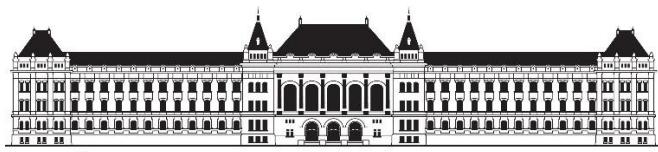


ROHRSETZER RÓBERT

THESIS



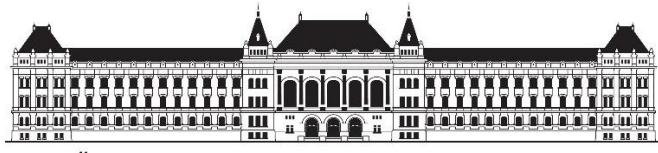
M Ś E G Y E T E M 1 7 8 2

BUDAPEST MŰSZAKI ÉS GAZDASÁGTUDOMÁNYI EGYETEM

GÉPÉSZMÉRNÖKI KAR

MŰSZAKI MECHANIKAI TANSZÉK





BUDAPEST MŰSZAKI ÉS GAZDASÁGTUDOMÁNYI EGYETEM
GÉPÉSZMÉRNÖKI KAR
MŰSZAKI MECHANIKAI TANSZÉK

Rohrsetzer Róbert

Thesis

Examining various electric vehicle powertrain configurations and comparing their efficiency in different driving scenarios.

Advisor:

Csörgő Dániel

AVL Mentor

Supervisor:

Dr. Miklós Ákos

assistant professor

Budapest, 2023.

Copyright Jog © Rohrsetzer Róbert, 2023.



Budapesti Műszaki és Gazdaságtudományi Egyetem

Gépészsmérnöki Kar

Műszaki Mechanikai Tanszék

<https://www.mm.bme.hu>

SZAKDOLGOZAT-FELADAT

ZÁRT KEZELÉSŰ

| | | | |
|------------|---|---|--|
| AZONOSÍTÁS | Név: Rohrsetzer Róbert Richard | | Azonosító: 71654043279 |
| | Képzéskód: 2N-AM0 | Specializáció kódja: 2N-AM0-GM-2017 | Feladatkiírás azonosítója: GEMM:2024-1:2N-AM0:VN48J4 |
| | Szak: Mechatronikai mérnöki alapszak (BSc) | | |
| | Szakdolgozatot kiadó tanszék: Műszaki Mechanikai Tanszék | Záróvizsgát szervező tanszék: Műszaki Mechanikai Tanszék | |
| | Témavezető: Dr. Miklós Ákos (71404907625), adjunktus | | |

| | | |
|---------|---------------------|--|
| FELADAT | Cím | Jármű hajtásláncok hatásfokának összehasonlítása különböző vezetési szituációkban Examining various electric vehicle powertrain configurations and comparing their efficiency in different driving scenarios |
| | Részletes feladatok | Categorize the different types of electric motors and gathering their pros and cons in relation to different use-case scenarios. Analyze currently utilized E-motor configurations in the market. Analyze the efficiency tests based on the WLTP (Worldwide Harmonized Light Vehicles Test Procedure) standard. Compare the efficiency of a single E-motor drive concept and a combined E-motor drive concept. Test and analyze the different E-motor configurations in the MATLAB Simulink environment. Summarize your results in English and Hungarian. |
| | Heyt | A szakdolgozat készítés helye: AVL Hungary Kft. 2030 Érd, Mühlgang utca 5. Konzulens: Csörgő Dániel, Software and function engineer (daniel.csoergoe@avl.com) |

| ZÁRÓVIZSGA | 1. záróvizsga tantárgy(csoport) | 2. záróvizsga tantárgy(csoport) | 3. záróvizsga tantárgy(csoport) |
|------------|---------------------------------|--|--|
| | ZVEGEMIBMIE Irányításelmélet | ZVEGEMMBMRD Robotmechanizmusok dinamikája | ZVEGEÁTBM04 Áramlások numerikus modellezése |

| | | | |
|-------------|---|--|--|
| HITELESÍTÉS | Feladat kiadása: 2023. szeptember 4. | | Beadási határidő: 2023. december 8. |
| | Összeállította: Dr. Miklós Ákos (71404907625) témavezető | Ellenőrizte: Dr. Inspéger Tamás s.k. tanszékvezető | Jóváhagyta: Dr. Györke Gábor s.k. dékánhelyettes |
| | Alulírott, a feladatkiírás átvételével egyúttal kijelentem, hogy a Szakdolgozat-készítés c. tantárgy előkötetelményeit maradéktalanul teljesítettem. Tudomásul veszem, hogy jogosulatlan tantárgyfelvétel esetén a jelen feladatkiírás hatálytalan. | | |
| | Rohrsetzer Róbert Richard | | |

Declaration of self-employment

The undersigned, Rohrsetzer Róbert Richard (VN48J4), a student at the Budapest University of Technology and Economics, aware of my criminal and disciplinary responsibility, I declare and certify with my own handwritten signature that I prepared this thesis by myself without authorized help, and in my thesis, I used only the sources provided. All parts that were taken verbatim or in the same meaning, but paraphrased from another source, were clearly marked with the source, in accordance with the regulations in force.

Budapest, December 10, 2023

Rohrsetzer Róbert

Table of Contents

| | | |
|-------|---|----|
| 1. | Electric Motor types and trends | 8 |
| 1.1 | Introduction..... | 8 |
| 1.2 | Categorization and fundamentals..... | 9 |
| 1.3 | Pros, Cons and use case scenarios | 11 |
| 1.4 | Analyzing E-motor configurations | 12 |
| 2. | Electric motor basics | 14 |
| 2.1 | Magnetic Flux and Magnetic Force | 14 |
| 2.2 | Air Gap and its influence | 15 |
| 2.3 | Backwards E.M.F..... | 18 |
| 2.4 | Equivalent circuit..... | 19 |
| 2.5 | Behavior under constant voltage..... | 20 |
| 2.5.1 | Behavior without mechanical load | 20 |
| 2.5.2 | Behavior with mechanical load | 20 |
| 2.6 | Efficiency variation..... | 21 |
| 2.7 | Conclusion | 22 |
| 3. | Electric Motors working principle | 24 |
| 3.1 | DC Motor | 24 |
| 3.2 | Induction Motor | 25 |
| 3.3 | AC Permanent Magnet Synchronous Motor (AC PMSM) | 28 |
| 4. | Simulink Implementation..... | 30 |
| 4.1 | The model | 30 |
| 4.2 | Simulink model of PMSM | 31 |
| 4.3 | Results of the PMSM simulation | 33 |
| 4.4 | Simulink model of car with 2 PMSM | 34 |
| 4.5 | Results of the 2 PMSM simulation concept..... | 36 |
| 4.6 | Conclusion | 36 |
| 4.7 | Conclusion in Hungarian | 37 |
| 4.8 | Ideas for alternative E-motor configurations | 38 |
| | Citations..... | 39 |

Preface

The goal of the thesis is to look at the effects of the different electric motor configurations by comparing their efficiency. We do this by simulating a car in MATLAB Simulink, the speed of which is controlled according to a WLTP cycle. It's enough to use a very simple car model since the focus is on the electric motor. We will look at the history and basics of electric motors, categorize them and look at their pros and cons in different use case scenarios. By the end of the thesis the reader will have a basic understanding of electric motors principles and how the efficiency of them is influenced by their configuration.

~~~

## Acknowledgement

I would like to thank my parents for all their support throughout all the semesters and that they created an environment for me where I could only focus on my studies and not have to worry about anything else. I also want to say thank you to Kántor Bence, Csörgő Dániel and my other colleges at AVL for helping me with my questions throughout the thesis.

Budapest, December 10. 2023.

Rohrsetzer Róbert

# Chapter 1

## 1. Electric Motor types and trends

### 1.1 Introduction

An electric motor is a machine which converts electrical energy into mechanical energy. This can be achieved with electromagnetic induction. The interaction between the rotors and the stators magnetic field creates a force which then spins the rotor, thus creating the mechanical energy.

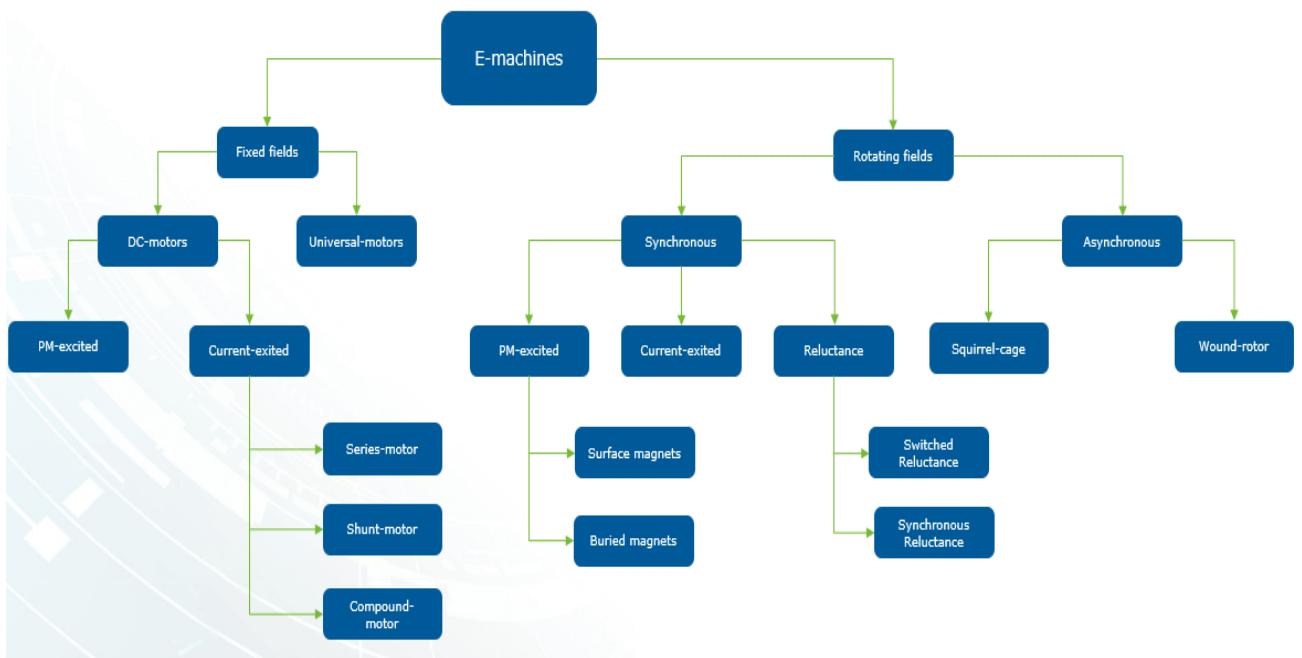
The first discovery after electricity which helped in the creation of electric motors is the discovery of electromagnetism from Hans Oersted, in 1820. The first DC motor was invented by Michael Faraday in 1821, but this wasn't capable of giving any application value. He also discovered electromagnetic induction in 1831 which was crucial in creating the first practical E-motor. After his discovery many scientists joined the research of electrical motors such as William Sturgeon and Thomas Davenport. The first AC motor was created by Nikola Tesla in 1887. The most used motor today, the induction motor with three phases, was invented by Mikhail Dolivo-Dobrovolsky during 1889 and 1890.

(Tong, 2014)

The awareness of global warming and the realization that the fossil fuels are a big part of the problem, more and more people want to be a part of the solution and are considering buying an electric vehicle. The rapid demand in the EVs motivated manufacturers to use more resources on the development to produce better and more efficient EVs. The main objective of this thesis is to introduce the different Electric motors used in the industry and compare the efficiency of the dual and single drive concept. The most widely used electric motors are the AC permanent magnet synchronous motor, the induction motor, and the Brushless DC motor. The comparison will be carried out in Simulink using a simplified EV model and with the speed determined by a WLTP cycle.

## 1.2 Categorization and fundamentals

Today there are lots of electric motor types, the most used ones are the brushless DC motors (BLDC), the induction motors (IM) and the permanent magnet synchronous motors (PMSM). There are also other ones like the switched reluctance motors (SRM) and the less and less used brushed DC motor.



*Electric machines Categorization: 1. Figure*

(AVL Basic\_training, Kántor Bence)

Categorizing the electric motors by operating principle we can talk about fixed fields and rotating fields. By fixed field the magnetic field of the stator is fixed. Motors working with this principle are permanent magnet (PM) excited DC motors or current excited DC motors, the main difference lies in how the magnetic field at the stator is created.

If we talk about PM excited DC motors, then the stator's magnetic field is achieved with a permanent magnet and the rotor has windings in which the current flows. In case of current excited DC motor there are wires wrapped around the stator (and in the rotor) and the current flowing through these wires create the magnetic field of the stator (and the rotor). In both cases the rotation is achieved with a commutator which switches the polarities of the wires of the rotor thus behaving as a permanent magnet of which the poles are swapped at every polarity switch.

In case of rotating field electric motors, the magnetic field of the stator also rotates. If it rotates with the same speed as the rotor, then we talk about synchronous motors and if there is a difference in the rotation speed (slip) then we talk about asynchronous motors.

Let's look at the synchronous motors first. Here we can distinguish three different types, PM excited, current excited and reluctance motors. Current excited synchronous motors are very similar to current excited DC motors, the only difference is that the wires on the stator get an AC current instead of a DC current thus creating the rotating magnetic field.

PM excited synchronous motors have a rotating magnetic field which is achieved by putting an AC signal on the windings of the stator, the rotor contains permanent magnets move with the rotating magnetic field thus creating the rotation of the rotor. In case the magnets of the rotor are surface mounted we talk about Surface Mounted Permanent Magnet Excited Synchronous Motor (SMPMSM) and if the magnets are embedded within the rotor, we talk about Buried Permanent Magnet Excited Synchronous Motor (BPMSM or IPM).

The reluctance motors use the magnetic reluctance (magnetic resistance) effect. In this case there are no windings or magnets within the rotor made of ferromagnetic material, so we use the reluctance effect to align the rotor with the stators rotating magnetic field. As we energize the stator windings the poles of the rotor are pulled towards the stator poles due to magnetic attraction, once they are aligned the current is switched off at that position and turned on at the next winding causing the rotor to move to the next position. The frequent changing of the energized winding creates the rotation of the rotor.

The other rotating field electric motors are the asynchronous motors. We distinguish two different types of rotors, squirrel-cage rotor and wound-rotor thus creating the two different types of asynchronous motors. The squirrel-cage rotor got its name from its look, in the rotor core there are conducting bars which ends are short circuited creating the resemblance of a squirrel cage. By energizing the stator windings there is current induced in the cage which has its own magnetic field. This magnetic field tries to align itself with the field of the stator, but it can't. The speed difference between the stator and rotor is called slip and is the reason why it doesn't require additional starting devices.

The most popular electric motors in the automobile industry are the induction motor and the permanent magnet synchronous motor.

(Keyes, 2007)

### 1.3 Pros, Cons and use case scenarios

In this chapter we look at the pros and cons as well as the use case scenarios of the Induction machine (IM), Permanent Magnet Synchronous Machine (PMSM), Synchronous Reluctance machine (SynRM), DC machine and the Brushless DC machine (BLDC).

The IM has a power factor of 0.75-0.9 which is good, and it has a high pull-out torque which defines the maximum torque the motor can deliver without an abrupt drop in speed thus allowing fast acceleration. The power factor is the ratio of the useful power and the apparent power (demand). But we need a high startup current for this acceleration and it has also a lower efficiency when used at a constant speed. Another advantage is that the IM is more efficient in field weakening operation, meaning we can operate the motor at higher speeds than its rated speed by weakening the magnetic field in the stator. The efficiency of the IM is lower than the SynRM or the PMSM because it has additional rotor losses. The IM is used in variable speed drive because of the high acceleration capabilities, such as trains, pumps, and general automation.

The PMSM has the highest speed in the base speed operation and the highest torque/weight ratio, where base speed operation refers to the maximum speed at which the motor can operate forever at its rated voltage, current and torque. But because of the magnetic material used in the rotors the PMSM is more expensive than the IM. Because of these properties both the aerospace and the automotive industry favors these motors, but we can also find PMSMs in household items which contain cheaper ferrite magnets. As the manufacturing complexity and the cost of the PMSM is dropping, its popularity is rising.

The SynRM has lower rotor temperatures and higher efficiency in base speed operation than the IM due to the negligible rotor losses, but the SynRM has a very low ratio between the base speed and the maximum speed meaning it has a limited region of constant power. The rotor losses mostly consist of copper losses, which happen due to the resistance of the rotor windings, and eddy current losses, which are circulating currents in the rotor produced by the changing magnetic field of the stator. So, these machines are used in applications with a single operation point, a fan would be a good example for this. Since there are no permanent magnets in the rotor, it is also immune to demagnetization and during maintenance there is no electrical induction resulting in a simpler machine handling. But it has a lower power factor than the PMSM, so it needs a bigger inverter. The use of this machine is not widely spread yet since it is relatively new to the market, but it's a perfect replacement for induction machines in pumps and fans which operate at a single operating point.

The DC machines are the oldest and simplest machines that were used for variable speed drives but nowadays they are replaced with IMs since they are a cheap alternative which do not require maintenance since they don't have slip rings and their efficiency is also higher. Because of the brushes the DC machines are mostly used in an environment with clean air to reduce the risk of brush failure. The advantages of the DC machines were that they can be controlled easily resulting in a minimal need of microcontroller and power electronics. But the various possibilities created by the digitalization, resulted in the replacement of the DC machines. Nowadays the series DC machines are used where a high starting torque is required in household appliances, such as vacuum cleaner and hand-held power tools. The separately excited DC machines are still used in position control and low power servo applications.

In the BLDC motor the commutation is done electronically with a controller, meaning that there is no need for mechanical brushes. It has higher efficiency, longer lifespan, and lower maintenance than the brushed DC machine, but its cost is higher and a controller for the complex circuits is also required. They are mostly used in CPU cooling fans, air-conditioning fans, and CD/DVD players.

|                            | IM | PMSM | SynRM | DC | BLDC |
|----------------------------|----|------|-------|----|------|
| Efficiency base-speed      | +  | ++   | +     | -- | +    |
| Efficiency field-weakening | ++ | +    | o     | o  | --   |
| High-speed capability      | ++ | +    | -     | o  | +    |
| Torque density             | o  | ++   | +     | o  | ++   |
| Power density              | o  | ++   | +     | -  | ++   |
| Control effort             | +  | -    | o     | ++ | +    |
| Maintenance demand         | +  | +    | +     | -  | +    |
| Power factor               | +  | ++   | o     | +  | ++   |
| Cost                       | o  | -    | +     | -  | -    |
| Rotor inertia              | -  | o    | +     | -  | -    |
| Noise                      | ++ | ++   | +     | +  | -    |
| Torque ripple              | ++ | ++   | +     | ++ | +    |

*Electric machines compared: 2. Figure*

(Müller, 2019)

## 1.4 Analyzing E-motor configurations

Let's look at some electric motor configurations in the nowadays used electric vehicles. I looked at the owner's manual of different EVs manufactured by Tesla, Porsche, Hyundai and KIA. All

the cars I looked at have a dual motor drive concept with a different front and rear motor. The Tesla Model S/3 has an IM in the front and a PMSM motor in the back while the Tesla Model S 2012-2020 and Tesla Model X 2015-2020 have an IM in the back and a PMSM in the front. The Tesla Model S plaid/ X plaid, Porsche Taycan Turbo, Hyundai Ioniq 5, and the KIA EV6 GT 2023 have a PMSM both in the front and in the back. These motor configurations seem logical since as we saw on the previous chapter the IM is designed to have a high acceleration while the PMSM has highest speed in the base speed operation and the highest torque/weight ratio. If we look at the electric motor configuration of one of the fastest electric vehicles, the Tesla Roadster (2010 edition), we see that it has only a single IM. So, it is no surprise to us that it can reach 0-100km/h in 3.9 seconds, since the IM is designed for fast accelerations, but it can only reach a top speed of 201 km/h which is around the same what the Tesla Model 3 is capable of. But if we look at the Tesla Model S which consists of two PMSM without an IM we see that it can reach a top speed of 250km/h. Of course, comparing them isn't so simple since there are a lot of other factors that play a crucial role, such as road condition, tires, weight or if it's all wheel drive configuration or not and so on, but we can see that if the goal is fast acceleration, the IM is a perfect choice while PMSM is a better choice if we want to hold a constant speed, since they are a good choice for precise speed control and have a higher efficiency at rated speeds. The combination of the two motors allows a well-balanced drive.



*Considered Tesla models: 3. Figure*

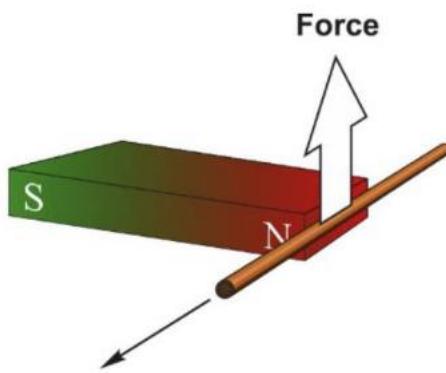
(<http://tesla.com>)

# Chapter 2

## 2. Electric motor basics

### 2.1 Magnetic Flux and Magnetic Force

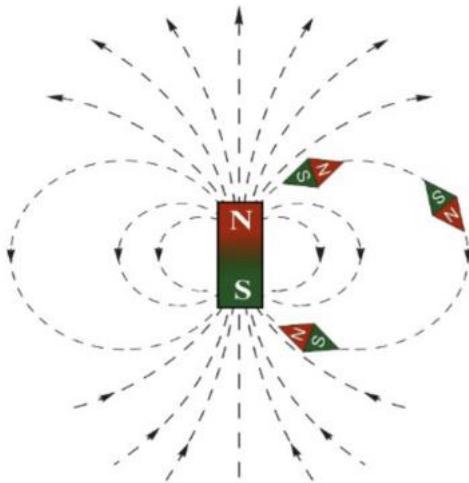
To be able to talk about electric motors we first must understand the basics of magnetic field, magnetic flux ( $\Phi$ ), and the concept of how the rotation of the shaft is achieved. Let's imagine a bar magnet with current flowing through a copper wire perpendicular to it.



*Force acting on current carrying conductor: 4. Figure*

(Austin Hughes, 2019)

The lines in the figure 4. are the magnetic flux lines which represent the magnetic field and points from the north pole the south pole of the magnet. These lines are also present inside the magnet, meaning that they have no clear starting and ending point, making them continuous. These flux lines can also show us the intensity/strength of the magnetic field with the concept of magnetic flux density ( $B$ ). This tells us the density of the Magnetic flux lines in a given area. For example, we can see, that inside the copper wire tube the flux lines are closer to each other (denser) than on the outside of the tube.



*Magnetic flux lines: 5. Figure*

(Austin Hughes, 2019)

This means the magnetic flux density is higher at the top of the magnet, than at the outside of the magnet, therefore the strength of the magnetic field is also higher. If we induce a current in the copper wire, then we experience an upwards force acting on the wire. This force depends on the strength of the magnetic field (flux density) emitted by the magnet and the current flowing in the wire. Using Fleming's left-hand rule, we can see that the direction of the force is perpendicular both to the magnetic flux density and the current flowing in the copper wire. Using the following formula, we can determine the value of the force acting upon the wire.

$$F = B \cdot I \cdot l$$

where  $B$  is the magnetic flux density,  $I$  is the current flowing in the wire and  $l$  is the length of the wire. The equation is simple, but the drawback is that its only applicable when the current is perpendicular to the field. To achieve the biggest force possible, we need a good magnetic circuit and a lot of copper wires densely packed into the magnetic field which carry as much current as possible.

(Austin Hughes, 2019)

## 2.2 Air Gap and its influence

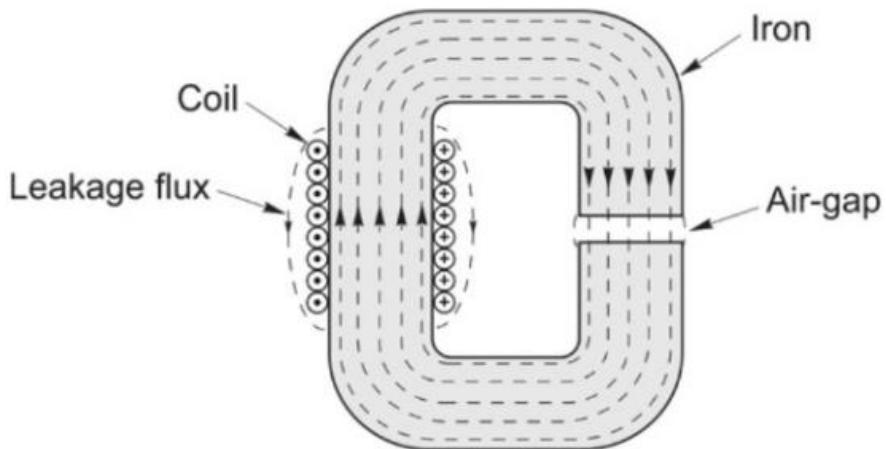
In electric motors the magnetic field is usually produced with current flowing through coils instead of magnets, with this we can change the strength of the magnetic field to our liking. Since we want the biggest force, we want to increase the flux density as much as possible. To achieve this, we either increase the number of turns of the coil or the current flowing in the coil. The flux producing

ability of the coil is called magnetomotive force (m.m.f.) which is a product of the current ( $I$ ) in the wire and the number of turns ( $N$ ) of the coil. Here we can detect an analogy between Ohm's law and the "magnetic Ohm's law".

$$I = \frac{V}{R}$$

$$\Phi = \frac{N \cdot I}{R_m}$$

Where  $R_m$  is the magnetic resistance which tells us how hard it is for the flux to move in the magnetic circuit analog to the electric resistance in an electric circuit. To keep the flux as high as possible we need to decrease the reluctance.



*Flux lines in an iron with air gap: 6. Figure*

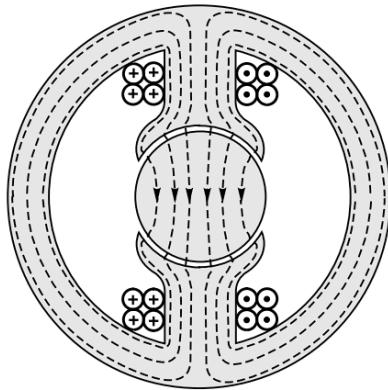
(Austin Hughes, 2019)

This means, instead of air with a high reluctance we want to use iron with low reluctance. Since the flux can move easier through iron than air, most of it stays in the iron. In order to be able to use this high flux density we need to have an air gap, in which we can place the current-carrying conductor. This airgap increases the reluctance, but we need to have it to be able to produce force on the conductor. Since the reluctance of iron is much smaller, we can neglect its reluctance compared to air. To calculate the flux density in the airgap we can use the following equation:

$$B = \frac{\Phi}{A} = \frac{\mu_0 \cdot N \cdot I}{g}$$

where  $g$  the length of the airgap is and  $\mu_0$  is the 'permeability of air'.

Unfortunately, there is a limit how much we can increase the flux density and if we go beyond this limit the reluctance of the iron significantly increases thus making it very inefficient. In practice the airgap is between the rotor and the stator as shown in figure 7.



*Simplified motor configuration: 7. Figure*

(Austin Hughes, 2019)

The conductors will be placed in the rotor. If there is a positive current on the top conductors and a negative current on the bottom conductors, then there will be a force on each side which causes the rotation of the rotor. This can be imagined as if the rotor was a magnet with two poles.

The magnitude of the torque depends on the radius of the rotor and the force acting on the rotor and can be calculated using the following equation:

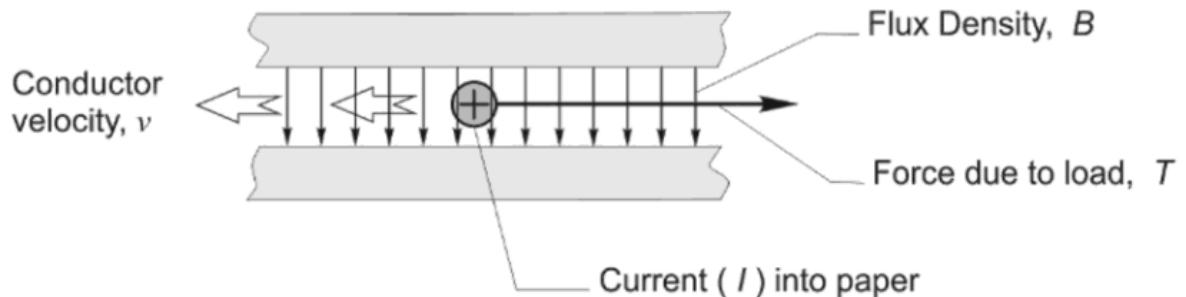
$$T = F \cdot r$$

The conductors are placed in the rotor since if they were mounted on the rotor then the air gap would have to be at least as wide as the diameter of the conductor, thus increasing the reluctance. The sinking of the conductors into the rotor, called “slotting”, also enabled a better transfer of torque between the conductor and the rotor shaft. It’s also important that if the conductors are too wide then the volume between them (also known as teeth) is too small then it can lead to saturation since the flux has to flow through a smaller area. To achieve balance between the radius of the conductor and the volume of the teeth is a task of the designers.

(Austin Hughes, 2019)

## 2.3 Backwards E.M.F

It's also important to talk about the backwards E.M.F. To make the explanation easier, let's picture a primitive linear dc motor.



*Primitive DC machine: 8. Figure*

(Austin Hughes, 2019)

The setup consists of two permanent magnets parallel to each other and a conductor between them on which a Force ( $T$ ) is applied. If the conductor is held stationary than it means that the electromagnetic force must be equal to the force due to the load.

$$T = B \cdot I \cdot l$$

Here we can see that the current depends on the mechanical load, which means the greater the load the greater current must be flowing to maintain the stationary position. If we look at work done, we can see that all the input power is used to produce heat in the conductor.

$$V_1 \cdot I = I^2 \cdot R$$

where  $V_1$  is the voltage applied to the conductor and  $R$  is the resistance of the conductor.

If the conductor were to move with a constant speed ( $v$ ) in opposite directions of the load, then we see a change in the power balance equation.

$$V_2 \cdot I = I^2 \cdot R + (B \cdot I \cdot l) \cdot v$$

Here we can see that the electrical input power supplied is not only used to produce heat in the conductor but now it also must supply the mechanical output power. This means that the voltage  $V_2$  must be bigger than  $V_1$  since the current stays unmodified. If we subtract the two power balance equations, we get the following term:

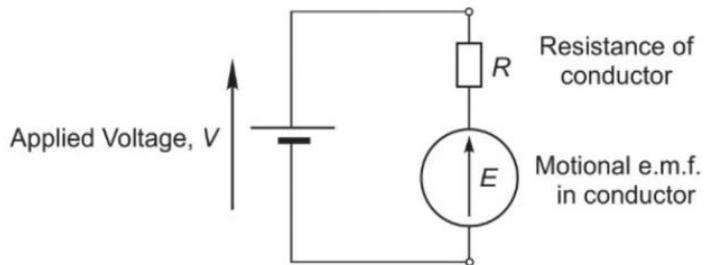
$$V_2 - V_1 = B \cdot l \cdot v = E$$

We can see that whenever the conductor is moving with speed  $v$ , a greater voltage is needed. This increase in voltage shows us that there is an electromotive voltage ( $E$ ) induced in the conductor which the source voltage also must overcome along with  $V_1$ , the source voltage at steady state. This means that the induced voltage always acts against the source voltage which lead to the name “back E.M.F”. Note that if we push the conductor and a greater backwards E.M.F is induced than the supplied voltage then the motor behaves as a generator since current will be negative meaning it flows back into the supply.

(Austin Hughes, 2019)

## 2.4 Equivalent circuit

We can represent our primitive dc motor with the following circuit.



*Equivalent circuit of primitive DC machine: 9. Figure*

(Austin Hughes, 2019)

We obtain the following equation of the current if we apply Kirchoff's law:

$$I = \frac{V - E}{R}$$

Where  $V$  is the applied source voltage,  $E$  is the back E.M.F and  $R$  is the resistance of the conductor. The power equation then looks like this:

$$V \cdot I = E \cdot I + I^2 \cdot R$$

Where  $VI$  the electrical input power,  $EI$  the mechanical output power and  $I^2R$  the copper losses are. Since the goal is to convert as much of the electrical input power, into mechanical output power, as possible, we need  $EI$  to be greater than the copper losses  $I^2R$ . This means we want  $E$  high and  $R$  as low as possible.

(Austin Hughes, 2019)

## 2.5 Behavior under constant voltage

Let's see how our primitive DC motor behaves under constant voltage with and without mechanical load since what we will see is almost the same as if we looked at a real DC motor.

### 2.5.1 Behavior without mechanical load

Let's look at the no mechanical load situation first. If we apply a voltage  $V$  to the stationary conductor in the magnetic field the current will rise to its maximum value  $V/R$ , since there is no back E.M.F reducing it. The current will develop a Force  $F = B \cdot I \cdot l$ , which means there will be an acceleration of the conductor ( $F = m \cdot a$ ). As the speed increases the back E.M.F will rise, meaning the current will be smaller and smaller depending on the speed. This means that the force and the acceleration will also fall, while the speed continues to rise until we reach the steady state where the back E.M.F is equal to the source voltage. In this state there is no current flowing and the speed is constant. Since there is no mechanical load there is also no mechanical power being produced. This situation is very similar to the 'no-load' condition in a motor, with the only difference being that the motor will have some friction therefore drawing a small amount of current. To calculate the steady-state speed of our primitive DC motor we can use the following equations:

$$E = V = B \cdot l \cdot v_0 \text{ and } v_0 = \frac{V}{B \cdot l}$$

we can see that the no-load speed is controllable with the applied voltage. If we reduce the field strength ( $B$ ) then we need a higher speed to achieve the same back E.M.F. This means that if we half the field strength then the steady-state speed will be doubled, but to reach this state we need more time since the growth of the back E.M.F is smaller. This technique is called 'field weakening'.

(Austin Hughes, 2019)

### 2.5.2 Behavior with mechanical load

If we put a load on the conductor in its steady state, then there will be a force in the opposing direction of the motion. The conductor will deaccelerate, but as the speed falls the back E.M.F will also fall, leading to a difference between the back E.M.F and the source voltage. This difference induces a current in the conductor producing a force opposing the load, which rises as the speed falls. As soon as the force becomes equal to the load the equilibrium conditions will be met, the conductor will continue to move at a constant speed smaller than the no-load speed and the

conductor now produces a continuous mechanical output power therefore acting as a motor under load. We can see that as soon we applied a load on the conductor the speed fell just enough for the needed current to flow thus producing the force needed to balance the load. If we were to half the flux, then the speed drop would be 4 times as much, since to develop the same force twice as much current is needed which means the back E.M.F must fall twice as much as by full flux. We can easily see this if we modify the equations as following:

$$\frac{v_{0,halfedB}}{2} = \frac{V}{\frac{B}{2} \cdot l} = 2 \cdot \frac{V}{B \cdot l} = 2 \cdot v_0 \rightarrow \frac{v_{0,halfedB}}{4} = v_0$$

The left-hand side must be divided by two to include that the back E.M.F must fall twice as much, which that the speed is halved.

(Austin Hughes, 2019)

## 2.6 Efficiency variation

Obviously, we want to have machines with the highest efficiency possible. From the already discuss equation:

$$V \cdot I = E \cdot I + I^2 \cdot R$$

we can see that to achieve the highest efficiency possible, we want the mechanical power ( $EI$ ) big and the copper losses ( $I^2R$ ) low. This means that we want as much of the applied voltage to be used for the back E.M.F and not on the wasteful volt drop on the resistance of the wire. Let's look at some examples with specific numbers to see how things change based on the magnitudes of  $E$  and  $V$ . If the conductor with a resistance of  $1\Omega$  can carry a current of  $5A$ , while its moving at a speed which produces a back E.M.F of  $10V$  then the source voltage can be calculated as follows:

$$V = E + I \cdot R = 10 + (5 \cdot 1) = 15 V$$

Let's calculate the efficiency.

$$\begin{aligned} V \cdot I &= E \cdot I + I^2 \cdot R \\ 15 \cdot 5 &= 10 \cdot 5 + 5^2 \cdot 1 \rightarrow 75 = 50 + 25 \\ \eta &= \frac{EI}{VI} = \frac{50}{75} = 0.6666 \end{aligned}$$

We can see that the electrical input power ( $VI$ ) is  $75W$  while the mechanical output power is  $50W$  and the copper loss is  $25W$ , thus giving us an efficiency of around 66%. If we double the supply

voltage from 15V to 30V while the resistance, the steady state current and the load stays the same, then the back E.M.F changes to:

$$E = V - IR = 30 - 5 \cdot 1 = 25V$$

We can see that the back E.M.F more than doubled meaning the speed has also more than doubled.

To get the efficiency we substitute back into the power equation.

$$V \cdot I = E \cdot I + I^2 \cdot R$$

$$30 \cdot 5 = 25 \cdot 5 + 5^2 \cdot 1 \rightarrow 150 = 125 + 25$$

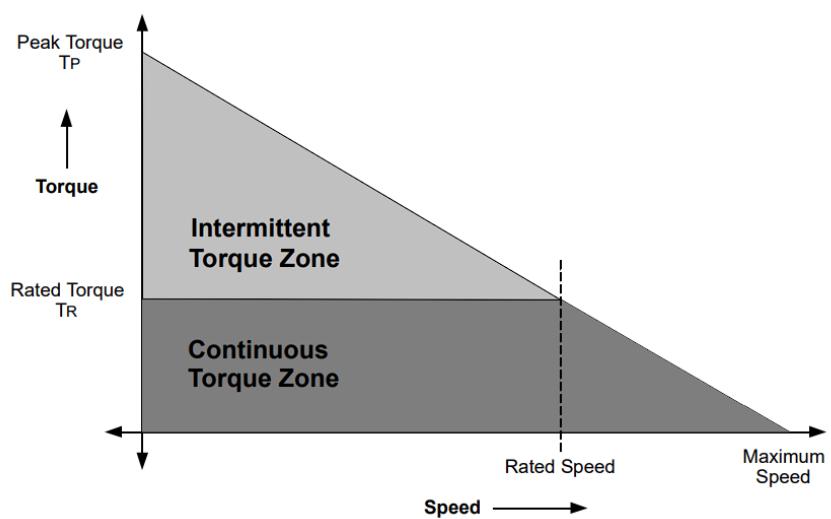
$$\eta = \frac{125}{150} = 0.8333$$

We can see that the efficiency is now around 83%, meaning that the energy conversion process gets better as the speed rises. The same is also true if we operate the machine as a generator.

(Austin Hughes, 2019)

## 2.7 Conclusion

We saw that machines can be used as motors, thus converting electrical power into mechanical power, and as generators, converting mechanical power into electrical power. From examples we could see that the efficiency rises with the speed thus making slow-speed motors not widely used. We also saw that our unloaded primitive motors speed was determined by the applied voltage and that the steady-state current drawn was determined by the mechanical load. Everything that we learned by looking at a primitive DC motor is going to help us understand how the different type of motors are being used in the industry work.



*Torque-Speed characteristics of BLDC: 10. Figure*

(Yedamale, 2003)

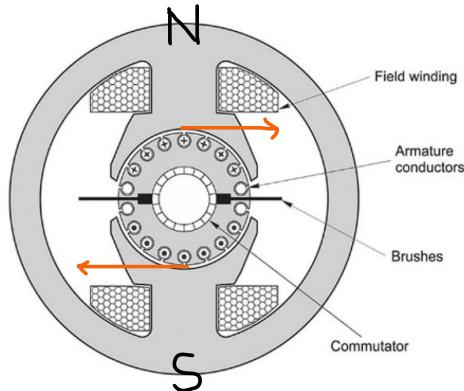
# Chapter 3

## 3. Electric Motors working principle

In this chapter we look at how a few of the most commonly used electric motors work.

### 3.1 DC Motor

Although the basic DC motor has less and less importance in the industry its worth to look at it since it can help us to understand how the rotation of the rotor is achieved. This will help us understand the different type of motors and their fundamentals. In the following figure we can see a conventional DC motor with 2 poles.

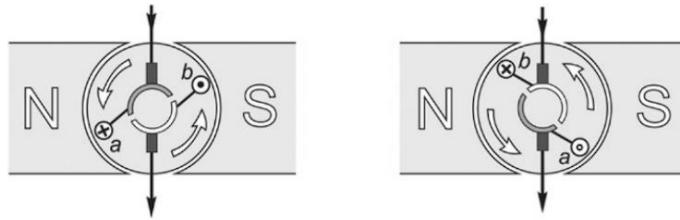


2 pole DC motor with commutator: 11. Figure

(Austin Hughes, 2019)

The field windings are the wires around each pole and are usually in series, the current flowing through these windings create a magnetic field and the poles direct the flux to the airgap. We can see that in the rotor there are also wires embedded on which the force will be induced if the current is flowing. Since the rotor is a rotating part, we need a way to feed current to the conductors embedded in the rotor. The solution for this is the commutator and the carbon brushes which press against it. The commutator is there to ensure that the flow of the current is always as shown in the figure regardless of how the rotor is positioned. The current flows in on one brush, flows through a commutator segment, or segments if the brush is bigger than one segment thus touching more at the same time, designated to a conductor and then flows out on the other brush. In the conductors under the N pole the current is flowing in one direction, creating a force that wants to pull the

conductor to the right and on the conductors by the S pole the current is flowing in the opposite direction thus creating a force pulling the conductors to the left. As the rotor is rotating the conductors move away from the pole and the brush is also not touching the commutator segment designated to that conductor. This means there will always be some conductors on which there is no current flow hence no force is being produced on them. The commutators also help us change the direction of the current flowing in one set of conductors since we can see that throughout a whole rotation the direction of the current flowing in side “a” and “b” is changing.



*Current-reversing function of the commutator: 12. Figure*

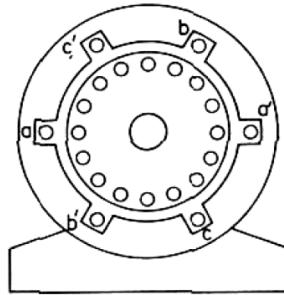
(Hughes and Drury, 2019)

Since we already discussed the back E.M.F on the primitive DC model we won't go into detail here, but it is worth mentioning that back E.M.F is induced will remain constant for half a revolution and then change direction. This is because the direction of the current is also changing. Now that we have a brief understanding of how the rotation of the rotor is created, we can move to the more complicated part and take a look at the electric motors used in the automotive industry, such as AC PMSM and induction motor.

(Hughes and Drury, 2019)

### 3.2 Induction Motor

Here we will look at the fundamentals of the induction motor. Let's briefly look at its construction and working principle first. In the squirrel cage induction motor the stator, which consists of lamination of sheet steel, is responsible for creating a rotating magnetic field.



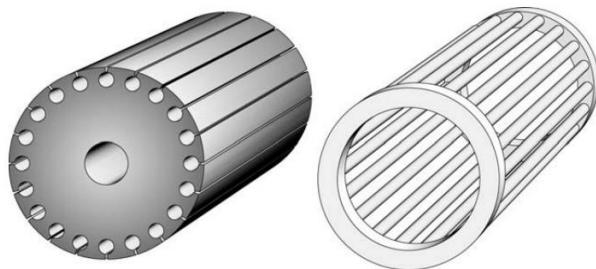
*Simplified construction of IM: 13. Figure*

(Thornton and Armintor, 2003)

The inner surface of the stator is slotted and in these slots are the windings which are supplied with a three-phase voltage to produce the rotating field. The speed of the rotating field depends on the frequency of the source and the number of poles. The rotor is also made of laminated iron and the windings can be squirrel-cage type or wound-rotor type.

We will only look at the squirrel cage type since it's the most used one. The squirrel-cage type rotor looks very similar to a squirrel cage hence the name. This means that the rotor has no permanent magnets, it only consists of conductive bars imbedded in the rotor slots, which are shorted with end rings at both sides, as we can see in the following figure.

(Thornton and Armintor, 2003)

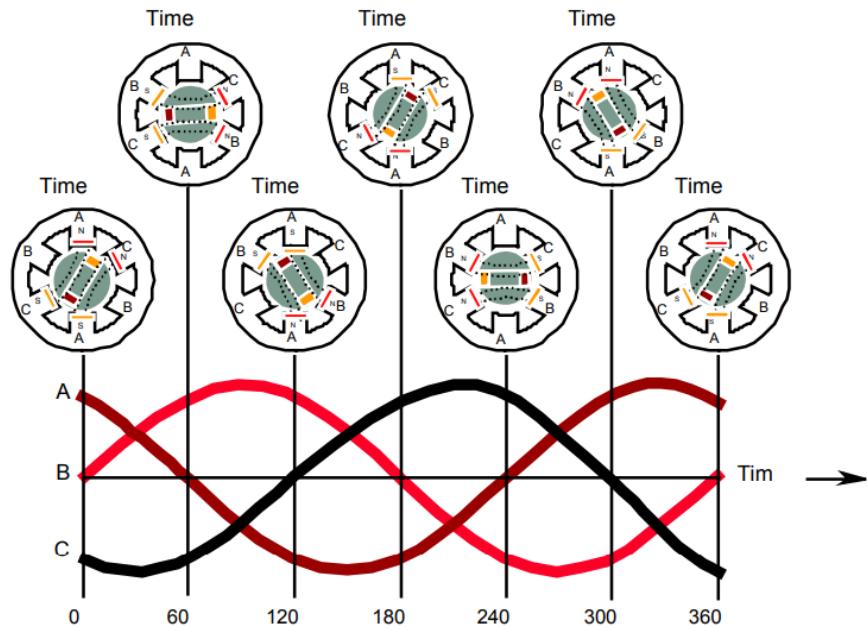


*IM rotor construction: 14. Figure*

(Hughes, 2006)

The rotating field created by the stator, induces a voltage in the bars of the rotor due to faraday's law (electromagnetic induction). This voltage causes a current flowing in the bars, also known as eddy current. As we know, if a current carrying conductor is placed in a magnetic field than there will be a force acting on it, hence creating the rotation of the rotor. The rotors rotation tries to

follow the rotation of the magnetic field, but it can never catch up, this difference is the reason why it's categorized as an asynchronous motor. In the following figure we can see how the rotation is achieved with three-phase currents.



*Rotation with 3-phase current: 15. Figure*

(Thornton and Armintor, 2003)

As we can see at a given time only two of the poles are active and one is neutral, meaning that one pole is pushing the rotor and the other is pulling it, resulting in a stable rotation.

(Thornton and Armintor, 2003)

Since there are no moving contacts like sliprings, brushes and commutators, the reliability is much higher compared to brushed DC motors and ac synchronous motors.

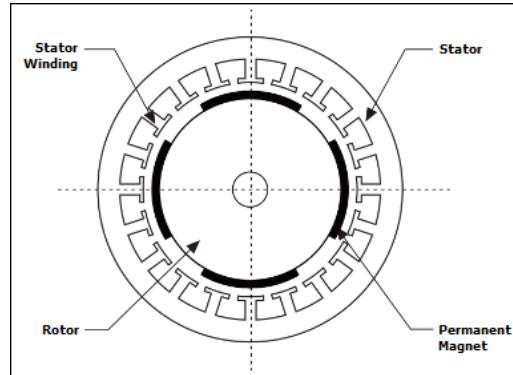
(Trzynadlowski, 2001)

But their efficiency is lower than the PMSM, they have a complex design, and they require an airgap as small as possible.

(AVL Basic\_Training, Kántor Bence)

### 3.3 AC Permanent Magnet Synchronous Motor (AC PMSM)

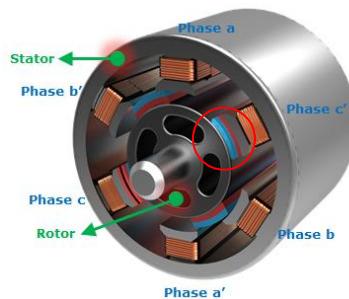
Here will look at the fundamentals of the AC PMSM. Let's briefly look at its working principle first. The PMSM is one of the most popular motors used in electric vehicles. It also consists of a rotor of which rotation is produced by the rotation of the magnetic field of the stator.



PMSM construction: 16. Figure

(AVL Basic\_Training, Kántor Bence)

It is called a synchronous motor because unlike the induction motor the rotors magnetic field is rotating with the magnetic field of the stator or in another words, they are in synch. The rotors magnetic field is produced by permanent magnets embedded in the rotor or mounted on the surface so there is no need for field windings. The stator, just like the induction's motors stator, consists of field windings that are energized using an inverter.



3D figure of a PMSM: 17. Figure

(AVL Basic\_Training, Kántor Bence)

The inverter supplies the 3-phase input AC to the stator thus creating the rotating magnetic field just like in case of an induction motor. At a given time two phases are active and one is inactive thus creating the effect that one pole is pushing the rotor and the other is pulling it. We can see this in the figure above, to rotate the rotor phase a and b is active while phase c is inactive. This

way the magnetic field created by phase a is going to push the magnet, circled with red in the figure, and the field created by phase b is going to pull the magnet.

(Hughes and Drury, 2019)

These motors are very efficient, have a high-power density and rely less on airgap size than other motors, but they are not self-starting motors, so they need a power supply with variable frequency.

(AVL Basic\_Training, Kántor Bence)

# Chapter 4

## 4. Simulink Implementation

### 4.1 The model

To be able to test the efficiency of the induction motor and the AC PMSM, we are going to need a car model. Since the focus of the thesis is the electrical motor, a very basic and simplified car model is enough for us. In the car model we use the inertia of the rotor and the tires, we also account for air resistance and rolling resistance of the car. The mass and geometry of the car is considered by the resistances. Let's look at the equations used for the resistances and the inertia and how they were modeled in Simulink. These forces are acting against the propelling force created by the electric motor, so we must subtract the two to get the actual force which accelerates the car.

The air resistance is proportional to the drag coefficient, frontal area of the car, density of the air and the square of the current speed of the car. The faster the car goes the bigger the resistance gets. Because it depends on the current speed of the car, we must use the speed as an input.

So, we calculate the air resistance as:

$$F_{air} = 0.5 \cdot C_d \cdot A \cdot \rho \cdot v^2 \quad (1)$$

$C_d$ : Drag coefficient. (0.3 [-])

A: area ( $2 \text{ [m}^2\text{]}$ )

$\rho$ : air density ( $1.3 \text{ [}\frac{\text{kg}}{\text{m}^3}\text{]}$ )

$v^2$ : square of speed

The rolling resistance is proportional to the rolling resistance coefficient, the mass of the car and the gravitation. Since we only use constants the implementation in Simulink doesn't depend on any outside factor.

$$F_{rolling} = C_{rr} \cdot m \cdot g \quad (2)$$

$C_{rr}$ : rolling resistance coefficient (0.01 [-])

(Jerzy Ejsmont, 2019)

$m$ : mass of the car (1500 [kg])

$$g: \text{gravitation } (9.81 \frac{\text{m}}{\text{s}^2})$$

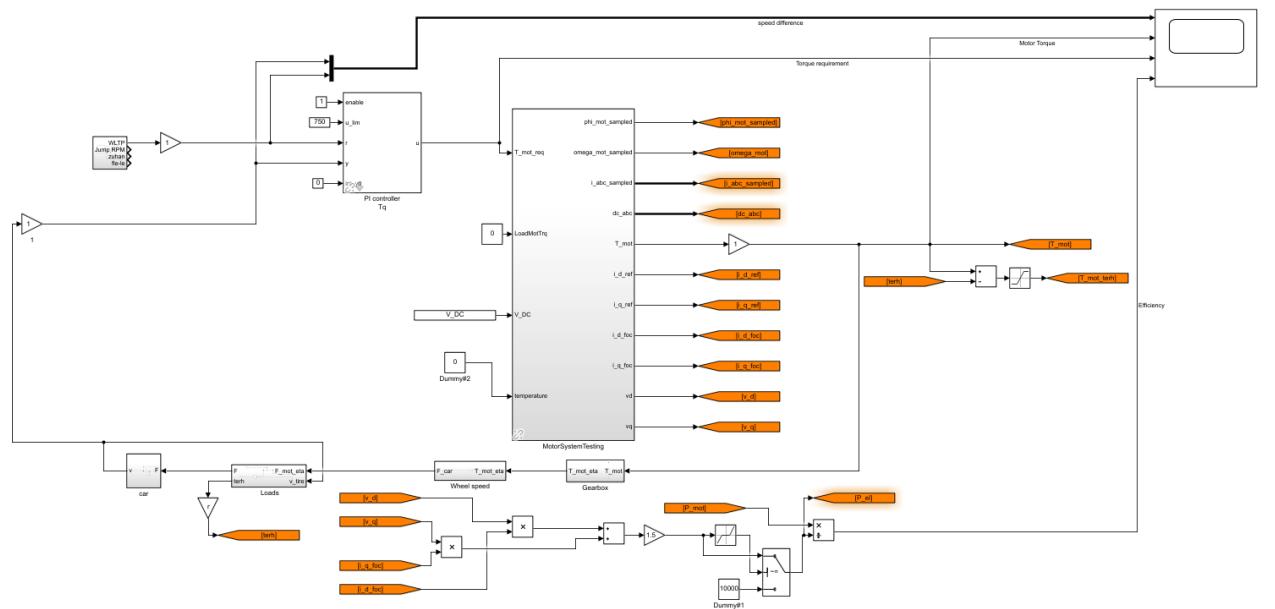
The inertia of the rotor is pre-determined by the model as  $0.1 \text{ [kgm}^2]$ , we need to calculate the inertia of the wheels and add it to the inertia of rotor. The inertia of the wheel is:

$$\frac{1}{2}mr^2 \quad (3)$$

where  $m$  is the mass and  $r$  is the radius of the wheel.

## 4.2 Simulink model of PMSM

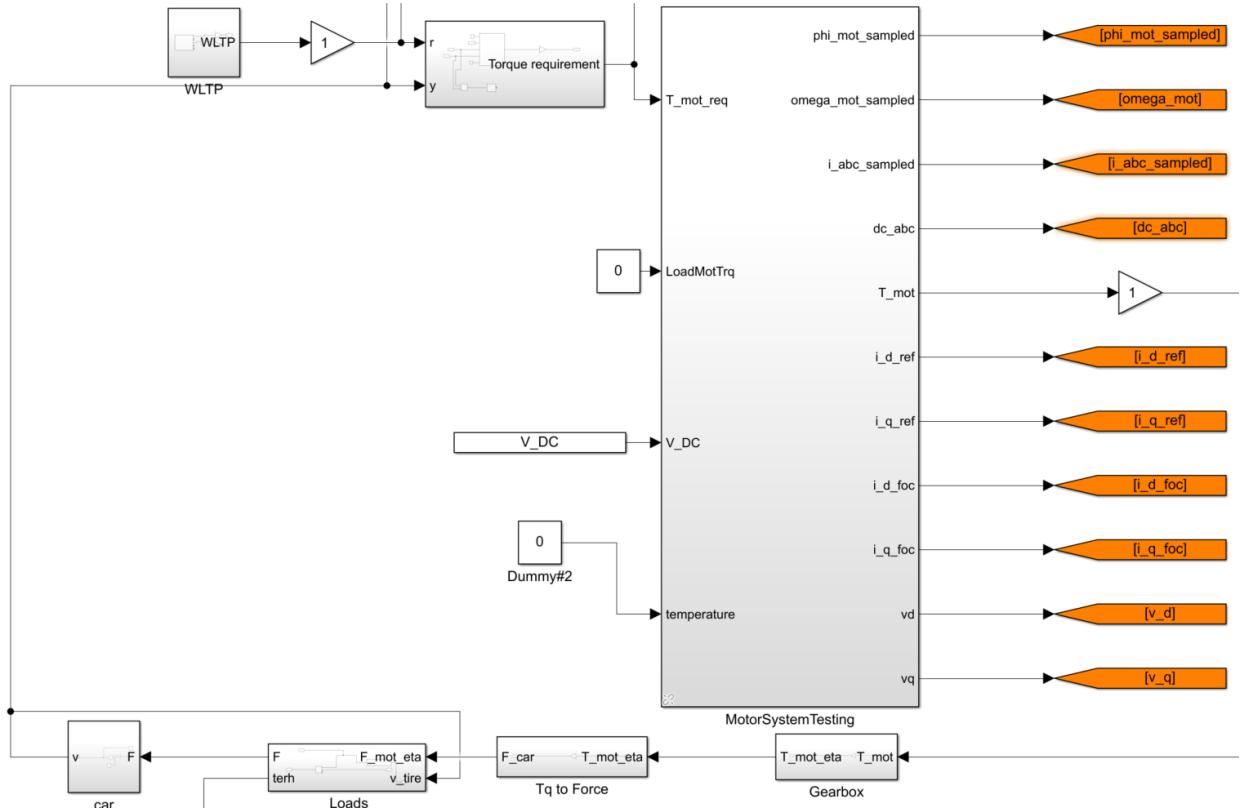
The inertia of the rotor and the tire together is  $J=2.2 \text{ [kgm}^2]$ , since the motor model is owned by AVL Hungary, most of the parameters are set by them. I got the model of the PMSM, and I built a system around it which allows us to model the car following the WLTP cycle. Note that even though the cycle is 600 steps long, I only simulate 100 steps because the PC I'm simulating on can't handle more steps and because 100 steps are more than enough to compare the efficiencies. The time step is the length of the simulation in seconds, the maximum step size is 0.0001s and MATLAB chooses the size of every step automatically, so if my simulation runs for 100 steps that would mean that the car follows the WLTP for 100 second.



PMSM driven car in Simulink: 18. Figure

Let's look at some of the components of the model, to understand the basics of the system. The PI controller has two inputs, the speed according to the WLTP cycle and the current speed of the car, with these two inputs the controller gives us a torque required to close the difference between the

two inputs, the maximum torque is limited to 750Nm. The “gearbox” subsystem takes the transmission (1:5) and its efficiency (95%) into account, the “Tq to Force” subsystem calculates the force propelling the car forward if there were no resistances, while the “Loads” subsystem subtracts the forces, created by the air- and rolling resistances shown by equation (1) and (2), from it thus creating the actual accelerating force of the car. All these subsystems can be seen in figure 19.



Main components of the model: 19. Figure

To calculate the efficiency of the PMSM we can use the following equations.

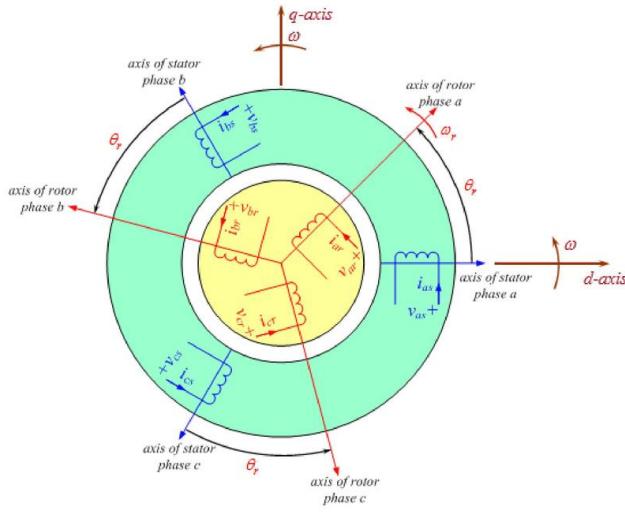
First, we need the mechanical power,

$$P_{mot} = T_{mot} \cdot \omega_{mot} \quad (4)$$

and the electrical power.

$$P_{el} = 1.5 \cdot (I_d \cdot V_d + I_q \cdot V_q) \quad (5)$$

where  $I_d, V_d, I_q, V_q$  are the current and voltage in the d-q plane obtained from the simulation and  $T$  and  $\omega$  are outputs of the motor. As we already know the 3-phase electrical systems has three voltages and current waveforms which have a  $120^\circ$  phase difference, denoted as A,B and C. To make the calculations easier we use a mathematical technique (Park's transformation) to transform the 3-phase variables to a new reference frame called the d-q plane. In this coordinate system the “d” axis is aligned with the magnetic field and the “q” axis is perpendicular to the magnetic field.



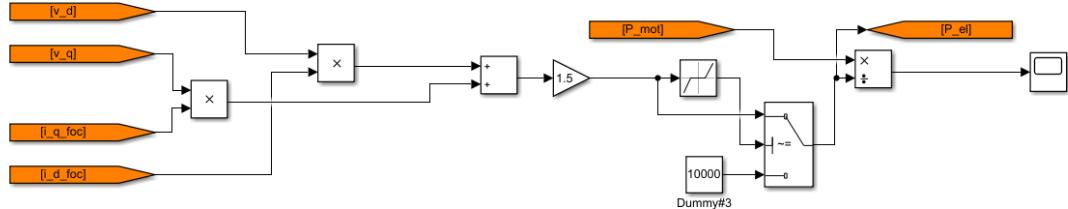
d-q and abc axis: 20. Figure

(Anshuman Bhattacharjee, 2017)

To calculate the efficiency, we use the following equation.

$$\eta = \frac{P_{mot}}{P_{el}} \quad (6)$$

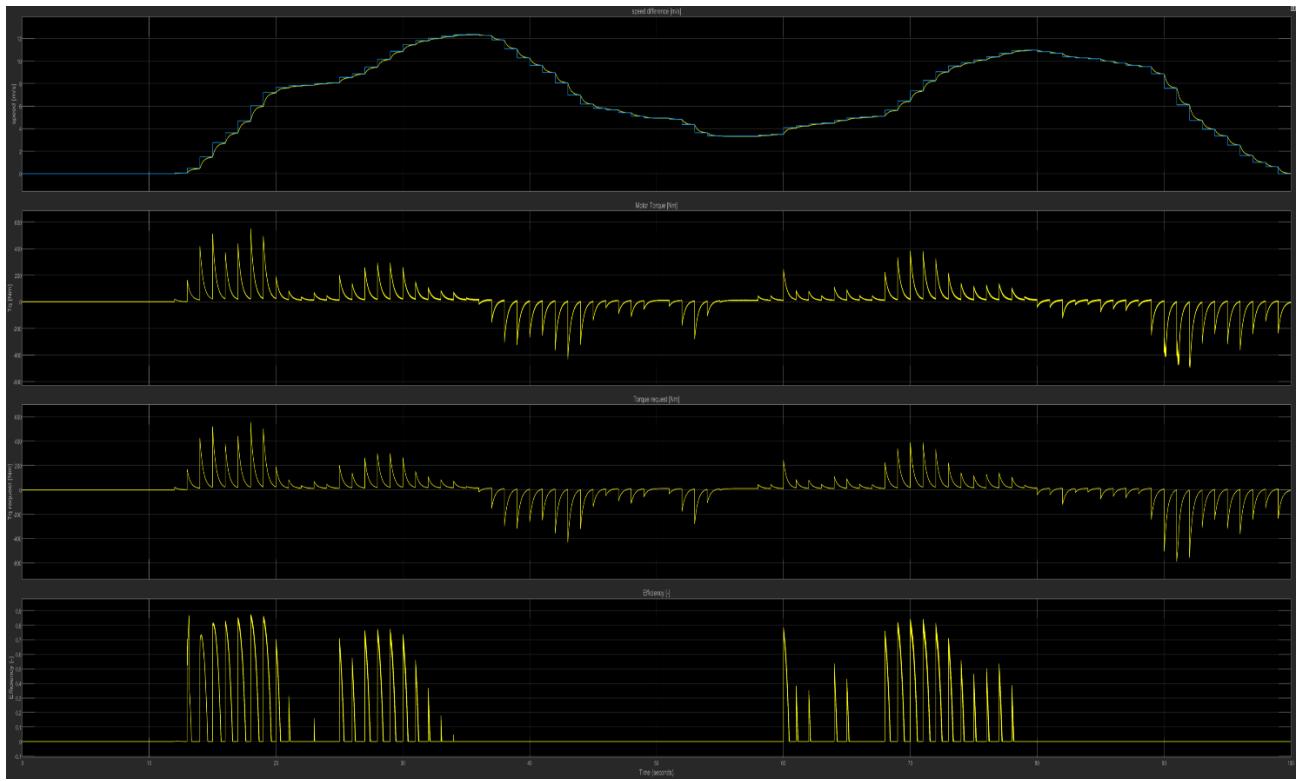
The Simulink implementation for the efficiency calculation looks like this.



Efficiency calculation: 21. Figure

### 4.3 Results of the PMSM simulation

To be able to see the results I put scopes on the speed difference between the WLTP cycle and the actual speed, the torque request given by the PI controller, the torque output of the motor and the efficiency of the motor. If we look at the scope, we can see all of them at once. All of them are plotted in relation to the time elapsed, meaning the x axis is the step from 0 to a 100.

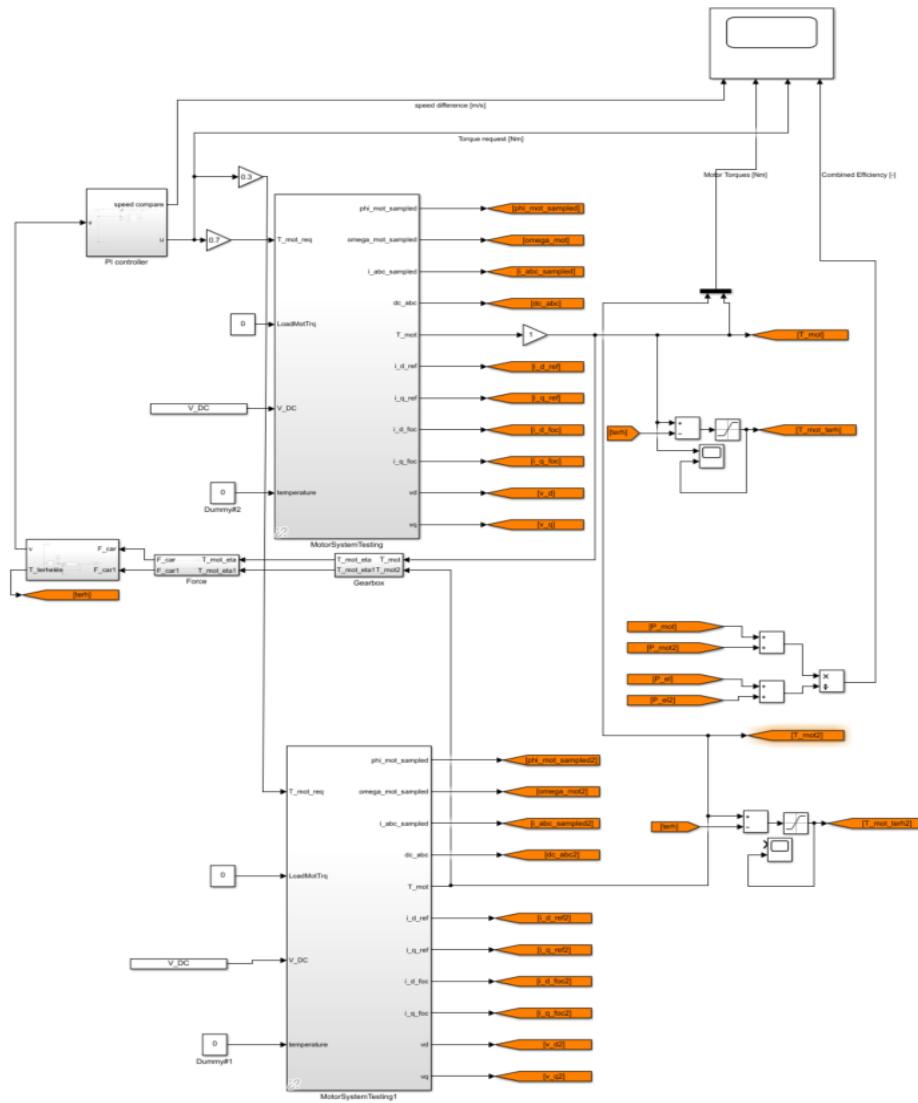


*Results of single drive concept: 22. Figure*

We can say that the car model follows our WLTP with a good enough accuracy. The efficiency of the motor jumps around because the motor torque jumps around as well, this all can be explained by the characteristics of the WLTP graph. Since the WLTP is not a continuous signal but rather a discrete signal with steps the motor and the controller act accordingly thus creating this jumpy behavior. We could solve this by having a WLTP input with a continuous signal. The maximum efficiency is around 83%, we can see that we also have 0% efficiency at some stages of the WLTP, this is because the motor is either breaking or not producing any torque.

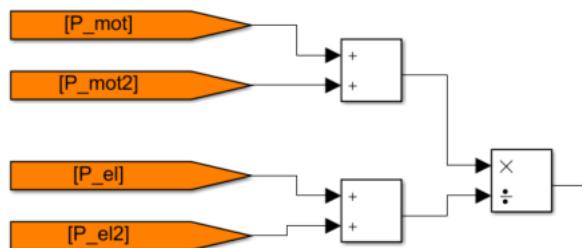
#### 4.4 Simulink model of car with 2 PMSM

The model of the combined drive PMSM is very similar to the model with 1 PMSM, the difference is that we have two motor subsystems, and the torque request is divided into two signals, each going into one of the motors. I choose the division so, that the first motor gets 70% of the request and the second motor gets 30% of the torque request. The mechanical and the electrical powers were calculated with the same equations as before, equation (4) and (5), but since we have two motors, we are going to get mechanical and electrical powers for both.



2PMSM Configuration: 23. Figure

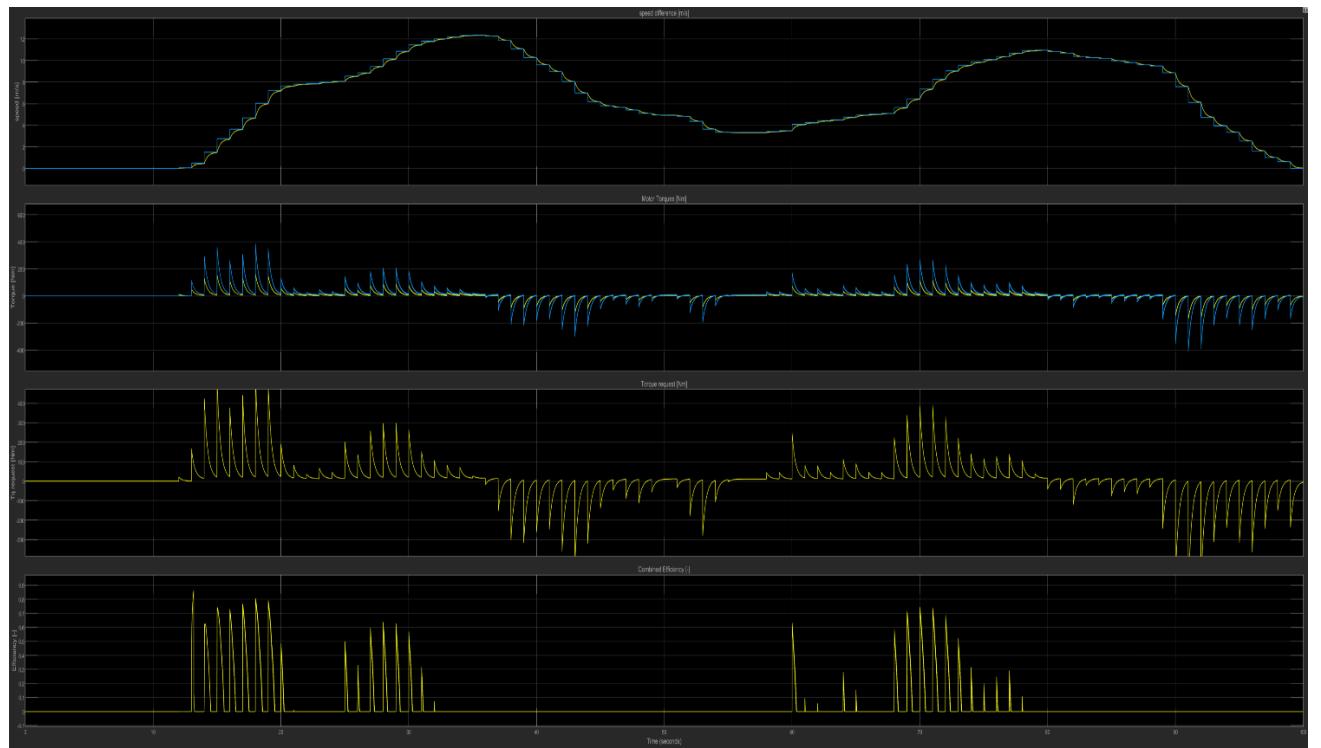
To calculate the efficiency of this configuration the same equations and subsystems were used as in the previous model, with the only difference being that the mechanical power was summed to get the mechanical power of the whole configuration and the electrical power of the motors were also summed to get the electrical power of the whole configuration, we can see this summation in figure 24.



Summ of power: 24. Figure

## 4.5 Results of the 2 PMSM simulation concept

In this case I put scopes on the speed difference between the WLTP cycle and the actual speed, the torque request given by the PI controller, the torque output of both motors and the combined efficiency of the motors. If we look at the scope, we can see all of them at once.



*Results of dual drive concept: 25. Figure*

In this case the car follows the WLTP again with high accuracy. The torque request is very similar to the previous case, but we can see that the torque provided by the motors are different, which is natural since they both got a different torque request. The efficiency increased by around 5% compared to the single drive concept.

## 4.6 Conclusion

In conclusion the goal of the thesis was to give a brief overview of the electric motors, categorize them, sum up their history and their working principle and simulate some motor configurations to analyze their efficiency. In chapter 1 I categorized the electric motors and looked at its pros and cons and their different use case scenarios, just so we can have some basic knowledge before going into chapter 2 where I describe the basics of the electric motors and explain the different behaviors of them with the help of a simplified DC motor. In chapter 1 I also looked at some of the motor configurations used by some popular company's such as Tesla, Porsche, and Hyundai. Chapter 2 is the longest since it gives us a very good explanation of the basics of the electric motors, here I talk about the basics of magnetic flux, magnetic force, the influence of an air gap between the rotor and the stator, the creation and use of backwards E.M.F and the behavior of a primitive DC motor with and without mechanical load. In chapter 3 I described the working principle of the three most

common electric motors such as the DC motor, the Induction motor, and the AC PMSM. The focus of the thesis was on chapter 4 where I created a basic car model around the PMSM model and look at its efficiency while it follows the WLTP cycle with very good accuracy. As the EV industry develops so does the need for electric cars with longer range, so the technology to be able to have higher efficiency is highly sought after. I simulated the single drive and dual drive concept in MATLAB Simulink, but I only used PMSM motors in both scenarios. From the result of the simulation, we can see that we have higher efficiency if we use the dual drive concept, but without further investigation we can't say for certain which concept has a higher efficiency. Due to the non-continuous nature of the WLTP cycle, fluctuations in the torque output are observed. Consequently, during each transition to a different speed regime, such as accelerating from 10 m/s to 13 m/s, the motor imparts abrupt changes in torque. Therefore, to attain the desired velocity, the accelerator pedal is depressed maximally, presenting a discrepancy from real-world scenarios.

To get better and more accurate results of the impact of dual drive vs single drive concept we would need a more accurate model with more motor combinations such as IM-PMSM or SynRm-IM and transform the WLTP into a continuous signal.

#### 4.7 Conclusion in Hungarian

Összegésként a dolgozat célja az volt, hogy rövid áttekintést adjon az elektromos motorokról, kategorizálja azokat, összefoglalja történetüket és működési elvüket, valamint néhány motorkonfigurációt szimuláljon a hatásfok elemzéséhez. Az 1. fejezetben kategorizáltam az elektromos motorokat, és megvizsgáltam előnyeiket és hátrányaikat, valamint különböző használati eseteiket, hogy alapismeretekhez jussunk, mielőtt belemennénk a 2. fejezetbe, ahol leírom az elektromos motorok alapjait, és elmagyarázom az elektromos motorok különböző viselkedésétegy egyszerűsített DC motor segítségével. Az 1. fejezetben megnéztem néhány olyan motorkonfigurációt is, amelyeket néhány népszerű cég, például a Tesla, a Porsche és a Hyundai használ az autóiban. A 2. fejezet a leghosszabb, hiszen részletesen elmagyarázza a villanymotorok alapjait, azaz szó volt a mágneses fluxus alapjairól, a mágneses erőről, a forgórész és az állórész közötti légrés hatásáról, az indukált feszültség létrehozásáról és egy primitív egyenáramú motor viselkedéséről mechanikai terhelés alatt és anélkül. A 3. fejezetben ismertettem a három legelterjedtebb villanymotor működési elvét, azaz az egyenáramú motorét, az indukciós motorét és az AC PMSM motorét. A szakdolgozat középpontjában a 4. fejezet állt, ahol a PMSM modell köré egy alap autómodellt készítettem, és megvizsgáltam annak hatásfokát, miközben nagyon jó pontossággal követi a WLTP ciklust. Ahogy az elektromos autóipar fejlődik, úgy nő a nagyobb hatótávolságú elektromos autók iránti igény is, így a nagyobb hatásfokot lehetővé tévő technológia nagyon keresett. MATLAB Simulinkben szimuláltam az egy- és kéthajtásos koncepciót, de mindenkorban csak PMSM motorokat használtam. A szimuláció eredményéből látható, hogy a kettős meghajtású koncepció alkalmazása esetén nagyobb a hatásfokunk, de további vizsgálat nélkül nem tudjuk biztosra megállapítani, hogy melyik koncepciónál ténylegesen nagyobb a hatásfok. A WLTP-ciklus nem folytonos jellege miatt a nyomatékkimenetben ingadozások figyelhetők meg. Következésképpen a motor minden egyes eltérő sebességre való áttérésekor, például a 10 m/s-ról 13 m/s-ra gyorsulásnál, a motor nyomatéka ugrásszerűen megugrik, ami olyan mintha a kívánt sebesség eléréséhez a gázpedált minden alkalommal tövig

nyomnánk, ami eltér a valóságtól, hiszen egy kisebb változás a gázpedál állásában is elég a sebesség megváltoztatására.

Ahhoz, hogy jobb és pontosabb eredményeket kapjunk a kettős hajtás kontra egyhajtás koncepció hatásáról, pontosabb modellre van szükségünk több motorkombinációval, mint például az IM-PMSM vagy a SynRm-IM motor kombinációk, és a WLTP-t folyamatos jellé kell átalakítanunk.

#### 4.8 Ideas for alternative E-motor configurations

In the simulations we could see that the efficiency increases if we have a dual drive system. I think it would also be interesting to look at motor configurations that include a SynRM since it has the highest efficiency if operating and constant speeds. This could mean a better drive of EVs on Highways where most of the time the speed is held constant. This could be achieved for example if we switch out one of the PMSM for a SynRM or add it in as a 3<sup>rd</sup> motor which turns on at constant speeds. We could also look at configurations which include an IM since they are very efficient when it comes to accelerating. An interesting example would be to simulate a car which uses IM at times where rapid acceleration is needed, and switches to a PMSM/ SynRM combination at constant speeds. It would be also beneficial to include recuperation in the model.

# Citations

- Anshuman Bhattacharjee, A. C. (2017). *Modeling and simulation of inverter fed electric drive.* India: VIT university.
- Austin Hughes, B. D. (2019). *Electric Motors and Drives: Fundamentals, Types and Applications.* Oxford, United Kingdom: Elsevier Ltd.
- Hughes, A. (2006). *Electric Motors and Drives: Fundamentals, Types and Applications.* Burlington, USA: Elsevier Ltd.
- Jerzy Ejsmont, W. O. (2019). Engineering method of tire rolling resistance evaluation. *Measurement*, 144-149.
- Keyes, C. (2007). *Electric Motors: Energy Efficiency Reference Guide.* Canada: CEA Technologies Inc.
- Müller, D. C. (2019). *Motor Handbook.* Germany, München: Infineon Technologies AG.
- Tong, W. (2014). *Mechanical Design of Electric Motors.* Boca Raton: CRC Press.
- Trzynadlowski, A. (2001). *Control of Induction Motors.* San Diego, USA: Academic Press.
- Yedamale, P. (2003). *Brushless DC (BLDC) Motor Fundamentals.* USA: Microchip Technology Incorporated. Retrieved from [https://electrathonoftampabay.org/www/Documents/Motors/Brushless%20DC%20\(BLD%20Motor%20Fundamentals.pdf](https://electrathonoftampabay.org/www/Documents/Motors/Brushless%20DC%20(BLD%20Motor%20Fundamentals.pdf)
- V.K., Arun Shankar & Subramaniam, Umashankar & Paramasivam, Sathesh & Venkatesh, Yamini & Kommanaboina, Venkatesh & Sailesh, Dileep. (2017). Mathematical modelling and investigation of reduced order induction motor drive system. 1-8. 10.1109/IPACT.2017.8244997.
- Thornton, Edward J.; Armintor, J. Kirk (2003). The Fundamentals Of Ac Electric Induction Motor Design And Application. Texas A&M University. Turbomachinery Laboratories. Available electronically from <https://hdl.handle.net/1969.1/164010>.