consider a LP transfer func designed for ~~[illegible]~~ continuous angular freq axis W^c, with cutoff freq W_c^c

1. $$H^{LP}(s) = \frac{(W_c^c)^2}{s^2 + \frac{W_c^c}{Q} s + (W_c^c)^2}$$

2. sample frequency axis with the following relations $\theta_c^d = W_c^c \cdot T_s$

$$H_d^{LP}(s) = \frac{(W_c^c \cdot T_s)^2}{s^2 + \frac{W_c^c \cdot T_s}{Q} \cdot s + (W_c^c \cdot T_s)^2}$$

3. Use bilinear transform to move to z-domain

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \qquad \theta^z = \cancel{\frac{2}{T}} tg\left(\frac{\theta^d}{2}\right) = \cancel{\frac{2}{T}} tg\left(\frac{W_c^c T_s}{2}\right)$$

$$H_d^{LP}(z) = H_d^{LP}\left(s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)\right) = \frac{\cancel{\left(\frac{2}{T}\right)^2} tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}{\cancel{\left(\frac{2}{T}\right)^2} \cdot \left(\frac{z-1}{z+1}\right)^2 + \frac{\cancel{\frac{2}{T}} tg\left(\frac{W_c^c \cdot T_s}{2}\right)}{Q} \cancel{\left(\frac{2}{T}\right)} \left(\frac{z-1}{z+1}\right) + \cancel{\left(\frac{2}{T}\right)^2} tg^2\left(\frac{W_c^c T_s}{2}\right)}$$

$$= \frac{tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}{\left(\frac{z-1}{z+1}\right)^2 + \frac{tg\left(\frac{W_c^c \cdot T_s}{2}\right)}{Q}\left(\frac{z-1}{z+1}\right) + tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}$$

$$= \left(\frac{z+1}{z+1}\right)^2 \cdot \frac{tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}{\left(\frac{z-1}{z+1}\right)^2 + \frac{tg\left(\frac{W_c^c \cdot T_s}{2}\right)}{Q}\left(\frac{z-1}{z+1}\right) + tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}$$

$$= \frac{tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)(z+1)^2}{(z-1)^2 + \frac{tg\left(\frac{W_c^c T_s}{2}\right)}{Q}(z^2-1) + (z+1)^2 tg^2\left(\frac{W_c^c \cdot T_s}{2}\right)}$$

$$H_d^{LP}(z) = \frac{tg^2\left(\frac{w_c^c \cdot T_s}{2}\right)(z^2+2z+1)}{z^2-2z+1+\frac{tg\left(\frac{w_c T_s}{2}\right)}{a}z^2-\frac{tg\left(\frac{w_c T_s}{2}\right)}{a}+tg^2\left(\frac{w_c T_s}{2}\right)z^2+2tg^2\left(\frac{w_c T_s}{2}\right)z+tg^2\left(\frac{w_c T_s}{2}\right)}$$

$$k = tg\left(\frac{w_c^c \cdot T_s}{2}\right)$$

$$H_d^{LP}(z) = \frac{k^2 \cdot z^2 + 2k^2 z + k^2}{\left(1+\frac{k}{a}+k^2\right)z^2 + 2(k^2-1)z + \left(k^2-\frac{k}{a}+1\right)}$$

$$H_d^{LP}(z) = \left(\frac{z^2}{z^2}\right) \cdot \frac{k^2 z^{-2} + 2k^2 z^{-1} + k^2}{\left(k^2-\frac{k}{a}+1\right)z^{-2} + 2(k^2-1)z^{-1} + \left(1+\frac{k}{a}+k^2\right)}$$

{ consider a system with $a = \frac{1}{\sqrt{2}}$
{ devide by $\left(k^2 + \frac{k}{a} + 1\right)$ to normalize b_0 to 1

$$H_d^{LP}(z) = \frac{\frac{k^2}{k^2+\sqrt{2}k+1} + \frac{2k^2}{k^2+\sqrt{2}k+1}z^{-1} + \frac{k^2}{k^2+\sqrt{2}k+1}z^{-2}}{1 + \frac{2(k^2-1)}{k^2+\sqrt{2}k+1}z^{-1} + \frac{k^2-\sqrt{2}k+1}{k^2+\sqrt{2}k+1}z^{-2}}$$

$\blacksquare$