IE-411 **2018 Fall**

Lecture 2: The Simplex Method

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2.1 The optimality of extreme points

Consider the following linear programming problem

$$\begin{aligned} \min_{x} \quad & c^T x \\ s.t \quad & x \in P = \{x | Ax = b, x \geq 0\}, \end{aligned} \tag{2.1}$$

where P is non-empty.

By the **representation theorem**, we know that for a $x^* \in P$, we can write x^* as $\sum_{i=1}^{k} \lambda_i x_i + \sum_{j=1}^{l} \mu_j d_j$, where $x_1, .., x_k$ are the extreme points and extreme directions of of P$\sum_i \lambda_i = 1, \lambda_i \geq 0, \mu_j \geq 0$.

Consider

$$c^T x_0 = \sum_{i=1}^{k} \lambda_i (c^T x_i) + \sum_{j=1}^{l} \mu_j (c^T d_j).$$

If $\exists i$ s.t $c^T d_j < 0$, then we can set u_j to arbitray large. As a result, 2.1 is unbounded, hence no optimal solution.

If there doesn't exist such a j, we can set $u_j = 0, \forall j$. So the optimal value of 2.1 attains at one of the extreme points, which is $x_i = \arg\min_{i=1,2,\dots,k} c^T x_i$.

The above discussion can be generalized to the following theorem.

Theorem 2.1 For linear programming problem, it either has a optimal value or is unbounded.

2.2 Simplex Method

2.2.1 Algebra

The simplex method, the "one non-basic in , one basic out" process guarantees there are still n active linear active constraints, that is, you move from one basic feasible solution to another. (See *p109 of BHS*.)