Use of DFT to simultaneously transform 2 sigs

$$\begin{cases} x(n) = x_R(n) + j\,x_I(n) \\ y(n) = y_R(n) + j\,y_I(n) \end{cases}$$

$$z(n) = x(n) + j\,y(n) = x_R(n) + j x_I(n) + j\left(y_R(n) + j\,y_I(n)\right)$$
$$= \left(x_R(n) - y_I(n)\right) + j\left(x_I(n) + y_R(n)\right)$$

$$Z_N^f(k) = X_R^f(k) - Y_I^f(k) + j\left(X_I^f(k) + Y_R^f(k)\right)$$

$$Z_N^f(N-k) = \left(Z_N^f(k)\right)^* = \left(X_N^f(k)\right)^* + j\left(Y_N^f(k)\right)^* = X_R^f(k) - jX_I^f(k) + j\left(Y_R^f(k) - j\,Y_I^f(k)\right)$$
$$X_R^f(k) + Y_I^f(k) + j\left(-X_I^f(k) + Y_R^f(k)\right)$$

$$\begin{cases} Z_N^f(k) + Z_N^f(N-k) = 2X_R^f(k) + j\left(2Y_R^f(k)\right) \\ Z_N^f(k) - Z_N^f(N-k) = -2Y_I^f(k) + j(2X_I^f(k) \end{cases}$$

$$\begin{cases} X_R^f = \frac{1}{2}\,\mathrm{Re}\left\{Z_N^f(k) + Z_N^f(N-k)\right\} \\ X_I^f = \frac{1}{2}\,\mathrm{Im}\left\{Z_N^f(k) - Z_N^f(N-k)\right\} \\ Y_R^f = \frac{1}{2}\,\mathrm{Im}\left\{Z_N^f(k) + Z_N^f(N-k)\right\} \\ Y_I^f = -\frac{1}{2}\,\mathrm{Re}\left\{Z_N^f(k) - Z_N^f(N-k)\right\} \end{cases}$$

$$\begin{cases} X^f(k) = \frac{1}{2}\,\mathrm{Re}\left\{Z_N^f(k) + Z_N^f(N-k)\right\} + j\cdot\frac{1}{2}\,\mathrm{Im}\left\{Z_N^f(k) - Z_N^f(N-k)\right\} \\ Y^f(k) = \frac{1}{2}\,\mathrm{Im}\left\{Z_N^f(k) + Z_N^f(N-k)\right\} + j\cdot\left(-\frac{1}{2}\right)\mathrm{Re}\left\{Z_N^f(k) - Z_N^f(N-k)\right\} \end{cases}$$