# 18

# Astronomical Lunar Calendars

*He spent his days and half his nights writing a book on the history of calendars.*
Isaac Bashevis Singer: *The Family Moskat* (1950)

In this chapter, we apply the methods of Chapter 14 to compute the old Babylonian calendar, the proposed uniform date of Easter, the observational Islamic lunar calendar, the classical Hebrew lunisolar calendar, and the Samaritan calendar. All but the calculation of Easter and the Samaritan calendar share the feature that the start of the month is determined by the first visibility of the crescent moon after new moon.

Around the time of the new moon, when the sun and moon are close to each other in the sky, the moon cannot be seen with the naked eye. Leading up to that time, a crescent moon is visible in the morning sky near the eastern horizon, while shortly after the new moon conjunction a crescent moon appears in the evening just after sunset, low in the western sky. On very rare occasions, the moon can be seen in the morning one day and in the evening the next [5]; usually, it is invisible for 1 to 3 days.

## 18.1 The Babylonian Calendar

*In the house of history studying chronology is like puttering about the basement working on the plumbing or furnace instead of joining the conversation in the dining room. But it is occasionally useful to check the basic apparatus.*
Leo Depuydt: "On the Consistency of the Wandering Year as Backbone of Egyptian Chronology," *Journal of the American Research Center in Egypt* (1995)

The classical Babylonian calendar, from about 380 B.C.E., or earlier, was of the lunisolar type, with a fixed 19-year Metonic cycle. Prior to that date, leap years were irregular (see [6], [9]). The month names are

| | |
|---|---|
| (1) Nisanu | (7) Tashritu |
| (2) Ayaru | (8) Arakhsamna |
| (3) Simanu | (9) Kislimu |
| (4) Du'uzu | (10) Tebetu |
| (5) Abu | (11) Shabatu |
| (6) Ululu | (12) Adaru |



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The day of the new moon was often determined by an approximate calculation based on the lag time between sunset and moonset. The lag time is simply the difference between the times of the setting of the moon (14.84) and the sun (14.77). Taking into account the possibility of the nonoccurrence of sunset or moonset, we have:

$$\textbf{moonlag}\ (\textit{date}, \textit{location}) \stackrel{\text{def}}{=} \begin{cases} \textbf{bogus} & \textbf{if } \textit{sun} = \textbf{bogus} \\ 24^{\text{h}} & \textbf{if } \textit{moon} = \textbf{bogus} \\ \textit{moon} - \textit{sun} & \textbf{otherwise} \end{cases} \tag{18.1}$$

where

$$\begin{aligned} \textit{sun} &= \textbf{sunset}\,(\textit{date}, \textit{location}) \\ \textit{moon} &= \textbf{moonset}\,(\textit{date}, \textit{location}) \end{aligned}$$

We take Babylon

$$\textbf{babylon} \stackrel{\text{def}}{=} \boxed{32.4794^\circ \mid 44.4328^\circ \mid 26\text{ m} \mid 3\tfrac{1}{2}^{\text{h}}} \tag{18.2}$$

as the determining location. The precise method of prediction seems to have varied [9]. Requiring that the month be at least a day old and that the lag be at least 48 minutes [3], we have

$$\textbf{babylonian-criterion}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{18.3}$$

$$\begin{aligned} &\textbf{new} < \textit{phase} < \textbf{first-quarter} \text{ and} \\ &\textbf{new-moon-before}\,(t) \leqslant t - 24^{\text{h}} \text{ and} \\ &\textbf{moonlag}\,(\textit{date} - 1, \textbf{babylon}) > 48^{\text{m}} \end{aligned}$$

where

$$\begin{aligned} \textit{set} &= \textbf{sunset}\,(\textit{date} - 1, \textbf{babylon}) \\ t &= \textbf{universal-from-standard}\,(\textit{set}, \textbf{babylon}) \\ \textit{phase} &= \textbf{lunar-phase}\,(t) \end{aligned}$$

Now, the start of the new month is found by linear search, in a similar fashion to **phasis-on-or-before** (page 252):

$$\textbf{babylonian-new-month-on-or-before}\,(\textit{date}) \stackrel{\text{def}}{=} \tag{18.4}$$

$$\underset{d \geqslant \tau}{\text{MIN}} \left\{ \textbf{babylonian-criterion}\,(d) \right\}$$

where

$$\textit{moon} = \textbf{fixed-from-moment}\,(\textbf{lunar-phase-at-or-before}\,(\textbf{new}, \textit{date}))$$

$$age = date - moon$$

$$\tau = \begin{cases} moon - 30 & \textbf{if } age \leqslant 3 \text{ and not } \textbf{babylonian-criterion}\,(date) \\ moon & \textbf{otherwise} \end{cases}$$

We use the beginning of the Seleucid era, April 3, 311 B.C.E. (Julian), as the calendar's epoch:

$$\textbf{babylonian-epoch} \stackrel{\text{def}}{=} \tag{18.5}$$

$$\textbf{fixed-from-julian}\left(\boxed{311 \text{ B.C.E.} \mid \textbf{april} \mid 3}\right)$$

The leap-year rule follows the same pattern as that of the Hebrew calendar (8.14), but the cycle is shifted 7 years:

$$\textbf{babylonian-leap-year?}\,(b\text{-}year) \stackrel{\text{def}}{=} ((7 \times b\text{-}year + 13) \bmod 19) < 7 \tag{18.6}$$

The last month of the year, Adaru, was intercalated in years 1, 4, 7, 9, 12, and 15 of the cycle; the sixth month, Ululu, was intercalated instead during the 18th year. Taking this anomaly into account, the conversions are straightforward:

$$\textbf{fixed-from-babylonian} \tag{18.7}$$

$$\left(\boxed{year \mid month \mid leap \mid day}\right) \stackrel{\text{def}}{=}$$

$$\textbf{babylonian-new-month-on-or-before}\,(midmonth) + day - 1$$

where

$$month_1 = \begin{cases} month & \textbf{if } leap \text{ or } \{(year \bmod 19) = 18 \text{ and } month > 6\} \\ month - 1 & \textbf{otherwise} \end{cases}$$

$$months = \left\lfloor \frac{1}{19} \times ((year - 1) \times 235 + 13) \right\rfloor + month_1$$

$$midmonth = \textbf{babylonian-epoch} + \text{round}\,(\textbf{mean-synodic-month} \times months) + 15$$

In the other direction,

$$\textbf{babylonian-from-fixed}\,(date) \stackrel{\text{def}}{=} \tag{18.8}$$

$$\boxed{year \mid month \mid leap \mid day}$$

where

$$crescent = \textbf{babylonian-new-month-on-or-before}\,(date)$$

$$months = \text{round}\left(\frac{crescent - \textbf{babylonian-epoch}}{\textbf{mean-synodic-month}}\right)$$

$$
\begin{aligned}
year &= \left\lfloor \frac{1}{235} \times (19 \times months + 5) \right\rfloor + 1 \\
approx &= \textbf{babylonian-epoch} \\
&\quad + \text{round}\left( \left\lfloor \tfrac{1}{19} \times ((year - 1) \times 235 + 13) \right\rfloor \right. \\
&\qquad\qquad \left. \times\, \textbf{mean-synodic-month} \right) \\
new\text{-}year &= \textbf{babylonian-new-month-on-or-before}\,(approx + 15) \\
month_1 &= \text{round}\left( \frac{1}{29.5} \times (crescent - new\text{-}year) \right) + 1 \\
special &= (year \bmod 19) = 18 \\
leap &= \begin{cases} month_1 = 7 & \textbf{if } special \\ month_1 = 13 & \textbf{otherwise} \end{cases} \\
month &= \begin{cases} month_1 - 1 & \textbf{if } leap \text{ or } \{special \text{ and } month_1 > 6\} \\ month_1 & \textbf{otherwise} \end{cases} \\
day &= date - crescent + 1
\end{aligned}
$$

Since it is not always certain how the evening of the occurrence of the new moon was actually determined, these dates should be considered approximate. See [6].

## 18.2 Astronomical Easter

*Snout: Doth the moon shine that night we play our play?*
*Bottom: A calendar, a calendar! look in the almanac;*
*find out moonshine, find out moonshine.*
*Quince: Yes, it doth shine that night.*

William Shakespeare: *A Midsummer Night's Dream,*
Act III, scene i (1600)

In 1997, the World Council of Churches [1] proposed a uniform date for Easter for the Eastern and Western churches (see Chapter 9). With the algorithms of Chapter 14, the proposed astronomical determination of Easter is straightforward. We need to find the first Sunday in Jerusalem[1] after the first true full moon after the true vernal equinox:

$$\textbf{astronomical-easter}\,(g\text{-}year) \stackrel{\text{def}}{=} \textbf{kday-after}\,(\textbf{sunday}, paschal\text{-}moon) \tag{18.9}$$

where

$$
\begin{aligned}
equinox &= \textbf{season-in-gregorian}\,(\textbf{spring}, g\text{-}year) \\
paschal\text{-}moon &= \big\lfloor\, \textbf{apparent-from-universal} \\
&\qquad (\textbf{lunar-phase-at-or-after}\,(\textbf{full}, equinox), \textbf{jerusalem}) \,\big\rfloor
\end{aligned}
$$

[1] "Astronomical observations, of course, depend upon the position on Earth which is taken as the point of reference. This consultation believes that it is appropriate to employ the meridian of Jerusalem ..." [1].

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Table 9.1 in Chapter 9 (page 151) gives the traditional dates of Passover and Easter along with those obtained by the preceding astronomical calculations.

## 18.3 The Observational Islamic Calendar

*It is He who gave the sun its radiance, the moon its luster, and appointed its stations so that you may compute years and numbers. God did not create them but with deliberation. He distinctly explains His signs for those who can understand.*
*Koran* (X, 5)

Muslims in India, Pakistan, and Bangladesh base their calendar on reported moon sightings. In Egypt, they require moonset to be at least 5 minutes after sunset on the first day of the month. In the United States, according to S. K. Shaukat (who was national coordinator and consultant for America): "A confirmed crescent sighting report in North America will be accepted as long as such a report does not contradict indisputable astronomical information." In Saudi Arabia and most of the Gulf countries, the rule is that the moon must set after the sun on the last day of the month as seen from Mecca.

With the functions of Section 14.9, we can approximate the observation-based Islamic calendars that are used in practice. Suppose that we take Cairo, site of Al-Azhar University, a major Islamic religious center, as the location of observation:[2]

$$\textbf{islamic-location} \stackrel{\text{def}}{=} \boxed{30.1^\circ \mid 31.3^\circ \mid 200\text{ m} \mid 2^{\text{h}}} \tag{18.10}$$

Then we calculate the calendar as follows:

$$\textbf{fixed-from-observational-islamic}\left(\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}\right) \stackrel{\text{def}}{=} \tag{18.11}$$

$$\textbf{phasis-on-or-before}\,(\mathit{midmonth}, \textbf{islamic-location}) + \mathit{day} - 1$$

where

$$\begin{aligned} \mathit{midmonth} = {} & \textbf{islamic-epoch} \\ & + \left\lfloor \left( (\mathit{year} - 1) \times 12 + \mathit{month} - \frac{1}{2} \right) \times \textbf{mean-synodic-month} \right\rfloor \end{aligned}$$

---

[2] In our *Calendrical Tabulations*, we made the less-than-obvious choice of Los Angeles as the location for the Islamic calendar based on the following advice of S. K. Shaukat [8]:

> The reason I pick Los Angeles is that according to the known practices these dates would be closest to Middle Eastern countries' practices although the visibility would not be in the Middle East. Moreover, in many cases, if the visibility is not in Los Angeles then most of the world would see it the next day and that would be reflected in the calculated dates for Los Angeles. The dates for Los Angeles would also be good for the rest of North America if an aided eye is used, which will also be in line with actual practice and I think these dates would be the closest to practices all around the world.

In other words, the actual observance of Ramadan, and other Islamic events frequently precedes dates as calculated astronomically, for various nonscientific reasons. Thus, choosing Los Angeles gave dates that are both scientifically and religiously reasonable for the United States *and* in good agreement with actual observance in the Middle East.

In the other direction,

$$\textbf{observational-islamic-from-fixed}\,(date) \stackrel{\text{def}}{=} \boxed{\,year \mid month \mid day\,} \qquad (18.12)$$

where

$$\begin{aligned}
crescent &= \textbf{phasis-on-or-before}\,(date, \textbf{islamic-location})\\
elapsed\text{-}months &= \text{round}\left(\frac{crescent - \textbf{islamic-epoch}}{\textbf{mean-synodic-month}}\right)\\
year &= \left\lfloor \frac{1}{12} \times elapsed\text{-}months \right\rfloor + 1\\
month &= (elapsed\text{-}months \bmod 12) + 1\\
day &= date - crescent + 1
\end{aligned}$$

These functions for the Islamic calendar are approximate at best for many reasons: The phenomenon of visibility is still an area of astronomical research and is not yet fully understood; this criterion is just one of many suggestions. It ignores the variation in the distance to the moon and also in the clarity of the atmosphere, which depends on location and season as well as on unpredictable factors. Muslim countries base the calendar on reported observations, not calculated observability. The best location for seeing the new moon varies from month to month (western locations are always better), and different religious authorities accept testimony from within different regions.

The above functions allow for a 31st day of an Islamic month, which is longer than is actually allowed by the rules.[3] Instead, that day would be the first of the following month—were the moon actually observed when the simple criterion we are using says it becomes visible. This shift can cascade for several months. We have not taken this into account because there is no way to determine when in fact the new moons are actually observed, and which months are affected.

Imagining that the functions precisely capture observability, the following functions do take this rule into account by checking month after month:

$$\textbf{month-length}\,(date, location) \stackrel{\text{def}}{=} moon - prev \qquad (18.13)$$

where

$$\begin{aligned}
moon &= \textbf{phasis-on-or-after}\,(date + 1, location)\\
prev &= \textbf{phasis-on-or-before}\,(date, location)
\end{aligned}$$

$$\textbf{early-month?}\,(date, location) \stackrel{\text{def}}{=} \qquad (18.14)$$

[3] It is possible for there to be 31 days from first visibility to first visiblity. For example, using Yallop's criterion (page 251), there would be have been a 31-day observation-based lunar month in Babylon extending from August 27, 2006 through September 26, 2006. As R. H. van Gent points out [2], there is also a 31-day month in the 10th year of Darius I according to the tables of [6, p. 30].

$$date - start \geqslant 30 \text{ or } \textbf{month-length}\,(prev, location) > 30 \text{ or}$$
$$\left\{ \textbf{month-length}\,(prev, location) = 30 \text{ and } \textbf{early-month?}\,(prev, location) \right\}$$

where

$$start = \textbf{phasis-on-or-before}\,(date, location)$$
$$prev = start - 15$$

$$\textbf{alt-fixed-from-observational-islamic}\left(\boxed{year \mid month \mid day}\right) \stackrel{\text{def}}{=} \qquad (18.15)$$

$$\begin{cases} date - 1 & \textbf{if early-month?}\,(midmonth, \textbf{islamic-location}) \\ date & \textbf{otherwise} \end{cases}$$

where

$$\begin{aligned} midmonth &= \textbf{islamic-epoch} \\ &\quad + \left\lfloor \left( (year - 1) \times 12 + month - \frac{1}{2} \right) \times \textbf{mean-synodic-month} \right\rfloor \\ moon &= \textbf{phasis-on-or-before}\,(midmonth, \textbf{islamic-location}) \\ date &= moon + day - 1 \end{aligned}$$

$$\textbf{alt-observational-islamic-from-fixed}\,(date) \stackrel{\text{def}}{=} \qquad (18.16)$$

$$\boxed{year \mid month \mid day}$$

where

$$\begin{aligned} early &= \textbf{early-month?}\,(date, \textbf{islamic-location}) \\ long &= early \text{ and } \textbf{month-length}\,(date, \textbf{islamic-location}) > 29 \\ date' &= \begin{cases} date + 1 & \textbf{if } long \\ date & \textbf{otherwise} \end{cases} \\ moon &= \textbf{phasis-on-or-before}\,(date', \textbf{islamic-location}) \\ elapsed\text{-}months &= \text{round}\left( \frac{moon - \textbf{islamic-epoch}}{\textbf{mean-synodic-month}} \right) \\ year &= \left\lfloor \frac{1}{12} \times elapsed\text{-}months \right\rfloor + 1 \\ month &= (elapsed\text{-}months \bmod 12) + 1 \\ day &= date' - moon - \left\{ \begin{array}{ll} -2 & \textbf{if } early \text{ and not } long \\ -1 & \textbf{otherwise} \end{array} \right\} \end{aligned}$$

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Saudi Arabia employs the *Umm al-Qura* calendar for some secular purposes, as an approximation of the observational Islamic calendar. The rule—since March 2002—is that the month begins on the first evening after the conjunction on which the moon sets after the sun.[4] This criterion can be expressed as

$$\textbf{saudi-criterion}\,(date) \stackrel{\text{def}}{=} \tag{18.17}$$

$$\textbf{new} < phase < \textbf{first-quarter} \text{ and } \textbf{moonlag}\,(date-1, \textbf{mecca}) > 0$$

where

$$\begin{aligned} set &= \textbf{sunset}\,(date-1, \textbf{mecca}) \\ t &= \textbf{universal-from-standard}\,(set, \textbf{mecca}) \\ phase &= \textbf{lunar-phase}\,(t) \end{aligned}$$

$$\textbf{saudi-new-month-on-or-before}\,(date) \stackrel{\text{def}}{=} \tag{18.18}$$

$$\underset{d \geqslant \tau}{\textbf{MIN}} \left\{ \textbf{saudi-criterion}\,(d) \right\}$$

where

$$\begin{aligned} moon &= \textbf{fixed-from-moment}\,(\textbf{lunar-phase-at-or-before}\,(\textbf{new}, date)) \\ age &= date - moon \\ \tau &= \begin{cases} moon - 30 & \textbf{if } age \leqslant 3 \text{ and not } \textbf{saudi-criterion}\,(date) \\ moon & \textbf{otherwise} \end{cases} \end{aligned}$$

The functions **fixed-from-saudi-islamic** and **saudi-islamic-from-fixed** below are analogous to **fixed-from-observational-islamic** and **observational-islamic-from-fixed**, respectively, except that **saudi-new-month-on-or-before** is used:

$$\textbf{fixed-from-saudi-islamic}\left( \boxed{year \mid month \mid day} \right) \stackrel{\text{def}}{=} \tag{18.19}$$

$$\textbf{saudi-new-month-on-or-before}\,(midmonth) + day - 1$$

where

$$\begin{aligned} midmonth = {} & \textbf{islamic-epoch} \\ & + \left\lfloor \left( (year-1) \times 12 + month - \frac{1}{2} \right) \times \textbf{mean-synodic-month} \right\rfloor \end{aligned}$$

In the other direction,

$$\textbf{saudi-islamic-from-fixed}\,(date) \stackrel{\text{def}}{=} \boxed{year \mid month \mid day} \tag{18.20}$$

[4] See `www.kacst.edu.sa/en/services/ummalqura` on the King Abdulaziz City for Science and Technology web site.

where

$$
\begin{aligned}
\textit{crescent} &= \textbf{saudi-new-month-on-or-before}\,(\textit{date})\\
\textit{elapsed-months} &= \text{round}\left(\frac{\textit{crescent} - \textbf{islamic-epoch}}{\textbf{mean-synodic-month}}\right)\\
\textit{year} &= \left\lfloor \frac{1}{12} \times \textit{elapsed-months} \right\rfloor + 1\\
\textit{month} &= (\textit{elapsed-months} \bmod 12) + 1\\
\textit{day} &= \textit{date} - \textit{crescent} + 1
\end{aligned}
$$

## 18.4 The Classical Hebrew Calendar

*O, swear not by the moon, th' inconstant moon,*
*That monthly changes in her circle orb . . .*

William Shakespeare: *Romeo and Juliet*, Act II, scene ii (1591)

In classical times, the Hebrew month began with the reported observation of the crescent new moon, just like the Islamic religious calendar of the previous section.[5] Unlike in the Islamic calendar, leap months were intercalated in such a way that the spring equinox always fell before the onset of Nisan 16 [4, 4:2]. The exact method of determining the day of the equinox and the exact cutoff date are uncertain; also, the courts had leeway to declare a leap year when spring came late.

We will take Haifa, a city at the western edge of Israel, as the location from which observations are made (being at the west makes visibility more likely):

$$\textbf{hebrew-location} \stackrel{\text{def}}{=} \boxed{32.82^\circ \mid 35^\circ \mid 0\text{ m} \mid 2^{\text{h}}} \tag{18.21}$$

With the methods of this chapter, it is straightforward to convert dates for this classical Hebrew observational calendar. The first of Nisan is determined on the basis of the vernal equinox:

$$\textbf{observational-hebrew-first-of-nisan}\,(\textit{g-year}) \stackrel{\text{def}}{=} \tag{18.22}$$

$$\textbf{phasis-on-or-after}\left(\lfloor \textit{equinox} \rfloor - \begin{cases} 14 & \textbf{if } \textit{equinox} < \textit{set} \\ 13 & \textbf{otherwise} \end{cases}\right\}, \textbf{hebrew-location}\right)$$

where

$$
\begin{aligned}
\textit{equinox} &= \textbf{season-in-gregorian}\,(\textbf{spring}, \textit{g-year})\\
\textit{set} &= \textbf{universal-from-standard}\\
&\qquad \left(\textbf{sunset}\,(\lfloor \textit{equinox} \rfloor, \textbf{hebrew-location}),\right.\\
&\qquad \left.\textbf{hebrew-location}\right)
\end{aligned}
$$

[5] Karaite Jews still use this form of the Hebrew calendar and intercalate based on the state of the barley crop.

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The start of each month is determined by the observability (visibilty) of the new moon:

$$\textbf{observational-hebrew-from-fixed}\,(date) \stackrel{\text{def}}{=} \boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}} \tag{18.23}$$

where

$$\begin{aligned}
\mathit{crescent} &= \textbf{phasis-on-or-before}\,(\mathit{date}, \textbf{hebrew-location})\\
\mathit{g\text{-}year} &= \textbf{gregorian-year-from-fixed}\,(\mathit{date})\\
\mathit{ny} &= \textbf{observational-hebrew-first-of-nisan}\,(\mathit{g\text{-}year})\\
\mathit{new\text{-}year} &= \begin{cases} \textbf{observational-hebrew-first-of-nisan}\,(\mathit{g\text{-}year}-1) & \textbf{if } \mathit{date} < \mathit{ny}\\ \mathit{ny} & \textbf{otherwise}\end{cases}\\
\mathit{month} &= \text{round}\left(\frac{1}{29.5} \times (\mathit{crescent} - \mathit{new\text{-}year})\right) + 1\\
\mathit{year} &= (\textbf{hebrew-from-fixed}\,(\mathit{new\text{-}year}))_{\textbf{year}} + \begin{Bmatrix} 1 & \textbf{if } \mathit{month} \geqslant \textbf{tishri}\\ 0 & \textbf{otherwise}\end{Bmatrix}\\
\mathit{day} &= \mathit{date} - \mathit{crescent} + 1
\end{aligned}$$

The inverse computation is

$$\textbf{fixed-from-observational-hebrew}\left(\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}\right) \stackrel{\text{def}}{=} \tag{18.24}$$

$$\textbf{phasis-on-or-before}\,(\mathit{midmonth}, \textbf{hebrew-location}) + \mathit{day} - 1$$

where

$$\begin{aligned}
\mathit{year}_1 &= \begin{cases} \mathit{year} - 1 & \textbf{if } \mathit{month} \geqslant \textbf{tishri}\\ \mathit{year} & \textbf{otherwise}\end{cases}\\
\mathit{start} &= \textbf{fixed-from-hebrew}\left(\boxed{\mathit{year}_1 \mid \textbf{nisan} \mid 1}\right)\\
\mathit{g\text{-}year} &= \textbf{gregorian-year-from-fixed}\,(\mathit{start} + 60)\\
\mathit{new\text{-}year} &= \textbf{observational-hebrew-first-of-nisan}\,(\mathit{g\text{-}year})\\
\mathit{midmonth} &= \mathit{new\text{-}year} + \text{round}\,(29.5 \times (\mathit{month} - 1)) + 15
\end{aligned}$$

Using the above functions, we can approximate the classical date of Passover Eve (Nisan 14) in any given Gregorian year:

$$\textbf{classical-passover-eve}\,(\mathit{g\text{-}year}) \stackrel{\text{def}}{=} \tag{18.25}$$

$$\textbf{observational-hebrew-first-of-nisan}\,(\mathit{g\text{-}year}) + 13$$

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As we did for the observational Islamic calendar of the previous section, we can take into account the rule disallowing 31-day months:

$$\textbf{alt-observational-hebrew-from-fixed}\,(date) \stackrel{\text{def}}{=} \tag{18.26}$$

$$\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}$$

where

$$\begin{aligned}
early &= \textbf{early-month?}\,(date, \textbf{hebrew-location}) \\
long &= early \text{ and } \textbf{month-length}\,(date, \textbf{hebrew-location}) > 29 \\
date' &= \begin{cases} date + 1 & \textbf{if } long \\ date & \textbf{otherwise} \end{cases} \\
moon &= \textbf{phasis-on-or-before}\,(date', \textbf{hebrew-location}) \\
g\text{-}year &= \textbf{gregorian-year-from-fixed}\,(date') \\
ny &= \textbf{observational-hebrew-first-of-nisan}\,(g\text{-}year) \\
new\text{-}year &= \begin{cases} \textbf{observational-hebrew-first-of-nisan}\,(g\text{-}year - 1) \\ \qquad \textbf{if } date' < ny \\ ny \quad \textbf{otherwise} \end{cases} \\
month &= \text{round}\left(\frac{moon - new\text{-}year}{29.5}\right) + 1 \\
year &= (\textbf{hebrew-from-fixed}\,(new\text{-}year))_{\textbf{year}} \\
&\quad + \left\{\begin{array}{ll} 1 & \textbf{if } month \geqslant \textbf{tishri} \\ 0 & \textbf{otherwise} \end{array}\right\} \\
day &= date' - moon - \left\{\begin{array}{ll} -2 & \textbf{if } early \text{ and not } long \\ -1 & \textbf{otherwise} \end{array}\right\}
\end{aligned}$$

In the other direction,

$$\textbf{alt-fixed-from-observational-hebrew}\left(\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}\right) \stackrel{\text{def}}{=} \tag{18.27}$$

$$\begin{cases} date - 1 & \textbf{if early-month?}\,(midmonth, \textbf{hebrew-location}) \\ date & \textbf{otherwise} \end{cases}$$

where

$$year_1 = \begin{cases} year - 1 & \textbf{if } month \geqslant \textbf{tishri} \\ year & \textbf{otherwise} \end{cases}$$

$$\begin{aligned}
start &= \textbf{fixed-from-hebrew}\left(\begin{array}{|c|c|c|}\hline year_1 & \textbf{nisan} & 1 \\ \hline\end{array}\right) \\
g\text{-}year &= \textbf{gregorian-year-from-fixed}\,(start + 60) \\
new\text{-}year &= \textbf{observational-hebrew-first-of-nisan}\,(g\text{-}year) \\
midmonth &= new\text{-}year + \text{round}\,(29.5 \times (month - 1)) + 15 \\
moon &= \textbf{phasis-on-or-before}\,(midmonth, \textbf{hebrew-location}) \\
date &= moon + day - 1
\end{aligned}$$

## 18.5 The Samaritan Calendar

*The last of such tables ever written was sent by Shalmah … in 1820 … As it is, most probably, the last document of its kind that ever will be drawn up by a Samaritan priest, I shall here subjoin it.*

John Mills: *Three Months' Residence at Nablus, and an Account of the Modern Samaritans* (1864)

The Samaritan calendar is lunisolar, like the Hebrew. Months are numbered, as in most of the Bible. The first day of each month is that on which the new moon occurs, unless it occurs after apparent noon, in which case the next day is the first of the month. The moment of new moon is determined according to their traditional method, referred to as the "True Reckoning," which agrees with the medieval tables of al-Battānī [7] for finding the true positions of the sun and moon.

Time is measured in temporal hours beginning at sunset and sunrise. The critical time for determining the beginning of the month is apparent noon on Mount Gerizim, for which we have

$$\textbf{samaritan-location} \stackrel{\text{def}}{=} \begin{array}{|c|c|c|c|}\hline 32.1994^\circ & 35.2728^\circ & 881\text{ m} & 2^{\text{h}} \\ \hline\end{array} \tag{18.28}$$

and

$$\textbf{samaritan-noon}\,(date) \stackrel{\text{def}}{=} \textbf{midday}\,(date, \textbf{samaritan-location}) \tag{18.29}$$

Rather than replicate these traditional approximations of the true times, we use our astronomical code to find the actual day of the new moon:

$$\textbf{samaritan-new-moon-after}\,(t) \stackrel{\text{def}}{=} \left\lceil \begin{array}{l} \textbf{apparent-from-universal} \\ \quad (\textbf{new-moon-at-or-after}\,(t), \textbf{samaritan-location}) \\ - 12^{\text{h}} \end{array} \right\rceil \tag{18.30}$$

and

$$\textbf{samaritan-new-moon-at-or-before}\,(t) \stackrel{\text{def}}{=} \left\lceil \begin{array}{l} \textbf{apparent-from-universal}\,(\textbf{new-moon-before}\,(t), \textbf{samaritan-location}) \\ - 12^{\text{h}} \end{array} \right\rceil \tag{18.31}$$

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(Since we are working with high precision reals, we can ignore the possibility of a new moon occurring precisely at noon.)

The first month of the year is that which begins on or after March 12 (Julian); this ensures that the Festival of the Unleavened Bread, which runs from the 15th through the 21st of the first month, occurs after the Julian vernal equinox, which was March 25 when the Julian calendar was instituted. A leap month is added when necessary at the end of the year. Years begin with the *sixth* lunar month. (The Hebrew calendar of Chapter 8 begins its calendar year with the *seventh* month.) They are counted from the summer of 1639 B.C.E., the traditional year when the Israelites entered the Promised Land. As epoch, we take month 1, day 1 of year 0 A.S.:[6]

$$\textbf{samaritan-epoch} \stackrel{\text{def}}{=} \tag{18.32}$$

$$\textbf{fixed-from-julian}\left(\boxed{1639 \text{ B.C.E.} \mid \textbf{march} \mid 15}\right)$$

The conversions are not difficult:

$$\textbf{samaritan-new-year-on-or-before}\,(\mathit{date}) \stackrel{\text{def}}{=} \tag{18.33}$$

$$\textbf{samaritan-new-moon-after}\left(\textbf{samaritan-noon}\left(\mathit{dates}_{[n]}\right)\right)$$

where

$$\begin{aligned}
\mathit{g\text{-}year} &= \textbf{gregorian-year-from-fixed}\,(\mathit{date})\\
\mathit{dates} &= \textbf{julian-in-gregorian}\,(\textbf{march}, 11, \mathit{g\text{-}year} - 1)\\
&\quad \parallel \textbf{julian-in-gregorian}\,(\textbf{march}, 11, \mathit{g\text{-}year}) \parallel \langle \mathit{date} + 1 \rangle\\
n &= \underset{i \geqslant 0}{\mathbf{MAX}} \left\{ \begin{array}{l} \textbf{samaritan-new-moon-after}\\ \quad \left(\textbf{samaritan-noon}\left(\mathit{dates}_{[i]}\right)\right) \leqslant \mathit{date} \end{array} \right\}
\end{aligned}$$

We search for the relevant March 11 from the list *dates*.

$$\textbf{fixed-from-samaritan}\left(\boxed{\mathit{year} \mid \mathit{month} \mid \mathit{day}}\right) \stackrel{\text{def}}{=} \tag{18.34}$$

$$\mathit{nm} + \mathit{day} - 1$$

where

$$\begin{aligned}
\mathit{ny} &= \textbf{samaritan-new-year-on-or-before}\\
&\quad \left( \left\lfloor \textbf{samaritan-epoch} + 50 \right.\right.\\
&\quad\quad \left.\left. + 365.25 \times \left(\mathit{year} - \left\lceil \frac{\mathit{month} - 5}{8} \right\rceil\right) \right\rfloor \right)\\
\mathit{nm} &= \textbf{samaritan-new-moon-at-or-before}\,(\mathit{ny} + 29.5 \times (\mathit{month} - 1) + 15)
\end{aligned}$$

[6] Anno Samaritanorum.

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In the other direction,

$$\textbf{samaritan-from-fixed}\,(date) \stackrel{\text{def}}{=} \tag{18.35}$$

$$\boxed{\begin{array}{c|c|c} year & month & day \end{array}}$$

where

$$\begin{aligned}
moon &= \textbf{samaritan-new-moon-at-or-before}\,(\textbf{samaritan-noon}\,(date))\\
new\text{-}year &= \textbf{samaritan-new-year-on-or-before}\,(moon)\\
month &= \operatorname{round}\left(\frac{moon - new\text{-}year}{29.5}\right) + 1\\
year &= \operatorname{round}\left(\tfrac{1}{365.25} \times (new\text{-}year - \textbf{samaritan-epoch})\right)\\
&\quad + \left\lceil \frac{month - 5}{8} \right\rceil\\
day &= date - moon + 1
\end{aligned}$$

The term $\lceil (month - 5)/8 \rceil$ serves to adjust for the fact that the calendar year begins with the sixth month.

The major holidays are those listed in the Pentateuch: Passover (month 1, day 14), Festival of the Unleavened Bread (month 1, days 15–21), Festival of Pentecost (the eighth Sunday after Passover), Festival of the Seventh Month (month 7, day 1), Day of Atonement (month 7, day 10), Festival of Tabernacles (month 7, days 15–21), and the Eighth Day (month 7, day 22). All holidays begin on the prior evening. There are two additional, preparatory feast days: Ṣimmut of Passover, which occurs on the Sabbath that falls seven weeks before Passover, and Ṣimmut of Tabernacles, which occurs on the Sabbath seven weeks before Tabernacles. On these days, the semi-annual calendar is delivered to the community by the high priest.

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Pottery figurines of the 12 traditional Chinese calendrical animals (terrestrial branches) excavated from a Táng Dynasty (618–907 C.E.) tomb. These figures, shown left to right in the order given on page 319, have animal faces on human bodies with long robes; such funerary use of the 12 animals is still in practice. (Image © The Metropolitan Museum of Art, New York. Image source: Art Resource, New York.)