STAT-511 **2018 Spring**

Lecture 4: Sign and Rank Tests

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4.1 Sign Test

A type of nonparmametric testing without any assumption on the distribution of samples. It is more robust in the sense that the performance is more stable among a wide variety of distributions.

4.1.1 Symmetry Tests: Median

Setup: $Z_1, ..., Z_N \overset{iid}{\sim} F \in \mathcal{F}$, where $\mathcal{F}$ is the set of continuous distribution functions that are symmetric about their median η. We test

$$H_0 : F \in \mathcal{F} \text{ with } \eta = 0 \quad H_A : F \in \mathcal{F} \text{ with } \eta \neq 0.$$

Test Statistics: $T(\mathbf{Z}) = \sum_{i=1}^{n} sign(z_i)$

Distribution of the Test Statistics: Under H_0, $\frac{n+T(\mathbf{Z})}{2} \sim \text{Binomial}(n, \frac{1}{2})$.

4.1.2 Asymmetry Tests: Quantile

We can generalize the notion of $sign(x)$ to general sign function $\phi_\tau(z)$.

Definition 4.1 The general sign function $\phi_\tau(z)$ is defined as

$$\phi_\theta^\tau(z) = \begin{cases} \tau, & z \geq \theta \\ \tau - 1, & z < \theta \end{cases}.$$

Remark: The θ serves as the cut-off and τ is the weight.

Setup: $Z_1, ..., Z_N \overset{iid}{\sim} F \in \mathcal{F}$, where $\mathcal{F}$ is the set of continuous distribution functions. Denote $Q_\tau(F)$ as the $\tau-$th quantile of the F. We test

$$H_0 : Q_\tau(F) = \theta \quad H_A : Q_\tau(F) \neq \theta.$$

Test Statistics: $T(\mathbf{Z}) = \sum_{i=1}^{n} \phi_\tau^\theta(z_i)$

Distribution of the Test Statistics: Under $H_0, n(1-\tau) + T(\mathbf{Z}) \sim \text{Binomial}(n, 1-\tau)$.