LATIN SQUARE DESIGN (LSD)

A relevant question on the problem on tyre brands is whether there is a possible position effect. Experience shows that rear tyres get worn out faster than front tyres and even different sides of the same car show different amounts of tread wear. In the RBD, the 4 brands were randomized to the 4 wheels of each car with no regard for position.
The effect of position on tread wear can be balanced out by rotating the tyres every 6666.7 km giving each brand 6666.7 km on each wheel. However the easiest randomization is to have the positions imposing another restriction on the randomization in such a way that each brand is not only used once on each car but only once on each of the 4 possible positions.
A design in which each treatment appears once and only once in each row and once and only once in each column is called a Latin Square Design.

Our interest is still centered on one factor which are our treatments (tyre types) but 2 restrictions are placed on the randomization.

			Car		
Positions		**I**	**II**	**III**	**IV**
LF	1	C	D	A	B
RF	2	B	C	D	A
LR	3	A	B	C	D
RR	4	D	A	B	C

Note that in the above table, each row and each column of the 4x4 square represents a complete replication of the 4 treatment levels. The model is now

$$y_{ijk} = \mu + \alpha_i + \beta_j + \theta_k + \epsilon_{ijk}$$ where θ_k represents the position effect.

Such a design is only possible when the number of levels of both restrictions equals the number of treatments, that is, r = c = treatment levels (t).In other words, it must be a **square.**

Standard Square (definition)

A Latin square is said to be a standard square if the first row and the first column are arranged alphabetically or numerically.

The figures below show examples of standard squares.

A	B
B	A

2x2

A	B	C
B	C	A
C	A	B

3x3

A	B	C	D
B	A	D	C
C	D	B	A
D	C	A	B

4x4

Advantages

1. The major advantage of LSD is that it has greater power compared to CRD and RBD since this design permits the investigator to isolate the variation attributable to three (3) variables; the rows, columns and treatments.

   $y_{ijk} = \mu + \alpha_i + \beta_j + \theta_k + \epsilon_{ijk}$

   θ_k – Position effect

2. Simplicity in data analysis especially in computations.

Disadvantages

1. The number of treatment levels, rows and columns must be the same. As a result of this condition, Latin squares larger than 8x8 are rarely used. Randomisation becomes more complicated.

   The randomisation is therefore relatively more complex compared to CRD and RBD.

2. Squares smaller than 3x3 are not practical because of the small number of degrees of freedom for the experimental error.

Analysis

$$\text{SST} = \sum\sum\sum y_{ijk}^2 - \frac{y_{ooo}^2}{N}$$

$$\text{SSR} = \sum \frac{y_{ioo}^2}{n} - \frac{y_{ooo}^2}{N}$$

$$\text{SSC} = \sum \frac{y_{ojo}^2}{n} - \frac{y_{ooo}^2}{N}$$

$$\text{TSS} = \sum \frac{y_{ook}^2}{n} - \frac{y_{ooo}^2}{N}$$

SSE = SST – SSR – SSC – TSS

Sources	d – f
Rows	n – 1
Columns	n – 1
Positions	n – 1
Error	(n – 1) (n – 2)
Total	**N – 1 or $n^2 – 1$**

The analysis of the data in a Latin Square Design is a simple extension of the previous analysis for the RBD except that now the data is added in a 3rd direction, that is, for the positions.

If data in table 3 was superimposed on the Latin square table 8, the data would look as shown in Table 9 below:

LSD for the tyre-type example

Car

Position	**I**	**II**	**III**	**IV**	y_{ioo}
1	C(12)	D(11)	A(13)	B(8)	44
2	B(14)	C(12)	D(11)	A(13)	50
3	A(17)	B(14)	C(10)	D(9)	50
4	D(13)	A(14)	B(13)	C(9)	49
y_{ojo}	**56**	**51**	**47**	**39**	y_{oo} **= 193**

SSC (car types) = $\frac{56^2+51^2+47^2+39^2}{4} - \frac{193^2}{16}$ = 38.69

SSR (positions) = $\sum \frac{y_{ioo}^2}{n} - \frac{y_{ooo}^2}{N}$ = $\frac{[44^2+50^2+50^2+49^2]}{4} -$ C.F = 6.19

TSS (tyre types) = $\sum \frac{y_{ook}^2}{n} - \text{C.F} = \frac{[57^2+49^2+43^2+44^2]}{4} -$ C.F = 30.69

SST = $\sum_i^r \sum_j^c \sum_k^t y_{ijk}^2 - \text{C.F}$ = 80.94

SSE = 80.94 – 38.69 – 6.19 – 30.69 = 5.37

ANOVA table

Source of variation	**df**	**SS**	**MS**	**F-ratio**
Tyre type	3	38.69	10.2	F_{1c} = 11.33
Car type	3	30.69	12.9	F_{2c} = 14.33
Position	3	6.19	2.1	F_{3c} = 2.33
Error	6	5.37	0.9	-
Total	**15**	**80.94**	**-**	**-**

Note that another restriction placed on the randomisation has further reduced the experimental error although the position effect is not significant at the 5% level of significance.

$F_{T\ 0.05(3,6)}$ = 4.76

But the further reduction in the error is attained at the expense of d.f since now the estimate of variation is σ^2 = MSE is based on only 6 degrees of freedom instead of 9 in the RBD.

This means less precision in estimating this variance. After discovering that position had no significant effect on tread loss, the investigator may pool the position sum of squares with the error sums of squares and obtain a more precise estimate of the common variance σ^2 namely 1.3 as in the RBD.