proof: $$H^{LP}_{(s)} = \frac{W_c^2}{s^2 + \frac{W_c}{Q}s + W_c^2}$$

use bilinear transform: $$\begin{cases} s = \left(\frac{2}{T}\right)\left(\frac{z-1}{z+1}\right) \\ W_c^1 = \left(\frac{2}{T}\right) tg\left(\frac{W_c \cdot T}{2}\right) \end{cases}$$

$$H_{(z)} = H^{LP}_{(s)}\Big|_{s=\left(\frac{2}{T}\right)\left(\frac{z-1}{z+1}\right)} = \frac{\left(\frac{2}{T}\right)^2 tg^2\left(\frac{W_c^d \cdot T}{2}\right)}{\left(\frac{2}{T}\right)^2\left(\frac{z-1}{z+1}\right)^2 + \frac{\left(\frac{2}{T}\right) tg\left(\frac{W_c^d T}{2}\right)}{Q} \cdot \left(\frac{2}{T}\right)\left(\frac{z-1}{z+1}\right) + \left(\frac{2}{T}\right)^2 tg^2\left(\frac{W_c^d T}{2}\right)}$$

$$= \left(\frac{(z+1)^2}{(z+1)^2}\right) \cdot \frac{\cancel{\left(\frac{2}{T}\right)^2} tg^2\left(\frac{W_c^d \cdot T}{2}\right)}{\cancel{\left(\frac{2}{T}\right)^2}\left(\frac{z-1}{z+1}\right)^2 + \frac{\cancel{\left(\frac{2}{T}\right)} tg\left(\frac{W_c^d \cdot T}{2}\right)}{Q}\left(\frac{z-1}{z+1}\right) + \cancel{\left(\frac{2}{T}\right)^2} tg^2\left(\frac{W_c^d \cdot T}{2}\right)}$$

$$= \frac{tg^2\left(\frac{W_c^d \cdot T}{2}\right) \cdot (z+1)^2}{(z-1)^2 + \frac{tg\left(\frac{W_c^d \cdot T}{2}\right)}{Q}(z^2-1) + tg^2\left(\frac{W_c^d \cdot T}{2}\right)(z+1)^2}$$

$$= \frac{tg^2\left(\frac{W_c^d \cdot T}{2}\right)(z^2 + 2 \cdot z + 1)}{\left\{tg^2\left(\frac{W_c^d \cdot T}{2}\right) + \frac{tg\left(\frac{W_c^d \cdot T}{2}\right)}{Q} + 1\right\}z^2 + \left\{2 \cdot tg^2\left(\frac{W_c^d \cdot T}{2}\right) - 2\right\}z + \left\{tg^2\left(\frac{W_c^d T}{2}\right) - \frac{1}{Q} tg\left(\frac{W_c^d T}{2}\right) + 1\right\}}$$

let $K = tg\left(\frac{W_c^d \cdot T}{2}\right)$

$$= \frac{\underbrace{tg^2\left(\frac{W_c^d \cdot T}{2}\right)}_{K}(z^2 + 2z + 1)}{\left\{K^2 + \frac{K}{Q} + 1\right\}z^2 + \left\{2(K^2-1)\right\}z + \left\{K^2 - \frac{K}{Q} + 1\right\}}$$

$$= \left(\frac{\cancel{z^2}}{\cancel{z^2}}\right) \cdot \frac{K^2 + 2K^2 z^{-1} + K^2 z^{-2}}{\left\{K^2 + \frac{K}{Q} + 1\right\} + \left\{2(K^2-1)\right\}z^{-1} + \left\{K^2 - \frac{K}{Q} + 1\right\}z^{-2}}$$

normalize with respect to b_0:

- assume $a = \frac{1}{\sqrt{2}}$

$$H(z) = \frac{\frac{K^2}{K^2+\sqrt{2}K+1} + \left(\frac{2K^2}{K^2+\sqrt{2}K+1}\right)z^{-1} + \left(\frac{K^2}{K^2+\sqrt{2}K+1}\right)z^{-2}}{1 + \frac{2(K^2-1)}{K^2+\sqrt{2}K+1}z^{-1} + \left(\frac{K^2-\sqrt{2}K+1}{K^2+\sqrt{2}K+1}\right)z^{-2}}$$

$\blacksquare$.