number of elapsed days and fractional days until the desired occurrence of a molad. Assuming that we want the first occurrence since the Hebrew epoch, we add this to the epoch—adjusted 876 parts (expressed in days) backward to the moment of the initial molad **beharad**,

$$\textbf{hebrew-epoch} - \frac{876}{25920} = \textbf{molad}(1, \textbf{tishri})$$

giving

$$\textbf{fixed-from-molad}\,(moon) \stackrel{\text{def}}{=} \tag{8.29}$$

$$\textbf{fixed-from-moment}\,(\textbf{molad}\,(1, \textbf{tishri}) + r \times 765433)$$

where

$$r = \left(74377 \times moon - \frac{2879}{2160}\right) \bmod 7$$

(This calculation requires exact rational arithmetic and 64-bit integers.) One could just as easily choose an arbitrary starting point, **molad**$(year, month)$, instead of **molad**(1, **tishri**), by replacing the offset 2879/2160 in (8.29) with

$$[74377 \times \textbf{molad}(year, month)] \bmod 7$$

Then the calculation would compute the first occurrence of a molad at the given time starting from that point onward.

The time of the molad is nowadays specified as the day of the week, d, together with h hours measured from midnight, m minutes, and p parts. To convert such a molad to a point in the weekly cycle, we express the time as a fraction of a day, using mixed-radix notation (Section 1.10):

$$moon = \langle d, h, m, p\rangle \xleftarrow{\text{rad}} \langle ; 24, 60, 18\rangle$$

For example, a molad of Wednesday, 18 hours, 35 minutes, 11 parts is specified by

$$\textbf{wednesday} + \frac{18}{24} + \frac{35}{24 \times 60} + \frac{11}{24 \times 60 \times 18} = \frac{97841}{25920}$$

and **fixed-from-molad**(97841/25920) is R.D. 735913 = Kislev 28, 5776 A.M. = November 11, 2015 (Gregorian), the day of the molad of the coming month, Tevet, 5776 A.M. To display a molad occurring at moment t in this traditional format, we use the inverse radix operation

$$(t \bmod 7) \xrightarrow{\text{rad}} \langle ; 24, 60, 18 \rangle$$

Traditionally the molad was specified by the day of the week, d, together with h hours measured from sunset not midnight, and p parts of an hour. To convert such a molad to a point in the weekly cycle, we would use

$$moon = \langle d, h-6, p \rangle \xleftarrow{\text{rad}} \langle ; 24, 1080 \rangle$$

because sunset is 6 hours before midnight. For example, the undated Oxford Bodleian manuscript, Pococke 368, folio 221 recto, refers to a traditional molad on a Sunday, at 2 hours and 240 parts:

$$\mathbf{sunday} - \frac{4}{24} + \frac{240}{24 \times 1080} = -\frac{17}{108}$$

and **fixed-from-molad**(−17/108) is R.D. 292452 = Elul 28, 4561 A.M. = September 11, 801 C.E. (Julian), meaning that this is the molad of the coming month, Tishri, 4562 A.M. For the moment t of a molad in traditional format, one computes the inverse:

$$(t + 6^{\text{h}}) \bmod 7 \xrightarrow{\text{rad}} \langle ; 24, 1080 \rangle$$

The easiest way to extract the coming Hebrew month and year from the result of **fixed-from-molad** is to apply **hebrew-from-fixed** to a few days afterwards, since the molad often precedes the first day of a month:

$$\begin{aligned} year &= \textbf{hebrew-from-fixed}(\textbf{fixed-from-molad}(moon) + 5)_{\textbf{year}} \\ month &= \textbf{hebrew-from-fixed}(\textbf{fixed-from-molad}(moon) + 5)_{\textbf{month}} \end{aligned}$$

Because **fixed-from-molad** inverts **molad**, we have the identity

$$\textbf{molad}(year, month) = moon$$

The year and month can also be derived directly from $k = n/765433$, the number of elapsed months. Applying formula (1.90) with $c = 19$, $\ell = 7$, $\Delta = 11$, and $L = 12$, we get

$$year = \lfloor 19k + 253 - ([7 \times 11] \bmod 19)/235 \rfloor = \lfloor (19k + 252)/235 \rfloor$$

The number of months unaccounted for is

$$m = \lfloor ([19k + 252] \bmod 235)/19 \rfloor = \lfloor ([19k + 17] \bmod 235)/19 \rfloor$$

To obtain the corresponding month number, considering that years begin with the seventh month, Tishri, we need to adjust m:

$$month = (\textbf{tishri} + m) \bmod [1 \,..\, \textbf{last-month-of-hebrew-year}(year)]$$

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8.4 Holidays and Fast Days

In the days of wicked Trajan, a son was born to him on Tishah be-Av and they fasted; his daughter died on Hanukkah and they lit candles. His wife sent to him and said, rather than conquer the Barbarians, come and conquer the Jews who have revolted ... He came ... and the blood flowed in the sea until Cyprus.

Jerusalem Talmud (Succah 5:1)

As throughout this book, we consider our aim to be the determination of holidays that occur in a specified Gregorian year. Because the Hebrew year is, within thousands of years of the present, consistently aligned with the Gregorian year, each Jewish holiday occurs just once in a given Gregorian year (with a minor exception noted below). The major holidays of the Hebrew year occur on fixed days on the Hebrew calendar but only in fixed seasons on the Gregorian calendar. They are easy to determine on the Gregorian calendar with the machinery developed above provided that we observe that the Hebrew year beginning in the Gregorian year y is given by

$$\text{Hebrew New Year occurring in the fall of Gregorian year } y = y + 1 - \textbf{gregorian-year-from-fixed}(\textbf{hebrew-epoch})$$

The Hebrew year that began in the fall of 1 (Gregorian) was A.M. 3762. This implies that holidays occurring in the fall and early winter of the Gregorian year y occur in the Hebrew year $y + 3761$, but holidays in the late winter, spring, and summer occur in Hebrew year $y + 3760$. For example, to find the R.D. date of Yom Kippur (Tishri 10) in a Gregorian year, we would use

$$\textbf{yom-kippur}\,(g\text{-}year) \stackrel{\text{def}}{=} \tag{8.30}$$

$$\textbf{fixed-from-hebrew}\left(\boxed{h\text{-}year \mid \textbf{tishri} \mid 10}\right)$$

where

$$h\text{-}year = g\text{-}year - \textbf{gregorian-year-from-fixed}\,(\textbf{hebrew-epoch}) + 1$$

The R.D. dates of Rosh ha-Shanah (Tishri 1), Sukkot (Tishri 15), Hoshana Rabba (Tishri 21), Shemini Aẓeret (Tishri 22), and Simḥat Torah (Tishri 23, outside Israel) are determined identically.[13] As on the Islamic calendar, all Hebrew holidays begin at sunset the prior evening.

The dates of the other major holidays—Passover (Nisan 15), the ending of Passover (Nisan 21), and Shavuot (Sivan 6)—are determined similarly but, because these holidays occur in the spring, the year corresponding to Gregorian year y is $y + 3760$. Conservative and Orthodox Jews observe two days of Rosh ha-Shanah—Tishri 1 and 2. Outside Israel, they also observe Tishri 16, Nisan 16, Nisan 22, and Sivan 7 as holidays.

[13] See [1, p. 800] for another way to determine the date of Rosh ha-Shanah.

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Thus, for example, we determine the R.D. date of Passover by

$$\textbf{passover}\,(\textit{g-year}) \stackrel{\text{def}}{=} \textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textbf{nisan} \mid 15}\right) \tag{8.31}$$

where

$$\textit{h-year} = \textit{g-year} - \textbf{gregorian-year-from-fixed}\,(\textbf{hebrew-epoch})$$

Gauss [9] developed an interesting alternative formula to determine the Gregorian date of Passover in a given year.

The 7-week period beginning on the second day of Passover is called the *omer* (sheave offering); the days of the omer are counted from 1 to 49, and the count is expressed in completed weeks and excess days. The following function tells the omer count for an R.D. date, returning a list of weeks (an integer 0–7) and days (an integer 0–6) if the date is within the omer period and returning **bogus** if not:

$$\textbf{omer}\,(\textit{date}) \stackrel{\text{def}}{=} \begin{cases} \left\langle \left\lfloor \dfrac{c}{7} \right\rfloor, c \bmod 7 \right\rangle & \textbf{if } 1 \leqslant c \leqslant 49 \\ \textbf{bogus} & \textbf{otherwise} \end{cases} \tag{8.32}$$

where

$$c = \textit{date} - \textbf{passover}\,(\textbf{gregorian-year-from-fixed}\,(\textit{date}))$$

The minor holidays of the Hebrew year are the "intermediate" days of Sukkot (Tishri 16–21) and of Passover (Nisan 16–20); Hanukkah (8 days, beginning on Kislev 25); Tu-B'Shevat (Shevat 15); and Purim (Adar 14 in normal years, Adar II 14 in leap years). Hanukkah occurs in late fall or early winter, and thus Hanukkah of the Gregorian year y occurs in the Hebrew year $y + 3761$, whereas Tu-B'Shevat occurs in late winter or early spring, and hence Tu-B'Shevat of Gregorian year y occurs in Hebrew year $y + 3760$. Thus, these two holidays are handled as were Yom Kippur and Passover, respectively. Purim also always occurs in late winter or early spring, in the last month of the Hebrew year (Adar or Adar II); hence its R.D. date is computed by

$$\textbf{purim}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.33}$$

$$\textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textit{last-month} \mid 14}\right)$$

where

$$\begin{aligned} \textit{h-year} &= \textit{g-year} - \textbf{gregorian-year-from-fixed}\,(\textbf{hebrew-epoch}) \\ \textit{last-month} &= \textbf{last-month-of-hebrew-year}\,(\textit{h-year}) \end{aligned}$$

The Hebrew year contains several fast days that, though specified by particular Hebrew calendar dates, are shifted when those days occur on a Saturday. The fast

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days are Tzom Gedaliah (Tishri 3), Tzom Tevet (Tevet 10), Ta'anit Esther (the day before Purim), Tzom Tammuz (Tammuz 17), and Tishah be-Av (Av 9). When Purim is on a Sunday, Ta'anit Esther occurs on the preceding Thursday and thus we can write

$$\textbf{ta-anit-esther}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.34}$$

$$\begin{cases} \textit{purim-date} - 3 & \textbf{if day-of-week-from-fixed}\,(\textit{purim-date}) = \textbf{sunday} \\ \textit{purim-date} - 1 & \textbf{otherwise} \end{cases}$$

where

$$\textit{purim-date} = \textbf{purim}\,(\textit{g-year})$$

Each of the other fast days, as well as Shushan Purim (the day after Purim, celebrated in Jerusalem), is postponed to the following day (Sunday) when it occurs on a Saturday. Because Tzom Gedaliah is always in the fall and Tzom Tammuz and Tishah be-Av are always in the summer, their determination is easy. For example,

$$\textbf{tishah-be-av}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.35}$$

$$\begin{cases} \textit{av}_9 + 1 & \textbf{if day-of-week-from-fixed}\,(\textit{av}_9) = \textbf{saturday} \\ \textit{av}_9 & \textbf{otherwise} \end{cases}$$

where

$$\begin{aligned} \textit{h-year} &= \textit{g-year} - \textbf{gregorian-year-from-fixed}\,(\textbf{hebrew-epoch}) \\ \textit{av}_9 &= \textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year}}\boxed{\textbf{av}}\boxed{9}\right) \end{aligned}$$

Tzom Tevet, which can never occur on Saturday, must be handled with (8.42) in Section 8.5 below, because Tevet 10 can fall on either side of January 1, and thus a single Gregorian calendar year can have 0, 1, or 2 occurrences of Tzom Tevet. For example, Tzom Tevet occurred twice in 1982 but not at all in 1984. We leave it to the reader to work out the details. For the foreseeable future, other Jewish holidays and fasts occur exactly once in each Gregorian year, because the Hebrew leap months and Gregorian leap days keep the two calendars closely aligned.

Yom ha-Shoah (Holocaust Memorial Day) is Nisan 27, unless that day is a Sunday (it cannot be a Saturday), in which case it is postponed by 1 day.[14] Yom ha-Zikkaron (Israel Memorial Day), nominally on Iyyar 4, is advanced to Wednesday if it falls on a Thursday or Friday, and delayed to Monday if it falls on a Sunday.[15]

[14] This exception was introduced by the Israeli Knesset in May 1997.

[15] This delay was instituted by the Israeli government in 2004, when it was decided that Yom ha-Zikkaron, as well as Israel Independence Day (normally on Iyyar 5), should be postponed by one day whenever Iyyar 4 falls on a Sunday.

Since Iyyar 4 can never fall on Monday, Wednesday, or Saturday, Yom ha-Zikkaron falls on Iyyar 4 only if the latter is a Tuesday. Thus, we write

$$\textbf{yom-ha-zikkaron}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.36}$$

$$\begin{cases} \textbf{kday-before}\,(\textbf{wednesday}, \textit{iyyar}_4) & \\ \qquad\qquad \textbf{if day-of-week-from-fixed}\,(\textit{iyyar}_4) \in \{\textbf{thursday}, \textbf{friday}\} & \\ \textit{iyyar}_4 + 1 & \textbf{if sunday} = \textbf{day-of-week-from-fixed}\,(\textit{iyyar}_4) \\ \textit{iyyar}_4 & \textbf{otherwise} \end{cases}$$

where

$$\textit{h-year} = \textit{g-year} - \textbf{gregorian-year-from-fixed}\,(\textbf{hebrew-epoch})$$

$$\textit{iyyar}_4 = \textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textbf{iyyar} \mid 4} \right)$$

On the Hebrew calendar, the first day of each month is called Rosh Ḥodesh and has a minor ritual significance. When the preceding month has 30 days, Rosh Ḥodesh includes also the last day of the preceding month. The determination of these days is elementary (except for the months of Kislev and Tevet, because of the varying length of the months that precede those two).

Some other dates of significance depend on the Julian-Coptic approximation of the tropical year (equinox to equinox), in which each of the four seasons is taken to be $91\frac{5}{16}$ days long: The beginning of *sh'ela* (request for rain) outside Israel, meant to correspond to the start of the sixtieth Hebrew day after the autumnal equinox, corresponds to Athōr 26 on the Coptic calendar and follows the same leap-year structure. (See Chapter 4.) Hence, we write

$$\textbf{sh-ela}\,(\textit{g-year}) \stackrel{\text{def}}{=} \textbf{coptic-in-gregorian}\,(3, 26, \textit{g-year}) \tag{8.37}$$

which is either December 5 or 6 (Gregorian) during the twentieth and twenty-first centuries (see [22]). As with most other Jewish holidays and events, sh'ela actually begins on the prior evening. In Israel, sh'ela begins on Marḥeshvan 7.

By one traditional Hebrew reckoning, attributed to the second century scholar Samuel of Nehardea, the vernal equinox of A.M. 5685 was at 6 p.m. on the eve of Wednesday, Paremotep 30, 1641, which is March 26, 1925 C.E. (Julian). It recurs on that day of the Coptic and Julian calendars and at that hour of the week every 28 years in what is called the *solar cycle* and is celebrated as *birkath haḥama*. Because 1641 mod 28 = 17, we can write

$$\textbf{birkath-ha-hama}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.38}$$

$$\begin{cases} \textit{dates} & \textbf{if } \textit{dates} \neq \langle\,\rangle \text{ and} \\ & \quad \left(\left(\textbf{coptic-from-fixed}\,\left(\textit{dates}_{[0]}\right)\right)_{\textbf{year}} \bmod 28 \right) = 17 \\ \langle\,\rangle & \textbf{otherwise} \end{cases}$$

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where

$$\textit{dates} = \textbf{coptic-in-gregorian}\,(7, 30, \textit{g-year})$$

(The bracketed subscript 0 extracts the first element of a list.) This function returns an empty list for the 27 out of 28 years in which this event does not occur.

These two functions, **sh-ela** and **birkath-ha-hama**, could alternatively be implemented as part of a Hebrew solar calendar, thereby avoiding the use of the Coptic calendar. First, we find when spring occurs according to Samuel of Nehardea's reckoning:

$$\textbf{samuel-season-in-gregorian}\,(\textit{season}, \textit{g-year}) \stackrel{\text{def}}{=} \tag{8.39}$$
$$\textbf{cycle-in-gregorian}\Big(\textit{season}, \textit{g-year}, Y, \textbf{fixed-from-hebrew}\Big(\boxed{1 \mid \textbf{adar} \mid 21}\Big) + 18^{\text{h}} + \textit{offset}\Big)$$

where

$$Y = 365 + 6^{\text{h}}$$
$$\textit{offset} = \frac{\textit{season}}{360^\circ} \times Y$$

Then it is an easy matter to check whether it meets the criteria for *birkath haḥama*:

$$\textbf{alt-birkath-ha-hama}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{8.40}$$
$$\begin{cases} \langle \textbf{fixed-from-moment}\,(\textit{moments}_{[0]}) \rangle & \textbf{if } \textit{moments} \neq \langle\,\rangle \text{ and} \\ & \quad \textbf{day-of-week-from-fixed}\,(\textit{moments}_{[0]}) = \textbf{wednesday} \text{ and} \\ & \quad \textbf{time-from-moment}\,(\textit{moments}_{[0]}) = 0^{\text{h}} \\ \langle\,\rangle & \textbf{otherwise} \end{cases}$$

where

$$Y = 365 + 6^{\text{h}}$$
$$\textit{season} = \textbf{spring} + 6^{\text{h}} \times \frac{360^\circ}{Y}$$
$$\textit{moments} = \textbf{samuel-season-in-gregorian}\,(\textit{season}, \textit{g-year})$$

A similar function can be constructed for sh'ela.

Another traditional Hebrew determination of seasons is attributed to one Rabbi Adda bar Ahava by Savasorda. It derives the year length from the assumption that the Metonic cycle provides a perfect correspondence between 19 solar years and 225 lunar months of length $29^{\text{d}}12^{\text{h}}44^{\text{m}}3\frac{1}{3}^{\text{s}}$. That gives a value of $365^{\text{d}}5^{\text{h}}55^{\text{m}}25\frac{25}{57}^{\text{s}}$

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for the length of one year. Taking 6 p.m. in the evening on Adar 28, 1 A.M., to be a spring equinox leads to the following:

$$\textbf{adda-season-in-gregorian}\,(\mathit{season}, \mathit{g\text{-}year}) \stackrel{\text{def}}{=} \tag{8.41}$$

$$\textbf{cycle-in-gregorian}\left(\mathit{season}, \mathit{g\text{-}year}, Y, \textbf{fixed-from-hebrew}\left(\boxed{1 \mid \textbf{adar} \mid 28}\right) + 18^{\text{h}} + \mathit{offset}\right)$$

where

$$Y = 365 + 5\tfrac{3791}{4104}{}^{\text{h}}$$

$$\mathit{offset} = \frac{\mathit{season}}{360^\circ} \times Y$$

8.5 The Drift of the Hebrew Calendar

I've been on a calendar, but never on time.
Marilyn Monroe: *Look* (1957)

The average Hebrew year length of about 365.2468 days (page 116) is slightly too long, meaning that the Hebrew year will drift slowly through the Gregorian year (which closely approximates the mean tropical year). This drift means, for example, that Passover will get later and later in the Gregorian year, as illustrated in Figure 8.2, which shows the advancing difference between the first day of Passover and the spring equinox. Although this will not be of practical concern for millennia, in general, in determining when a given Hebrew date will fall in a given Gregorian year, one needs to consider three Hebrew years for the given Gregorian year, just as we did for Islamic dates at the end of the previous chapter. We thus write:

$$\textbf{hebrew-in-gregorian}\,(\mathit{h\text{-}month}, \mathit{h\text{-}day}, \mathit{g\text{-}year}) \stackrel{\text{def}}{=} \tag{8.42}$$

$$\{\mathit{date}_0, \mathit{date}_1, \mathit{date}_2\} \cap \textbf{gregorian-year-range}\,(\mathit{g\text{-}year})$$

where

$$\mathit{jan}_1 = \textbf{gregorian-new-year}\,(\mathit{g\text{-}year})$$

$$y = (\textbf{hebrew-from-fixed}\,(\mathit{jan}_1))_{\textbf{year}}$$

$$\mathit{date}_0 = \textbf{fixed-from-hebrew}\left(\boxed{y \mid \mathit{h\text{-}month} \mid \mathit{h\text{-}day}}\right)$$

$$\mathit{date}_1 = \textbf{fixed-from-hebrew}\left(\boxed{y+1 \mid \mathit{h\text{-}month} \mid \mathit{h\text{-}day}}\right)$$

$$\mathit{date}_2 = \textbf{fixed-from-hebrew}\left(\boxed{y+2 \mid \mathit{h\text{-}month} \mid \mathit{h\text{-}day}}\right)$$

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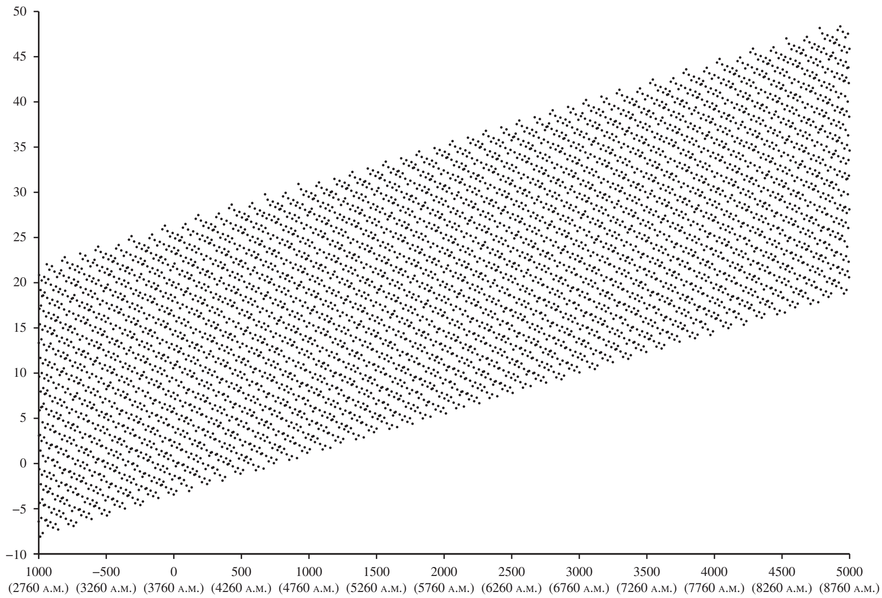

Figure 8.2 Number of days after the spring equinox that the first day of Passover occurs for Gregorian years −1000 to 5000 (= 2760–8760 A.M.). By Gregorian year 5000, Passover will occur, on the average, more than a full month after the spring equinox, whereas it should always occur within 30 days or so.

For example, in the Gregorian year 22336 (but not before), Yom Kippur occurs on January 11 and again on December 30, neither in the same Hebrew year as January 1.

Using the above robust function, we would compute the occurrences of the first day of Hanukkah as follows:

$$\textbf{hanukkah}\,(g\text{-}year) \stackrel{\text{def}}{=} \textbf{hebrew-in-gregorian}\,(\textbf{kislev}, 25, g\text{-}year) \qquad (8.43)$$

Because of the drift, there are no occurrences of Kislev 25 in the year 4999, but two in 5000.

8.6 Personal Days

Most modern calendars mar the sweet simplicity of our lives by reminding us that each day that passes is the anniversary of some perfectly uninteresting event.
Oscar Wilde: “A New Calendar,” *Pall Mall Gazette* (February 1887)

The Hebrew calendar contains what we might term “personal” days: one’s birthday according to the Hebrew calendar determines the day of one’s *Bat Mitzvah* (for girls) or *Bar Mitzvah* (for boys) (the 12th or 13th birthday). Dates of death determine when *Kaddish* is recited (*yahrzeit*, *naḥala*) for parents (and sometimes for

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other relatives). These are ordinarily just anniversary dates, but the leap-year structure and the varying number of days in some months require that alternative days be used in certain years, just as someone born on February 29 on the Gregorian calendar has to celebrate on an alternative day in common years.

The birthday of someone born in Adar of an ordinary year or Adar II of a leap year is also always in the last month of the year, be that Adar or Adar II. The birthday in an ordinary year of someone born during the first 29 days of Adar I in a leap year is on the corresponding day of Adar; in a leap year, the birthday occurs in Adar I, as expected. Someone born on the thirtieth day of Marḥeshvan, Kislev, or Adar I has his or her birthday postponed until the first of the following month, in years when that day does not occur. First, we write a function to determine the anniversary date in a given Hebrew year:

$$\textbf{hebrew-birthday}\left(\boxed{\textit{birth-year} \mid \textit{birth-month} \mid \textit{birth-day}}\,, \textit{h-year}\right) \stackrel{\text{def}}{=} \tag{8.44}$$

$$\begin{cases} \textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \begin{array}{l}\textbf{last-month-of-}\\ \textbf{hebrew-year}\\ (\textit{h-year})\end{array} \mid \textit{birth-day}}\right) \\ \quad \textbf{if } \textit{birth-month} \\ \qquad = \textbf{last-month-of-hebrew-year}\,(\textit{birth-year}) \\ \textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textit{birth-month} \mid 1}\right) + \textit{birth-day} - 1 \\ \quad \textbf{otherwise} \end{cases}$$

Unlike for the Islamic calendar, it will be many millennia before the Hebrew and Gregorian New Years coincide. Hence, a Gregorian year always comprises part of two (and eventually, millennia from now, three) Hebrew years. Thus we can collect a list of anniversaries in the possible Hebrew years:

$$\textbf{hebrew-birthday-in-gregorian}\,(\textit{birthdate}, \textit{g-year}) \stackrel{\text{def}}{=} \tag{8.45}$$

$$\{\textit{date}_0, \textit{date}_1, \textit{date}_2\} \cap \textbf{gregorian-year-range}\,(\textit{g-year})$$

where

$$\begin{aligned} \textit{jan}_1 &= \textbf{gregorian-new-year}\,(\textit{g-year}) \\ y &= (\textbf{hebrew-from-fixed}\,(\textit{jan}_1))_{\textbf{year}} \\ \textit{date}_0 &= \textbf{hebrew-birthday}\,(\textit{birthdate}, y) \\ \textit{date}_1 &= \textbf{hebrew-birthday}\,(\textit{birthdate}, y+1) \\ \textit{date}_2 &= \textbf{hebrew-birthday}\,(\textit{birthdate}, y+2) \end{aligned}$$

Similar functions for birthdays can be written for other calendars with variable-length years.

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The customary anniversary date of a death is more complicated and depends also on the character of the year in which the first anniversary occurs. There are several cases:

- If the date of death is Marḥeshvan 30, the anniversary in general depends on when the *first* anniversary occurs; if that first anniversary was not on Marḥeshvan 30, use the day before Kislev 1.
- If the date of death is Kislev 30, in general the anniversary again depends on the first anniversary—if that was not Kislev 30, use the day before Tevet 1.
- If the date of death is in Adar II, the anniversary is on the same day in the last month of the Hebrew year (Adar or Adar II).
- If the date of death is Adar I 30, the anniversary in a Hebrew year that is not a leap year (in which Adar has only 29 days) is on the last day in Shevat.
- In all other cases, use the normal (that is, same month number) anniversary of the date of death.

Perhaps these rules are best expressed algorithmically:

$$\textbf{yahrzeit}\left(\boxed{\textit{death-year} \mid \textit{death-month} \mid \textit{death-day}}\,, \textit{h-year}\right) \stackrel{\text{def}}{=} \tag{8.46}$$

$$\begin{cases}
\textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textbf{kislev} \mid 1}\right) - 1 \\
\quad \textbf{if } \textit{death-month} = \textbf{marheshvan} \text{ and } \textit{death-day} = 30 \text{ and} \\
\quad\quad \text{not } \textbf{long-marheshvan?}\,(\textit{death-year} + 1) \\
\textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textbf{tevet} \mid 1}\right) - 1 \\
\quad \textbf{if } \textit{death-month} = \textbf{kislev} \text{ and } \textit{death-day} = 30 \text{ and} \\
\quad\quad \textbf{short-kislev?}\,(\textit{death-year} + 1) \\
\textbf{fixed-from-hebrew} \\
\quad \left(\boxed{\textit{h-year} \mid \begin{array}{l}\textbf{last-month-of-}\\ \textbf{hebrew-year}\\ (\textit{h-year})\end{array} \mid \textit{death-day}}\right) \\
\quad \textbf{if } \textit{death-month} = \textbf{adarii} \\
\textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textbf{shevat} \mid 30}\right) \\
\quad \textbf{if } \textit{death-day} = 30 \text{ and } \textit{death-month} = \textbf{adar} \text{ and} \\
\quad\quad \text{not } \textbf{hebrew-leap-year?}\,(\textit{h-year}) \\
\textbf{fixed-from-hebrew}\left(\boxed{\textit{h-year} \mid \textit{death-month} \mid 1}\right) + \textit{death-day} - 1 \\
\quad \textbf{otherwise}
\end{cases}$$

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There are minor variations in custom regarding the anniversary date in some of these cases.[16] For example, Spanish and Portuguese Jews never observe the anniversary of a common-year date in Adar I.

As with birthdays, anniversaries all occurring in a given Gregorian year must be collected together:

$$\textbf{yahrzeit-in-gregorian}\,(\mathit{death\text{-}date}, \mathit{g\text{-}year}) \stackrel{\text{def}}{=} \tag{8.47}$$

$$\{\mathit{date}_0, \mathit{date}_1, \mathit{date}_2\} \cap \textbf{gregorian-year-range}\,(\mathit{g\text{-}year})$$

where

$$\begin{aligned}
\mathit{jan}_1 &= \textbf{gregorian-new-year}\,(\mathit{g\text{-}year})\\
y &= (\textbf{hebrew-from-fixed}\,(\mathit{jan}_1))_{\textbf{year}}\\
\mathit{date}_0 &= \textbf{yahrzeit}\,(\mathit{death\text{-}date}, y)\\
\mathit{date}_1 &= \textbf{yahrzeit}\,(\mathit{death\text{-}date}, y+1)\\
\mathit{date}_2 &= \textbf{yahrzeit}\,(\mathit{death\text{-}date}, y+2)
\end{aligned}$$

8.7 Possible Days of the Week

These budget numbers are not just estimates; these are the actual results for the fiscal year that ended February the 30th.

George W. Bush: President Bush Discusses
the Economy and Budget (October 2006)

As described on page 117, the Hebrew calendar rule *lo iddo rosh* precludes Tishri 1 (Rosh ha-Shanah) from occurring on a Sunday, Wednesday, or Friday. This restriction means that, throughout the year, some dates are precluded from occurring on certain weekdays. In this section, we examine the consequences of the restriction, developing a function that gives, for each Hebrew calendar date, a list of the possible weekdays on which it can occur. It turns out that, though a Hebrew year can begin on any of four weekdays, can be leap or ordinary, and can be long (355 for ordinary years and 385 for leap years), short (353 or 383), or regular (354 or 384), only 14 of the $4 \times 2 \times 3 = 24$ combinations are actually possible.

The Tishri 1 restriction means that that date can occur only on a Monday, Tuesday, Thursday, or Saturday. Because the lengths of the months Nisan through Tishri are unvarying, the $177 \equiv 2 \pmod{7}$ days separating the previous Nisan 1 from the following Tishri 1 mean that Nisan 1 occurs only on a Saturday, Sunday, Tuesday, or Thursday. Thus, we can determine the possible weekdays for a given Hebrew date by working forward from Nisan 1. The fixed lengths of the months Nisan through Tishri mean that for any date *h-month*, *h-day* from Nisan 1 through Marheshvan 29,

[16] The rules described accord with Ashkenazic practice as given in [22] and in the *Talmudic Encyclopedia: A Digest of Halachic Literature from the Tannaitic Period to the Present Time Alphabetically Arranged*, Talmudic Encyclopedia Publishing, Jerusalem, vol. I (1951), p. 93; vol. XXIII (1997), cols. 153–154. However, M. Feinstein (*Iggerot Moshe*, vol. 6, *Yoreh Deah*, part 3, p. 426) rules that *yahrzeit* anniversaries of the last day of a month follow the rules for birthdays.

the list of possible weekdays can be obtained by adding the number of days from Nisan 1 to *h-month*, *h-day* to each value in the list

$$\langle \textbf{sunday}, \textbf{tuesday}, \textbf{thursday}, \textbf{saturday} \rangle \tag{8.48}$$

and applying **day-of-week-from-fixed** to the sum. We use the function

$$\textbf{shift-days}\,(l, \Delta) \stackrel{\text{def}}{=} \tag{8.49}$$

$$\begin{cases} \langle\,\rangle & \textbf{if } l = \langle\,\rangle \\ \left\langle \left(l_{[0]} + \Delta\right) \bmod 7 \right\rangle \parallel \textbf{shift-days}\left(l_{[1..]}, \Delta\right) & \textbf{otherwise} \end{cases}$$

to shift a list such as (8.48) by a given increment.

Marheshvan 30 is exceptional, however. Although Marheshvan 29 can be a Thursday, Marheshvan 30 cannot fall on a Friday: for Marheshvan 29 to be on a Thursday, Tishri 1 must have been on a Tuesday. Marheshvan has 30 days only when the year is 355 or 385 days long. But if a 355-day year began on a Tuesday, the following year would start on a Sunday, violating *lo iddo rosh*. And for a leap year to be extended to 385 days, it would have to begin on a permissible day that is preceded by an excluded day, so that the molad of the year following the leap year—which is just under 384 days after the molad of the leap year—falls on the excluded day and is thereby delayed. Tuesday is not such a day, since Monday is also a permissible day, so a long leap year cannot begin on a Tuesday.

In a year in which Marheshvan has 30 days, there are $236 \equiv 2 \bmod 7$ days between Nisan 1 and Marheshvan 30; that date falling on a Friday would correspond to Nisan 1 falling on a Sunday. So, the possible weekdays for any Hebrew date Nisan–Marheshvan can be found by including **sunday** in the list (8.48) only for dates from Nisan 1 through Marheshvan 29, finding the number of days from Nisan 1 to *h-month*, *h-day*, and applying **shift-days** with that increment to the list.

Other dates in the year are affected by three factors: whether Marheshvan is long (30 days) or short (29 days), whether Kislev is long or short, and whether the year is a leap year. For example, the calculations described in the previous paragraph hold for days in Kislev when Marheshvan does not have 30 days. When Marheshvan does have 30 days, the calculation is off by one day, meaning that it is as though Nisan 1 occurred on a Wednesday, Friday, or Sunday, the days following the days in the list (8.48) with Sunday omitted. Thus, for dates in Kislev, we can find possible weekdays by augmenting (8.48) with

$$\langle \textbf{sunday}, \textbf{wednesday}, \textbf{friday} \rangle$$

and applying **shift-days** to the augmented list. Similar considerations apply for the months Tevet through Adar or Adar II in leap years.

We can calculate the interval between Nisan 1 and *h-month*, *h-day* in any leap year in which both Marheshvan and Kislev are long (a maximal 385-day Hebrew year, so that every month-day combination occurs), adjusting the contents of the list of weekdays equivalent to Nisan 1 as needed. We arbitrarily choose the 385-day Hebrew year 5–6 A.M. The resulting calculation is thus

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$$\textbf{possible-hebrew-days}\,(\textit{h-month}, \textit{h-day}) \stackrel{\text{def}}{=} \tag{8.50}$$

$$\textbf{shift-days}\,(\textit{basic} \parallel \textit{extra}, n)$$

where

$$\textit{h-date}_0 = \boxed{5 \mid \textbf{nisan} \mid 1}$$

$$\textit{h-year} = \begin{cases} 6 & \textbf{if } \textit{h-month} > \textbf{elul} \\ 5 & \textbf{otherwise} \end{cases}$$

$$\textit{h-date} = \boxed{\textit{h-year} \mid \textit{h-month} \mid \textit{h-day}}$$

$$n = \textbf{fixed-from-hebrew}\,(\textit{h-date}) - \textbf{fixed-from-hebrew}\,(\textit{h-date}_0)$$

$$\textit{basic} = \langle \textbf{tuesday}, \textbf{thursday}, \textbf{saturday} \rangle$$

$$\textit{extra} = \begin{cases} \langle\,\rangle & \textbf{if } \textit{h-month} = \textbf{marheshvan} \text{ and } \textit{h-day} = 30 \\ \langle \textbf{monday}, \textbf{wednesday}, \textbf{friday} \rangle & \\ & \textbf{if } \textit{h-month} = \textbf{kislev} \text{ and } \textit{h-day} < 30 \\ \langle \textbf{monday} \rangle & \textbf{if } \textit{h-month} = \textbf{kislev} \text{ and } \textit{h-day} = 30 \\ \langle \textbf{sunday}, \textbf{monday} \rangle & \textbf{if } \textit{h-month} \in \{\textbf{tevet}, \textbf{shevat}\} \\ \langle \textbf{sunday}, \textbf{monday} \rangle & \textbf{if } \textit{h-month} = \textbf{adar} \text{ and } \textit{h-day} < 30 \\ \langle \textbf{sunday} \rangle & \textbf{otherwise} \end{cases}$$

This function produces an unsorted list of possible weekdays for the specified Hebrew date. For example, it tells us that Tu B'Shevat (Shevat 15) can occur only on a Thursday, Saturday, Monday, Tuesday, or Wednesday; that is, it can never occur on a Sunday or Friday.

The above function combines those weekdays on which a given date can occur in leap years with those on which it can occur in nonleap years. In particular, there is a difference in possible dates during the twelfth month, depending on whether it is Adar in a plain (common) year or Adar I in a leap year, that is not reflected in **possible-hebrew-days**. One can write a similar function that gives weekdays for leap and nonleap years separately.

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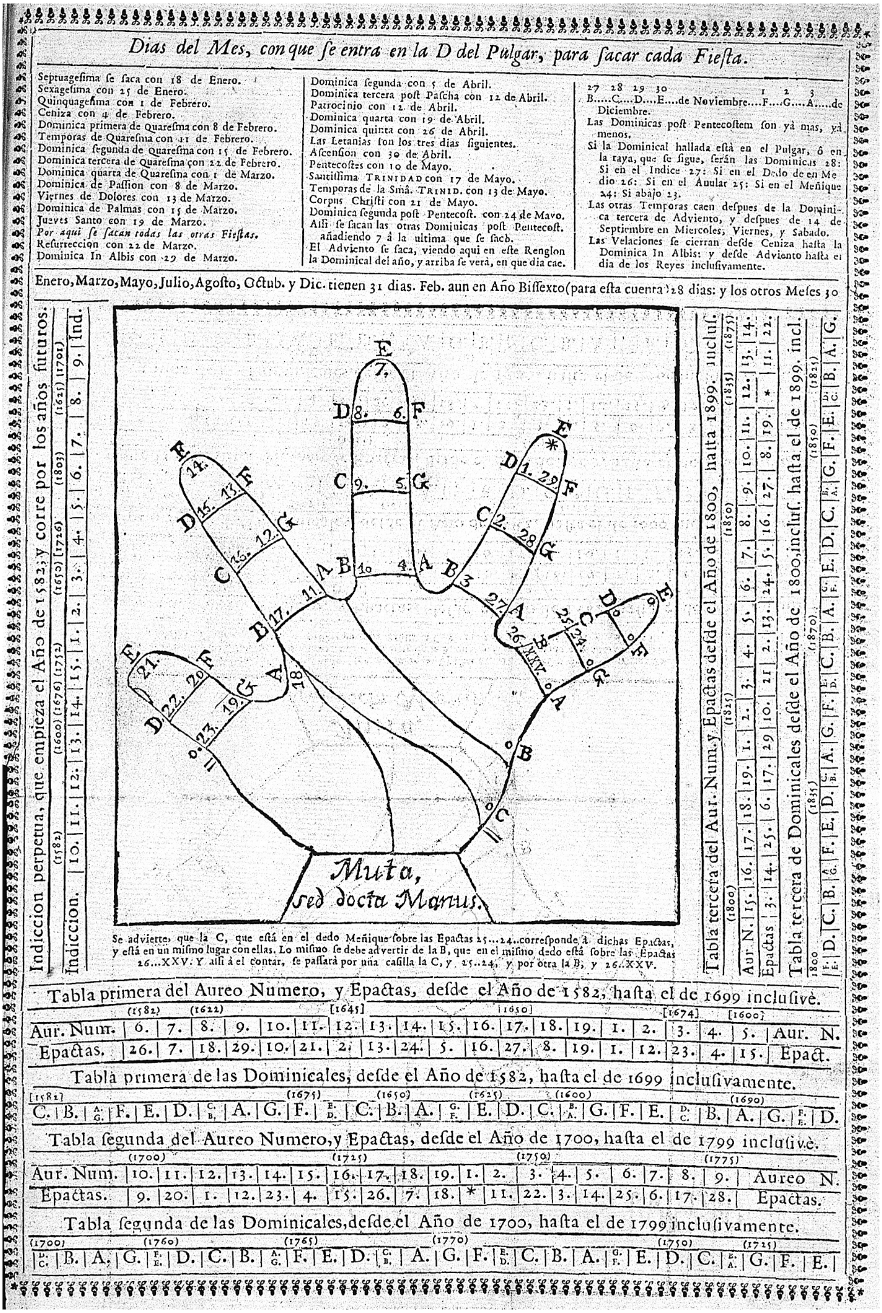

Dias del Mes, con que se entra en la D del Pulgar, para sacar cada Fiesta.

Septuagesima se saca con 18 de Enero.
Sexagesima con 25 de Enero.
Quinquagesima con 1 de Febrero.
Ceniza con 4 de Febrero.
Dominica primera de Quaresma con 8 de Febrero.
Temporas de Quaresma con 11 de Febrero.
Dominica segunda de Quaresma con 15 de Febrero.
Dominica tercera de Quaresma con 22 de Febrero.
Dominica quarta de Quaresma con 1 de Marzo.
Dominica de Passion con 8 de Marzo.
Viernes de Dolores con 13 de Marzo.
Dominica de Palmas con 15 de Marzo.
Jueves Santo con 19 de Marzo.
Por aqui se sacan todas las otras Fiestas.
Resurreccion con 22 de Marzo.
Dominica In Albis con 29 de Marzo.

Dominica segunda con 5 de Abril.
Dominica tercera post Pascha con 12 de Abril.
Patrocinio con 12 de Abril.
Dominica quarta con 19 de Abril.
Dominica quinta con 26 de Abril.
Las Letanias son los tres dias siguientes.
Ascension con 30 de Abril.
Pentecostes con 10 de Mayo.
Santissima TRINIDAD con 17 de Mayo.
Temporas de la Smã. TRINID. con 13 de Mayo.
Corpus Christi con 21 de Mayo.
Dominica segunda post Pentecost. con 24 de Mayo.
Assi se sacan las otras Dominicas post Pentecost. añadiendo 7 à la ultima que se sacò.
El Adviento se saca, viendo aqui en este Renglon la Dominical del año, y arriba se verà, en que dia cae.

27 28 29 30 1 2 3
B.....C.....D.....E.....de Noviembre.....F.....G.....A.....de Diciembre.
Las Dominicas post Pentecostem son yà mas, yà menos.
Si la Dominical hallada està en el Pulgar, ò en la raya, que se sigue, seràn las Dominicas 28: Si en el Indice 27: Si en el Dedo de en Medio 26: Si en el Anular 25: Si en el Meñique 24: Si abajo 23.
Las otras Temporas caen despues de la Dominica tercera de Adviento, y despues de 14 de Septiembre en Miercoles, Viernes, y Sabado.
Las Velaciones se cierran desde Ceniza hasta la Dominica In Albis: y desde Adviento hasta el dia de los Reyes inclusivamente.

Enero, Marzo, Mayo, Julio, Agosto, Octub. y Dic. tienen 31 dias. Feb. aun en Año Bissexto (para esta cuenta) 28 dias: y los otros Meses 30

Indiccion perpetua, que empieza el Año de 1582; y corre por los años futuros.

	(1582)			(1600)	(1676)	(1752)			(1650)	(1726)		(1803)		(1625)	(1701)	
Indiccion.	10.	11.	12.	13.	14.	15.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Ind.

Muta, sed docta Manus.

Se advierte, que la C, que està en el dedo Meñique sobre las Epactas 25...24..corresponde à dichas Epactas, y està en un mismo lugar con ellas. Lo mismo se debe advertir de la B, que en el mismo dedo està sobre las Epactas 26...XXV. Y assi à el contar, se passarà por una casilla la C, y 25..24, y por otra la B, y 26..XXV.

Tabla tercera del Aur. Num. y Epactas desde el Año de 1800, hasta 1899. inclus.

	(1800)						(1825)						(1850)				(1835)		(1875)
Aur. N.	15.	16.	17.	18.	19.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Epactas	3.	14.	25.	6.	17.	29	10.	21	2.	13.	24.	5.	16.	27.	8.	19.	*	11.	22.

Tabla tercera de Dominicales desde el Año de 1800, inclus. hasta el de 1899. incl.

1800							(1835)						(1870)						(1850)			(1825)		
F/E.	D.	C.	B.	A/G.	F.	E.	D.	C/B.	A.	G.	F.	E/D.	C.	B.	A.	G/F.	E.	D.	C.	B/A.	G.	F.	E.	D/C.

B.	A.	G.

Tabla primera del Aureo Numero, y Epactas, desde el Año de 1582, hasta el de 1699 inclusivè.

	(1582)		(1622)				[1645]					(1650)					[1674]		[1600]	
Aur. Num.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	1.	2.	3.	4.	5.	Aur. N.
Epactas.	26.	7.	18.	29.	10.	21.	2.	13.	24.	5.	16.	27.	8.	19.	1.	12.	23.	4.	15.	Epact.

Tabla primera de las Dominicales, desde el Año de 1582, hasta el de 1699 inclusivamente.

[1582]								(1675)			(1650)			(1625)			(1600)					(1690)			
C.	B.	A/G.	F.	E.	D.	C/B.	A.	G.	F.	E/D.	C.	B.	A.	G/F.	E.	D.	C.	B/A.	G.	F.	E.	D/C.	B.	A.	G.

F/E.	D.

Tabla segunda del Aureo Numero, y Epactas, desde el Año de 1700, hasta el de 1799 inclusivè.

	(1700)						(1725)						(1750)					(1775)		
Aur. Num.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Aureo N.
Epactas.	9.	20.	1.	12.	23.	4.	15.	26.	7.	18.	*	11.	22.	3.	14.	25.	6.	17.	28.	Epactas.

Tabla segunda de las Dominicales, desde el Año de 1700, hasta el de 1799 inclusivamente.

(1700)		(1760)						(1765)					(1770)							(1750)			(1725)		
D/C.	B.	A.	G.	F/E.	D.	C.	B.	A/G.	F.	E.	D.	C/B.	A.	G.	F.	E/D.	C.	B.	A.	G/F.	E.	D.	C.	E/A.	G.

F.	E.

Finger calculation for the date of Easter, according to the Gregorian rule, from page 14 of *Astronómica y Harmoniosa Mano* by Buenaventura Francisco de Ossorio, Bibliotheca Mexicana, Mexico, 1757. For an explanation of the calculation, see F. Cajori's "A Notable Case of Finger-Reckoning in America," *ISIS*, vol. 8, pp. 325–327, 1926. (Courtesy of the Bancroft Library, University of California, Berkeley.)