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Problem Chosen

E

2016
MCM/ICM
Summary Sheet

WATER IS LIFE, WATER IS DIGNITY

Water scarcity challenges people around the world. Global population growth and economic development suggest a future of increased demand. Scarcer water, however, creates new challenges because residency, agriculture and industry require massive amounts of freshwater. Reliable and sustainable water supply for future growth must be available to local population in sufficient quantity and quality, and without compromising local ecosystems.

Finding a typical region or country to study the water scarcity problem and then give detailed resolutions can provide good examples for other countries or regions. Therefore, we design several models to do the job.

First, we address the problem of ranking the shortage degree by using systematic cluster analysis. Considering the difficulty to find comprehensive data, we use simulated data to illustrate how these methods work and how to build a reliable ranking standard.

Second, we introduce our modified Cobb-Douglas production function to find the mechanism of how social and environmental drivers influence supply and demand. We explain the reason of a given region where water is heavily-exploited. To get a reliable prediction, we exclusively combine the Grey Model and the Fourier Series method to apply in data of Free State, South Africa. After drawing the prediction of each factor in the next 15 years, we apply the prediction of factors to forecast the tendency of demand-supply relationship in the following 15 years. The robustness of our algorithm is enough to accommodate various factors and different types of changing tendency. It can also be easily applied to any other places by just changing reference data.

Finally, we devise an intervention strategy based on the existing project (LHWP), which is easy and feasible to implement. With valid results, we prove that the intervention is highly eco-balanced and sustainable even if there is a heavier water scarcity.

Key Words: Cobb-Douglas function; Grey Model; Nonlinear Fitting

1 Introduction

Water is life, sanitation is dignity.

–Department of Water and Sanitation, South Africa

1.1 Background

Water is crucial to both life and human civilization. Water is the basic component of human body. Water nourishes fauna and flora. It holds the key to agriculture. And it is essential to almost all industries. However, although 70% of the earth surface are covered by water, only 2.553% is fresh water, and 97.01% of it is frozen in the polar areas or lying deep underground. Global trends suggest the next 25 years will see the increasing demand and competition for water resources. Today about 1.6 billion people are living in water scarcity and the world's population is projected to grow from 7 billion today to 9 billion by 2040 (UN 2015). That growth, together with trends in urbanization, mobility, economic development, international trade, cultural and technological advancement, and climate change, will aggravate water scarcity.

The increasing demand for water will force trade-offs in arid or water-stressed areas. After planning thoroughly for over 50 years, China started South-to-North Water Diversion project, aiming at solving serious water scarcity in northern China and supporting the development. Desalinization in Singapore and Saudi Arabia has been successfully commercialized. Isarel is expert at advanced irrigation technology.

Nevertheless, the situation is still very grave. Current technologies and investment of developed countries are not capable of solving water scarcity at present and confronting the future, let alone the developing and underdeveloped countreies.

1.2 Our Work

Based on the current situation, in order to analyze problems above and provide valid prediction in the following 15 years as well as giving a feasible solution, we build an assessment model and a prediction model among a series of submodels to analyze the following problems:

- Validate different factors considering the supply and demand of fresh water;
- Establish an assessment system using the factors above to assess the current situation of a Free State;
- Give a prediction of Free State in the next 15 years by existed data;
- Find a feasible solution to solve water scarcity in the region and make assessment to the solution.

In the whole analysis process, we give full consideration to validity, feasibility and cost-efficiency of our models.

2 Assumption, Definition and Notations

2.1 General Assumptions

We make the following general assumptions in this paper.

1. There are no coup or negative policy changes that prenvent our interventions to be implemented in the predictable future.
2. All the data found in Internet are accurate and relaible.
3. Only major factors will be considered and measured, while the minor factors are taken as constants in our model.

Additional assumptions are made to simplify analysis for individual sections. These assumptions will be stated at the corresponding locations.

2.2 Definitions and Notations

We give a series of definitions in following to make our model better undertood.

Definition 2.1 (**Supply**). *Supply means the quantity of water which can be used by current methods in a given region within a given period of time, such as river water,lake water, rainfall, groundwater and others.*

Definition 2.2 (**Demand**). *Demand means the quantity of water needed by domestic, agricultural and industrial purpose in a given region within a given period of time.*

Definition 2.3 (**Dynamic Factor**). *Dynamic factors are factors which will change by time or variation in space.*

Definition 2.4 (**Social Driver**). *Social drivers are factors relating to human society and its members which will affect the relationship between supply and demand of water.*

Definition 2.5 (**Environmental Driver**). *Environmental drivers are factors relating to the environment which will affect the relationship between supply and demand of water.*

3 Water Supply Capacity Assessment Model

To measure the ability of a region providing clean water to meet the needs of its population, we'd better determine the supply and demand of clean water in the chosen region. Then apply the following formula:

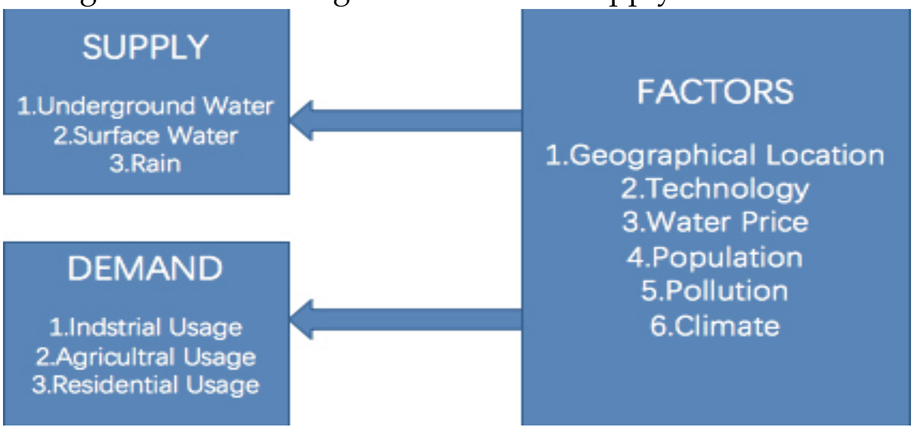
$$\Delta = Supply - Demand \tag{3.1}$$

Hence, we need to figure out some dynamic factors which determine supply and demand. Accoring to the exisiting literature(Mohamad Hejazi et al.2013),as well as taking

Notations	Explaination
$C_i, \alpha_i, \beta_i, \gamma_i, \eta_i, \omega, \theta$,$i = 1,2$	parameters of Cobb-Dogulas production function
S	Water Supply
D	Water Demand
RA,CL,PO	Rainfall, Climate, Pollution
PE,RD,WP	Population, R&D, Water Price
Δ	Measurement of gap between water supply and demand
T	Time interval from 2016 to 2030
V	The whole capability of the tunnels
$\bar{\Delta}$	Baseline of the scarcity level
$f_n(t)$	Fourier series with n terms

the physcial and economic aspects into consideration, we choose some factors shown in Figure 1

Figure 1: Factors might contribute to supply and demand



Then we work out the mechanism that how those factors influnce each indicator of supply and demand respectively by appling regrssion method. For any given country/region, climate and geographical location are fixed, so we only need to consider how to estimate water price, population, pollution and technology.

Data about water price and population can easily obtained from World Bank website, however, data related to pollution and technology are hard to either directly quantify or find historical data, so we choose other factors which can measure pollution and technology to help determine the analytical expression of our assesment model.

- How to quantify pollution?

  Actually, various pollution can lead to the water scarcity. For simplicity purpose, as well as to find the most influential factor which directly influences the supply of clean water, total waste water discharge is a ideal representative(Ziying Zhang et al. 2013)[1].

- How to quantify technology?

  Technology plays an important part in determining the ability of a region to provide clean water. Research and Development (R&D), also known as RTD (research and technical development) in Europe, is widely used among governments and corporations to measure their technological capabilities. It is measured by the direct investment of innovation activities, including research phases commercialization phases.

After quantifing all indicators, we use them to determine the quantity of both supply and demand by appling our modified Cobb-Dogulas production function. This function is a particular functional form of the production function which is widely used to represent the technological relationship between the amounts of two or more inputs, particularly the amount of output that can be produced by those inputs.

Considering the fact that different factor has different impacts on supply and demand, first we have the Table 1 :

Table 1: Impacts of factors on supply and demand

	Supply	Demand
Geographical Location	+	-
Climate	+	+
Population	+	+
Water Price	-	+
Technology	+	+
Pollution	+	+

$+$ means there is an impact
$-$ means there is no impact.

Then we can have the model functions as follows.
The supply function:

$$S = \alpha_1 \times PO(t)^{\beta_1} \times RD(t)^{\gamma_1} \times CL(t)^{\eta_1} + C_1 \tag{3.2}$$

The demand function:

$$D = \alpha_2 \times PE(t)^{\theta_2} \times PO(t)^{\beta_2} \times WP(t)^{\omega} \times RD(t)^{\gamma_2} \times Cl(t)^{\eta_2} + C_2 \tag{3.3}$$

where C_1, C_2 are constants. Both supply function and demand function have take the dynamic nature of each factor into account, therefore indictors also rely on time. In this paper, we set 2001 to be the base-year, hence t=1. At last, we can obtain the assessment formula:

$$\Delta = S - D \tag{3.4}$$

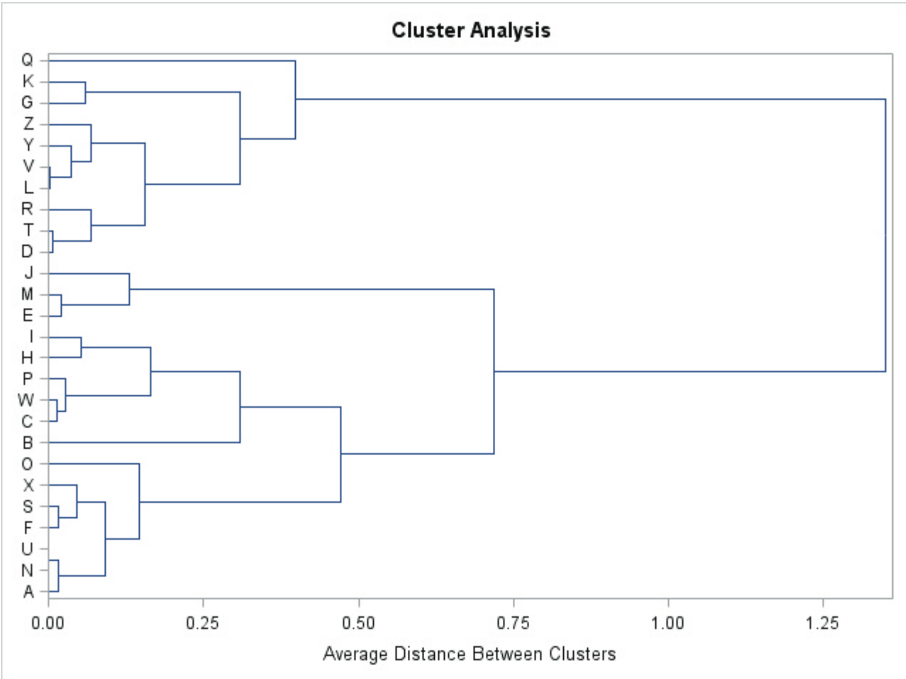
Based on the sign and magnitude of Δ of different countries and regions, by applying cluster anlysis method, we can build a standard to determine the degree of water shortage. Then once we obtain the value of Δ of a specific region, we can classify it into proper category. Considering truth that it consumes countless time to collect sufficient data, we decide to simulated some data to demonstrate how our assessment model works.

Table 2: Cluster Results

Standard	Elements	Mean	Approximate Range
I	D,G,K,L,Q,R,V,Y,Z	0.15994	[0.054,0.248]
II	B,C,H,I,P	-0.12914	[-1.806,-0.051]
III	A,F,N,O,U,X,S	-0.26449	[-0.301,-0.236]
IV	E,J,M	-0.407	[-0.4232,-0.381]

I sufficent water; II slight short of water;
III moderate short of water; IV heavy short of water

Figure 2: Cluster Analysis



From the Table 2, we can see clearly that 26 regions are divided into 4 groups,ranging from Standard I to Standard IV. Hence, for any given new region or country, we can approximately evaluate its water shortage degree.

4 South Africa, South Africa!

According to the UN water scarcity map[7], we decide to choose Free State in South Africa to analyze the reason of water scarcity as it is, in general, a heavily-exploited region.Free State experiences a continental climate, characterised by warm to hot summers and cool to cold winters. Almost all precipitation falls in the summer months as brief afternoon thunderstorms, with aridity increasing towards the west.The capital, Bloemfontein, experiences hot, moist summers and cold, dry winters frequented by severe frost.(Wikipedia)

4.1 The General Situation of South Africa

Water scarcity is a urgent problem in South Africa. Low precipitation, high evaporation, heavy pollution, poor management and other factors cause severe water scarcity to the whole country. 14 million people in South Africa suffered from water scarcity. And 21 million people (about half the poplulation) lived without sanitaion facilities. According to the Department of Water Affairs(DWA), the situation got better after the long efforts of two decades.

However, in 2008, it was estimated that there were still 5 million people need clean water and 15 million residents need sanitation facilities. According to the official report, there are several foundamental causes of the water scarcity in modern South Africa. We can classify them by physical factors (including geographical properties and climate) and ecomonic factors (including population, pollution, technological factors and the water price).

The physical factors are:

1. The annual rainfall level (about $495mm/year$) is about only half of the world average, which is about $1,033mm/year$; the evaporation rate is also very high;
2. The main vegetation type is Savanna, so the water retention capacity of the land is poor;

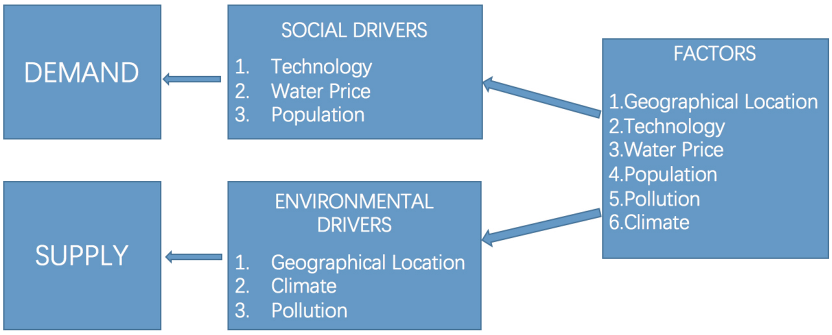
The economic factors are:

1. Trained engineers and technicians are not enough to support the country;
2. Construction of related infrastructure is far behind the demand;
3. Maintanance of the current infrastructures is very poor, about 36.8% of municipal water is wasted;
4. Municipal water is polluted, especially in West Cape State, more than 50% are polluted by Escherichia coli;

4.2 Assessment Model application

Use the supply-demand relationship function (Equation 3.4) to estimate the influence by addressing physical and economic drivers. It is clear that the social drivers influence the water demand and the environmental drivers influence the water supply. We consider labeling population, technology and water price as social drivers and geographical location, climate and pollution as environmental drivers (in particular, geographical properties are usually fixed in a given region).

Figure 3: Factors Catogorized By Social Drivers and Environmental Drivers



By this way, the results of estimation can be more clear and persuasive.
Now, we analyze the situation of Free State using modified estimation model:

- Environmental Drivers:
  Considering environmental drivers, by replacing $CL(t)$ with $RA(t$, we can get the modified supply function:

$$S = \alpha_1 \times PO(t)^{\beta_1} \times RD(t)^{\gamma_1} \times RA(t)^{\eta} + C_1 \tag{4.1}$$

- Social Drivers:
  Considering social drivers, we can get the modified demand function social drivers:

$$D = \alpha_2 \times PE(t)^{\theta_2} \times PO(t)^{\beta_2} \times WP(t)^{\omega} \times RD(t)^{\gamma_2} + C_2 \tag{4.2}$$

- Supply-Demand function by social drivers and environmental drivers:

$$\Delta = S - D \tag{4.3}$$

We collect detailed data of each indicator from World Bank website and Water Affairs of South Africa, shown in Table 3 and then use 1stOp(software) to determine all parameters.

Table 3: Data of Each Indicator from 2000 to 2014[6,8]

year	demand /bcm	supply /bcm	population /capita	pollution /cm	price /cent	RD /(br/mp)	rainfall $/mm.Y^{-1}$
2000	0.713858943	0.425874684	2482987.628	2217.744083	11.83	20.2896	92.5
2001	0.70934442	0.391487288	2534325.541	2202.402754	11.55	23.3696	108.75
2002	0.705394212	0.460262081	2570250.253	2244.547733	12.1	23.7472	76.25
2003	0.70200832	0.322712494	2603019.256	2540.485988	11.1	24.1308	67.5
2004	0.698622428	0.285679912	2636915.593	2556.262797	11.9	24.5207	42.5
2005	0.696365167	0.179872538	2671977.579	2906.576372	13.8	24.9169	17.5
2006	0.694107905	0.740651625	2708245.338	3220.058583	17	25.3195	97.45
2007	0.692979274	0.687747938	2745761.418	3326.047713	14.84	25.7286	89.32
2008	0.692414959	0.105807375	2784570.628	3529.310993	18.29	26.1443	15
2009	0.692979274	0.63484425	2824720.143	3968.787874	35.68	26.5667	26.4
2010	0.69467222	0.63484425	2866259.792	4139.822867	51.43	26.9959	50
2011	0.696929482	0.21161475	2909242.057	4709.516783	67.5	27.4321	35
2012	0.700879689	0.285679912	2953722.298	7915.016314	78.89	27.8753	92.5
2013	0.705394212	0.391482788	2999758.863	10920.59141	75.33	28.3257	80
2014	0.711037366	0.33858135	3047413.208	10568.04698	83.7	28.7834	86.25

1.The groundwater remains stable at about 24 t$/km^2$, so we take it as a constant coefficient in the function.
2.'bcm' stands for 'billion cubic meters'; 'cm' stands for 'cubic meters'; 'cent' is South Africa 's currency unit; 'br/mp' stands for billion rand per million persons; $mm.Y^{-1}$ stands for millimeter per year.

The results are:

$$\begin{cases} D &= 313.02 \times PE^{-0.43} \times PO^{0.04} \times PR^{0.01} \times RD^{-0.01601} \\ S &= 1.6^{-8} \times PO^{-0.89} \times RD^{5.64} \times RA^{0.54} + 0.693 \end{cases} \tag{4.4}$$

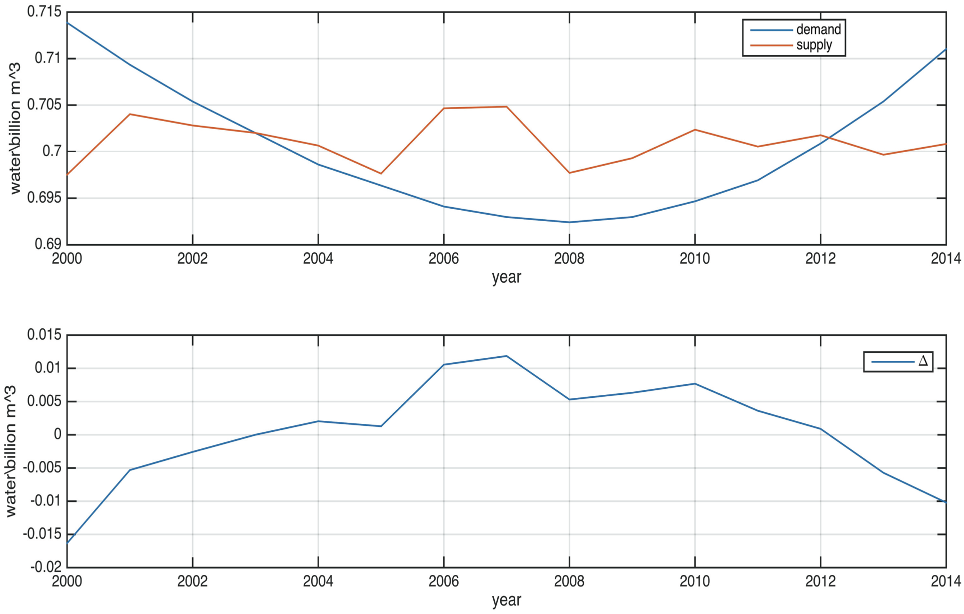
Table 4: Nonlinear fit Results

Item	R^2	Fiting method
demand	0.812	Robust simple plane climbing method
supply	0.360	Robust simple plane climbing method

The fitting results of the demand function is desirable. However, considering that the climate changes dramatically in different years, our approximation of supply is also acceptable.

Then by taking all coefficients and variables in to the formula Equation 4.3, by using date in Table 2 we can get more intuitionistic results shown in Figure 4:

Figure 4: The Relationship of Supply-Demand and Δ-Measure



We can draw the following conclusion:

1. Water supply in Free State is highly relying on the rainfall (climate) and is unstable;
2. Water supply measured by environmental drivers is also affected by the growing sewage disposal;
3. As the economy among other factors like technology and population is growing the water demand measured by social drivers is growing in recent years;
4. Δ-Measure shows that people in Free State are lacking in water after 2013.

5 Predicting the future

The future situation is always of our concerns. Since the current situation in Free State is negative and still looking blue, we have to consider what will happen in the future.

First, we predict the change of dynamic factors in the following 15 years.
The prediction job is very tough because we have to forecast every aspects (all factors) of the following 15 years. But we only get the data from 2000 to 2014. So the fifteen-year data is too less to forecast the fifteen-year trends of future. Ordinary forecasting models,such as linear models is unreliable. In view of current situation, we use a Grey Forecasting Model to get data with higher reliability, thus successfully overcoming the weakness of Function Fit Model.

5.1 Buiding Grey Forecasting Model[2,3,5]

First we assume that different factors are mutually independent, then we can construct GM(1,1) model.

We define $X^{(0)}$ as the original data sequence of a given factor from 2000 to 2014:

$$X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), \ldots, X^{(0)}(n)\}$$

Then we can get the whitened equation of GM(1,1) model:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{5.1}$$

where the $X^{(1)}$ is the accumulated generating operation sequence of $X^{(0)}$.

Next we use the least square methods (LS) to get the parameter a and b as:

$$\hat{\mu} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y \tag{5.2}$$

While

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad Y = \begin{bmatrix} X^{(0)}(2) & 1 \\ X^{(0)}(3) & 1 \\ \vdots & \vdots \\ X^{(0)}(n) & 1 \end{bmatrix} \tag{5.3}$$

$$z^1(k) = 0.5\left[X^{(1)}(k) + X^{(1)}(k-1)\right] \tag{5.4}$$

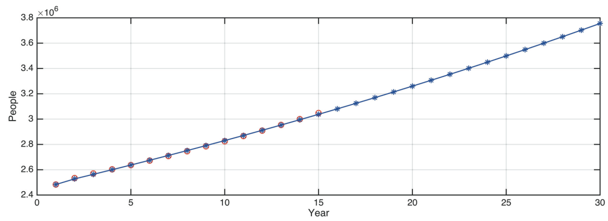
The respective time response sequence of GM(1,1) model is:

$$\hat{X}^{(1)}(k+1) = \left(X^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right) e^{-\hat{a}k} + \hat{b}\hat{a},\ k = 1, 2, 3 \ldots, n-1 \tag{5.5}$$

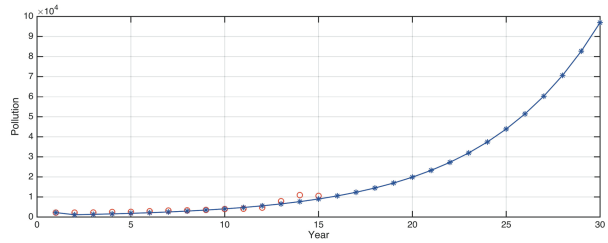
Then we can get the reduced $\hat{X}^{(0)}$ by repeated decreasing:

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k) \tag{5.6}$$

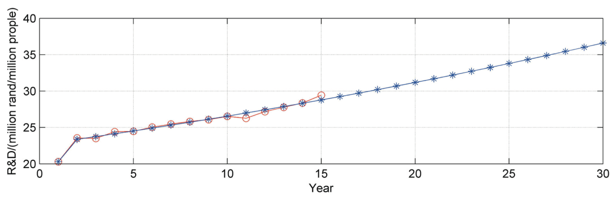
The predictions of pollution, population, R&D and water price by GM(1,1) are as follows:



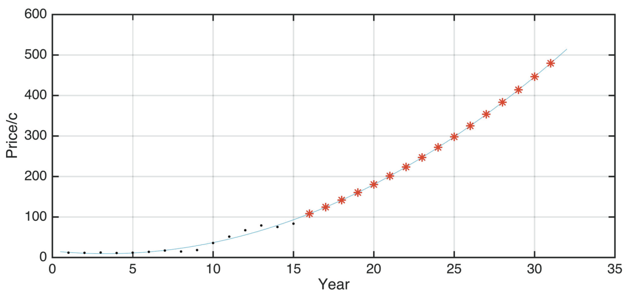
(a) Population Prediction



(b) Pollution Prediction



(c) R& D Prediction



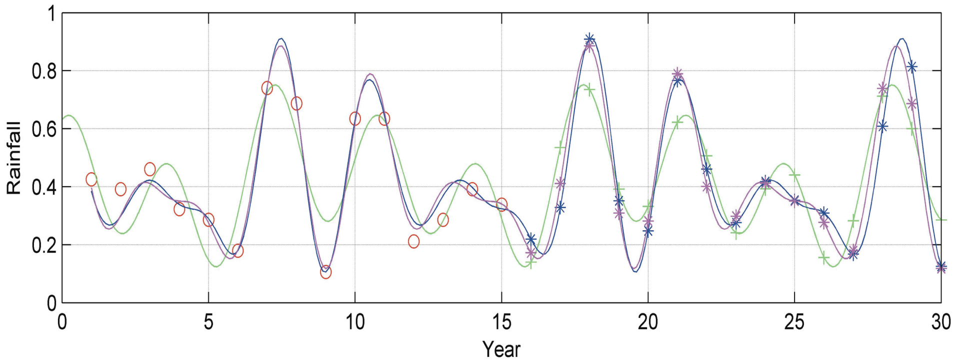
(d) Water Price Prediction

Figure 5: Predictions of 15 Years

The prediction results shows the trend of the dynamic factors. However, GM(1,1) model behaves ill in predicting oscillating variables like rainfall, so we use Fourier-Series-Fitting Method to study the rainfall pattern. To get an accurate prediction and aviod contingency factors, we use 3 typical different kinds of Fourier series to make a prediction:

$$f_n(t) = a_0 + \sum_{i=1}^{n} \left[a_i \cdot cos(i\omega t) + b_i \cdot sin(i\omega t) \right], \quad n = 4, 5, 6 \tag{5.7}$$

Figure 6: Three Fourier Series Prediction



We can see that the prediction of different Fourier series are stable, and thus we average three predictions' results to measure rainfall.

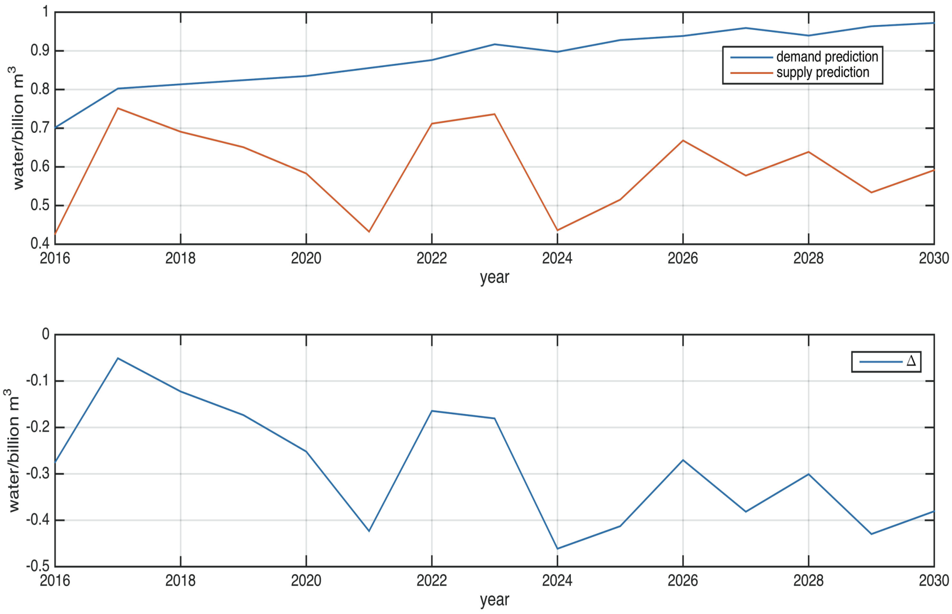
The prediction results of all dynamic factors are:

Table 5: Predictions of Dynamic Factors

Year	Pollution	Population	Water Price	R&D	Climate
2016	10500	3081000	108.1234	29.2484	17.70913246
2017	12400	3125000	124.3036	29.721	42.46001328
2018	14670	3169000	141.7114	30.2012	84.29551253
2019	17340	3214000	160.3466	30.6892	35.06515987
2020	20490	3260000	180.2094	31.185	28.69963007
2021	24210	3307000	201.2997	31.6889	72.58905431
2022	28610	3354000	223.6175	32.2009	45.57725512
2023	33810	3401000	247.1629	32.7212	27.18980157
2024	39950	3450000	271.9357	33.2498	40.71974096
2025	47210	3499000	297.9361	33.7871	38.13770019
2026	55800	3549000	325.164	34.333	24.72391733
2027	65940	3599000	353.6194	34.8877	20.95872939
2028	77920	3651000	383.3024	35.4514	68.6456159
2029	72080	3703000	414.2129	36.0242	70.00083926
2030	79640	3755000	446.3508	36.6062	17.62393299

Then we can apply the prediction of all factors to the demand function Equation 4.2 and supply function Equation 4.1, and we can get the relationship of supply and demand in the following 15 years:

Figure 7: Prediction of Supply and Demand from 2016 to 2030



The quantity of Δ prediction is:

Table 6: Δ Prediction from 2016 to 2030 ($billon\ m^3$)

2016	2017	2018	2019	2020	2021	2022	2023
-0.275	-0.0509	-0.1226	-0.1734	-0.252	-0.4232	-0.1644	-0.1806
2024	2025	2026	2027	2028	2029	2030	-
-0.4613	-0.4127	-0.2703	-0.3816	-0.3008	-0.4297	-0.3806	-

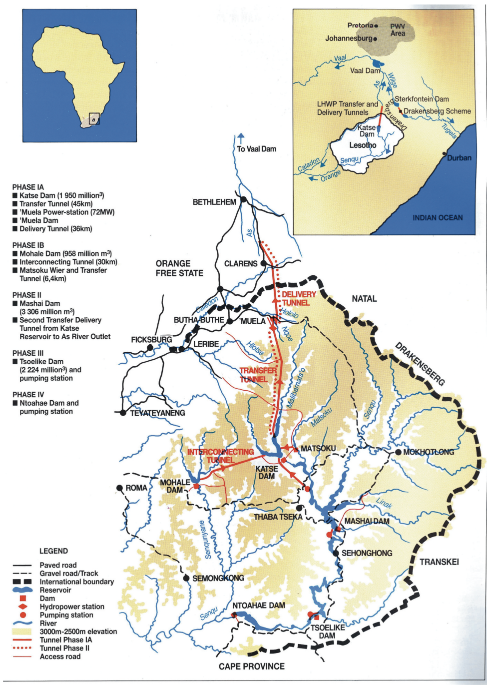
So far, we can safely draw the conclusion that in the next 15 years from now, all people in Free State will suffer from constant severe water scarcity. Accoring to our results, water scarcity will reach its peak, which is $0.46\ billon\ m^3$ in 2024 and the average gap is about $0.28\ billon\ m^3$.If the government doesn't employ sufficient water policies, the society of Free State will finally collapse as the long-lasting water scarcity.

6 Water-Transfering Intervention Strategy

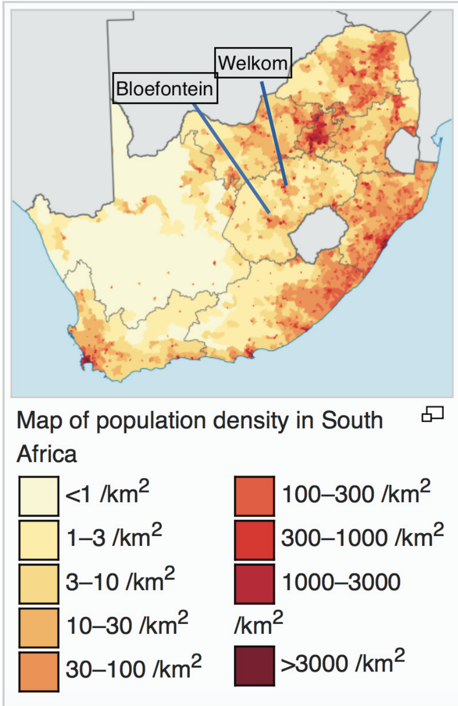
Noticing that available water resources in Free State are already heavily exploited, with the unbalanced distribution of water within this coutry and the geographical advantage of the Free state, we decide to design a water transfer project based on the Lesotho Highlands Water Project[4]. Lesotho Highlands Water Project is an ongoing water supply project with hydropower dams, developed in partnership between the government of Lesotho and South Africa. It comprises a system of several large dams and tunnels throughout Lesotho to South Africa. In Lesotho, it involves the rivers Malibamatso, Matsoku, Senqunyane and Senqu. In South Africa, it involves the Vaal River. It is

South Africa's largest water transfer scheme(Wikipedia).

Figure 8: Hydrologic Regime



(a) Lesotho Highlands Water Project

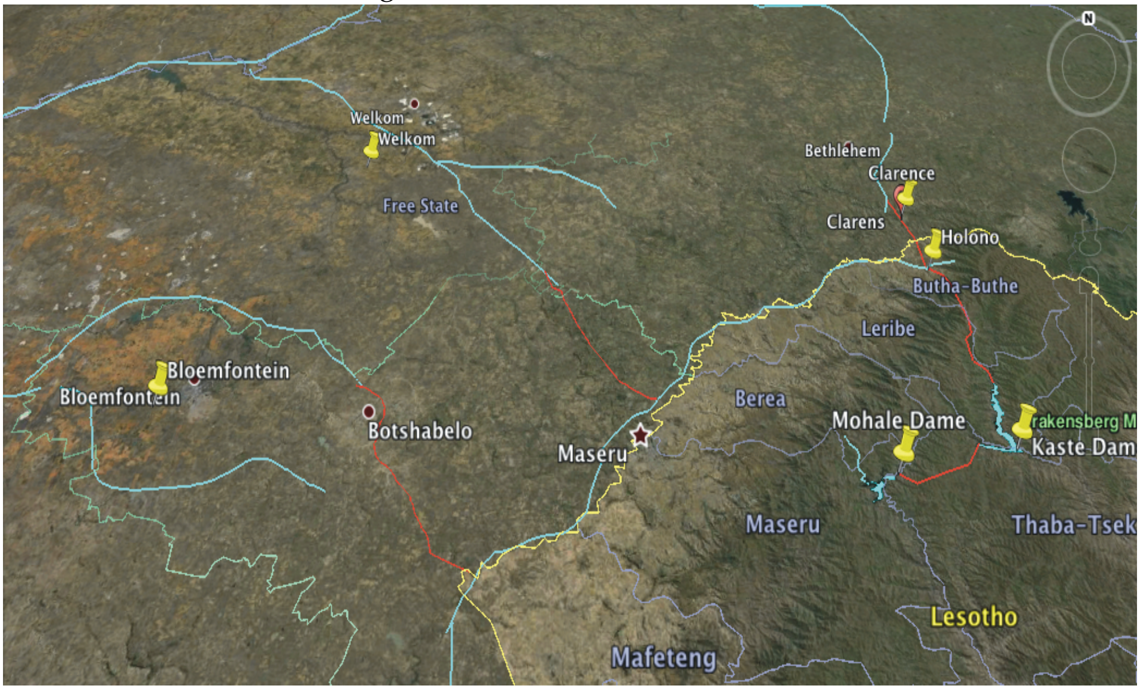


(b) South Africa Population Density

The figures are cited from www.wikipedia.org/south_africa .

In our intervention strategy, taking the population density of Free state into account, we plan to build 2 more water tunnels based on the existing projects to alleviate short supply of water in Free state. Based on the Figure 7(b), the two tunnels are aimed to supply sufficient water to the two most populous regions– Bloemfontein and Welkom.

Figure 9: Two Planned Tunnels



- The first tunnel is 72.4 km long, starting from ($27°2'E$, $29°36'S$), end at ($26°40'E$, $29°6'14S$), through Botshabelo, providing water for Bloemfontein area.
- The second tunnels is 60.6 km long, staring from Maseru($27°32'E$, $29°10'S$), end at Winburg($27°11'E$, $28°43'S$),suppling water for Welkom area.
- There will also be 2 water reservoirs at the end of each tunnel to reserve water from Lesotho.

According to the water-supply-ability estimation, when the whole project is completed in 2020(Wikipedia), total amount of water being transfered from Lesotho to South Africa is approximately $0.22\ billion\ m^3$ per year, which means the demand gap can almost be eliminated according to Table 6. However, the project aims to provide water to Witwatersrand and Johannesburg, our project will inevitably influnce those two regions as well as places where water transfer tunnels pass through. So we have to expand original tunnels' capacity in Lesotho twice as the original. Namely, elevate the orginal capacity from $70\ m^3/s$ to $140m^3/s$.

6.1 Prediction with Intervention

6.1.1 Assumption

The rainfall of Lesotho is over $700\ mm/year$, thus we can safely predict that the water resource of the transportation is always enough to meet the demand of $140m^3/s$.

6.1.2 Assumption

At this moment, we can predict the new water avalibilty by implementing our intervention strategy.

First, we consider the capacity of the two new water reservoirs in Free State. Considering storage abilities, future demand and maxium water supply ability of Lesotho, the capacity of new reservoir should at least be $0.15\ billion\ m^3$. If we assume that the project is of high priority in South Africa infrastructure construction, all projects can be finished by 2020.

If the total capacity of the two new tunnels are $70\ m^3/s$, then they can provide up to $0.221\ billion\ m^3/year$ water. Since reservoirs are large enough, we can use the $average-integration$ method to make the new prediction based on the previous Δ-prediction:

$$\begin{cases} \Delta_1(t) & , \Delta_1(t)\ is\ the\ linear\ interpolation\ of\ \Delta(t)\ (Table\ 6) \\ V(t) & = 0.221, \quad T \in [2020, 2030] \end{cases} \tag{6.1}$$

$$\delta = \frac{1}{|T|} \int_T \left(V(t) + \Delta_1\right) dt \tag{6.2}$$

The result is $64.3\ km^3/year$. Comparing to the former average gap of $280\ km^3/year$, our intervention strategy is adequate. The small gap can be easily filled by other methods, such as wastewater recycling.

6.1.3 Positive Effects

1. **Sustainable Investment**
   Once the whole project is finished by 2020, people in Free State can get up to 0.221 $billion\ m^3$ water supply per year, which means the major water scarcity problem will be solved by the two tunnels.

2. **Environmental Aspect**
   The transportation of water is from streams of Senqu River to Vaal River and Calender River. All of them are tributaries of Orange river. In other word, the transportation is within the ecosystem. The project also makes the system more balanced.

3. **Economic Aspect**
   Both Lasotho and South Africa will benifit a lot from the tunnels and hydropower stations. Both algriculture and industry of Lasotho and Free State will enjoy the aboudant water and electricity.

6.1.4 Negative Effects

1. **Years of Construction**
   The whole project won't be finished by 2020, which means that in the following 5 years, there will still be serious water scarcity in Free State.

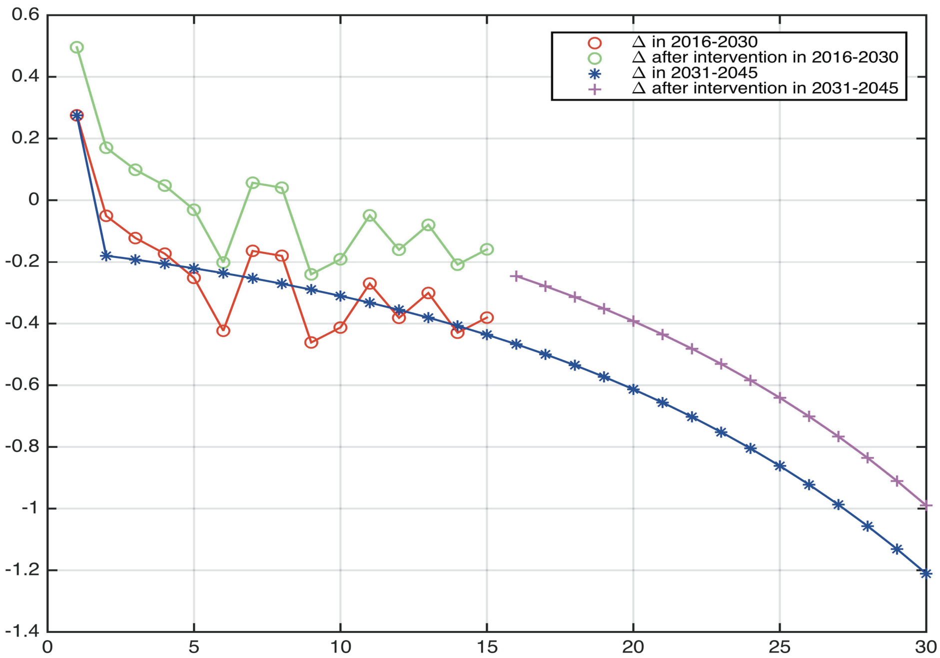
2. **Environmental Aspect**
Reservoirs will change the basic hydrological situation (raise the watet level, slow down speed of water flow, lead to the lack of ecological water in the lower reaches of the river and reduce water temperature), hence making aquatic life endangered. More than $2,000$ people (1% of population in Lasotho) will have to move and Lasotho will loss more than 20% of its framland and ranch.

3. **Economic Aspect**
Building dams may inundate fertile cultivated land, causeing deforestation, inaccessibility and transport hurdles. People in the lower reache will be under the stress of poverty.

7 Looking Further

Considering the supply is limited but demand is always on the increase, we use the results of Δ prediction in 2016-2030 and GM(1,1) to forecast Δ from 2031 to 2045,then use water-transfering intervention strategy to project water availability.

Figure 10: A Longer Prediction with Intervention



We set $\bar{\Delta}(t) = -0.28,\ t \in [2016, 2030]$ as a baseline. If the quantity of new Δ prediction is more than $0.28\ billion\ m^3$ after intervention, we can say that people in Free State will suffer from severe water scarcity again. According to results, in 2033, the water scarcity as heavy as the one in 2020s occurs again. The authority should develop new methods to prevent the scarcity from happening before that.

8 Strengths and Weaknesses

Like any model, our analysis and results present above have strengths and weaknesses. Some of the major points are presented below.

8.1 Strengths

1. **Comprehensive Assessment Model**
   Our assessment model applies the modified Cobb-Dogulas production function, which can evaluate factors comprehensively contributing to supply and demand, and reflect thoroughly on the impact by related factors.

2. **Reliable Coeffiecients Calculation**
   Based on aboundant data (except climate), by fitting the Cobb-Dogulas production function by the hill-climbing algorithm and universal global optimization, we successfully quantify coefficients of various factors and get satisfying results.

3. **Strong Prediction**
   Our forecasting models are based on different prediction methods based on the properties of the variables supported by quantitative analysis, the results are highly consistent. So the prediction results are objective and valid.

4. **Eco-balanced and Sustainable Intervention**
   The water transportation project is based on the LHWP project, so it is both a cost-saving and time-saving project. Besides, it is highly sustainable and eco-balanced.

8.2 Weakness

1. **Simplified Function**
   In the assessment model, we define several restrictions to simplify the function. ThereforeïijŇrelevent relationship of predictor-predictor and predictor-response are ignored.

2. **Numerical Errors**
   Although all of our results are stable and meet the real situation, there are still minor numerical errors caused by the computer.

3. **Unpredictable Factor**
   In forecasting model, we are restricted by limited data in climate. And climate forecasting is still a challange.

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Appendices

Appendix A First Appendix

Below is the our simulated data for cluster analysis.

Table 7: simulated data

region	delta	region	delta
A	-0.275	N	-0.2703
B	-0.0509	O	-0.3008
C	-0.122	P	-0.1278
D	0.13	Q	0.054
E	-0.4232	R	0.1076
F	-0.252	S	-0.247
G	0.23	T	0.1278
H	-0.1644	U	-0.2703
I	-0.1806	V	0.176
J	-0.3806	W	-0.1176
K	0.248	X	-0.236
L	0.1765	Y	0.165
M	-0.4172	Z	0.1524

Appendix B Second Appendix

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
clc
clear
x0=[108.75  76.25   67.5    42.5    17.5    16.25   2.5 15  15  50  67.5    92.5];
n=length(x0);
a_x0=diff(x0)';
B=[-x0(2:end)',ones(n-1,1)];
disp('The least squares fitting parameters')
u=B\a_x0
disp('Symbolic solution of second order differential equations ')
x=dsolve('D2x+a*Dx=b','x(0)=c1,Dx(0)=c2');
x=subs(x,{'a','b','c1','c2'},{u(1),u(2),x0(1),x0(1)});
yuce=subs(x,'t',0:n-1);
x=vpa(x,6)
disp('The projections for known number stronghold ')
x0_hat=double( [yuce(1),diff( double(yuce) )] )
disp('calculate residual ')
```

```
epsilon=double( x0-x0_hat )
disp('calculate the relative error ')
delta=double( abs(epsilon./x0) )

res=GM11(x0',1);
plot(1:length(x0),x0,'r',1:length(res),res,'g')
```

```
syms w x y z
y= 0.4222 + 0.06314*cos(x*0.5968) -0.08527*sin(x*0.5968) + -0.04488*cos(2*x*0.5968) + 0.
z=0.4267 + 0.07104*cos(x*0.5931) -0.08292*sin(x*0.5931) -0.03208*cos(2*x*0.5931) + 0.061
w= 0.4261 + 0.07522*cos(x*0.5981) -0.08117*sin(x*0.5981) -0.03859*cos(2*x*0.5981) + 0.06

x=[1:14];
x1=x;
y1=[92.5    108.75  76.25   67.5    42.5    17.5    16.25   2.5 15   15   50   67.5    92.5

x=[0:0.1:30];
x2=x;
y2=eval(y);

x3=[16:30];
x=x3;%(x3-15)./1.5+15;
y3=eval(y)

x4=[1:0.1:30];
x=x4;%(x4-15)./1.5+15;
y4=eval(z);

x5=[16:30];
x=x5;%(x3-15)./1.5+15;
y5=eval(z)

x6=[1:0.1:30];
x=x6;%(x4-15)./1.5+15;
y6=eval(w);

x7=[16:30];
x=x7;%(x3-15)./1.5+15;
y7=eval(w)

plot(x1,y1,'ro',x2,y2,'g',x3,y3,'g+',x4,y4,'b',x5,y5,'b*',x6,y6,'m',x7,y7,'m*');
grid on;
```

```
function GM1N(datalist)
n=size(datalist,1);
for i=1:n
    datalist(i,:)=datalist(i,:)/datalist(i,1);
end
data=datalist;
ck=data(1,:);m1=size(ck,1);
bj=data(2:n,:);m2=size(bj,1);
r=[];
t=[];
for i=1:m1
    for j=1:m2
        t(j,:)=bj(j,:)-ck(i,:);
    end
    jc1=min(min(abs(t')));jc2=max(max(abs(t')));
```

```
    rho=0.5;
    ksi=(jc1+rho*jc2)./(abs(t)+rho*jc2);
    rt=sum(ksi')/size(ksi,2);
    r(i,:)=rt;
end
r
[rs,rind]=sort(r,'descend')
```

```
function res=GM11(inputDataList,nexti) %Input n rows and n column, nexti is the number o
n=length(inputDataList);
lamda=inputDataList(1:n-1)./inputDataList(2:n);
lamda=lamda';
range=minmax(lamda) %Actual stepwise ratio of minimum and maximun
X=[exp(-2/(n+1)),exp(2/(n+2))] %The range of stepwise ratio, rangeąÊX
t1=cumsum(inputDataList);
n=length(t1);
B=[-0.5*(t1(1:end-1)+t1(2:end)),ones(n-1,1)];
Y=inputDataList(2:end);
r=B\Y
y=dsolve('Dy+a*y=b','y(0)=y0');
y=subs(y,{'a','b','y0'},{r(1),r(2),t1(1)});
yuce1=subs(y,'t',[0:n+nexti]);
res(1)=inputDataList(1);
for i=2:n+nexti
    res(i)=yuce1(i)-yuce1(i-1);
end
digits(6)
res=vpa(res); %To improve the prediction accuracy, calculate procedure firstly and then
epsilon=inputDataList'-res(1:n)  %Calculate residual
delta=abs(epsilon./(inputDataList')) %Calculate relative error
rho=1-(1-0.5*r(1))/(1+0.5*r(1))*lamda % Calculate the deviation value of stepwise ratio.
```



```
function res=GM11(inputDataList,nexti) %Input n rows and n column, nexti is the number o
n=length(inputDataList);
lamda=inputDataList(1:n-1)./inputDataList(2:n);
lamda=lamda';
range=minmax(lamda) %Actual stepwise ratio of minimum and maximun
X=[exp(-2/(n+1)),exp(2/(n+2))] %The range of stepwise ratio, rangeąÊX
t1=cumsum(inputDataList);
n=length(t1);
B=[-0.5*(t1(1:end-1)+t1(2:end)),ones(n-1,1)];
Y=inputDataList(2:end);
r=B\Y
y=dsolve('Dy+a*y=b','y(0)=y0');
y=subs(y,{'a','b','y0'},{r(1),r(2),t1(1)});
yuce1=subs(y,'t',[0:n+nexti]);
res(1)=inputDataList(1);
for i=2:n+nexti
    res(i)=yuce1(i)-yuce1(i-1);
end
digits(6)
res=vpa(res); %To improve the prediction accuracy, calculate procedure firstly and then
epsilon=inputDataList'-res(1:n)  %Calculate residual
delta=abs(epsilon./(inputDataList')) %Calculate relative error
rho=1-(1-0.5*r(1))/(1+0.5*r(1))*lamda % Calculate the deviation value of stepwise ratio.
```

```
function res=GMVerhulst(inputDataList,nexti)
x1=inputDataList;
```

```
n=length(x1);
year=0:n-1;
x0=diff(x1);
x0=[x1(1),x0];
for i=2:n
    z1(i)=0.5*(x1(i)+x1(i-1));
end
B=[-z1(2:end)',z1(2:end)'.^2];
Y=x0(2:end)';
abvalue=B\Y;
x=dsolve('Dx+a*x=b*x^2','x(0)=x0');
x=subs(x,{'a','b','x0'},{abvalue(1),abvalue(2),x1(1)});
res=subs(x,'t',0:n-1+nexti);
plot(year,x1,'b+',0:n-1+nexti,res,'g-');
axis tight;
```

```
t=2002:1:2014;
F=[12.1 11.1 11.9 13.8 17 14.84 18.29 35.68 51.43 67.5 78.89 75.33 83.7 ];
M=[11.25 11.1 12.84 13.8 13.75 15.55 20.83 27.16 49.66 70.25 75.54 71.33 72.53];
plot(t,M,'r-',t,F,'b-')
```

```
function res=learnAndSolve(demoinput,demooutput,testinput)
inputnum=size(demoinput,1);
outputnum=size(demooutput,1);
sprintf('input%d output%d',inputnum,outputnum)
pr=minmax(demoinput);
%net=newff(pr,[inputnum+1,(inputnum+1)*outputnum+1,outputnum],{'logsig','logsig','logsig
net=newff(pr,[inputnum+1,10,outputnum],{'logsig','logsig','logsig'});
net.trainParam.show = 10;
net.trainParam.lr = 0.05;
net.trainParam.goal = 1e-10;
net.trainParam.epochs = 50000;
net = train(net,demoinput,demooutput);
res=sim(net,testinput);
```

```
testdata=[
27.28955781
23.54802106
29.50721321
24.42278192
28.4623027
27.04410278
29.06055387
28.82031753
28.07752866
27.52133743
25.26694912
26.18331997
27.76264024
26.36490958
26.43086103
]';
%testdata=[0.025 0.0123 0.029 0.044 0.084 0.164 0.332 0.521 0.97 1.6 2.45 3.11 3.57 3.76
res=GMVerhulst(testdata,2)
```

```
testin=[
2217.744083
```

```
2202.402754
2244.547733
2540.485988
2556.262797
2906.576372
3220.058583
3326.047713
3529.310993
3968.787874
4139.822867
4709.516783
7915.016314
10920.59141
10568.04698
]
res=GM11(testin,15)
plot(1:length(testin),testin,'ro',1:length(res),res,'b*',1:length(res),res,'b')
grid on
```

```
testin=[
2482987.628
2534325.541
2570250.253
2603019.256
2636915.593
2671977.579
2708245.338
2745761.418
2784570.628
2824720.143
2866259.792
2909242.057
2953722.298
2999758.863
3047413.208

]
res=GM11(testin,15)
plot(1:length(testin),testin,'ro',1:length(res),res,'b*',1:length(res),res,'b')
grid on
```

```
testdata=[
906.25  650 586.25  387.75  190 187.5   150 188.75  207.5   473.5   667.5   753.75;
    87.5    70  70  50  30  30  30  40  37.5    63.75   70  72.5;
108.75  76.25   67.5    42.5    17.5    16.25   2.5 15  15  50  67.5    92.5;
142.5   90  75  35  15  10  1.25    6.25    15  62.5    83.75   110;
135 96.25   90  55  16.25   22.5    22.5    27.5    55  83.5    117.5   110;
112.5   82.5    75  41.25   7.5 7.5 5   2.5 11.25   42.5    92.5    107.5;
153.75  95  81.25   47.5    10  7.5 6.25    7.5 20  70  117.5   123.75;
32.5    37.5    30  30  17.5    15  10  10  7.5 16.25   15  26.25;
110 77.5    68.75   42.5    13.75   10  0   5   10  40  60  81.25;
23.75   25  28.75   44  62.5    68.75   72.5    75  36.25   45  43.75   30;
]
GM1N(testdata);
```

Appendix C Third appendix

sas codes
proc cluster data=work.sample method=ave std pseudo ccc outtree=o;
var delta;
id region;
proc tree data=o1 horizontal graphics;
title 'Average Method';
run;

1stOp codes Variable supply,poll,RDïijŇrainfall;
Parameters a,b,c,d;
Function $supply = a * poll^b * RD^c * rainfall^d$;
Data;
0.425874684 2217.744083 20.2896 92.5
0.391487288 2202.402754 23.3696 108.75
0.460262081 2244.547733 23.7472 76.25
0.322712494 2540.485988 24.1308 67.5
0.285679912 2556.262797 24.5207 42.5
0.179872538 2906.576372 24.9169 17.5
0.740651625 3220.058583 25.3195 76.4
0.687747938 3326.047713 25.7286 74.32
0.105807375 3529.310993 26.1443 15
0.63484425 3968.787874 26.5667 26.4
0.63484425 4139.822867 26.9959 50
0.21161475 4709.516783 27.4321 35
0.285679912 7915.016314 27.8753 92.5
0.391482788 10920.59141 28.3257 80
0.33858135 10568.04698 28.7834 86.25
Variable supply,poll,RDïijŇrainfall;
Parameters a,b,c,d;
Function $supply = a * poll^b * RD^c * rainfall^d$;
Data;
0.425874684 2217.744083 20.2896 92.5
0.391487288 2202.402754 23.3696 108.75
0.460262081 2244.547733 23.7472 76.25
0.322712494 2540.485988 24.1308 67.5
0.285679912 2556.262797 24.5207 42.5
0.179872538 2906.576372 24.9169 17.5
0.740651625 3220.058583 25.3195 76.4
0.687747938 3326.047713 25.7286 74.32
0.105807375 3529.310993 26.1443 15
0.63484425 3968.787874 26.5667 26.4
0.63484425 4139.822867 26.9959 50
0.21161475 4709.516783 27.4321 35
0.285679912 7915.016314 27.8753 92.5
0.391482788 10920.59141 28.3257 80

0.33858135 10568.04698 28.7834 86.25