March 27, 2024

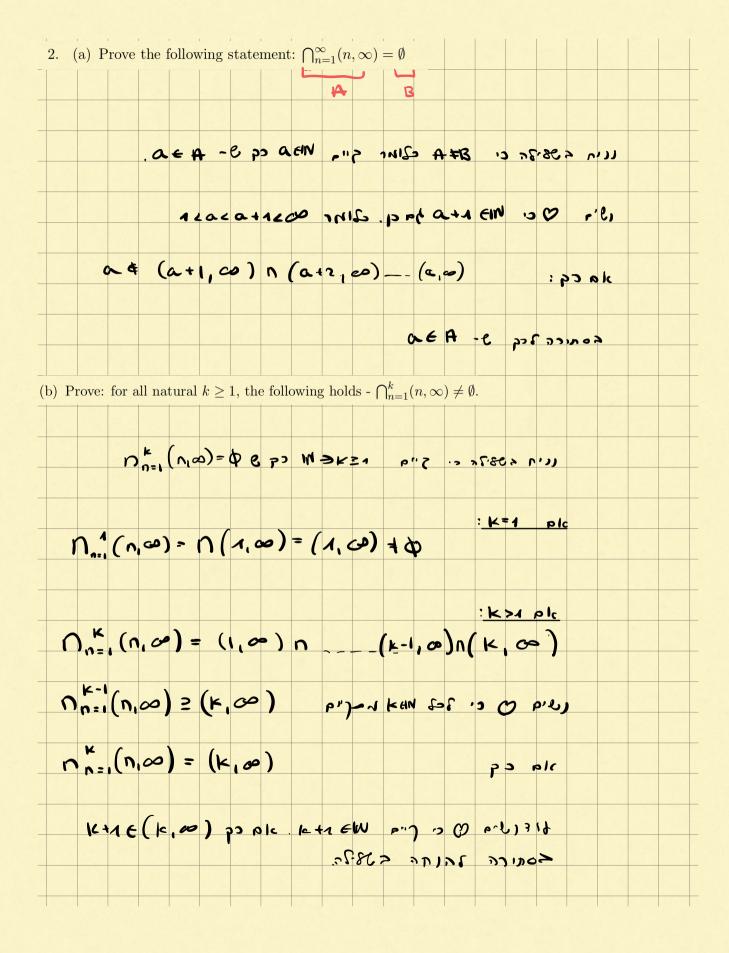
Keep the proofs in this exercise short and concise - For each section in questions 1,2,3,5 only the first 10 lines will be checked.

- 1. (a) Write all the elements of $\bigcap_{k=1}^4 A_k$, for A_i defined as follows:
 - A_1 The set of all integers (\mathbb{Z}).
 - A_2 The open interval (-700, 100).
 - A_3 The set of all integers whose sum of digits is 8.
 - A_4 The set of integers with '6' on the second rightmost position.
 - (b) Define $A = \{S \in P(\mathbb{N}) | 2 \in S\}$. Write the sets $\bigcup A, \bigcap A$.
- 2. (a) Prove the following statement: $\bigcap_{n=1}^{\infty} (n, \infty) = \emptyset$
 - (b) Prove: for all natural $k \geq 1$, the following holds $\bigcap_{n=1}^{k} (n, \infty) \neq \emptyset$.
- 3. Prove: The set of strictly positive real numbers $\mathbb{R}^+ = \bigcup_{n=1}^{\infty} (\frac{1}{n}, \infty)$
- 4. **Prove** that the Axiom of Separation is provable by the rest of the axioms.

Guideline: Define some set A and some predicate P(x), and consider the following two cases: 1. No element in A satisfies the predicate; 2. There exists at least one element that satisfies the predicate. For the first case, use the Axiom of the Existence of the Empty Set, and for the second, the Axiom of Replacement.

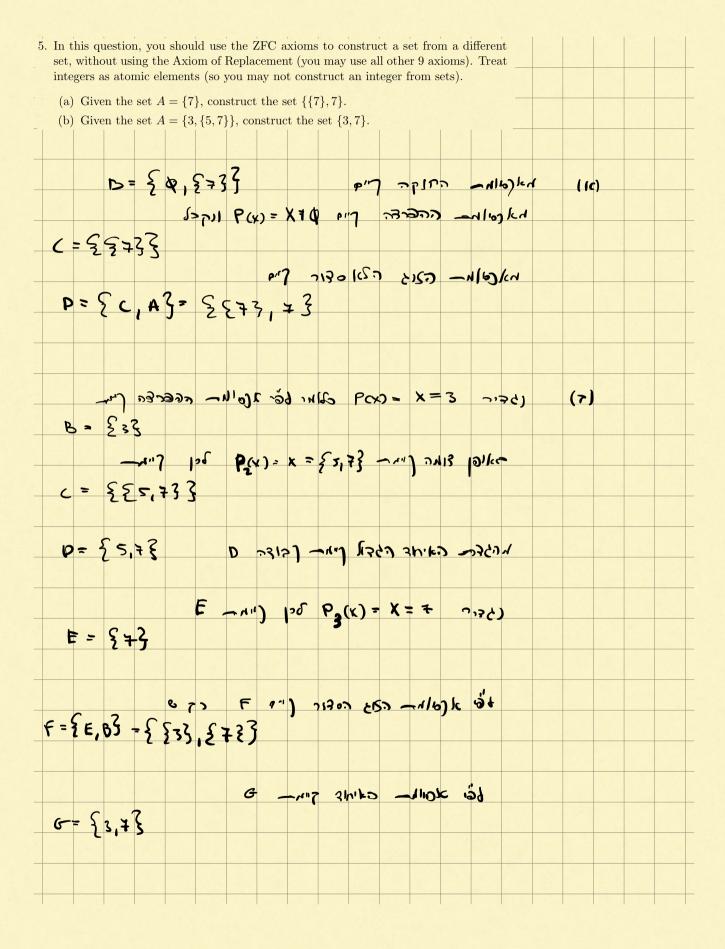
- 5. In this question, you should use the ZFC axioms to construct a set from a different set, without using the Axiom of Replacement (you may use all other 9 axioms). Treat integers as atomic elements (so you may not construct an integer from sets).
 - (a) Given the set $A = \{7\}$, construct the set $\{\{7\}, 7\}$.
 - (b) Given the set $A = \{3, \{5, 7\}\}$, construct the set $\{3, 7\}$.
- 6. For each of the following, determine whether it is a valid representation of an ordered pair. If so, prove it using the ordered pair property. If not, show a counter example:
 - (a) $\langle x, y \rangle = \{ \{x, \emptyset\}, \{y, \{\emptyset\}\} \} \}.$
 - (b) $\langle x, y \rangle = \{ \{x, \emptyset\}, \{\emptyset, y\} \}$

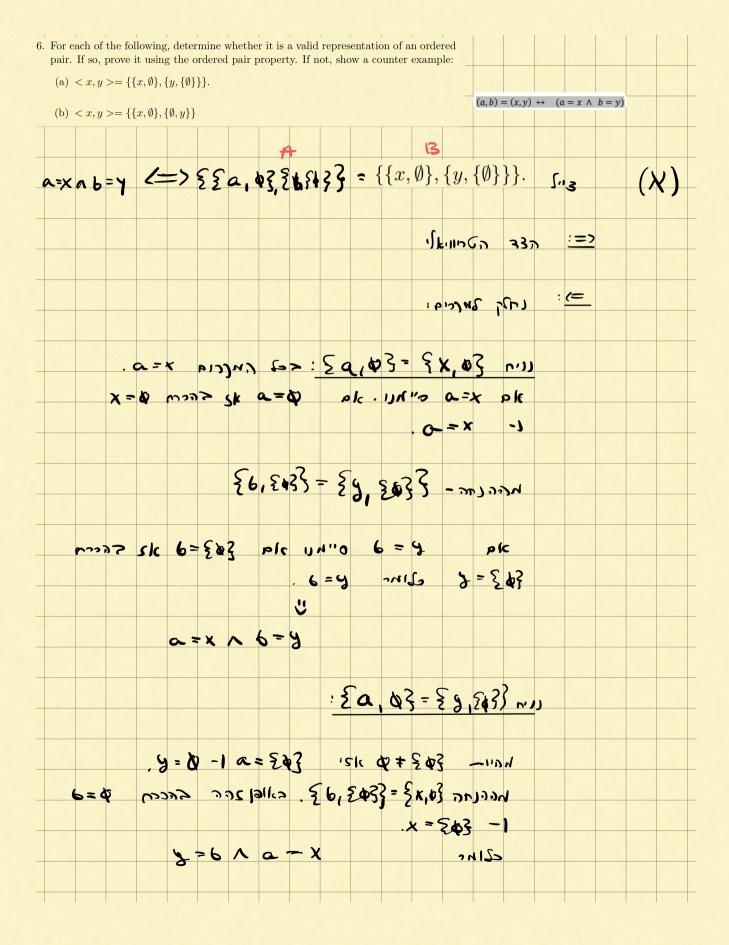
(a)				$\begin{array}{c} { m eleme} \\ {\it of all} \end{array}$				for A	$d_i \operatorname{def}$	ined a	as foll	ows:								
				n inte		,	*													
										its is										
	A_4 -	Th	$e \ set$	$of\ int$	egers	with	'6' or	$i \; the$.	secon	$d\ right$	itmos	$t \; posi$	tion.							
	1.1																			
1) c =	, 1	4k	- /	7, 1	A	, n	A	n.	Aч	=									
										128			, 10		0		ری.		,	
		6				7,1) [7]		2	136	+	2 5								
-	4,0	Az	=	AZ																
								*	n, c	Azl	1A	301	44	21	nik=	6.	۸)	ນ1 ວ		
																		っろり		
	0	1.		1220						ľ										
	. 6	<i>J</i> C"	, ,	1 100	,) N	120	ויאמ	1.4.	, y	وعر		4614	M 19.	,,,	5 %	J-9/0)		رم		
						.3	= ک ح	• 6	2	730	(,ne	د ک	2	a,	חיובי	って	حرارع)		
										10		10				~• C ~ C	0			
								•		60	•	16	' '	-62	- p	9 8	~			
	4															0				
(JKT	, 1	k:	- A	, nA	hn	A3	n A	4 =	٤	62.	- G Z	, - la	1, -	260	yc {	حا			
																-				
) D	efine	e A	$= \{S$	$S \in F$	$P(\mathbb{N})$	$2 \in \mathcal{A}$	S }. V	W rit ϵ	the	sets	$\bigcup A$	$, \bigcap A$	4.							
			2	- 216		0.1	1		-								n			
			•	ا ام د	((۱	OK.	b, .45	, (v	90	عم ه	- رن	~131A	م را	3 ~	3127	9)4) /			
														- -	とろう	lc	34)			
	UF	7	=	M																
	J. 1	•																		
				C	2															
1	\cap	4	=	2	5															



3. Prove: The set of st	rictly positive real numbers $\mathbb{R}^+ = \bigcup_{n=1}^{\infty} (\frac{1}{n}, \infty)$
	BA
	R+= (0, co)
	נטור התלה בו כיוונה.
$\left(\frac{1}{2}R\right)CR^{+}$	שבה נוים מי שב חבוא בהרח מתנים
N, 6 / 2 %	(13 4) 2151 (2) (2) (3) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1
1100 (1 m) = B + 1416 4 1019 604 1019 6
Un=1 (n, ce	
	יה (ס, ס) ארכיאביות הפוכה
X ∈ (+, 00)	א קיים שושת כק ש- ה כ X. כאותר
	אמכן הפכם גבלבר נאימוד נישבול
XE Une (in	
	$R \subseteq U_{n=1}^{\infty}(1, \infty)$, then
	Rt = Une (n, c)

Em	ipty S	Set, an	nd for	the s	econd,	the A	1xiom	of Re	eplace	ment.								
						014	1	<u></u>	2-7		215	2		3,				
					•	B	- - {	∪) × <i>∈</i> r	75 /a) 3+ ∶ () (x)	31r, }		ני א	" "				
			.Pc) E	F	٬٬٬	א ת	X	E A		ጉየ	12	לניח	20.	7	:1		
		20		5 6		_	. 2		. 3	0		21-		٠.٠٨				
	520				EA:													
	3, (,,,		, ,,	3110	,,,		11.03.7		٦	2	-	,,,,,	J., 100 t				
	53	<u>s</u> A	19	٧ - ،	7 F	دار	31 ,	ŋſc	הין.	η -	ગ્કાર	7	ノつる	そうい				
В	= '	ξxe	A:	P(x	,} -	→ ५ .	ין	121	((·) \		BE	A	ادر)			
				Pox) = 1	Γ	O	در	XE	A	f	7"?	כי	ריח)	2		
					C ~		0			щ	_	11911	1 7	136)				
		,	n(K)	2	{		70	(<i>k</i>)										
						1	7.	(X)										
-	~2'S	ر بر م م	6	n-1	קבונ		'لرم	7	פת	יש ען	, .	الدم	o)k	\$	r 2	r		
	ارم	محالد:	3 6	ڪ	В.	-317	2 ا و		~ ()	()- 7	f	•	25	ار	ه احر			
												PCK)	ادرم	יין			
		2	•	ς,	(eA	•		. 7							เก			
		0		5,	(en	•	L(*	, 2										





	e,6) = (x,y) = (ر درام	ا ح دایام	[a] (I)
الرم) اع ه و اد	(a,6)	, (x,y) 3	360, 29	0 116)	
(a, b) =					
7 2 9, 1	3, 22,455	-> (2,1) = ((x,y)	
	(1,2)	(2,1)	בנ מניו	הלני סתים	וכני