STAT-511 **2018 Spring**

Lecture 0: Preliminaries

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0.1 P-value

Definition 0.1 Consider the hypothesis test problem based on test statistic $T(\mathbf{X})$

$$H_0 : X \sim F_\theta,\ \theta \in \Theta_0 \qquad H_1 : X \sim F_\theta,\ \theta \in \Theta_A,$$

where $\mathbf{X}$ is the random sample. P-value is defined as the probability that x is generated by the distribution of $\mathbf{X}$ under H_0, where x is the observed sample. This is quantified by observing the test statistic $T(\mathbf{X})$. Therefore, the maximum probability that $T(\mathbf{X})$ is more extreme than $T(x)$ is taken as p-value.

$$\text{p-value} = \sup_{\theta \in \Theta_0} P(T(\mathbf{X}) > T(x)),$$

Lemma 0.2 If X is continuous, and $\Theta_0 = \theta_0$, then p-value $\sim U[0,1]$.

Proof: Suppose the c.d.f of $-T(X)$ is F, where F is invertible because it's monotone.

$$F(-T(x)) = P_{\theta_0}(-T(X) \le -T(x)) = P_{\theta_0}(T(X) \ge T(x)) = \text{p-value}.$$

So

$$P_{\theta_0}(\text{p-value} \le \alpha) = P_{\theta_0}(F(-T(x)) \le \alpha) = P_{\theta_0}(-T(x) \le F^{-1}(\alpha)) = F(F^{-1}(\alpha)) = \alpha.$$

Therefore p-value $\sim U[0,1]$. ■

Lemma 0.3 Define a test function

$$\phi(x) = \begin{cases} 1 & \text{p-value } \le \alpha \\ 0 & \text{p-value } > \alpha. \end{cases}$$

Then, this test is of size α, where $\alpha \in (0,1)$.

Proof: From Lemma 0.2, we take supremum over Θ_0, we obtain

$$\sup_{\theta \in \Theta_0} P_\theta(\text{p-value} \le \alpha) = \alpha.$$

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