

<pre> > a := evalf(25^{1/8}) </pre>	<pre> a := 1.495348781 </pre>	<p>(1)</p>
<pre> > evalf(a⁸) </pre>	<pre> 24.99999997 </pre>	<p>(2)</p>
<pre> > #hopa > x := 0.5 </pre>	<pre> x := 0.5 </pre>	<p>(3)</p>
<pre> > f := x ↦ x · (1 - x) </pre>	<pre> f := x ↦ x · (1 - x) </pre>	<p>(4)</p>
<pre> > for i from 1 to 200 do x := f(x) : psi(i) := x : print(x); od: </pre>	<pre> 0.01349095392 0.01330894808 0.01313181998 0.01295937528 0.01279142987 0.01262780919 0.01246834762 0.01231288793 0.01216128072 0.01201338397 0.01186906258 0.01172818793 0.01159063754 0.01145629466 0.01132504797 0.01119679126 0.01107142313 0.01094884672 0.01082896948 0.01071170290 0.01059696232 0.01048466671 0.01037473847 0.01026710327 0.01016168986 0.01005842992 0.009957257908 </pre>	

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0.009665652845
0.009572228001
0.009480600452
0.009390718667
0.009302533070
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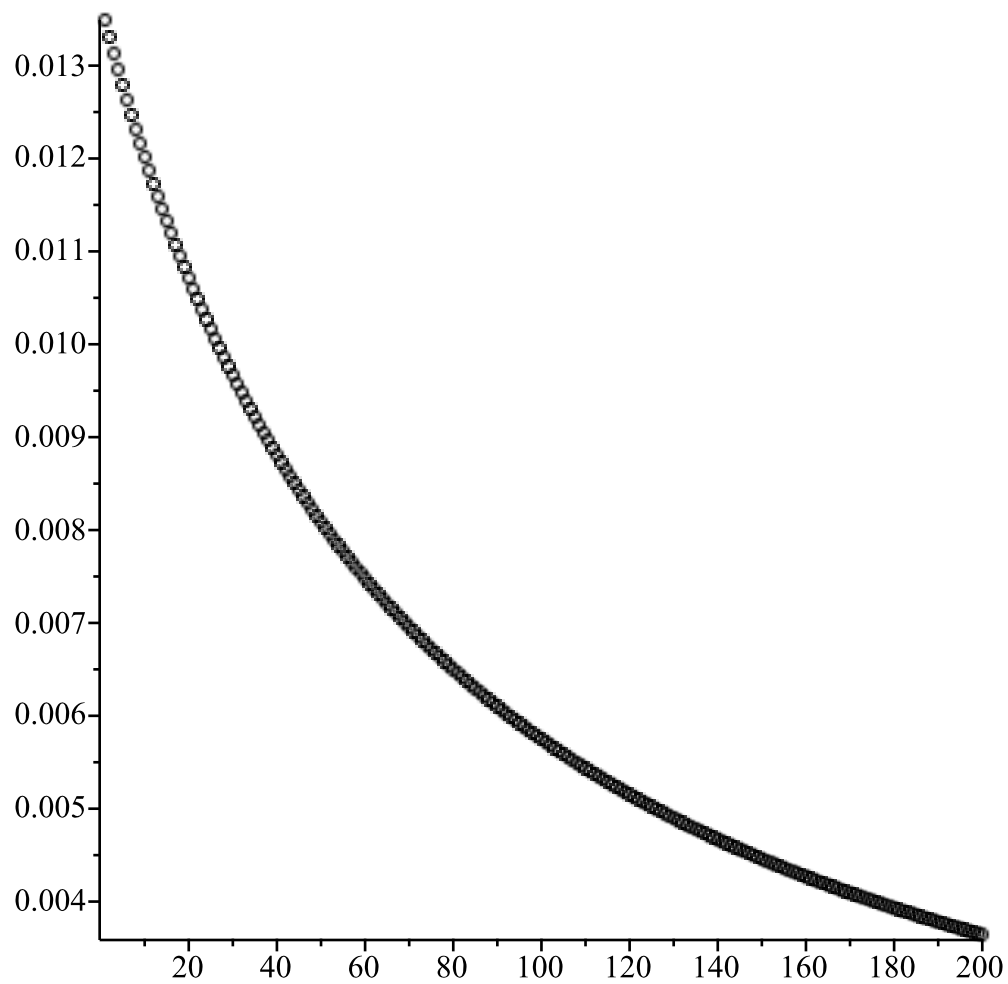
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0.004780902199
0.004758045173
0.004735406179
0.004712982107
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0.003725567503
0.003711687650
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0.003684236479
0.003670662880
0.003657189114
0.003643814082

(5)

```
> points := [ [n, psi(n)]$n = 1 ..200 ] : with(plots) : pointplot(points, symbol=circle);
```



```
> #now the same for x0=0.7
```

```
> x := 0.7
```

```
x := 0.7
```

(6)

```
> for i from 1 to 200 do x := f(x) : psi(i) := x : print(x);od:
```

```
0.21
```

```
0.1659
```

```
0.13837719
```

```
0.1192289433
```

```
0.1050134024
```

```
0.09398558772
```

```
0.08515229702
```

```
0.07790138333
```

```
0.07183275781
```

```
0.06667281272
```

```
0.06222754877
```

```
0.05835528094
```

```
0.05494994213
```

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0.04461857015
0.04262775335
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0.03761278016
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0.02786143249
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0.02635156647
0.02565716141
0.02499887148
0.02437392790
0.02377983954
0.02321435877
0.02267545232
0.02216127618
0.02167015402
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0.02032048684
0.01990756466
0.01951125353
0.01913056452
0.01876458602
0.01841247633
0.01807345705
0.01774680720
0.01743185803

0.01712798836
0.01683462037
0.01655121593
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0.01601232356
0.01575592905
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0.01416929938
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0.01194067191
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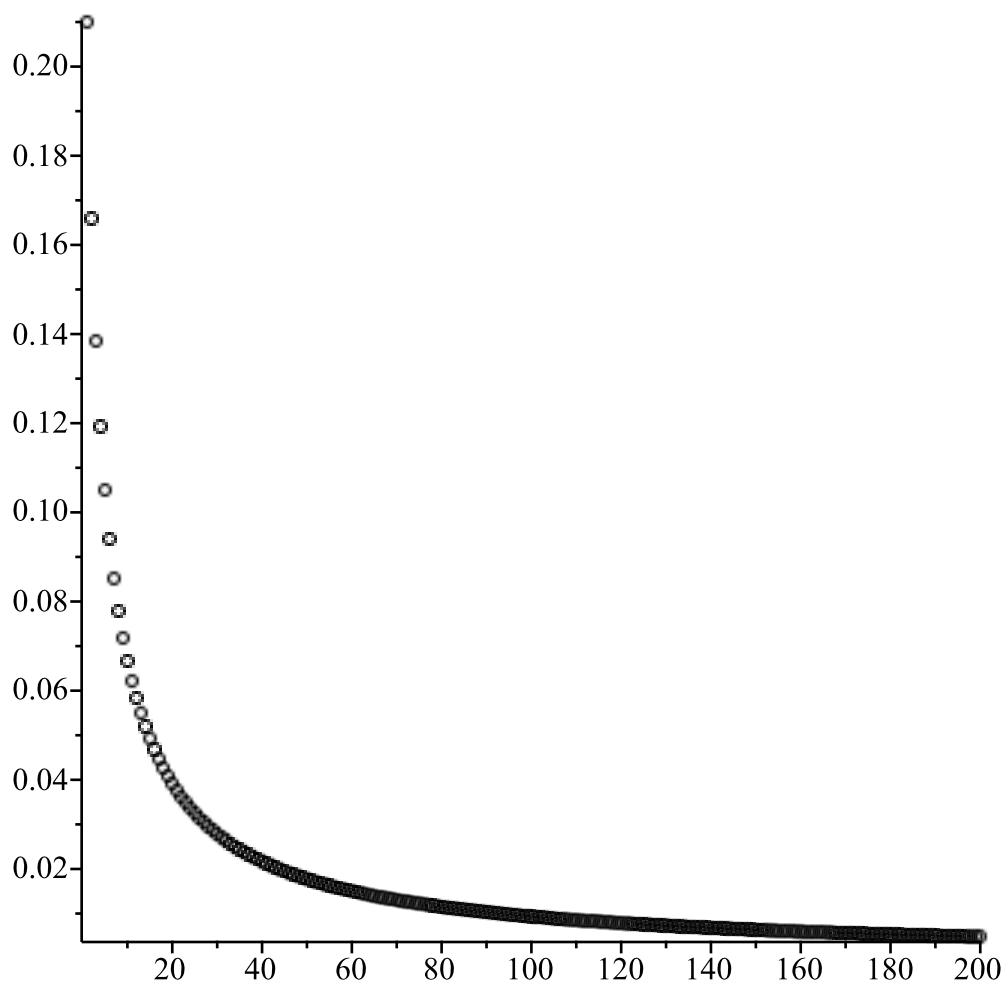
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0.01010971427
0.01000750795
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0.009618639542
0.009526121316
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0.004862646577
0.004839001245
0.004815585312

(7)

> *points* := [[*n*, *psi*(*n*)]\$*n* = 1 ..200] : *with*(*plots*) : *pointplot*(*points*, *symbol* = *circle*);



```
> x := 0; for i from 1 to 200 do x := f(x) : psi(i) := x : print(x); od; points := [[n, psi(n)]$n = 1
..200] : with(plots) : pointplot(points, symbol=circle);
```

```
      x := 0
```

```
      0
```

```
      0
```

```
      0
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      0
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      0
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      0
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      0
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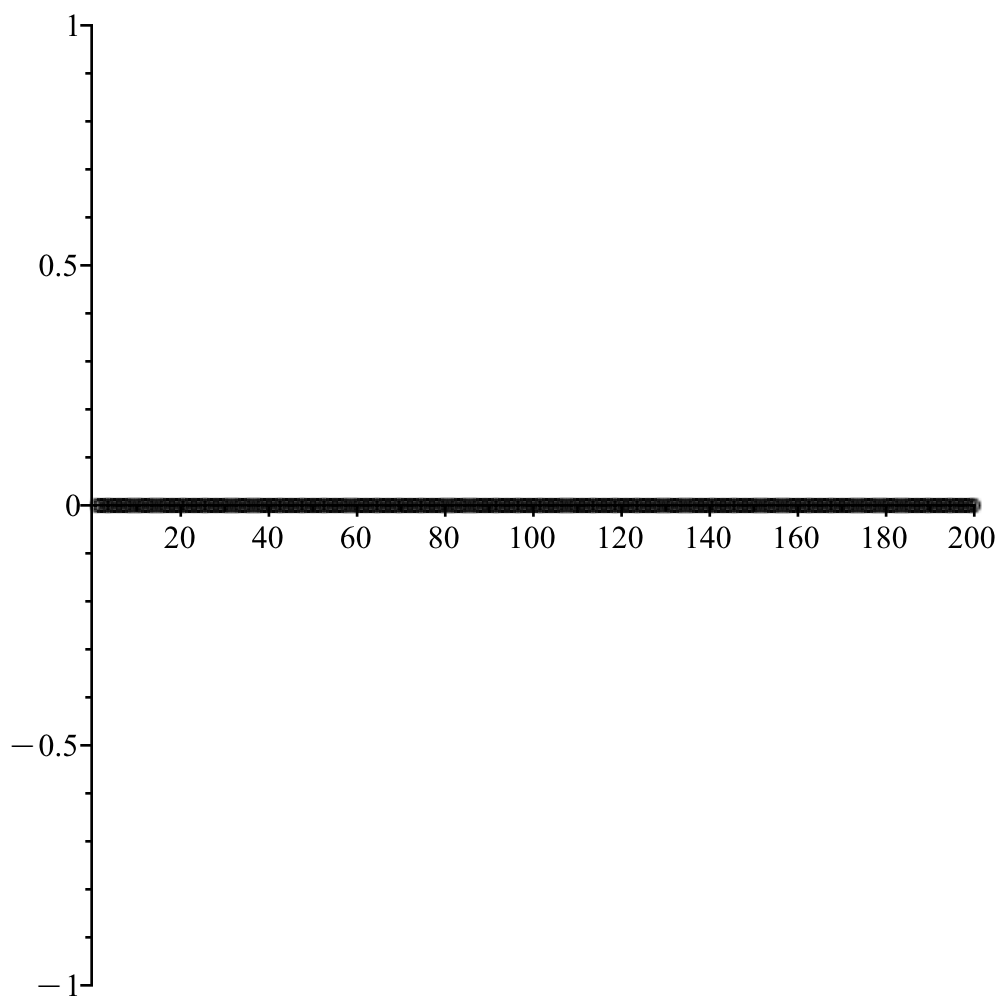
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]



$$\begin{aligned} &> f2 := x \mapsto 2 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f2 := x \mapsto 2 \cdot x \cdot (1 - x) \end{aligned} \tag{8}$$

$$\begin{aligned} &> f3point1 := x \mapsto 3.1 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f3point1 := x \mapsto 3.1 \cdot x \cdot (1 - x) \end{aligned} \tag{9}$$

$$\begin{aligned} &> f3point5 := x \mapsto 3.5 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f3point5 := x \mapsto 3.5 \cdot x \cdot (1 - x) \end{aligned} \tag{10}$$

$$\begin{aligned} &> f3point55 := x \mapsto 3.55 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f3point55 := x \mapsto 3.55 \cdot x \cdot (1 - x) \end{aligned} \tag{11}$$

$$\begin{aligned} &> f3point6 := x \mapsto 3.6 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f3point6 := x \mapsto 3.6 \cdot x \cdot (1 - x) \end{aligned} \tag{12}$$

$$\begin{aligned} &> f3point8 := x \mapsto 3.8 \cdot x \cdot (1 - x) \\ & \qquad \qquad \qquad f3point8 := x \mapsto 3.8 \cdot x \cdot (1 - x) \end{aligned} \tag{13}$$

$$\begin{aligned} &> x := 0.5 \\ & \qquad \qquad \qquad x := 0.5 \end{aligned} \tag{14}$$

$$\begin{aligned} &> x := 0.5; \text{ for } i \text{ from } 1 \text{ to } 200 \text{ do } x := f2(x) : \text{psi}(i) := x : \text{print}(x); \text{od}; \text{points} := [[n, \text{psi}(n)] \$ n \\ & \qquad \qquad \qquad = 1 .. 200] : \text{with}(\text{plots}) : \text{pointplot}(\text{points}, \text{symbol} = \text{circle}); \\ & \qquad \qquad \qquad x := 0.5 \end{aligned}$$

0.50

0.5000

0.50000000

0.5000000000

0.5000000000

0.5000000000

0.5000000000

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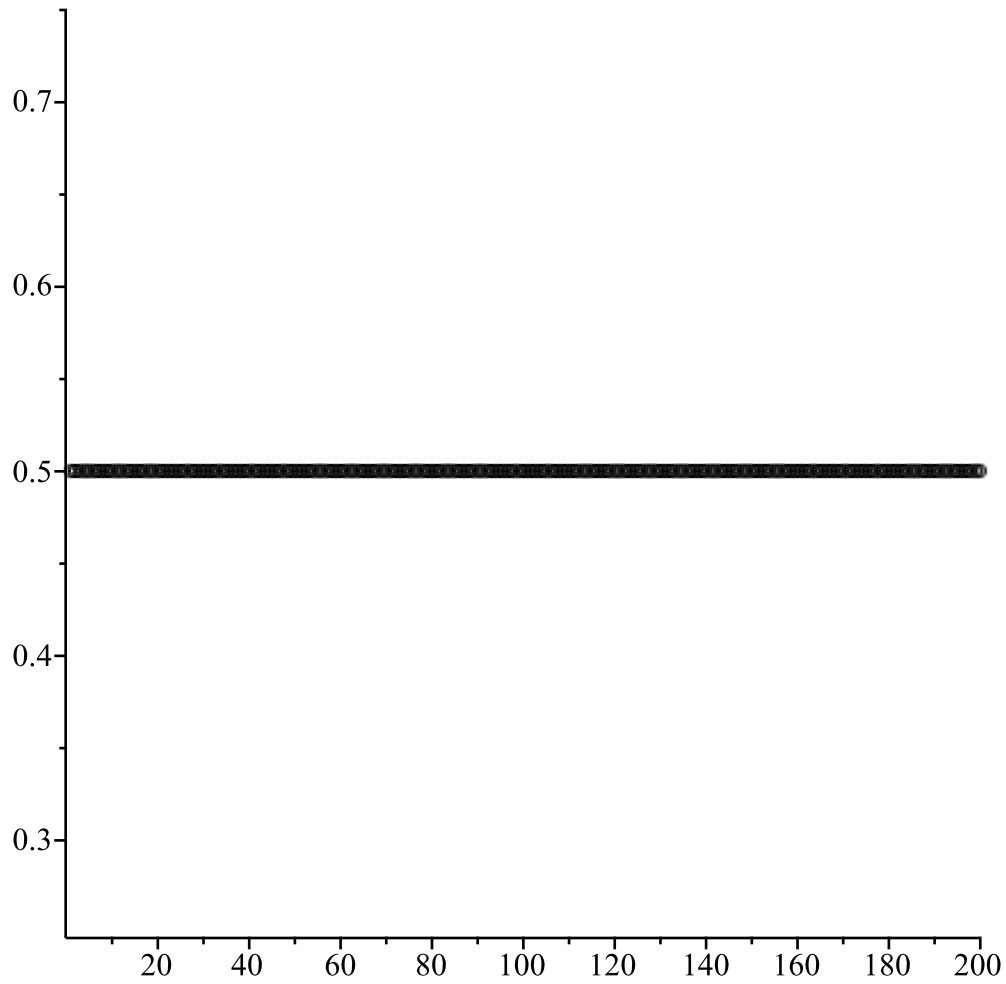
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[illegible]

[illegible]

[illegible]

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0.5000000000
0.5000000000



```
> x := 0.5; for i from 1 to 200 do x := f3point5(x) : psi(i) := x : print(x); od;; points := [[n,  
psi(n)]$n = 1..200] : with(plots) : pointplot(points, symbol = circle);  
x := 0.5
```

0.875

0.3828125

0.8269348145

0.5008976948

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0.8269407063

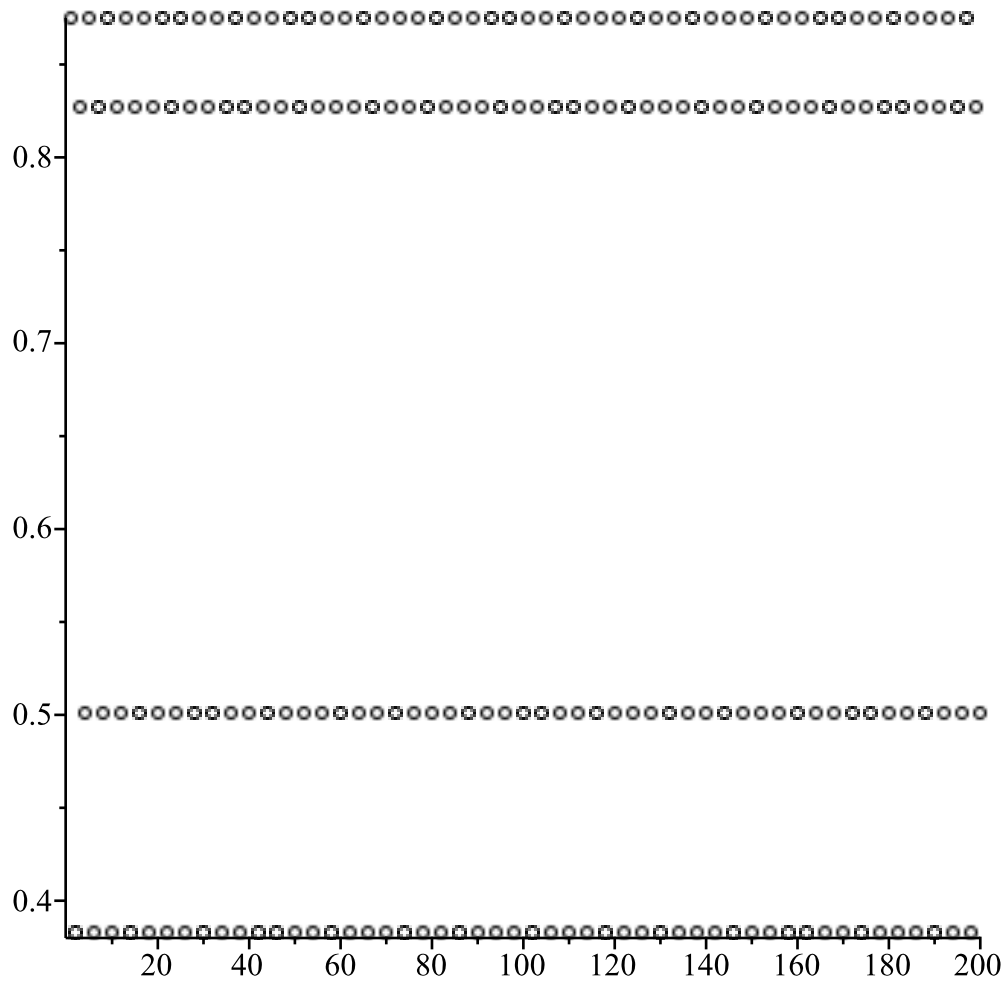
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0.8749972633

0.3828196839

0.8269407075

0.5008842082



```
> x := 0.5; for i from 1 to 200 do x := f3point55(x) : psi(i) := x : print(x); od;; points := [[n,  
psi(n)]$n = 1..200] : with(plots) : pointplot(points, symbol = circle);
```

$x := 0.5$

0.8875

0.3544453125

0.8122891067

0.5412880742

0.8814482968

0.3709650489

0.8283924342

0.5046623524

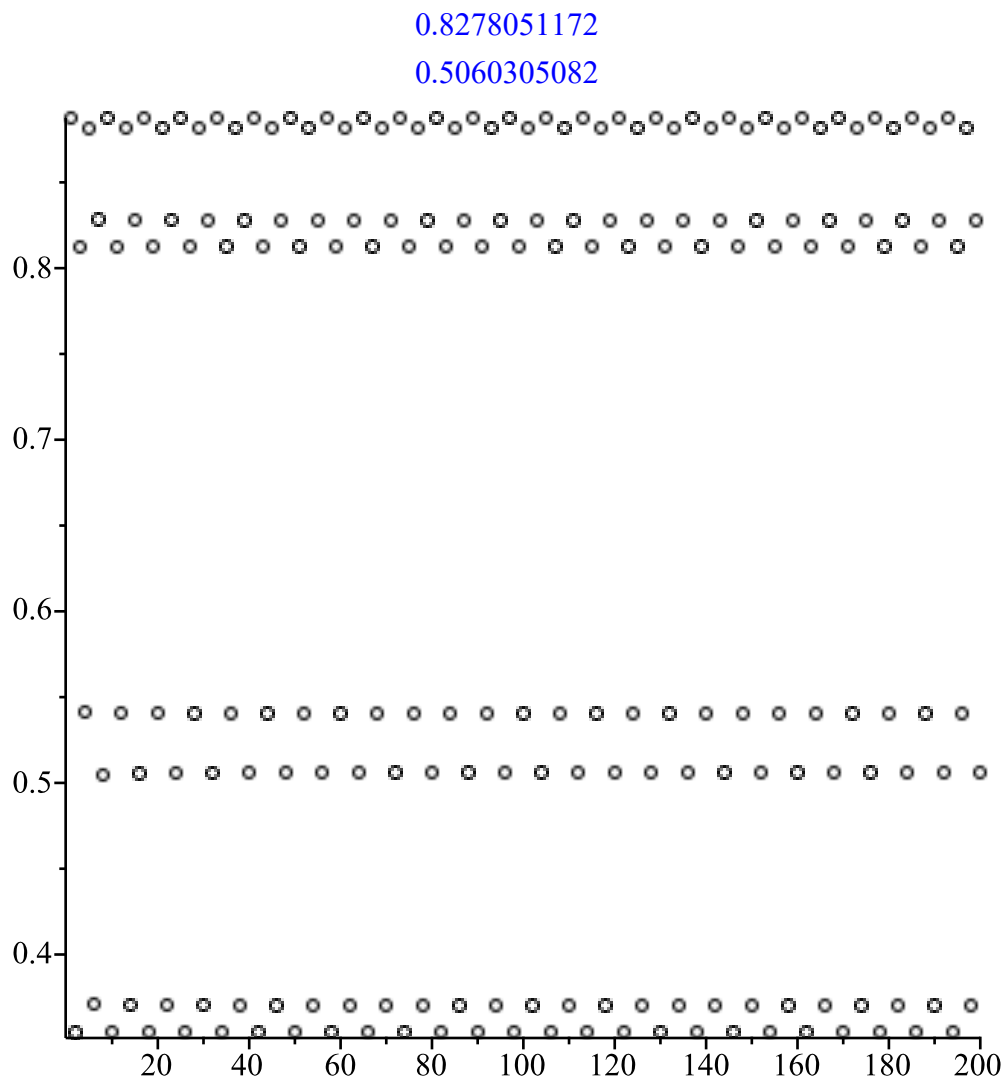
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0.8873932734
0.3547389037
0.8125922093
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0.8816437976
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0.8816842809
0.3703257395
0.8278052807
0.5060301276
0.8873709134
0.3548004029
0.8126556232
0.5404749375
0.8816843169
0.3703256419
0.8278051912
0.5060303361
0.8873709043
0.3548004279
0.8126556491
0.5404748800
0.8816843335
0.3703255969
0.8278051496
0.5060304328
0.8873709001
0.3548004394
0.8126556611
0.5404748534
0.8816843414
0.3703255755
0.8278051299
0.5060304787
0.8873708981
0.3548004449
0.8126556664
0.5404748417
0.8816843445
0.3703255671
0.8278051220
0.5060304970
0.8873708974
0.3548004469

0.8126556684
0.5404748372
0.8816843458
0.3703255637
0.8278051189
0.5060305043
0.8873708971
0.3548004477
0.8126556693
0.5404748352
0.8816843464
0.3703255620
0.8278051174
0.5060305078
0.8873708972
0.3548004474
0.8126556691
0.5404748356
0.8816843461
0.3703255628
0.8278051182
0.5060305060
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082

0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617
0.8278051172
0.5060305082
0.8873708970
0.3548004479
0.8126556697
0.5404748342
0.8816843465
0.3703255617



```
> x := 0.5; for i from 1 to 200 do x := f3point6(x) : psi(i) := x : print(x); od; points := [[n,
psi(n)]$n = 1..200] : with(plots) : pointplot(points, symbol = circle);
```

$x := 0.5$

0.900

0.3240000

0.7884864000

0.6003921493

0.8637170987

0.4237555396

0.8790724164

0.3826947712

0.8504621397

0.4578346391

0.8935994966

0.3422859706

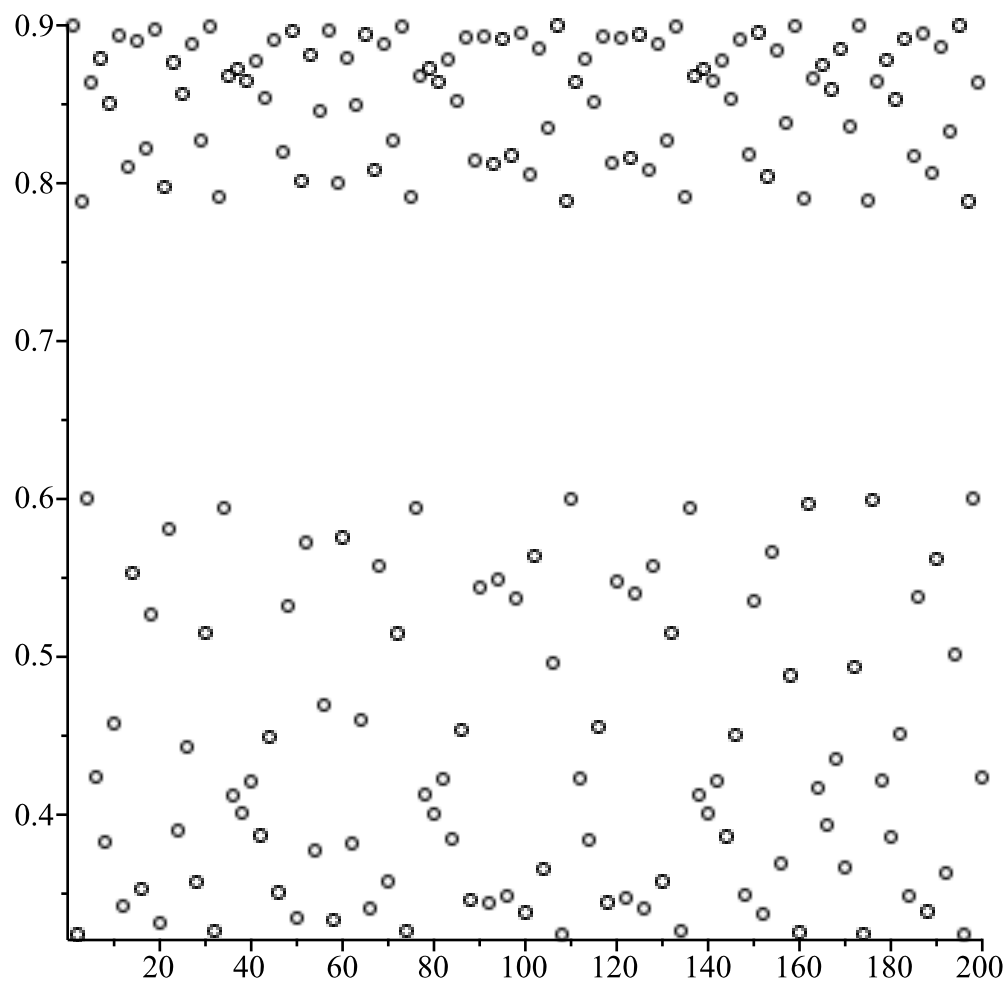
0.8104546256
0.5530245316
0.8898782367
0.3527818580
0.8219765473
0.5267919707
0.8974158853
0.3314182108
0.7976886493
0.5809732850
0.8763959776
0.3899738449
0.8564192829
0.4426750611
0.8881698650
0.3575669613
0.8269661864
0.5151352066
0.8991753320
0.3263725955
0.7914726880
0.5941572198
0.8680839045
0.4122512613
0.8722805721
0.4010658323
0.8647633095
0.4210118209
0.8775391231
0.3868711579
0.8539267140
0.4490491720
0.8906544471
0.3506007707
0.8196475333
0.5321716361
0.8962739490
0.3346810464

0.8016107169
0.5725115117
0.8810714904
0.3772242691
0.8457340318
0.4696847252
0.8966915430
0.3334889510
0.8001866543
0.5755967012
0.8794264994
0.3817279136
0.8496421689
0.4599012734
0.8942115315
0.3405501663
0.8084727021
0.5574405315
0.8881221071
0.3577004280
0.8271029946
0.5148130714
0.8992100625
0.3262727736
0.7913478629
0.5944191219
0.8679061060
0.4127223499
0.8725774027
0.4002698844
0.8641940546
0.4225056862
0.8783806726
0.3845810397
0.8520424690
0.4538379600
0.8923286378
0.3458816639

0.8144911385
0.5439431658
0.8930483935
0.3438466574
0.8122180812
0.5490715311
0.8913311454
0.3486957647
0.8175853022
0.5369024730
0.8950975470
0.3380325421
0.8055595536
0.5638800931
0.8853096013
0.3655314402
0.8349055433
0.4962177975
0.8999485018
0.3241483052
0.7886742534
0.6000018315
0.8639986811
0.4230178566
0.8786654987
0.3838047843
0.8513952184
0.4554770417
0.8928637422
0.3443690882
0.8128044698
0.5477521092
0.8917910498
0.3473991838
0.8161667674
0.5401388708
0.8941999358
0.3405830782

0.8085104825
0.5573566159
0.8881567869
0.3576035116
0.8270036645
0.5150469722
0.8991849191
0.3263450413
0.7914382394
0.5942295094
0.8680348785
0.4123811816
0.8723625937
0.4008459557
0.8646065121
0.4214235289
0.8777726575
0.3862361493
0.8534080303
0.4503699508
0.8911326896
0.3492547889
0.8181931728
0.5355111772
0.8954602427
0.3370003072
0.8043519606
0.5665315827
0.8840647747
0.3689792958
0.8382008703
0.4882326168
0.8995015030
0.3254347768
0.7902971379
0.5966192581
0.8663929882
0.4167222416

0.8750333340
0.3936599941
0.8592904915
0.4352772337
0.8849194685
0.3666132100
0.8359486713
0.4936985650
0.8998570509
0.3244116198
0.7890073945
0.5993090133
0.8644957916
0.4217141445
0.8779367693
0.3857896742
0.8530416053
0.4513018497
0.8914625646
0.3483254179
0.8171813558
0.5378255551
0.8948492184
0.3387387410
0.8063813031
0.5620697896
0.8861304286
0.3632518516
0.8326797982
0.5015669468
0.8999911606
0.3240254572
0.7885186571
0.6003251443
0.8637655153
0.4236287396



```
> x := 0.5; for i from 1 to 200 do x := f3point8(x) : psi(i) := x : print(x); od; points := [[n,
psi(n)]$n = 1..200] : with(plots) : pointplot(points, symbol=circle);
```

x := 0.5

0.950

0.1805000

0.5620950500

0.9353479781

0.2297941243

0.6725573820

0.8368510098

0.5188193093

0.9486541675

0.1850958643

0.5731744642

0.9296528916

0.2485138924

0.7096680032

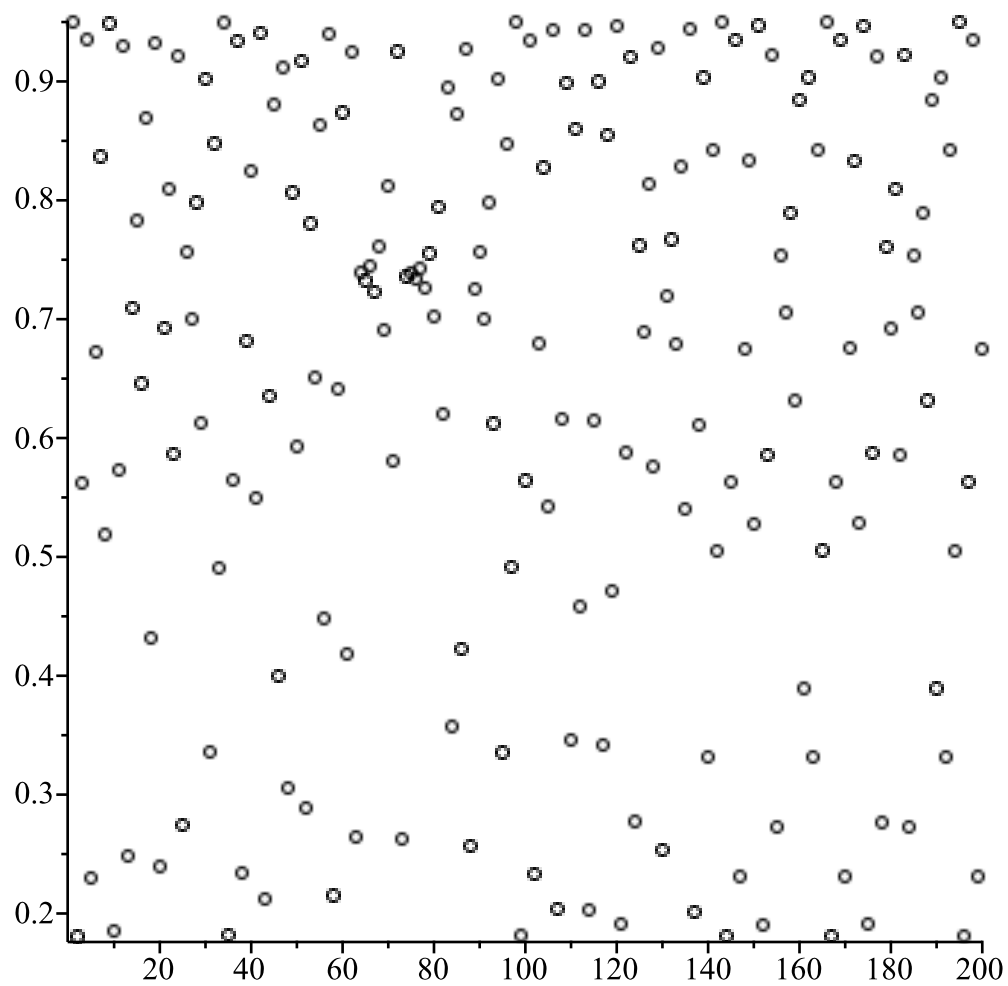
0.7829494480
0.6457705174
0.8692536337
0.4318766652
0.9323650029
0.2396299162
0.6923881940
0.8093497746
0.5863503243
0.9216657615
0.2743523452
0.7565159165
0.6999584215
0.7980631928
0.6124016658
0.9019902890
0.3359344687
0.8477135054
0.4905622091
0.9496615270
0.1816571423
0.5648997348
0.9339944927
0.2342653651
0.6816633945
0.8245939621
0.5496272871
0.9406411030
0.2121745897
0.6351948261
0.8805449641
0.3997050152
0.9117754810
0.3056756224
0.8065045377
0.5930088798
0.9171275234
0.2888175911

0.7805275625
0.6509562895
0.8634063549
0.4481561207
0.9397864065
0.2150340833
0.6414188200
0.8740027259
0.4184634518
0.9247368071
0.2644748498
0.7392060336
0.7325657993
0.7444699661
0.7228908555
0.7612147329
0.6907140806
0.8117869299
0.5805978594
0.9253151433
0.2626067098
0.7358488179
0.7386262734
0.7336185063
0.7426050954
0.7263425173
0.7553224465
0.7022797037
0.7945151016
0.6203912487
0.8949225995
0.3573373335
0.8726599814
0.4222732454
0.9270424964
0.2570118838
0.7256357464
0.7565363377

0.6999186082
0.7981236903
0.6122646080
0.9021072994
0.3355769352
0.8472672121
0.4917408369
0.9497407875
0.1813862514
0.5642440610
0.9343162625
0.2332036597
0.6795149087
0.8275427107
0.5423199362
0.9431942876
0.2035995291
0.6161576912
0.8987280851
0.3458604737
0.8597158244
0.4582971976
0.9433913299
0.2029356886
0.6146606206
0.9000411798
0.3418748069
0.8549864084
0.4711416694
0.9468353478
0.1912850535
0.5878413109
0.9206788354
0.2775114063
0.7618955376
0.6893607641
0.8137415040
0.5759518209

0.9280790192
0.2536437426
0.7193726589
0.7671274189
0.6788431799
0.8284574447
0.5400396867
0.9439079290
0.2011938522
0.6107165670
0.9034189990
0.3315618227
0.8421886055
0.5050464414
0.9499032269
0.1808309284
0.5628981942
0.9349665052
0.2310557296
0.6751421218
0.8334359013
0.5275178990
0.9471225078
0.1903095595
0.5855489580
0.9221892278
0.2726737726
0.7536265882
0.7055595042
0.7894321029
0.6316704197
0.8841190222
0.3893197917
0.9034495874
0.3314680356
0.8420685123
0.5053587050
0.9498908803

0.1808731441
0.5630005894
0.9349175179
0.2312176600
0.6754730041
0.8329950545
0.5286343160
0.9468842887
0.1911188435
0.5874512384
0.9209386675
0.2766804251
0.7604877961
0.6921552107
0.8096902251
0.5855494649
0.9221888986
0.2726748288
0.7536284125
0.7055559879
0.7894375962
0.6316583361
0.8841311136
0.3892844928
0.9034198908
0.3315590885
0.8421851053
0.5050555441
0.9499028778
0.1808321221
0.5629010897
0.9349651211
0.2310603050
0.6751514737



```
> #problem 3
```

```
> f4 := x → 4 · x · (1 − x)
```

$f4 := x \mapsto 4 \cdot x \cdot (1 - x)$

(15)

```
> x := 0.67; for i from 1 to 40 do x := f4(x) : psi(i) := x : print(x); od;; points := [[n, psi(n)]$n
= 1..40] : with(plots) : pointplot(points, symbol=circle);
```

$x := 0.67$

0.8844

0.40894656

0.9668370844

0.1282525465

0.4472153232

0.9888551116

0.04408271944

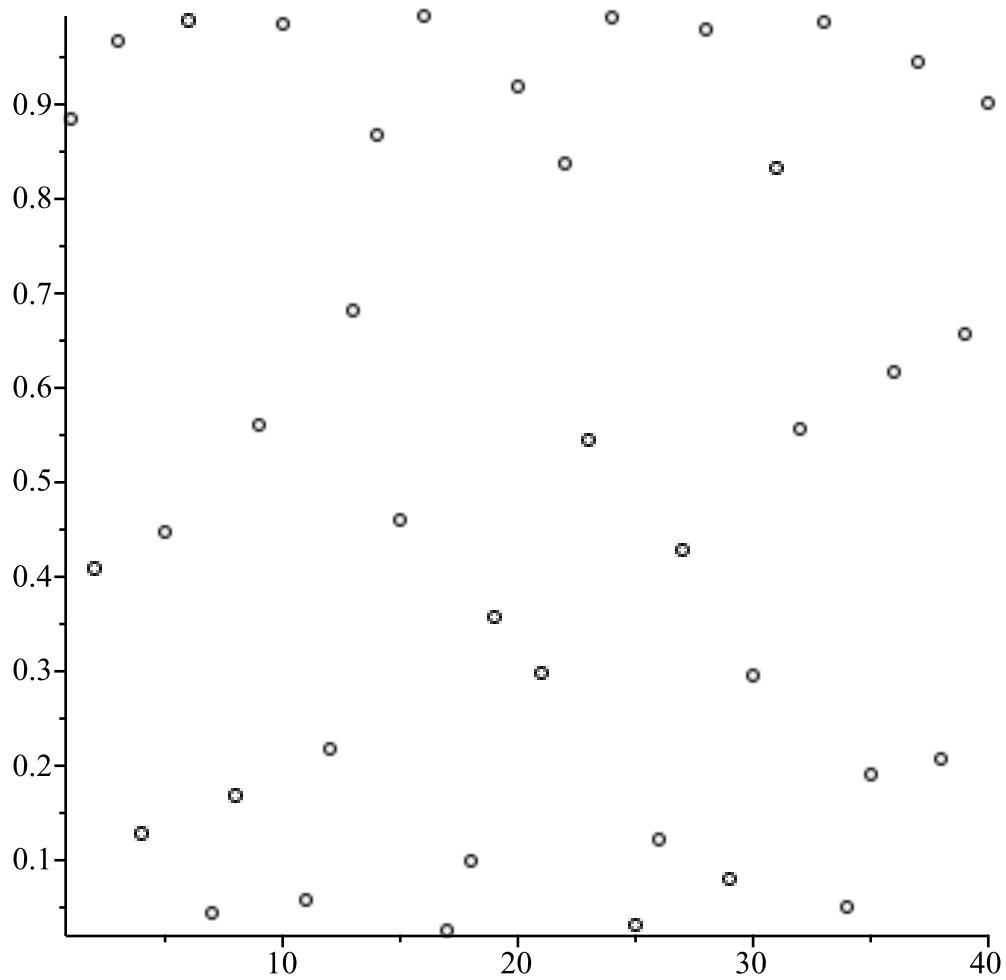
0.1685577332

0.5605840952

0.9853182696

0.05786470876

0.2180655370
0.6820518344
0.8674285184
0.4599851356
0.9935952424
0.02545494672
0.09922796964
0.3575271188
0.9188059124
0.2984064310
0.8374401316
0.5445366304
0.9920659544
0.03148438608
0.1219724780
0.4283807704
0.9794827440
0.08038519284
0.2956936545
0.8330356688
0.5563489732
0.9872991728
0.05015806476
0.1905689332
0.6170096596
0.9452349584
0.2070633273
0.6567524232
0.9017147112

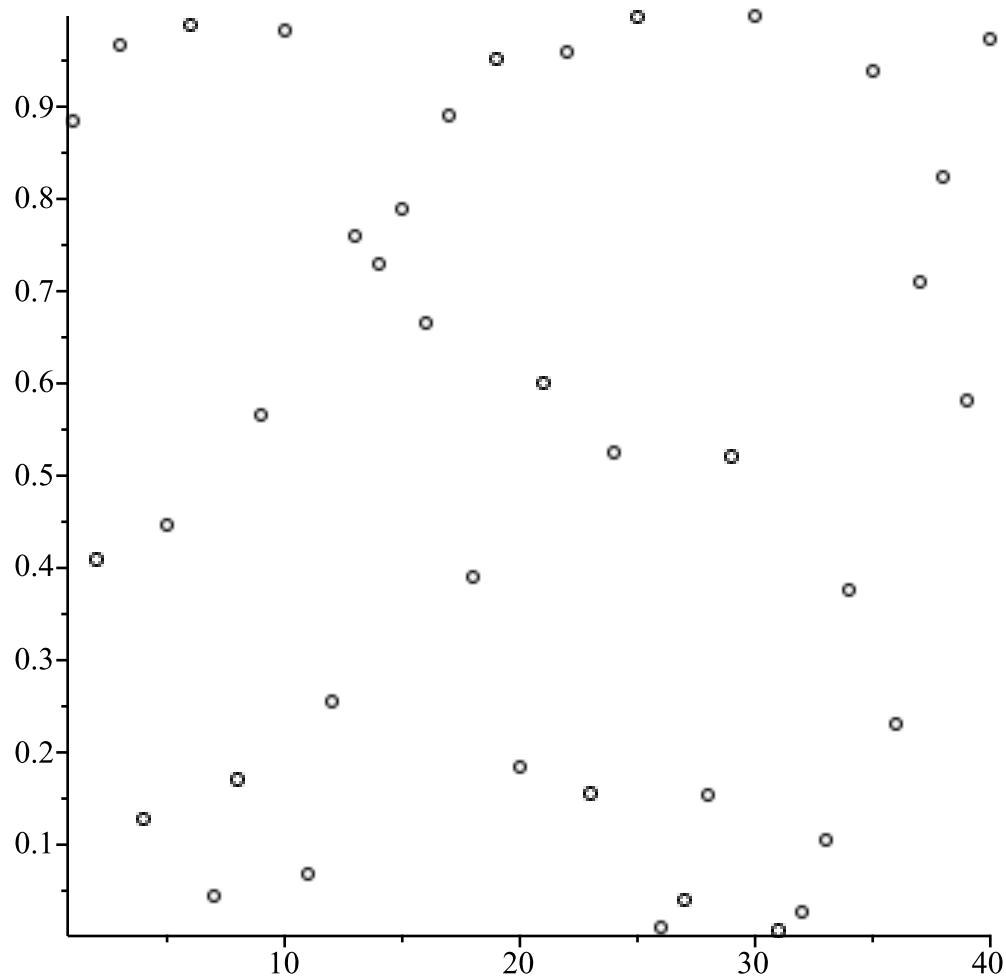


```

> x := 0.67001; for i from 1 to 40 do x := f4(x) : psi(i) := x : print(x); od; points := [ [n, psi(n)]
    $n = 1..40 ] : with(plots) : pointplot(points, symbol=circle);
    x := 0.67001
    0.8843863996
    0.4089883832
    0.9668675424
    0.1281387914
    0.4468769660
    0.9887117732
    0.04464321096
    0.1706007787
    0.5659846120
    0.9825841240
    0.06845025304
    0.2550592636
    0.7600161428
    0.7295664220

```

0.7891970316
0.6654603076
0.8904915464
0.3900654088
0.9516575428
0.1840218561
0.6006312504
0.9594934056
0.1554632408
0.5251776864
0.9974643364
0.01011693604
0.04005833460
0.1538146577
0.5206228352
0.9982987948
0.006793244404
0.02698838494
0.1050400481
0.3760265456
0.9385223304
0.2307926630
0.7101096388
0.8234157588
0.5816089880
0.9733598924



```
> f4alternative := x→4·x - 4·x2
```

```
f4alternative := x ↦ 4·x - 4·x2
```

(16)

```
> x := 0.67; for i from 1 to 40 do x := f4alternative(x) : psi(i) := x : print(x); od;; points := [[n,
psi(n)]$n = 1..40] : with(plots) : pointplot(points, symbol=circle);
```

```
x := 0.67
```

```
0.8844
```

```
0.40894656
```

```
0.9668370844
```

```
0.128252547
```

```
0.4472153248
```

```
0.9888551122
```

```
0.044082717
```

```
0.1685577242
```

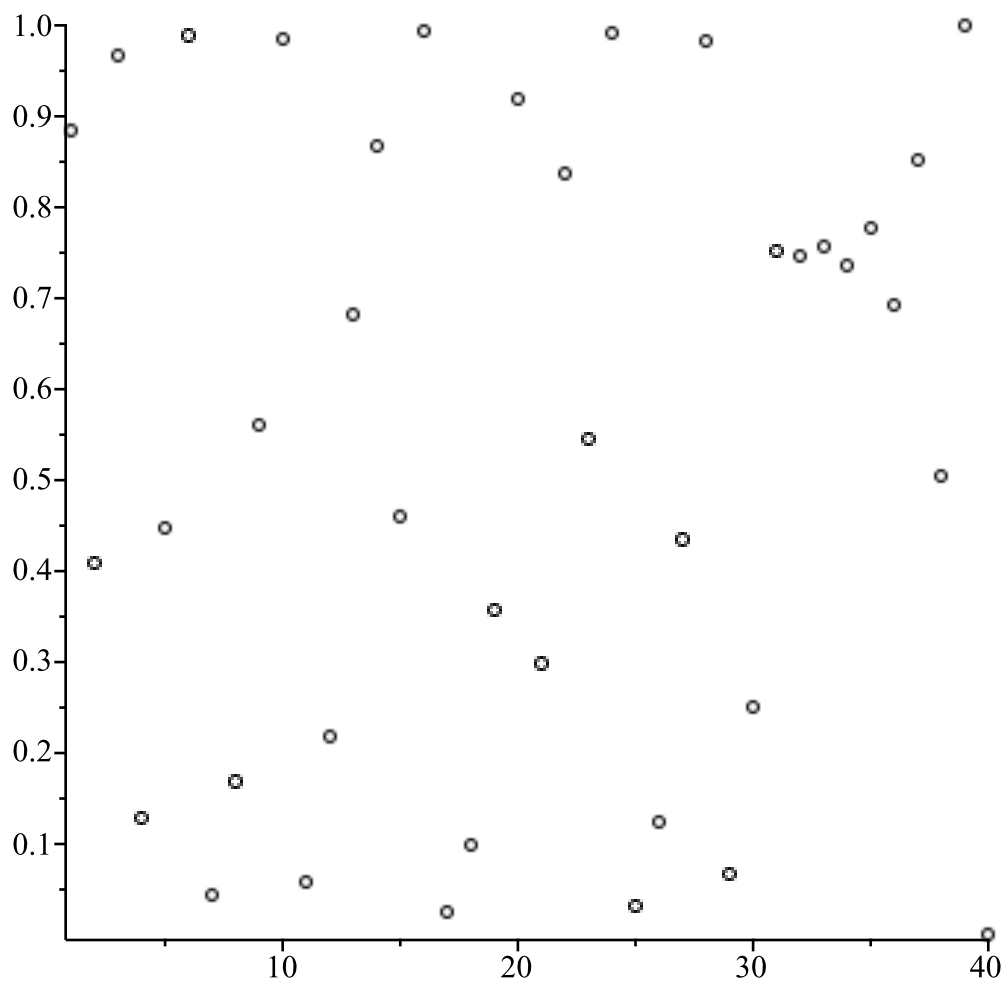
```
0.5605840712
```

```
0.985318281
```

```
0.057864664
```

```
0.2180653786
```

0.6820514770
0.867429039
0.459983605
0.9935947524
0.025456882
0.09923531664
0.3575506743
0.9188327582
0.298316483
0.8372950359
0.544928236
0.991925814
0.032035974
0.1240386815
0.4346123480
0.9828978200
0.067238782
0.2508709128
0.7517387914
0.746510324
0.756930641
0.735946583
0.777316840
0.692381481
0.851957463
0.504503777
0.999918864
0.000324518



```
> #problem 4
```

```
> restart :
```

```
> with(DEtools) :
```

```
> f := (x,y) -> 2 * x * y;
```

$$f := (x, y) \mapsto 2 \cdot y \cdot x$$

(17)

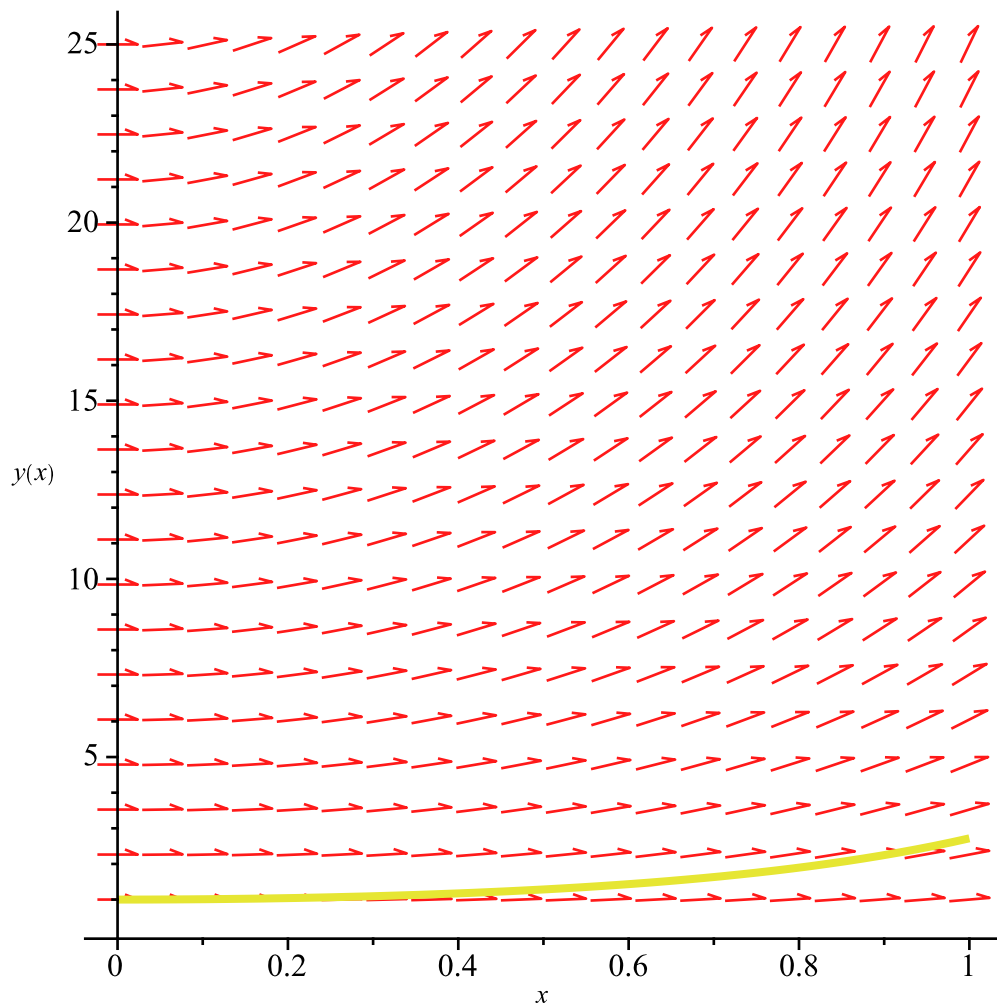
```
> dsolve( {diff(y(x), x) = f(x, y(x)), y(0) = 1 } ); phi := unapply(rhs(%), x);
```

$$y(x) = e^{x^2}$$

$$\phi := x \mapsto e^{x^2}$$

(18)

```
> DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0 .. 1, [[y(0) = 1]], y = 1 .. 25);
```



```
> h := 0.1; x := 0; y := 1
```

```
h := 0.1
```

```
x := 0
```

```
y := 1
```

(19)

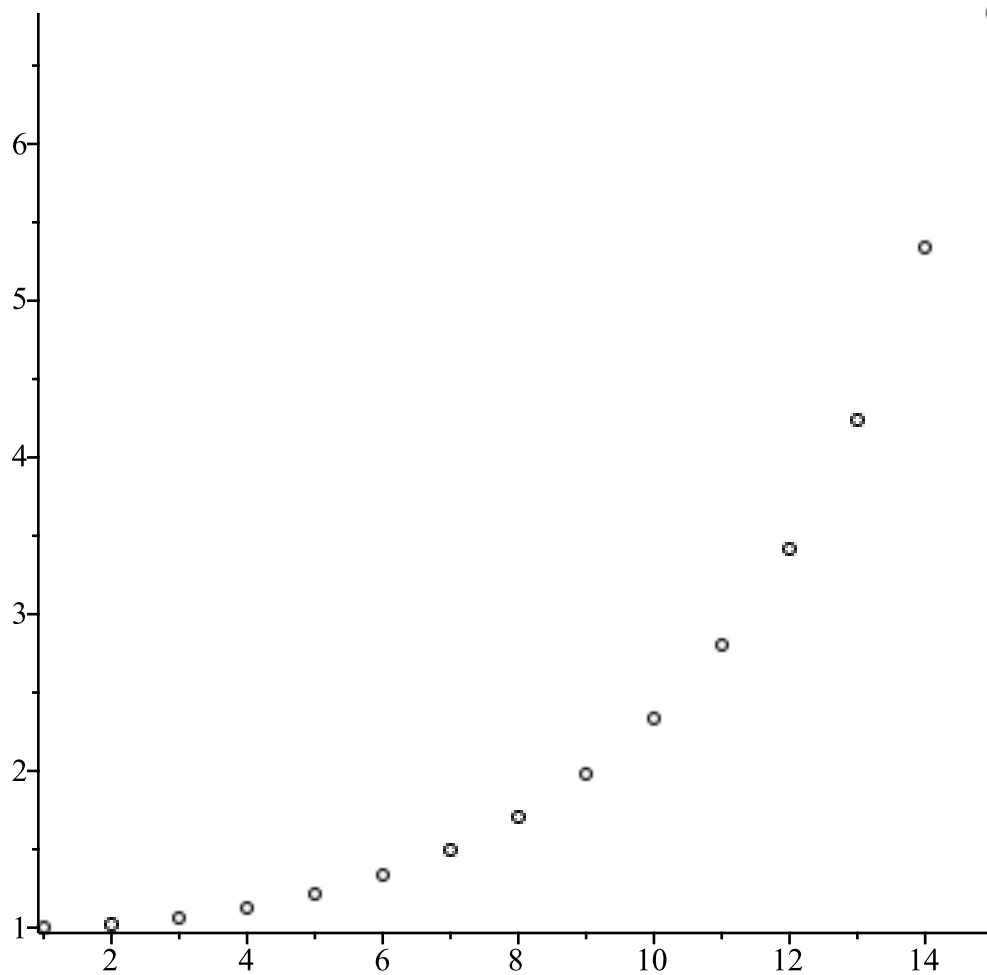
```
> for i from 1 to 15 do y := y + h*f(x, y) : psi(i) := y : x := x + h : print(x, y, phi(x), abs(y
- phi(x) )) ; od:
```

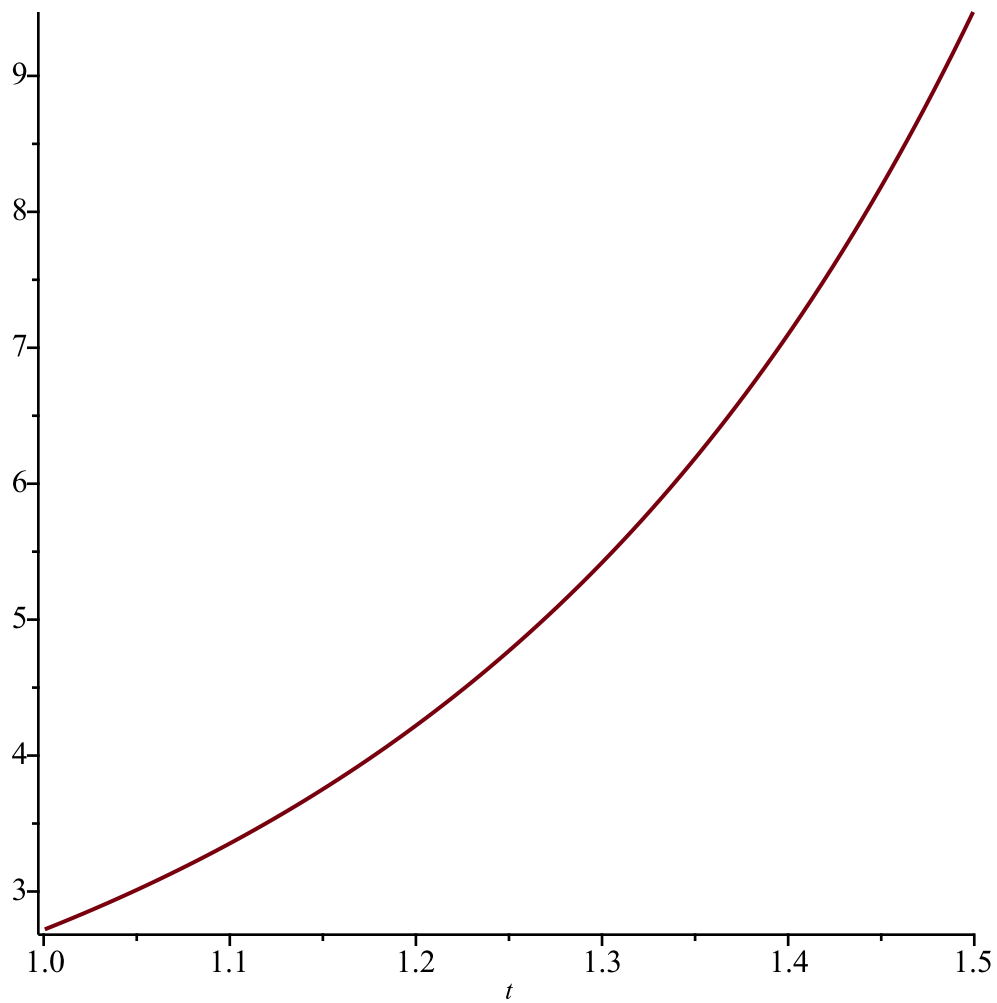
```
0.1, 1., 1.010050167, 0.010050167
0.2, 1.02, 1.040810774, 0.020810774
0.3, 1.0608, 1.094174284, 0.033374284
0.4, 1.124448, 1.173510871, 0.049062871
0.5, 1.21440384, 1.284025417, 0.069621577
0.6, 1.335844224, 1.433329415, 0.097485191
0.7, 1.496145531, 1.632316220, 0.136170689
0.8, 1.705605905, 1.896480879, 0.190874974
0.9, 1.978502850, 2.247907987, 0.269405137
1.0, 2.334633363, 2.718281828, 0.383648465
1.1, 2.801560036, 3.353484653, 0.551924617
```


1.2, 3.417903244, 4.220695817, 0.802792573
1.3, 4.238200023, 5.419480705, 1.181280682
1.4, 5.340132029, 7.099327065, 1.759195036
1.5, 6.835368997, 9.487735836, 2.652366839

(20)

```
> points := [ [n, psi(n)]$n = 1..15 ] : with(plots) : pointplot(points, symbol=circle); plot(phi(t),  
t = 1..1.5);
```





```
> #now euler's improved method
```

```
> restart :
```

```
> phi := x → exp(x^2);
```

$$\phi := x \mapsto e^{x^2}$$

(21)

```
> h := 0.1; x := 0; y := 1;
```

$$h := 0.1$$

$$x := 0$$

$$y := 1$$

(22)

```
> f := (x, y) → 2 * x * y;
```

$$f := (x, y) \mapsto 2 \cdot y \cdot x$$

(23)

```
> for i from 1 to 15 do y := y + h/2 * f(x, y) + h/2 * f(x + h, y + h * f(x, y)) : psi(i) := y :  
  x := x + h : print(x, y, phi(x), abs(y - phi(x))) : od:
```

0.1, 1.010000000, 1.010050167, 0.000050167

0.2, 1.040704000, 1.040810774, 0.000106774

0.3, 1.093988045, 1.094174284, 0.000186239

0.4, 1.173192779, 1.173510871, 0.000318092

```

0.5, 1.283472900, 1.284025417, 0.000552517
0.6, 1.432355756, 1.433329415, 0.000973659
0.7, 1.630593792, 1.632316220, 0.001722428
0.8, 1.893445511, 1.896480879, 0.003035368
0.9, 2.242596863, 2.247907987, 0.005311124
1.0, 2.709057011, 2.718281828, 0.009224817
1.1, 3.337558237, 3.353484653, 0.015926416
1.2, 4.193308169, 4.220695817, 0.027387648
1.3, 5.372466427, 5.419480705, 0.047014278
1.4, 7.018590140, 7.099327065, 0.080736925
1.5, 9.348762067, 9.487735836, 0.138973769

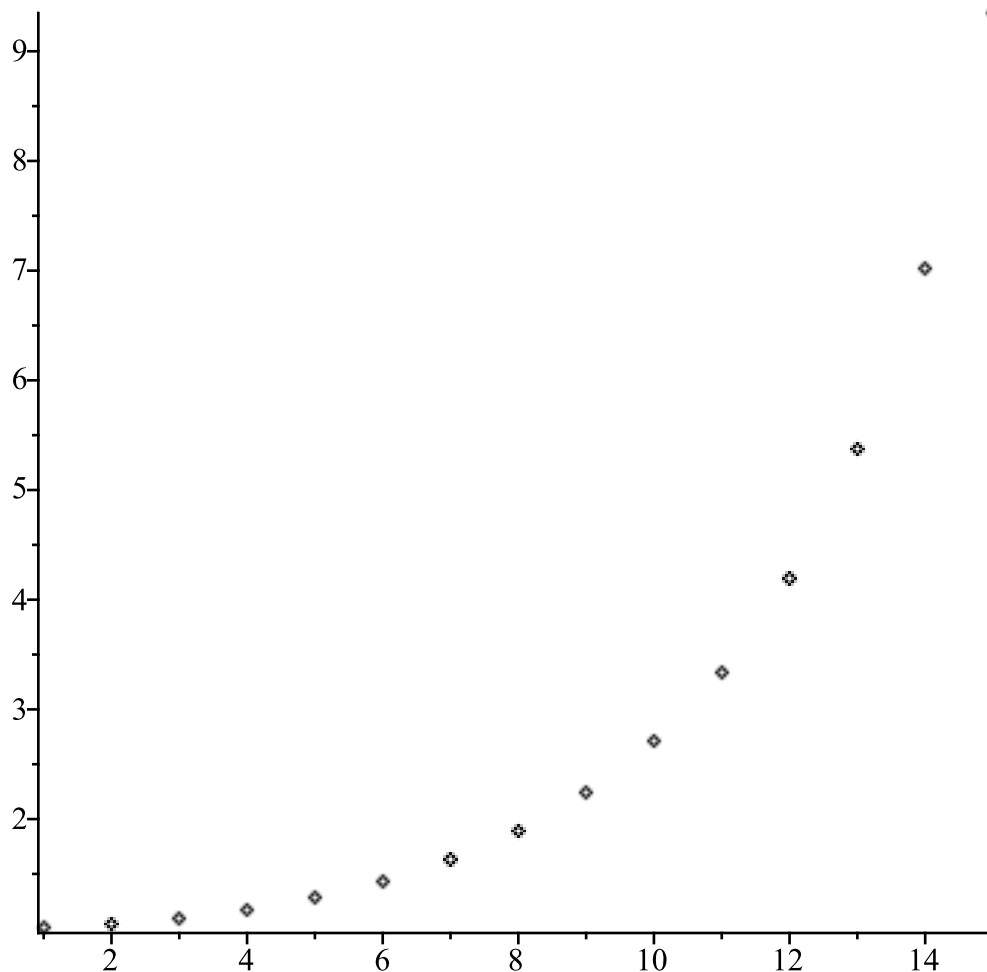
```

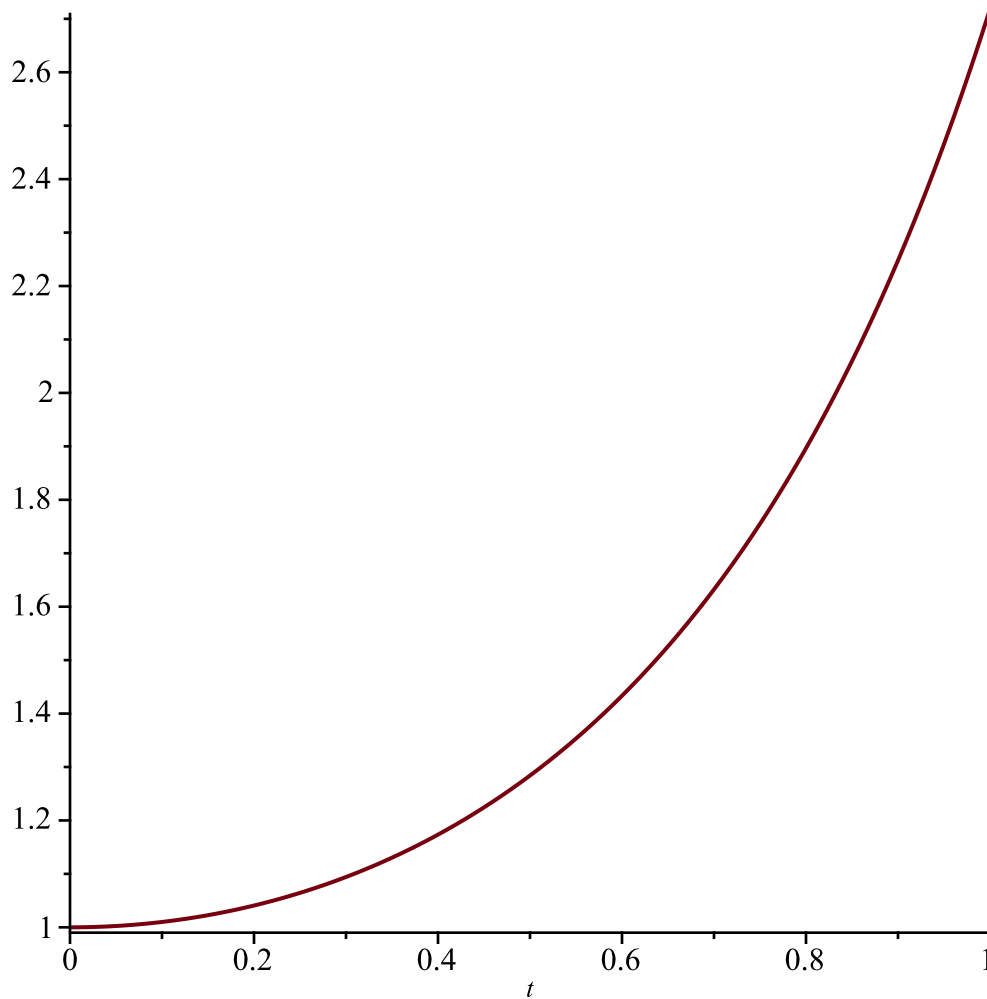
(24)

```

> points := [[n, psi(n)]$n = 1 ..15] : with(plots) : pointplot(points,
style = point); plot(phi(t), t = 0 ..1);

```





```
> #problem 5
```

```
> f := (x, y) → y2 + x2
```

$$f := (x, y) \mapsto y^2 + x^2$$

(25)

```
> #simple method
```

```
> restart :
```

```
> with(DEtools) :
```

```
> f := (x, y) → y2 + x2
```

$$f := (x, y) \mapsto y^2 + x^2$$

(26)

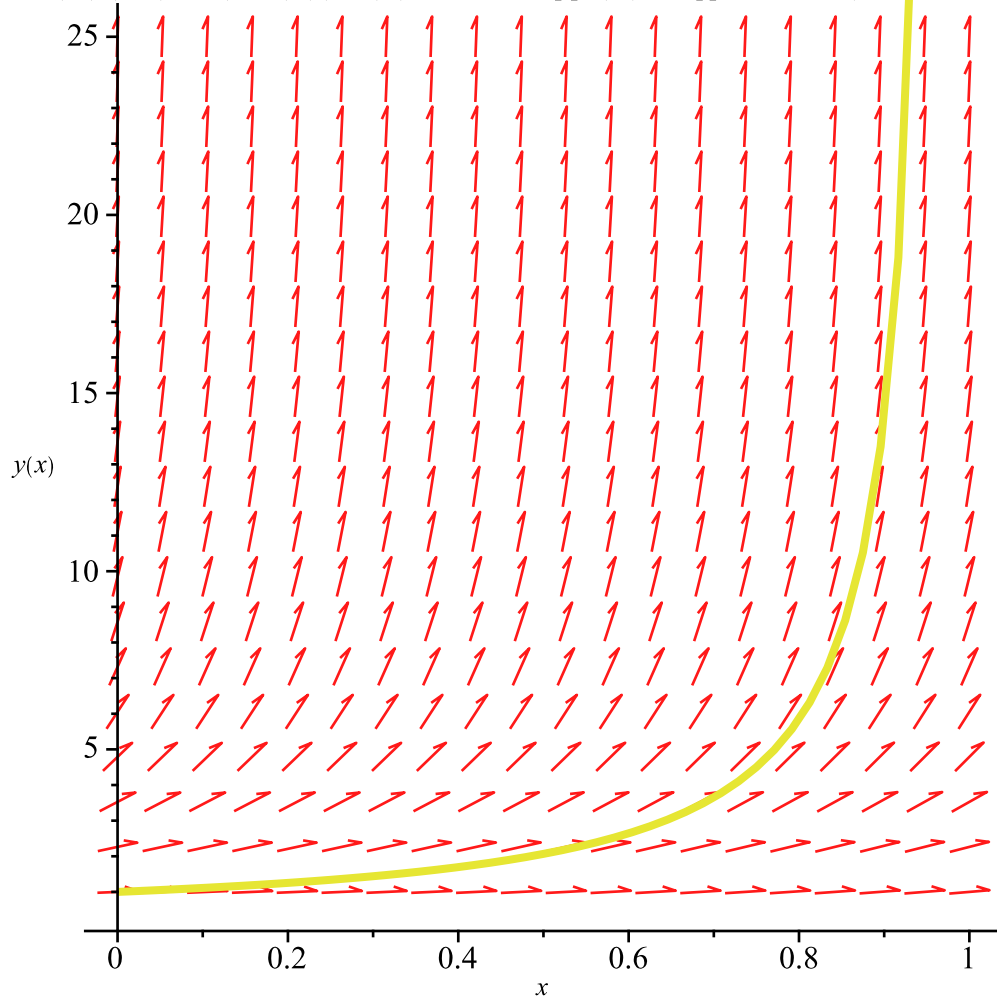
```
> dsolve( {diff(y(x), x) = f(x, y(x)), y(0) = 0}); phi := unapply(rhs(%), x);
```

$$y(x) = - \frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right) \right) x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

$$\phi := x \mapsto - \frac{\left(-\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right) \right) \cdot x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

(27)

```
> DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0..1, [[y(0) = 1]], y = 1..25);
```



```
> h := 0.1; x := 0; y := 0;
```

```
h := 0.1
```

```
x := 0
```

```
y := 0
```

(28)

```
> for i from 1 to 15 do y := y + h*f(x, y) : psi(i) := y : x := x + h : print(x, y, phi(x), abs(y - phi(x))) ; od:
```

```
0.1, 0., 0.00033333349060, 0.00033333349060
```

```
0.2, 0.001, 0.002666869814, 0.001666869814
```

```
0.3, 0.0050001, 0.009003473189, 0.004003373189
```

```
0.4, 0.01400260010, 0.02135938017, 0.00735678007
```

```
0.5, 0.03002220738, 0.04179114620, 0.01176893882
```

```
0.6, 0.05511234067, 0.07244786117, 0.01733552050
```

```
0.7, 0.09141607768, 0.1156598536, 0.02424377592
```

```
0.8, 0.1412517676, 0.1740802646, 0.0328284970
```

```
0.9, 0.2072469738, 0.2509066825, 0.0436597087
```

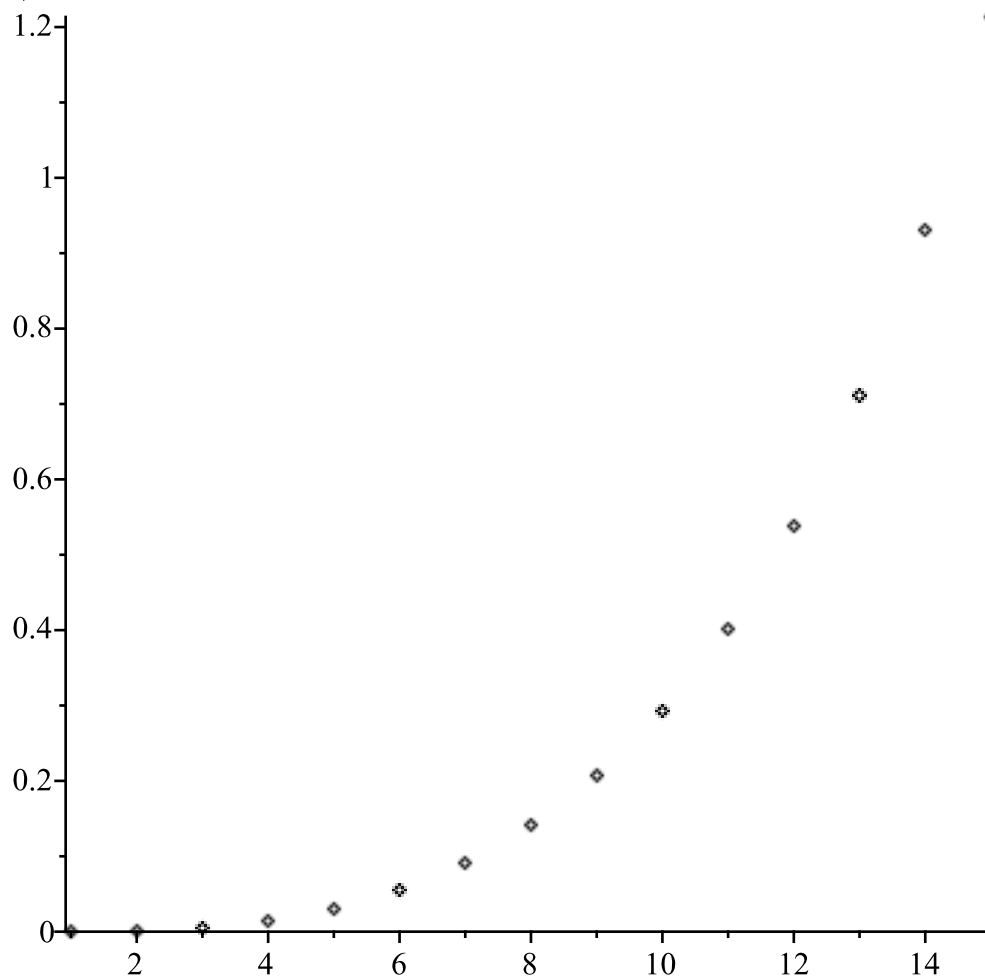
```
1.0, 0.2925421046, 0.3502318440, 0.0576897394
```

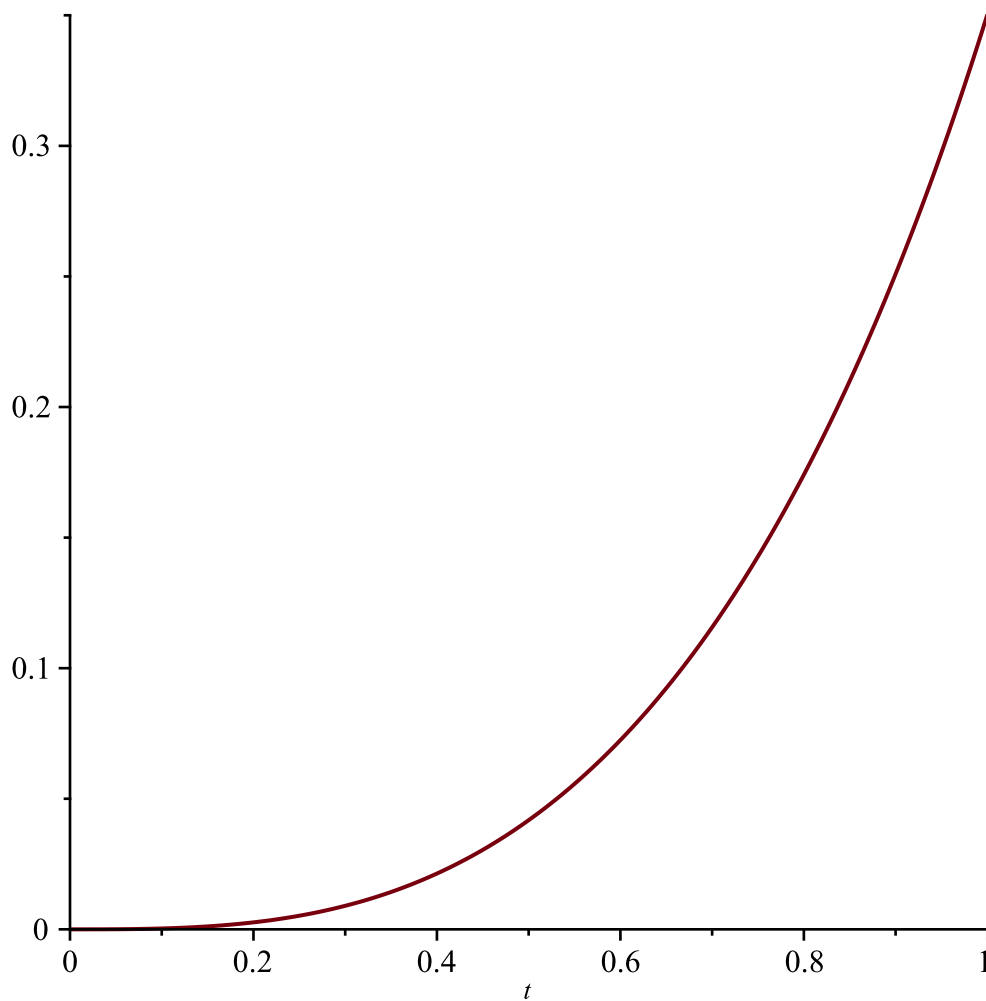
```
1.1, 0.4011001929, 0.4776170219, 0.0765168290
```

1.2, 0.5381883294, 0.6410767262, 0.1028883968
 1.3, 0.7111529972, 0.8528799930, 0.1417269958
 1.4, 0.9307268557, 1.133112675, 0.2023858193
 1.5, 1.213352104, 1.517447543, 0.304095439

(29)

> *points* := [[*n*, *psi*(*n*)]\$*n* = 1 ..15] : *with*(*plots*) : *pointplot*(*points*, *style* = *point*); *plot*(*phi*(*t*), *t* = 0 ..1);





```
> #now improved method
```

```
> restart :
```

```
> f := (x, y) -> y^2 + x^2; dsolve( {diff(y(x), x) = f(x, y(x)), y(0) = 0} ); phi := unapply(rhs(%), x)
```

$$f := (x, y) \mapsto y^2 + x^2$$

$$y(x) = - \left(\left(\begin{array}{ll} 0 & x=0 \\ \frac{\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right) \right) x}{\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} & \text{otherwise} \end{array} \right) \right)$$

$$\phi := x \mapsto - \left(\left(\begin{array}{ll} 0 & x=0 \\ \frac{\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right) \right) \cdot x}{\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} & \text{otherwise} \end{array} \right) \right)$$

(30)

```
> h := 0.1; x := 0; y := 0
```

$h := 0.1$

$x := 0$

$y := 0$

(31)

> **for** i **from** 1 **to** 15 **do** $y := y + h/2 * f(x, y) + h/2 * f(x + h, y + h * f(x, y)) :$
 $\text{psi}(i) := y : x := x + h : \text{print}(x, y, \text{phi}(x), \text{abs}(y - \text{phi}(x))) ;$ **od**:

0.1, 0.0005000000000, 0.00033333349060, 0.0001666650940

0.2, 0.003000125004, 0.002666869814, 0.000333255190

0.3, 0.009503025760, 0.009003473189, 0.000499552571

0.4, 0.02202467595, 0.02135938017, 0.00066529578

0.5, 0.04262140864, 0.04179114620, 0.00083026244

0.6, 0.07344210066, 0.07244786117, 0.00099423949

0.7, 0.1168165840, 0.1156598536, 0.0011567304

0.8, 0.1753963673, 0.1740802646, 0.0013161027

0.9, 0.2523742135, 0.2509066825, 0.0014675310

1.0, 0.3518301326, 0.3502318440, 0.0015982886

1.1, 0.4792938348, 0.4776170219, 0.0016768129

1.2, 0.6427029949, 0.6410767262, 0.0016262687

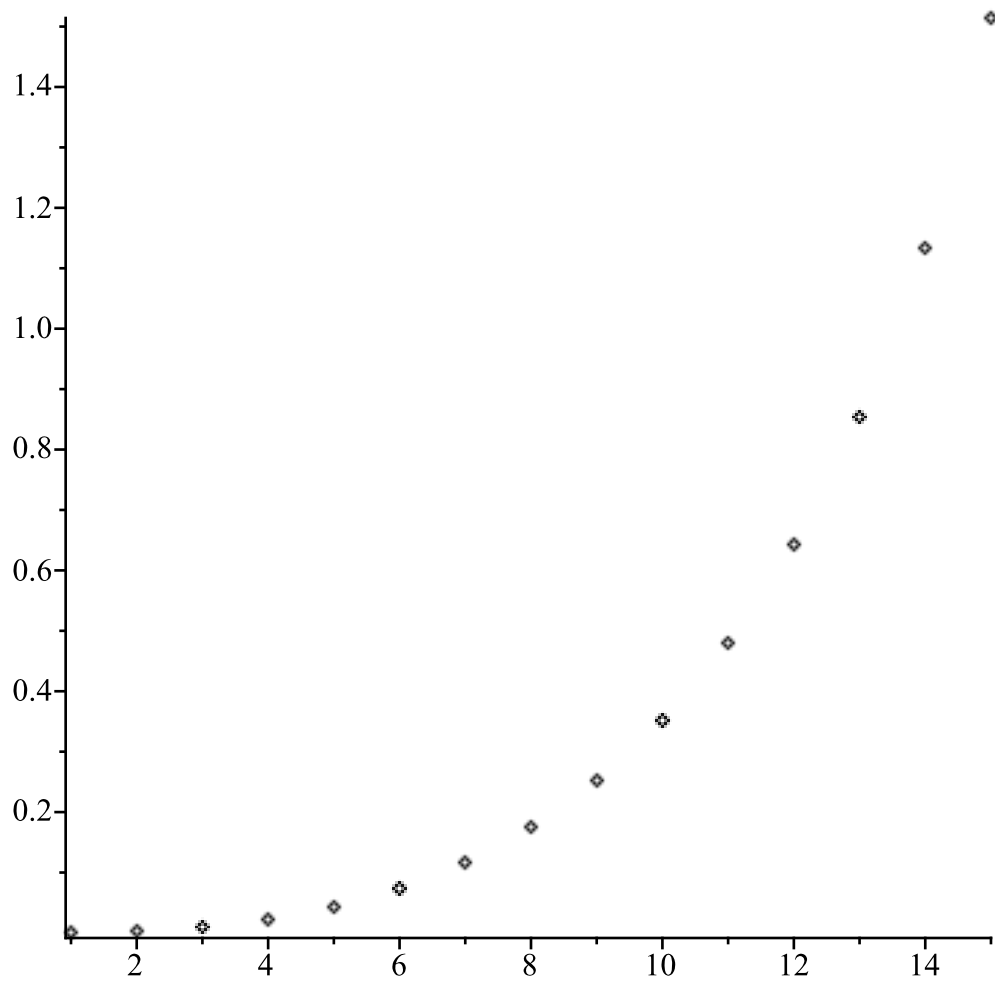
1.3, 0.8541363558, 0.8528799930, 0.0012563628

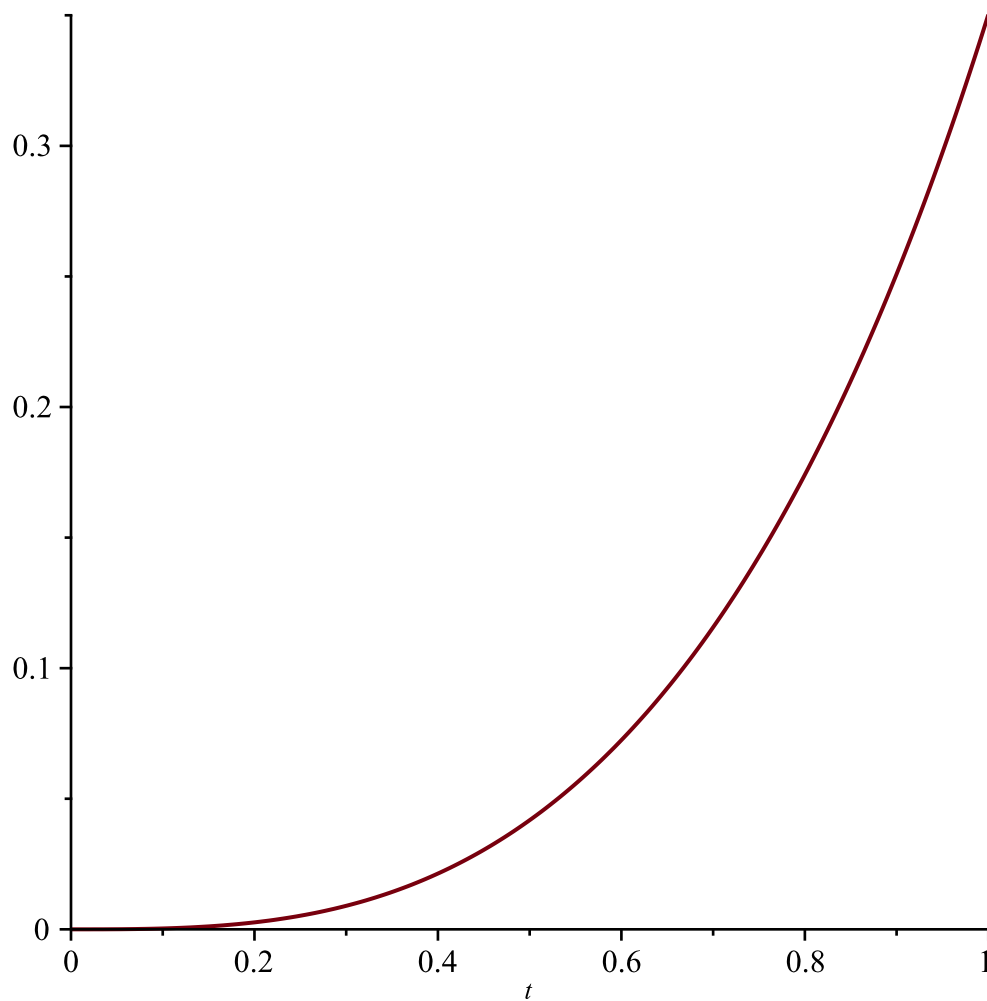
1.4, 1.133184603, 1.133112675, 0.000071928

1.5, 1.514119178, 1.517447543, 0.003328365

(32)

> $\text{points} := [[n, \text{psi}(n)] : n = 1 \dots 15] : \text{with}(\text{plots}) : \text{pointplot}(\text{points},$
 $\text{style} = \text{point}) ; \text{plot}(\text{phi}(t), t = 0 \dots 1) ;$





```
> #problem 6
```

```
> restart ;; f := (x, y) -> -250*y
```

$$f := (x, y) \mapsto -250 \cdot y$$

(33)

```
> dsolve( {diff(y(x), x) = f(x, y(x)), y(0) = 1 } ); phi := unapply(rhs(%), x);
```

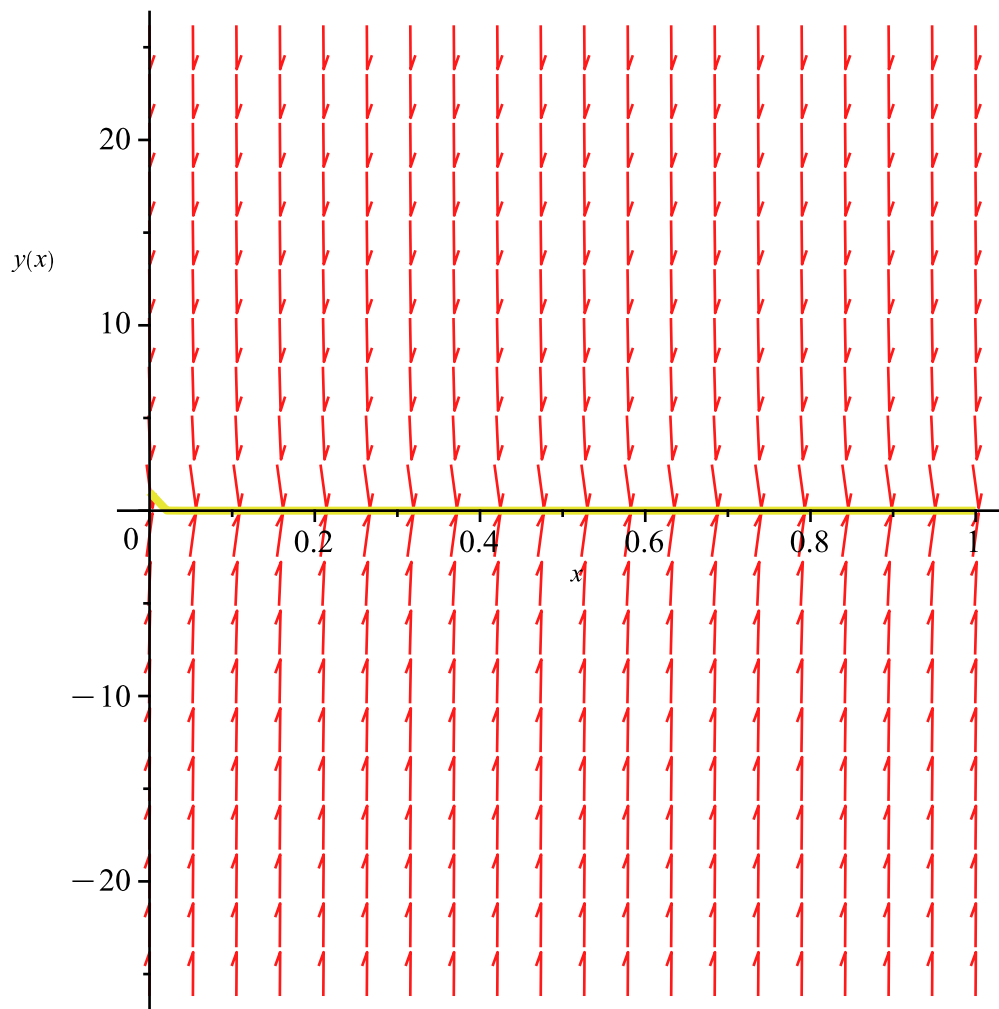
$$y(x) = e^{-250x}$$

$$\phi := x \mapsto e^{-250 \cdot x}$$

(34)

```
> with(DEtools) :
```

```
> DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0..1, [ [y(0) = 1 ] ], y = -25 .. 25);
```



```
> h := 0.1; x := 0; y := 0;
```

```
h := 0.1
```

```
x := 0
```

```
y := 0
```

(35)

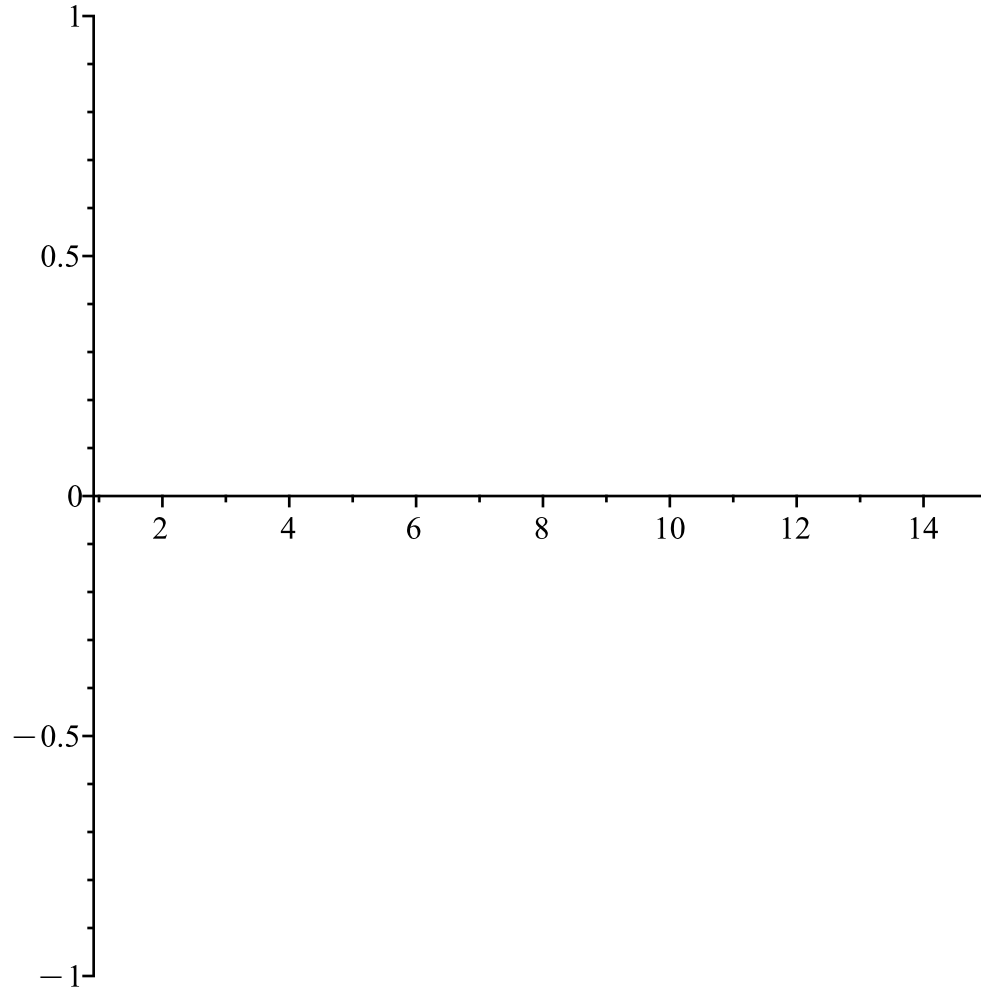
```
> for i from 1 to 15 do y := y + h*f(x, y) : psi(i) := y : x := x + h :  
print(x, y, phi(x), abs(y-phi(x))) ; od:
```

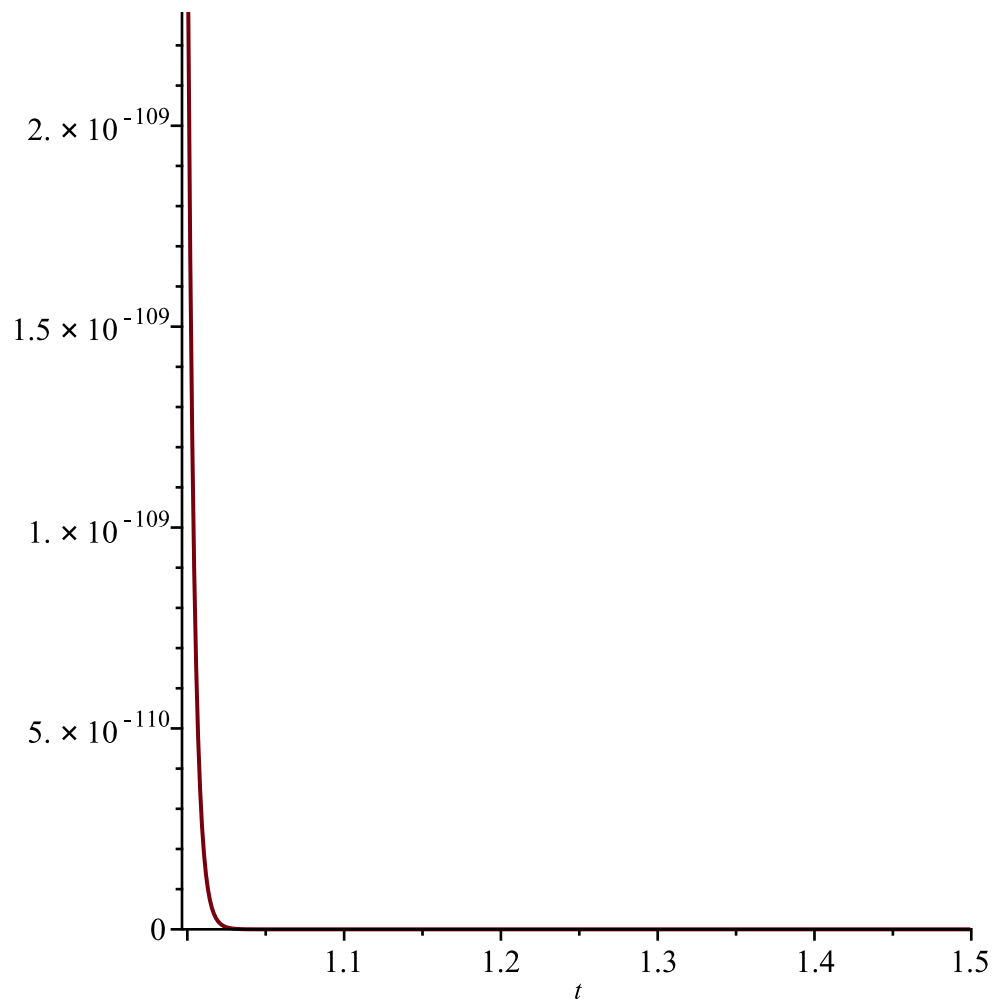
```
0.1, 0., 1.388794386 × 10-11, 1.388794386 × 10-11  
0.2, 0., 1.928749848 × 10-22, 1.928749848 × 10-22  
0.3, 0., 2.678636962 × 10-33, 2.678636962 × 10-33  
0.4, 0., 3.720075976 × 10-44, 3.720075976 × 10-44  
0.5, 0., 5.166420633 × 10-55, 5.166420633 × 10-55  
0.6, 0., 7.175095973 × 10-66, 7.175095973 × 10-66  
0.7, 0., 9.964733010 × 10-77, 9.964733010 × 10-77  
0.8, 0., 1.383896527 × 10-87, 1.383896527 × 10-87  
0.9, 0., 1.921947728 × 10-98, 1.921947728 × 10-98  
1.0, 0., 2.669190216 × 10-109, 2.669190216 × 10-109  
1.1, 0., 3.706956388 × 10-120, 3.706956388 × 10-120
```

$1.2, 0., 5.148200222 \times 10^{-131}, 5.148200222 \times 10^{-131}$
 $1.3, 0., 7.149791569 \times 10^{-142}, 7.149791569 \times 10^{-142}$
 $1.4, 0., 9.929590396 \times 10^{-153}, 9.929590396 \times 10^{-153}$
 $1.5, 0., 1.379015940 \times 10^{-163}, 1.379015940 \times 10^{-163}$

(36)

\triangleright `points := [[n, psi(n)]$n = 1 ..15]: with(plots): pointplot(points, symbol=point); plot(phi(t),
 $t = 1 ..1.5$);`





```
> #improved method
```

```
> restart : f := (x, y) -> -250 y
```

$$f := (x, y) \mapsto -250 \cdot y \quad (37)$$

```
> dsolve( {diff(y(x), x) = f(x, y(x)), y(0) = 1 } ); phi := unapply(rhs(%), x);
```

$$y(x) = e^{-250x}$$

$$\phi := x \mapsto e^{-250 \cdot x} \quad (38)$$

```
> h := 0.1; x := 0; y := 0;
```

$$h := 0.1$$

$$x := 0$$

$$y := 0$$

(39)

```
> for i from 1 to 15 do y := y + h/2 * f(x, y) + h/2 * f(x + h, y + h * f(x, y)) : psi(i) := y :  
  x := x + h : print(x, y, phi(x), abs(y - phi(x))) ; od:
```

$$0.1, 0., 1.388794386 \times 10^{-11}, 1.388794386 \times 10^{-11}$$

$$0.2, 0., 1.928749848 \times 10^{-22}, 1.928749848 \times 10^{-22}$$

$$0.3, 0., 2.678636962 \times 10^{-33}, 2.678636962 \times 10^{-33}$$

$$0.4, 0., 3.720075976 \times 10^{-44}, 3.720075976 \times 10^{-44}$$

$0.5, 0., 5.166420633 \times 10^{-55}, 5.166420633 \times 10^{-55}$
 $0.6, 0., 7.175095973 \times 10^{-66}, 7.175095973 \times 10^{-66}$
 $0.7, 0., 9.964733010 \times 10^{-77}, 9.964733010 \times 10^{-77}$
 $0.8, 0., 1.383896527 \times 10^{-87}, 1.383896527 \times 10^{-87}$
 $0.9, 0., 1.921947728 \times 10^{-98}, 1.921947728 \times 10^{-98}$
 $1.0, 0., 2.669190216 \times 10^{-109}, 2.669190216 \times 10^{-109}$
 $1.1, 0., 3.706956388 \times 10^{-120}, 3.706956388 \times 10^{-120}$
 $1.2, 0., 5.148200222 \times 10^{-131}, 5.148200222 \times 10^{-131}$
 $1.3, 0., 7.149791569 \times 10^{-142}, 7.149791569 \times 10^{-142}$
 $1.4, 0., 9.929590396 \times 10^{-153}, 9.929590396 \times 10^{-153}$
 $1.5, 0., 1.379015940 \times 10^{-163}, 1.379015940 \times 10^{-163}$

(40)

$\triangleright points := [[n, \psi(n)] \$ n = 1 .. 15] : with(plots) : pointplot(points, style=point); plot(\phi(t), t = 1 .. 1.5)$

