

with(Student[LinearAlgebra]) :  
with(LinearAlgebra) : with(linalg) :

#Exercise 1 - i

$A := \text{Matrix}([ [0, -2, 0], [1, -2, 0], [0, 0, -2] ])$

$$A := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (1)$$

$\det(A)$

$$-4 \quad (2)$$

$A^{-1}$

$$\begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad (3)$$

$\text{CharacteristicPolynomial}(A, x)$

$$x^3 + 4x^2 + 6x + 4 \quad (4)$$

$\text{Eigenvalues}(A)$

$$\begin{bmatrix} -2 \\ -1 - I \\ -1 + I \end{bmatrix} \quad (5)$$

$\text{lam}, P := \text{Eigenvectors}(A)$

$$\text{lam}, P := \begin{bmatrix} -1 + I \\ -1 - I \\ -2 \end{bmatrix}, \begin{bmatrix} 1 + I & 1 - I & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

# Exercise 1 - ii

$u1 := \text{Matrix}(\langle \langle 0, 0, 1 \rangle \rangle)$

$$u1 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

$$lambda1 := -2$$

$$\lambda1 := -2 \tag{8}$$

$$b := convert(A \bullet u1, Vector)$$

$$b := \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{9}$$

$$c := convert(lambda1 \cdot u1, Vector)$$

$$c := \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \tag{10}$$

$$b - c$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

# Exercise 1 - iii

$$u2 := Matrix(\langle \langle 1 + I, 1, 0 \rangle \rangle)$$

$$u2 := \begin{bmatrix} 1 + I \\ 1 \\ 0 \end{bmatrix} \tag{12}$$

$$\lambda2 := -1 + I$$

$$\lambda2 := -1 + I \tag{13}$$

$$b := convert(A \bullet u2, Vector)$$

$$b := \begin{bmatrix} -2 \\ -1 + I \\ 0 \end{bmatrix} \tag{14}$$

$$c := convert(\lambda 2 \cdot u2, Vector)$$

$$c := \begin{bmatrix} -2 \\ -1 + I \\ 0 \end{bmatrix} \tag{15}$$

$$b - c$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

$$\#Exercise\ 1 - iv$$

$$u3 := Matrix(\langle \langle 1 - I, 1, 0 \rangle \rangle)$$

$$u3 := \begin{bmatrix} 1 - I \\ 1 \\ 0 \end{bmatrix} \tag{17}$$

$$\lambda 3 := -1 - I$$

$$\lambda 3 := -1 - I \tag{18}$$

$$b := convert(A \bullet u3, Vector)$$

$$b := \begin{bmatrix} -2 \\ -1 - I \\ 0 \end{bmatrix} \tag{19}$$

$$c := convert(\lambda 3 \cdot u3, Vector)$$

$$c := \begin{bmatrix} -2 \\ -1 - I \\ 0 \end{bmatrix} \tag{20}$$

$$b - c$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

#Exercise 1 - v

$$P := \text{Matrix}(\langle \langle u1|u2|u3 \rangle \rangle)$$

$$P := \begin{bmatrix} 0 & 1 + \text{I} & 1 - \text{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (22)$$

#Exercise 1 - vi

$$J := \text{DiagonalMatrix}([\lambda 1, \lambda 2, \lambda 3])$$

$$J := \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 + \text{I} & 0 \\ 0 & 0 & -1 - \text{I} \end{bmatrix} \quad (23)$$

#Exercise 1 - vii

$$A - P \bullet J \bullet P^{-1}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$\text{MatrixExponential}(t \cdot J)$$

$$\begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t} \cos(t) + \text{I} e^{-t} \sin(t) & 0 \\ 0 & 0 & e^{-t} \cos(t) - \text{I} e^{-t} \sin(t) \end{bmatrix} \quad (25)$$

$$\text{MatrixExponential}(t \cdot A)$$

$$\begin{bmatrix} e^{-t} \cos(t) + e^{-t} \sin(t) & -2 e^{-t} \sin(t) & 0 \\ e^{-t} \sin(t) & e^{-t} \cos(t) - e^{-t} \sin(t) & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix} \quad (26)$$

Map(limit, MatrixExponential( $t \cdot A$ ),  $t = \text{infinity}$ )

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (27)$$

# Exercise 3 - i

$$eq1 := \text{diff}(x(t), t) = 1 - x(t)^2$$

$$eq1 := \frac{d}{dt} x(t) = 1 - x(t)^2 \quad (28)$$

$$sol := \text{rhs}(\text{dsolve}(\{eq1, x(0) = \text{theta}\}))$$

$$sol := \tanh(t + \text{arctanh}(\theta)) \quad (29)$$

$$\text{eval}(sol, \text{theta} = 1)$$

Error, (in arctanh) numeric exception: division by zero

$$x(t) = -1 \quad (30)$$

$$\text{limit}(sol, \text{theta} = 1)$$

$$1 \quad (31)$$

$$s1 := \text{convert}(\text{convert}(\tanh(t - \text{arctanh}(2)), \text{exp}), \text{exp})$$

$$s1 := \frac{e^{2t} + 3}{e^{2t} - 3} \quad (32)$$

$$s2 := \text{convert}(\text{convert}(\tanh(t - \text{arctanh}(0)), \text{exp}), \text{exp})$$

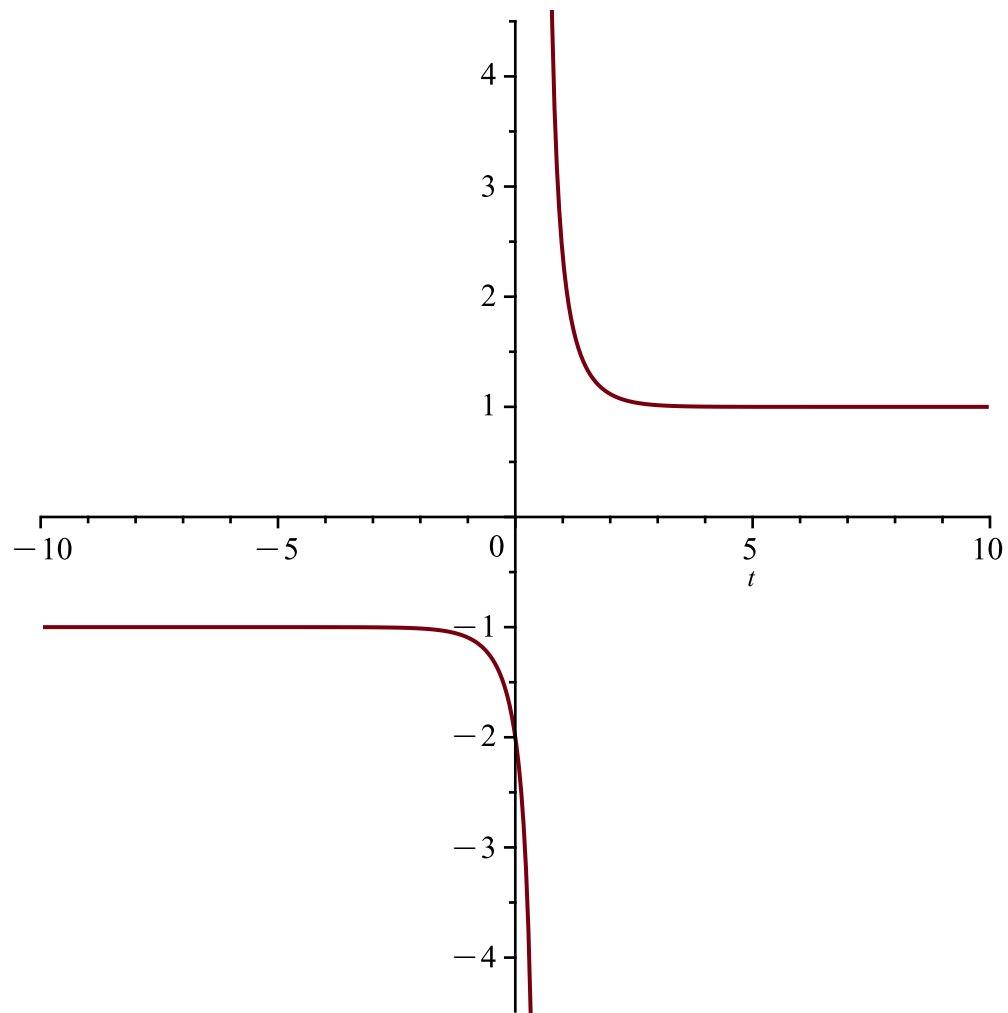
$$s2 := \frac{e^{2t} - 1}{e^{2t} + 1} \quad (33)$$

$$s3 := \text{convert}(\text{convert}(\tanh(t - \text{arctanh}(-2)), \text{exp}), \text{exp})$$

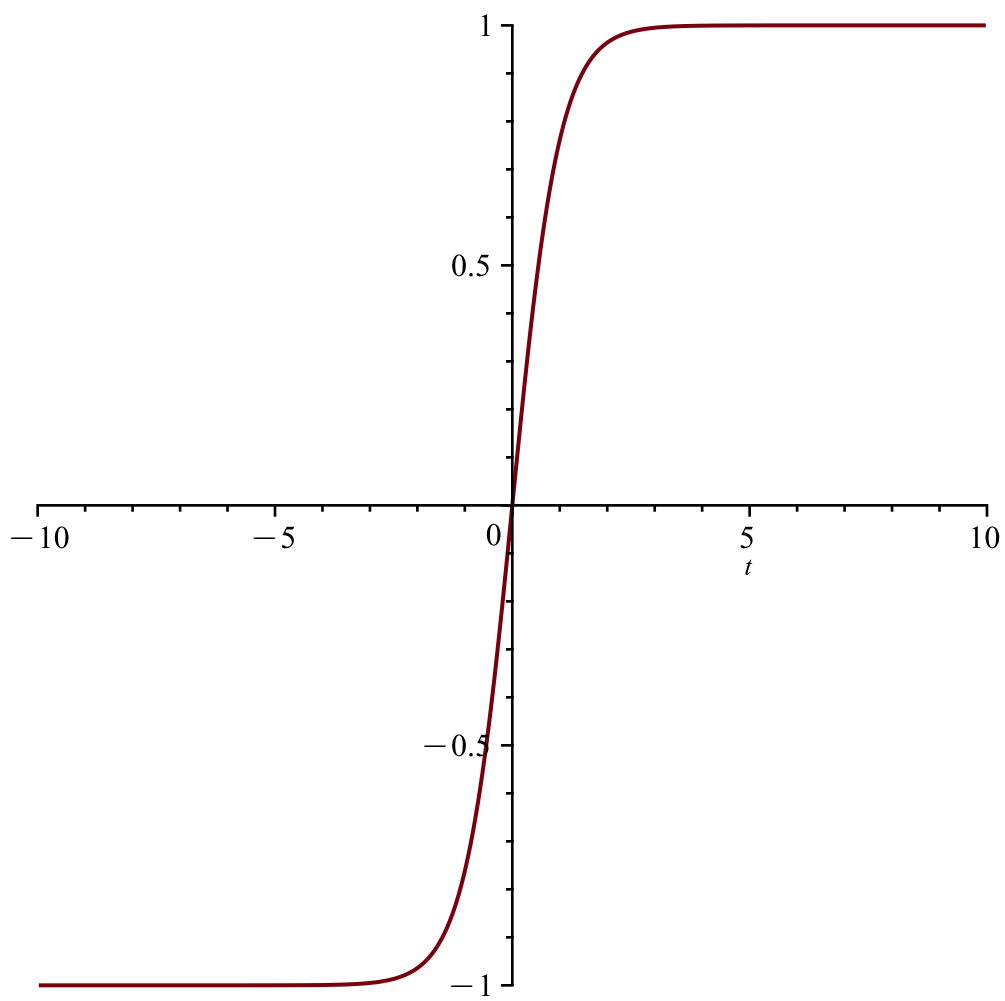
$$s3 := \frac{3e^{2t} + 1}{3e^{2t} - 1} \quad (34)$$

# iii

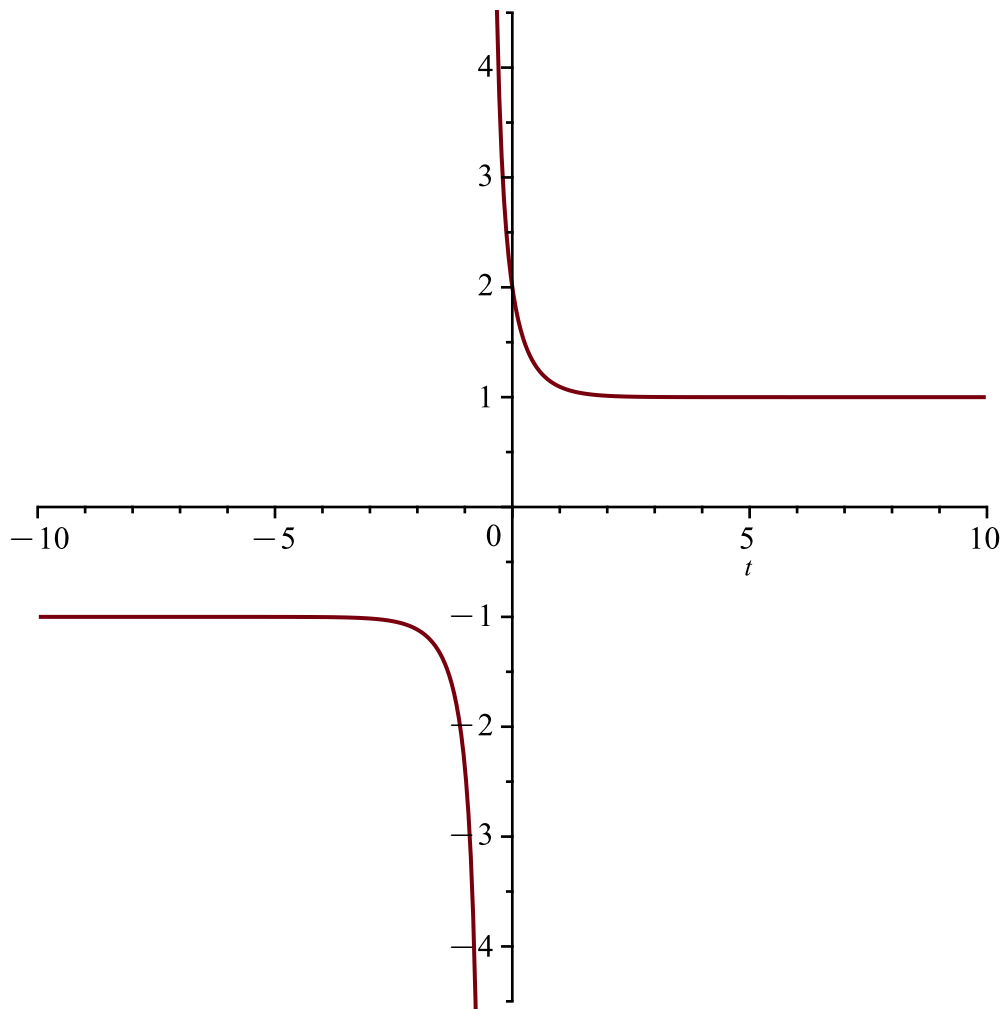
$$\text{plot}(s1)$$



$plot(s_2)$



$plot(s_3)$



#iv

$$sol41 := \tanh(t + \operatorname{arctanh}(-2))$$

$$sol41 := \coth\left(t - \operatorname{arctanh}\left(\frac{1}{2}\right)\right) \quad (35)$$

$$\lim(sol41, t = -\infty)$$

$$-1 \quad (36)$$

$$sol42 := \tanh(t + \operatorname{arctanh}(0))$$

$$sol42 := \tanh(t) \quad (37)$$

$$\lim(sol42, t = -\infty)$$

$$-1 \quad (38)$$

$$sol43 := \tanh(t + \operatorname{arctanh}(0))$$

$$sol43 := \tanh(t) \quad (39)$$

$$\lim(sol43, t = \infty)$$

$$1 \quad (40)$$



$$sol44 := \tanh(t + \operatorname{arctanh}(2))$$

$$sol44 := \coth\left(t + \operatorname{arctanh}\left(\frac{1}{2}\right)\right) \tag{41}$$

$$\operatorname{limit}(sol44, t = \text{infinity})$$

$$1 \tag{42}$$