מטלה 3 - שונות אקרצ״ין

שאלה 1

תוחלת המשערך הראשון:

(א) $E(\hat{\mu}_{spike}) = E\left(\frac{1}{N}\sum_{1}^{N} n_i\right) = \frac{1}{N}\sum_{1}^{N} E(n_i) = \frac{1}{N}\sum_{1}^{N}(\mu)$

$= \frac{1}{N}\cdot N\mu = \mu.$

הטיה: $B = \theta - E(\hat{\theta}_m) = \mu - \mu = 0$

(פרמטר ← θ ; תוחלת המשערך ← $E(\hat{\theta}_m)$)

המשערך חסר הטיה.

(ב) $E(\hat{\mu}_{axon}) = E\left(\frac{1}{N}\sum_{1}^{N}(n_i - \bar{n})^2\right)$

$= \frac{1}{N}\sum_{1}^{N} E((n_i - \mu) - (\bar{n} - \mu))^2$

$= \frac{1}{N}\left\{\sum_{1}^{N} E\left\{(n_i-\mu)^2 - 2(n_i-\mu)(\bar{n}-\mu) + (\bar{n}-\mu)^2\right\}\right\}$

$= \frac{1}{N}\left\{\sum_{1}^{N} E(n_i-\mu)^2 - 2(\bar{n}-\mu)\sum_{1}^{N} E(n_i-\mu) + \sum_{1}^{N} E(\bar{n}-\mu)^2\right\}$

$= \frac{1}{N}\left\{\sum_{1}^{N} E(n_i-\mu)^2 - 2(\bar{n}-\mu)\left(\sum_{1}^{N} E(n_i) - \sum_{1}^{N} E(\mu x)\right) + NE(\bar{n}-\mu)^2\right\}$

$= \frac{1}{N}\left\{\sum_{1}^{N} E(n_i-\mu)^2 - 2(\bar{n}-\mu)NE(\bar{n}-\mu) + N\cdot E(\bar{n}-\mu)^2\right\}$

$$= E\left\{\frac{1}{N}\sum_{1}^{N}(n_i-\mu)^2 - 2(\bar{n}-\mu)^2 + (\bar{n}-\mu)^2\right\}$$

$$= E\left\{\frac{1}{N}\sum_{1}^{N}(n_i-\mu)^2 - (\bar{n}-\mu)^2\right\}$$

$$= \frac{1}{N}\cdot\sum_{1}^{N}E(n_i-\mu)^2 - E(\bar{n}-\mu)^2$$

$$= \frac{1}{N}\cdot N\cdot Var(n_i) - \frac{Var(\bar{n})}{N} = \mu - \frac{\mu}{N} = \mu\left(1-\frac{1}{N}\right)$$

$$= \mu\left(\frac{N-1}{N}\right) = E(\hat{\mu}_{axon})$$

<u>הט״ה</u>: $B = \mu - \mu\left(\frac{N-1}{N}\right) = \mu\left(1-\frac{N-1}{N}\right)$

$$= \mu\left(\frac{N-N+1}{N}\right) = \frac{\mu}{N}$$

(ג) המשערך החדש מק״ים:

$$\hat{\mu}_{ax\text{-}fixed} = \frac{N}{N-1}\cdot\frac{1}{N}\sum(n_i-\bar{n})^2 = \frac{N}{N-1}\hat{\mu}_{axon}$$

התוחלת מק״מ:

$$E(\hat{\mu}_{ax\text{-}fixed}) = E\left(\frac{N}{N-1}\hat{\mu}_{axon}\right) = \frac{N}{N-1}E(\hat{\mu}_{axon})$$

$$= \frac{N}{N-1}\cdot\frac{N-1}{N}\mu = \mu$$

והמשערך יהיה חסר הטיה. $B' = \mu - \mu = 0$,,

(1) ידוע בקשר

$$\hat{\mu}_{axon\ fixed} = \frac{N}{N-1}\hat{\mu}_{axon}$$

ולכן:

$$Var(\hat{\mu}_{axon\ fixed}) = \left(\frac{N}{N-1}\right)^2 Var(\hat{\mu}_{axon})$$

$$\Rightarrow Var(\hat{\mu}_{axon}) = \left(\frac{N-1}{N}\right)^2 Var(\hat{\mu}_{axon\ fixed})$$

$$\Rightarrow Var(\hat{\mu}_{axon}) = \left(\frac{N-1}{N}\right)^2 \left(\frac{\mu}{N} + \frac{2\mu^2}{N-1}\right)$$

(2) עבור $\hat{\mu}_{spike}$

$$\hat{\mu}_{spike} = \frac{1}{N}\sum_{1}^{N} n_i \quad / Var(\ )$$

$$Var(\hat{\mu}_{spike}) = \frac{1}{N^2} Var\left(\sum_{1}^{N} n_i\right) = \frac{1}{N^2}\left\{\sum_{1}^{N} Var(n_i) + 2\sum_{1<i<j<N}\sum cov(n_i, n_j)\right\}$$

$$= \frac{1}{N^2}\sum_{1}^{N} Var(n_i) = \frac{1}{N^2}\sum_{1}^{N}\mu = \frac{N\mu}{N^2} = \frac{\mu}{N}$$