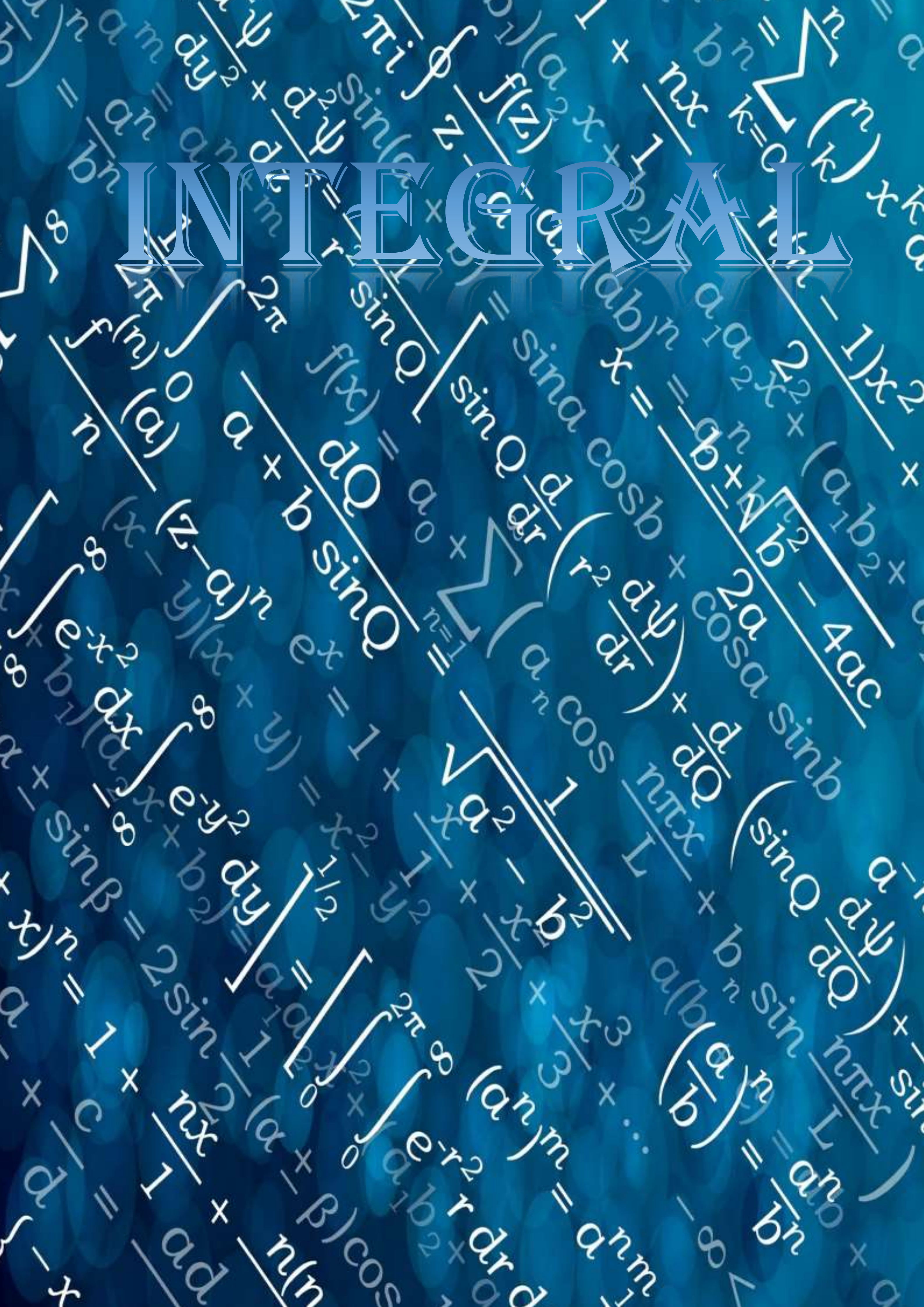
INTEGRAL



ផ្នែកលំហាត់

001. $I=\int_{-1}^{0}\frac{x(x+2)}{\log(x+1)}dx$

002. $J=\int_{-\infty}^{+\infty}\frac{\cos(x)}{x^2+1}dx$

003. $K=\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(x)}{1+2030^x}dx$

004. $I=\int_{-\infty}^{+\infty}\frac{\sin(x)}{x}dx$

005. $J=\int_{0}^{1}\frac{x^n-1}{\log(x)}dx \quad ,n\geq 0$

006. $K=\frac{1}{2\pi}\int_{0}^{2\pi}\frac{1}{a+\cos(x)}dx$

007. $I=\int_{0}^{+\infty}\frac{1}{x^4+x^2+1}dx$

008. $J=\int_{0}^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx$ -

009. $K=\int_{0}^{+\infty}\frac{\log(x^2+1)}{(x^2+1)}dx$

010. $I=\int_{0}^{1}\frac{\log(x+1)}{\left(x^2+1\right)}dx$

011. $I=\int_{0}^{\frac{\pi}{2}}\frac{x}{\tan(x)}dx$

012. $J=\int_{0}^{2\pi}\left(|\sin(x)|+|\cos(x)|\right)dx$

013. $K=\int_{-1}^{+1}x\arcsin(x)\arccos(x)dx$

014. $I=\int_{0}^{\frac{\pi}{4}}\frac{\log\left(\sqrt{1-\tan^2(x)}\right)}{\cos^2(x)\sqrt{1-\tan^2(x)}}dx$

015. $J=\int_{0}^{1}(1-x)^5\,dx$

016. $K=\int_{0}^{+\infty}\frac{x\tan^{-1}(x)}{x^4+x^2+1}dx$

017. $I=\int_{2}^{4}\frac{\binom{x}{1}\binom{x}{3}\binom{x}{5}}{\binom{x}{2}\binom{x}{4}\binom{x}{6}}dx$

018. $J=\int_{0}^{1}\frac{\log(x)}{x^2-1}dx$

019. $K=\int_{0}^{\pi}\frac{\sin^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx$

020. $I=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(x)}{e^{\frac{1}{x}}+1}dx$

021. $J=\int_{0}^{\frac{\pi}{2}}\sqrt{\sin(x)-\sin^2(x)}dx$

022. $K=\int_{0}^{\frac{\pi}{4}}e^x\left(\tan^2(x)+\log\left(\cos(x)\right)\right)dx$

023. $I=\int_{0}^{\frac{\pi}{2}}\frac{x}{\sin(x)+\cos(x)}dx$

024. $J=\int_{0}^{\pi}\frac{x}{\cos^2(x)+9\sin^2(x)}dx$

025. $K=\int_{-1}^{0}\frac{x^2+x}{\left(e^x+x+1\right)^2}dx$

026. $I=\int_{0}^{1}\frac{W\left(\frac{x\log(3)}{e^{-x\log(3)}}\right)}{e^{-x\log(3)}}dx$

027. $J=\int_{0}^{1}\frac{1}{\sqrt[3]{(1+x)^2(1-x)}}dx$

028. $K=\int_{0}^{+\infty}\frac{x^{n-1}}{e^x-1}dx$

029. $I=\int_0^{\pi}\frac{1}{\left(1+\sin(x)\right)^2}dx$

030. $J=\int_0^1\log\left(\Gamma(x)\right)dx$

031. $K=\int_0^1\Gamma\left(1+\frac{x}{2}\right)\Gamma\left(1-\frac{x}{2}\right)dx$

032. $J=\int_0^1\frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}}dx$

033. $K=\int_0^{\frac{\pi}{2}}\frac{\cos(x)\sqrt{\cos(x)}-\sin(x)\sqrt{\sin(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx$

034. $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^5(x)}dx$

035. $J=\sum_{n=0}^{\infty}\int_{2n}^{2n+1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx-\sum_{n=1}^{\infty}\int_{2n}^{2n-1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx$

036. $K=\int_0^1\frac{1}{1+x+x^2+x^3+\dots}dx$

037. $I=\int_{-1}^0\frac{1}{\sqrt{e^x}+\sqrt{e^{3x}}+\sqrt{e^{5x}}+\dots}dx$

038. $J=\int_0^1 x^x dx$

039. $K=\int_0^{+\infty}\frac{nx^{n-1}}{(x^2+1)^{\frac{n+2}{2}}}dx\quad,(n>0)$

040. $I=\int_0^{\infty}\left\lfloor ne^{-x}\right\rfloor dx\ ,n\in\mathbb{N}$

041. $J=\int_1^2(x+1)^2e^{\frac{x^2-1}{x}}dx$

042. $K=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx$

043. $I=\int_1^2 x^x\left(1+\log(x)\right)dx$

044. $J=\int_0^{\frac{\pi}{2}}\sqrt{\tan(x)}dx$

045. $K=\int_0^1(-1)^x e^{\frac{\pi}{2}i}dx$

046. $I=\int_1^0\frac{\log(1-x^2)}{(1+x^2)}dx$

047. $J=\int_0^{\frac{\pi}{4}}\log\left(\cos(x)\right)dx$

048. $K=\int_0^1\frac{\tan^{-1}(x)}{\left(1+x^2\right)^2}dx$

049. $I=\int_0^1\frac{\log(x^2)}{\left(1+x^2\right)^2}dx$

050. $J=\int_0^{\infty}\frac{1}{1+x+x^2+x^3+x^4+x^5}dx$

051. $K=\int_0^1 x\left\lfloor\frac{1}{x}\right\rfloor dx$

052. $I=\int_0^1\sin\left(\sqrt{-\log(x)}\right)dx$

053. $J=\int_0^1\frac{\log(x)}{1+x}dx$

054. $K=\int_1^e\frac{\log\left[\Gamma\left(1-\log(x)\right)\right]}{x}dx$

055. $I=\int_0^1\frac{1}{x\sqrt{1-x^2}}\log\left(\frac{2+x}{2-x}\right)dx$

056. $J=\int_0^{\infty}\frac{\sqrt{x}}{(x+9)^2}dx$

057. $K=\int_0^1\frac{1}{\sqrt{1-x^3}}dx$

058. $I=\int_0^1 \frac{\pi-4\tan^{-1}(x)}{1-x^2}dx$

059. $J=\int_0^1 \frac{x^p\log(x)}{x-1}dx$

060. $K=\int_0^1 \frac{1}{\sqrt{x(1-x)}}dx$

061. $I=\int_0^n \frac{\log(x)}{x^2+n^2}dx$

062. $J=\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\sin(x)}}dx$

063. $K=\int_0^{\frac{\pi}{2}} \frac{x}{\tan(x)}dx$

064. $K=\int_0^{\infty} \frac{\tan^{-1}(2x)}{\sqrt[3]{x^2}}dx$

065. $I=\int_0^{2\pi} \lfloor 2023\sin(x)\rfloor dx$

066. $J=\int_1^e \frac{\log^2(x)}{x^3}dx$

067. $K=\int_0^1 \lfloor x\rfloor^{-1}dx$

068. $I=\int_0^1 \frac{x^{-1}}{\sqrt{x^{-1}-1}}dx$

069. $J=\int_e^{\infty} x^{1-\log(x)}dx$

070. $K=\int_0^1 x^{\log(x)-1}\log(x)dx$

071. $I=\int_0^1 \frac{\log(x)}{\sqrt{x(x-1)}}dx$

072. $J=\int_0^n \frac{\log(x+1)}{x}dx$

073. $K=\int_0^{\pi} x\sin^4(x)dx$

074. $I=\int_0^{\frac{\pi}{2}} \frac{\sin(x)+\cos(x)}{\sin^5(x)+\cos^5(x)}dx$

075. $J=\int_0^{e^e} x^{2e-\log(xe)}dx$

076. $I=\int_1^2 \frac{\log(x+1)-\log(2)}{(x^2-1)}dx$

077. $J=\int_{\frac{1}{2025}}^{2025} \frac{x^2+1}{x^2+x^{2025}}dx$

078. $K=\int_1^{\infty} \frac{1}{x^n(x^2+1)}dx$

079. $I=\int_0^1 \frac{x^{\pi}-x^2}{x\log(x)}dx$

080. $J=\int_{-\infty}^{+\infty} \frac{\arctan^2(x)}{x^2}dx$

081. $K=\int_0^{2\pi} (\sin(x)+\cos(x))^{11}dx$

082. $I=\int_0^{\pi} \sqrt[x]{\log(x)}dx$

083. $J=\int_0^1 \sqrt{\frac{1}{x}\log\left(\frac{1}{x}\right)}dx$

084. $K=\int_0^{+\infty} e^{-\lfloor x\rfloor}dx$

085. $I=\int_1^2 \frac{\log(x)}{x^2-2x+2}dx$

086. $J=\int_{-\infty}^{+\infty} \frac{1}{1+x^4}dx$

087. $K=\int_{-1}^{+1} \frac{e^x-1}{e^x+1}dx$

088. $I=\int_{-1}^{+1} \log\left(\frac{1-x}{1+x}\right)dx$

089. $J=\int_1^{\infty} \frac{\left(\frac{x-1}{x+1}\right)}{(x+1)(x-1)}\log^2\left(\frac{x+1}{x-1}\right)dx$

090. $K=\int_0^{\infty} \frac{t^n}{e^x-1}dx$

091. $I=\int_0^{e-1} \frac{x}{(x+1)\log(x+1)}dx$

092. $J=\int_{1}^{2024}\lfloor \log_{43}(x)\rfloor dx$

093. $K=\int_{-\infty}^{0}\frac{\log(x+1)-\log(x)}{(x+1)x}dx$

094. $I=\int_{0}^{1}\frac{\log^2(x)}{x^2-1}dx$

095. $J=\int_{0}^{1}\sqrt{1-x^{\pi}}\,dx$

096. $K=\int_{0}^{\frac{\pi}{2}}\frac{\sin(x)+1}{\sin(x)+\cos(x)+1}dx$

097. $I=\int_{0}^{9}\frac{x+\frac{x+\dots}{1+\dots}}{1+\frac{x+\dots}{1+\dots}}dx$

098. $J=\int_{0}^{\infty}\frac{x\log(x)}{x^4+1}dx$

099. $K=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^x}dx$

100. $I=\int_{0}^{\frac{\pi}{2}}\frac{x+\sin(x)}{1+\cos(x)}dx$

101. $J=\int_{0}^{1}\frac{\sqrt{x(1-x)}}{1+x}dx$

102. $K=\int\frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)\cos(x)}}dx$

103. $I=\int_{0}^{\frac{\pi}{2}}\frac{\cos(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx$

104. $J=\int_{-1}^{+1}\frac{1}{\sqrt{1+x}+\sqrt{1-x}+2}dx$

105. $K=\int_{0}^{2\pi}\frac{1}{e^{\sin(x)}+1}dx$

106. $I=\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}dx$

107. $J=\int_{-1}^{1}\left|3^x-2^x\right|dx$

108. $K=\int_{0}^{1013\pi}\left|\sin(1013x)\right|dx$

109. $I=\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1-x}{1+x}\right)dx$

110. $J=\int_{0}^{1}\frac{\sqrt{x}}{(x^2+1)\sqrt{1-x^2}}dx$

111. $K=\int_{0}^{\infty}\frac{x^2}{x^4+1}dx$

112. $I=\int_{0}^{\infty}\frac{x^2}{(x^4+1)^2}dx$

113. $J=\int_{0}^{1}\frac{x^4(x^2-1)}{(2x^3+1)^3}dx$

114. $K=\int_{1}^{\sqrt[4]{2}}\frac{x^8-1}{x(x^8+1)}dx$

115. $I=\int_{-1}^{+1}\frac{(x+\sqrt{x^2+1})^3}{\sqrt{x^2+1}}dx$

116. $J=\int_{e^{-1}}^{e^3}\frac{\sqrt{1+\log(x)}}{x\log(x)}dx$

117. $K=\int_{0}^{\pi}\sqrt{\frac{1+\cos(2x)}{3}}dx$

118. $I=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\sqrt{1+\sin(2x)}dx$

119. $J=\int_{0}^{\infty}\frac{1}{(1+x^{\phi})^{\phi}}dx$

120. $K=\int_{0}^{1}\frac{x^e-x^{\pi}}{\log(x)}dx$

121. $I=\int_{0}^{1}\left(x\log(x)\right)^n dx$

122. $J=\int_{0}^{1}\frac{\log(x+1)\log(x)}{x}dx$

123. $K=\int_0^{\infty}\left(x-\frac{x^2}{2}+\frac{x^5}{2\times4}-\frac{x^7}{2\times4\times6}+\dots\right)\left(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\times4^2}+\frac{x^6}{2^2\times4^2\times6^2}+\dots\right)dx$

124. $I=\int_0^1\log\left((x-1)!\right)dx$

125. $J=\int_{-\infty}^{+\infty}\Gamma(1+ix)\Gamma(1-ix)dx$

126. $K=\int_0^{\infty}\log\left(1+\frac{1}{x^2}+\frac{1}{x^4}\right)dx$

127. $I=\int_0^{\infty}\frac{x\sqrt{x}}{e^{2x}-1}dx$

128. $J=\int_0^1\frac{x^8+1}{x^{10}+1}dx$

129. $K=\int_{-\infty}^{+\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx$

130. $I=\int_0^{\infty}\frac{x\log(x)}{(x^2+1)^2}dx$

131. $I=\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{1}{(x^2+1)}dx$

132. $J=\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x)-\sin(x)}dx$

133. $K=\int_0^{\frac{\pi}{4}}\frac{1}{\cos(x)+\sin(x)}dx$

134. $I=\int_0^{90^\circ}\frac{1}{5\cos^2(x)+4\sin^2(x)-3}dx$

135. $J=\int_0^1 x^{-x}dx$

136. $K=\int_0^{90^\circ}\sin^3(2x)\cos(x)dx$

137. $I=\int_0^1\frac{1}{\sqrt{x}(1+\sqrt{x})\sqrt{1-x}}dx$

138. $J=\int_0^{\infty}\frac{e^{-2x}\sin(3x)}{x}dx$

139. $K=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{\left(\sin(x)+\cos(x)\right)^2}dx$

140. $I=\int_{-1}^1 x^{\frac{x}{\log(x)}}dx$

141. $J=\int_{-1}^1 x^{\frac{1}{\log(2x)}}dx$

142. $K=\int_e^{\pi}\sqrt{x-e}\sqrt{\pi-x}dx$

143. $I=\int_{\pi}^e x^{\frac{1-2x}{x}}\log(x/e)dx$

144. $J=\int_0^{\frac{\pi}{2}}\sin^4(x)\cos^5(x)dx$

145. $K=\int_0^{\pi}\frac{1}{1+e^{\tan(x)}}dx$

146. $I=\int_0^1\frac{\sin^{-1}(x)}{x}dx$

147. $J=\int_{e^{-1}}^{e^2}\left|\frac{\log(x)}{x}\right|dx$

148. $K=\int_0^2(1-x)\log(x)dx$

149. $I=\int_1^e\frac{x-1}{x^2-\log(x^x)}dx$

150. $J=\int_{-1}^1\sqrt{\frac{1-x}{1+x}}dx$

151. $K=\int_1^2\sqrt{\frac{x-1}{2-x}}dx$

152. $I=\int_{-1}^1\sqrt{\frac{x+1}{x-1}}dx$

153. $J=\int_{-\infty}^{\infty}\frac{1}{x^{12}+1}dx$

154. $K=\int_0^{45^\circ}\tan(x)\log\left(\tan(x)\right)dx$

155. $I=\int_1^2\left\lfloor x^2-x\right\rfloor dx$

156. $J=\int_0^{\log(2)}\frac{\lfloor e^x\rfloor}{\lfloor e^x-1\rfloor}dx$

157. $K=\int_0^2\sin\left(\lfloor x+1\rfloor\right)dx$

158. $I=\int_{-2}^2\frac{\sin(x)}{\lfloor x/\pi\rfloor+2}dx$

159. $J=\int_0^\infty\lfloor x\rfloor e^{-x}dx$

160. $K=\int_0^{\frac{\pi}{2}}\lfloor 1+\sin(x)\rfloor dx$

161. $I=\int_{\sqrt{2}}^{\sqrt{3}}\frac{\lfloor x^2\rfloor}{\lfloor x^2-1\rfloor}dx$

162. $J=\int_0^{\frac{\pi}{2}}\lfloor\cos(2x)\rfloor dx$

163. $K=\int_{-2}^2\frac{\left\lfloor\frac{x}{\pi}\right\rfloor}{\left\lfloor\frac{x}{\pi}\right\rfloor+\frac{1}{2}}dx$

164. $I=\int_0^{\log(3)}\lfloor e^x+1\rfloor dx$

165. $K=\int_e^{e^2}\lfloor\log(x)\rfloor^{\lfloor\log(x)+1\rfloor}dx$

166. $I=\int_{e-1}^{e^2-1}\lfloor\log(x+1)\rfloor dx$

167. $J=\int_0^4\frac{|x-1|}{|x-2|+|x-3|}dx$

168. $K=\int_0^{\frac{\pi}{2}}\log\left(9\cos^2(x)+\cos(x)\right)dx$

169. $I=\int_0^\pi\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx$

170. $J=\int_{-\infty}^\infty\frac{x^2+1}{x^8+1}dx$

171. $K=\int_{-1}^1x\sqrt{x^2}dx$

172. $I=\int_0^\infty\frac{\sqrt{x}}{(x+1)^2}dx$

173. $J=\int_0^1x^2(x-1)^3dx$

174. $K=\int_0^{\frac{\pi}{2}}\left(x\cos(x)+1\right)e^{\sin(x)}dx$

175. $I=\int_0^{\frac{\pi}{2}}\left(x\sin(x)-1\right)e^{\cos(x)}dx$

176. $J=\int_0^\infty\frac{x}{e^{\pi x}-1}dx$

177. $K=\int_{-2026}^{2026}x^{2026}\cot^{-1}(2026x)dx$

178. $I=\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor x^2-4x+4\rfloor+\lfloor x^2\rfloor}dx$

179. $J=\int_0^{45}\lfloor 45x\rfloor dx$

180. $K=\int_0^\infty\frac{(2-x)^{2023}}{(2+x)^{2025}}dx$

181. $I=\int_0^\infty\frac{x^6+1}{x^{12}+1}dx$

182. $J=\int_0^{\frac{\pi}{2}}\frac{x\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx$

183. $K=\int_0^{\frac{\pi}{2}}\frac{\sin(x)}{1+\sin(x)\cos(x)}dx$

184. $I=\int_0^\infty\frac{\log(x)}{x^2-x+1}dx$

185. $J=\int_0^1\frac{1}{\sqrt{-\log(x)}}dx$

186. $K=\int_0^1 (\log(1/x))^{n-1}dx$

187. $I=\int_0^{\pi/2} \tan^n(x)dx$

188. $J=\int_0^1 \frac{e^x-1}{x}dx$

189. $K=\int_0^{\infty} x^{-\log(x)}\log(x^x)dx$

190. $I=\int_0^{\infty} \frac{x^4}{(1+x^3)^2}dx$

191. $J=\int_{-1}^1 (1-x^2)^n dx$

192. $K=\int_0^4 \binom{x}{5}^{-1} dx$

193. $I=\int_0^{45^\circ} \arcsin\left(\frac{2x}{1+x^2}\right)dx$

194. $J=\int_0^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx$

195. $K=\int_{-\pi}^{2\pi} \left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)dx$

196. $I=\int_0^{\infty} \frac{1}{1+x^n}dx$

197. $J=\int_0^{\frac{\pi}{4}} \log\left(\cot(x)-1\right)dx$

198. $K=\int_0^{\infty} \left(\frac{\log(x)}{(1+x)}\right)dx$

199. $I=\int_0^1 \log(1+x)\log(1-x)dx$

200. $J=\int_0^{\infty} \frac{\sqrt{x}}{e^{\sqrt{x}}}dx$

201. $K=\int_0^{\infty} \frac{x}{e^x+e^{-x}}dx$

202. $I=\int_0^{\frac{\pi}{4}} \frac{x}{\left(\sin(x)+\cos(x)\right)\cos(x)}dx$

203. $J=\int_1^2 \frac{\log(x-1)}{x(2-x)}dx$

204. $K=\int_0^{\sqrt{2}} \lfloor x^2 \rfloor dx$

205. $I=\int_0^{\sqrt{2}} (\lfloor x \rfloor)^2 dx$

206. $J=\int_0^{\infty} \frac{x}{1+x^3}dx$

207. $K=\int_0^{\frac{\pi}{2}} \sin^2(x)\log(\tan(x))dx$

208. $I=\int_0^{\frac{\pi}{2}} \log\left(\lfloor \sin(x)+1 \rfloor\right)dx$

209. $I=\int_0^1 \frac{x-1}{(x+1)^3}e^x dx$

210. $J=\int_0^{\frac{\pi}{2}} \left(1-\sin(x)+\sin^2(x)-\sin^3(x)+..\right)dx$

211. $K=\int_1^2 \frac{\sqrt{x-1}\tan^{-1}\left(\sqrt{x-1}\right)}{x}dx$

212. $I=\int_0^{\frac{\pi}{2}} \frac{\sin(x)-\cos(x)}{e^x-\sin(x)}dx$

213. $J=\int_{-2}^2 \left[x^{2025}\cos\left(\frac{x}{2026}\right)+\frac{1}{2}\right]\sqrt{4-x^2}dx$

214. $K=\int_0^{\infty} \frac{\sin^2(x)}{x^2(x^2+1)}dx$

215. $I=\int_1^{\infty} \left(\frac{\log(x)}{x}\right)^{n+m} dx$

216. $J=\int_{-2}^{2}\frac{\lfloor x\rfloor}{|x+1|}dx$

217. $K=\int_{-2}^{2}\frac{\lceil x\rceil}{|x+1|}dx$

218. $I=\int_{0}^{1}x(-\log(x))^{3}\,dx$

219. $J=\int_{0}^{\pi}\log\left(|\tan(x)|\right)dx$

220. $K=\int_{0}^{\pi}\log\left(|\sin(x)|\right)dx$

221. $I=\int_{1}^{e}\left[(x/e)^{x}+(e/x)^{x}\right]\log(x)dx$

222. $J=\int_{0}^{1}\frac{\log(x^{2}+1)}{x}dx$

223. $K=\int_{0}^{\pi}\frac{\log(1-\sin(x))}{\sin(x)}dx$

224. $I=\int_{1}^{e}(x-1)\log^{2}(x)dx$

225. $J=\int_{\frac{\pi}{2}}^{\pi}\log^{2}\left(1+(e-1)\sin(x)\right)\sin(2x)dx$

226. $K=\int_{0}^{1}\frac{\log(x+1)}{x}dx$

227. $I=\int_{-1}^{1}\log\left(x+\sqrt{1+x^{2}}\right)dx$

228. $J=\int_{0}^{\infty}\frac{\tan^{-1}(ex)-\tan^{-1}(x)}{x}dx$

229. $K=\int_{0}^{1}\frac{x^{3}(1+x^{2})}{(1+x)^{10}}dx$

230. $I=\int_{0}^{\pi}\sin^{5}(x)(1-\cos(x))^{3}\,dx$

231. $J=\int_{0}^{1}\tan^{-1}\left(\sec(x)+\tan(x)\right)dx$

232. $K=\int_{0}^{\pi}\frac{x\sin^{2}(x)}{1+\cos^{2}(x)}dx$

233. $I=\int_{1}^{\infty}\frac{x-1}{x^{4}\log(x)}dx$

234. $I=\int_{0}^{\infty}\frac{x}{x^{8}+2x^{4}+1}dx$

235. $J=\int_{0}^{1}\frac{1}{1+\lfloor 1/x\rfloor}dx$

236. $K=\int_{0}^{1}\frac{\sin\left(\sqrt[x]{\log(x)}\right)}{\log(x)}dx$

237. $I=\int_{0}^{\infty}\frac{\sin(x)}{x+x\cos^{2}(x)}dx$

238. $J=\int_{0}^{\frac{\pi}{2}}\frac{\sin^{2}(x)}{\sin(x)+\cos(x)}dx$

239. $K=\int_{0}^{2022}\left(x^{2}-\lfloor x\rfloor\lceil x\rceil\right)dx$

240. $I=\int_{0}^{\pi}x\cos^{4}(x)\sin^{5}(x)dx$

241. $J=\int_{0}^{1}\frac{\log(x)}{\sqrt{x(x-1)}}dx$

242. $K=\int_{0}^{\infty}\frac{x^{n}}{e^{x}+1}dx$

243. $I=\int_{0}^{1}\frac{\log(x)}{\sqrt{x(x-1)}}dx$

244. $J=\int_{1}^{\int_{1}^{\int_{1}^{\cdots}(2x)dx}(2x)dx}(2x)dx$

245. $K=\int_{0}^{\infty}\frac{e^{-2x}\cos(x)-e^{-3x}}{x}dx$

246. $I=\int_{0}^{\infty}\left(\frac{\log^{2}(x)}{x(x+1)}\right)dx$

247. $J=\int_{0}^{\frac{\pi}{2}}\sin^{2}(x)\sin^{2}(2x)dx$

248. $K=\int_{0}^{\frac{\pi}{4}}\frac{\sin(x)+\cos(x)}{9+16\sin(2x)}dx$

249. $I=\int_{0}^{\infty}\frac{e^{-\pi x}-e^{-ex}}{x}dx$

250. $J = \int_{-1}^{1} x\tan(x)\tan\left(\frac{1}{x}\right)dx$

251. $K = \int_{0}^{\infty} \frac{\log(2e^x - 1)}{e^x - 1}dx$

252. $I = \int_{0}^{\pi} \frac{x\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)}dx$

253. $J = \int_{1}^{e} \left(\frac{\tan^{-1}(x)}{x} + \frac{\log(x)}{x^2 + 1}\right)dx$

254. $K = \int_{0}^{2} \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}\,dx$

255. $I = \int_{0}^{\infty} \frac{1}{\sqrt{e^{2x} + e^x + 1}}dx$

256. $J = \int_{0}^{1} \left\lfloor \frac{1}{\sqrt{x}} \right\rfloor dx$

257. $K = \int_{0}^{\infty} \log\left(\frac{e^x + 1}{e^x - 1}\right)dx$

258. $I = \int_{0}^{\infty} e^{-\lfloor x \rfloor (1 + \{x\})}dx$

259. $J = \int_{0}^{\infty} i^{ix^2}dx \ , i = \sqrt{-1}$

260. $K = \int_{0}^{1} \frac{1}{x^2 + 1}dx$

261. $I = \int_{0}^{1} \frac{\log^3(1 - x^2)}{x}dx$

262. $J = \int_{0}^{1} \cos\left(\log(x)\right)dx$

263. $K = \int_{0}^{\pi} e^x \sin(x)dx$

264. $I = \int_{0}^{1} \frac{\eta(x)}{\zeta(x)}dx$

265. $J = \int_{-1}^{\infty} \frac{9x + 4}{4x^5 + 3x^2 + x}dx$

266. $K = \int_{0}^{1} \log(x)\log(1 - x)dx$

267. $I = \int_{0}^{1} \frac{\tan^{-1}(x)}{x + 1}dx$

268. $J = \int_{0}^{\pi} \frac{x + \sin(x) - \cos(x) - 1}{e^x + \sin(x) + x}dx$

269. $K = \int_{0}^{\pi} \frac{\sin(x)}{\sin(x) + \cos(x)}dx$

270. $I = \int_{0}^{\log(2)} \sqrt{\frac{e^x - 1}{e^x + 1}}\,dx$

271. $J = \int_{0}^{\infty} \frac{x^n}{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots}dx$

272. $K = \int_{0}^{\infty} \frac{x^n}{x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots}dx$

273. $I = \int_{0}^{\infty} \frac{\log(x)}{x^2 + y^2}dx$

274. $J = \int_{0}^{1} \frac{\log(x)}{\sqrt{1 - x^2}}dx$

275. $K = \int_{0}^{1} \frac{\log(x)}{1 + x}dx$

276. $I = \int_{0}^{1} \frac{\log(x)}{1 - x}dx$

277. $J = \int_{0}^{\infty} \log\left(\frac{e^x + 1}{e^x - 1}\right)dx$

278. $K = \int_{0}^{\infty} x^2 e^{-x}\cos(x)dx$

279. $I = \int_{0}^{\infty} (-1)^{ix^2}dx \ , i = \sqrt{-1}$

280. $J = \int_{0}^{\frac{\pi}{2}} \frac{x\sin(x)}{\cos(2x) + 1}dx$

281. $K = \int_{-1}^{1} \frac{1}{2025^x + 1}dx$

282. $I=\int_1^{\infty}\frac{\log^3(x)}{x^2(x-1)}dx$

283. $J=\int_{-\pi}^{\pi}\frac{x(\sin(x)+1)}{\cos^2(x)+1}dx$

284. $K=\int_0^{\pi}x\sin^6(x)dx$

285. $I=\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2dx$

286. $J=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[(x)!+(x-1)!](-x)!dx$

287. $K=\int_0^{\infty}\frac{\log(x)}{x^2+2x+4}dx$

288. $I=\int_0^{\infty}e^{-x^2}dx$

289. $J=\int_0^{\infty}\frac{\log(x+x^{-1})}{x^2+1}dx$

290. $K=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{\tan^4(x)+1}}dx$

291. $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\sin(x)\sin(2x)\right)^2dx$

292. $J=\int_0^{\infty}\frac{1}{(x^9+1)(x^2+1)}dx$

293. $K=\int_0^{\infty}2^{-3x^2}dx$

294. $I=\int_0^{2\pi}\frac{1}{2+\cos(x)}dx$

295. $J=\int_0^1\log\left(1-x^4\right)dx$

296. $K=\int_0^{\pi}\sec(x)\log\left(1+\frac{1}{2}\cos(x)\right)dx$

297. $I=\int_0^3\sqrt{\frac{x}{3-x}}dx$

298. $J=\int_0^3\{x\}^{\lfloor x\rfloor}dx$

299. $K=\int_0^1\frac{x^4-1}{x(x^4-5)(x^5-5x+1)}dx$

330. $I=\int_0^1\{x\}^x dx$

301. $J=\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{\cos(2x)}}{\sin(x)}\right)dx$

302. $K=\int_1^{\infty}\left(\left(\frac{x}{x+1}\right)^2\left(\frac{x-1}{x+1}\right)\left(\frac{1}{x+1}\right)\right)^2dx$

303. $I=\int_{\frac{1}{2}}^2\sqrt{\log^2(x)}dx$

304. $J=\int_0^{\infty}\frac{\sin^{2n+1}}{x}dx \quad ,\forall n\in\mathbb{N}$

305. $K=\int_0^{\pi}\frac{\sin(x)+x/2}{e^x+\sin(x)+\cos(x)+x+1}dx$

306. $I=\int_0^2 Max\left\{x,x^2\right\}dx$

307. $J=\int_{-2}^2 Max\left\{2x^2,x^2+1\right\}dx$

308. $I=\int_{-1}^1 Max\left\{1-x^2,x^2\right\}dx$

309. $K=\int_{-1}^1 Max\left\{x^2,\lfloor x\rfloor+1\right\}dx$

ផ្នែកចម្លើយ
និងដំណោះស្រាយ

01, Calculate integral $I = \int_{-1}^{0} \frac{x(x+2)}{\log(x+1)}dx$

Answer

They give $I = \int_{-1}^{0} \frac{x(x+2)}{\log(x+1)}dx$

$$= \int_{-1}^{0} \frac{(x+1)^2 - 1}{\log(x+1)}dx$$

$$let: u = x+1 \Rightarrow du = dx, if: x \in (-1,0) \Rightarrow u \in (0,1)$$

$$= \int_{0}^{1} \frac{u^2 - 1}{\log(u)}dx$$

$$\Rightarrow I(t) = \int_{0}^{1} \frac{u^t - 1}{\log(u)}dx$$

$$\Rightarrow I'(t) = \int_{0}^{1} \frac{\partial}{\partial t}\left(\frac{u^t - 1}{\log(u)}\right)du$$

$$= \int_{0}^{1} \frac{u^t \log(u)}{\log(u)}du = \int_{0}^{1} u^t du = \frac{1}{t+1}$$

$$\Rightarrow I(t) = \log(t+1) + C$$

$$if: t = 0 \Rightarrow I(0) = 0 = \log(0+1) + C \Rightarrow C = 0$$

$$if: t = 2 \Rightarrow I(2) = I = \log(2+1) \Rightarrow I = \log(3)$$

SO, $\boxed{\int_{-1}^{0} \frac{x(x+2)}{\log(x+1)}dx = \log(3)}$

02, Calculate integral $J = \int_{-\infty}^{+\infty} \frac{\cos(x)}{x^2+1}dx$

Answer

They give $J = \int_{-\infty}^{+\infty} \frac{\cos(x)}{x^2+1}dx$

$$B(t) = \int_{-\infty}^{+\infty} \frac{\cos(tx)}{x^2+1}dx$$

$$\Rightarrow J'(t)=\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\cos(tx)}{x^2+1}\right)dx$$

$$=\int_{-\infty}^{+\infty}\left(\frac{x\sin(tx)}{x^2+1}\right)dx$$

$$=\int_{-\infty}^{+\infty}\left(\frac{x^2\sin(tx)}{x(x^2+1)}\right)dx$$

$$=\int_{-\infty}^{+\infty}\left(\frac{\left((x^2+1)-1\right)\sin(tx)}{x(x^2+1)}\right)dx$$

$$=-\int_{-\infty}^{+\infty}\left(\frac{\sin(tx)}{x}\right)dx+\int_{-\infty}^{+\infty}\left(\frac{\sin(tx)}{x(x^2+1)}\right)dx$$

$$=-\pi+\int_{-\infty}^{+\infty}\left(\frac{\sin(tx)}{x(x^2+1)}\right)dx \quad (*)$$

$$\Rightarrow J''(t)=\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}\left(\frac{\sin(tx)}{x(x^2+1)}\right)dx$$

$$=\int_{-\infty}^{+\infty}\left(\frac{\cos(tx)}{x^2+1}\right)dx$$

$$=J(t)$$

$$\Leftrightarrow \underbrace{J''(t)-J(t)=0\Rightarrow J(t)=me^{t}+ne^{-t}}_{\text{According to the differential equation}}$$

$$J'(t)=me^{t}-ne^{-t}$$

$$But: J(0)=\int_{-\infty}^{+\infty}\frac{1}{x^2+1}dx=\pi=m+n \ (1)$$

$$Take:(*) \qquad J'(0)=-\pi=m-n \qquad (2)$$

$$Take:(1)\,\&\,(2):\begin{cases}\pi=m+n\\-\pi=m-n\end{cases} \quad That \quad m=0, n=\pi$$

$$\Rightarrow J(t)=0+\pi e^{-t} \qquad \left(but: J(t)=B(1)\right)$$

$$\Rightarrow J(1)=B=\pi e^{-1}$$

$$SO, \quad \boxed{\int_{-\infty}^{+\infty}\frac{\cos(x)}{x^2+1}dx=\frac{\pi}{e}}$$

03, Calculate integral $K=\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(x)}{1+2030^x}dx$

Answer

They give $$K=\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(x)}{1+2030^x}dx \quad (1)$$

$$=\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(-x)}{1+2030^{-x}}dx \quad ,Because:\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$$

$$=\int_{-45^\circ}^{+45^\circ}\frac{2030^x\tan^2(x)}{1+2030^x}dx \quad (2)$$

$$Take:(1)+(2):They\ have\quad 2K=\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(x)}{1+2030^x}dx+\int_{-45^\circ}^{+45^\circ}\frac{2030^x\tan^2(x)}{1+2030^x}dx$$

$$\Rightarrow K=\frac{1}{2}\int_{-45^\circ}^{+45^\circ}\tan^2(x)dx=\frac{1}{2}\int_{-45^\circ}^{+45^\circ}\left(1+\tan^2(x)-1\right)dx$$

$$=\frac{1}{2}\int_{-45^\circ}^{+45^\circ}\left[\left(\tan(x)\right)'-(x)'\right]dx=\frac{1}{2}\left[\tan(x)-x\right]_{-45^\circ}^{+45^\circ}$$

$$=\frac{4-\pi}{4} \qquad Note:\int_a^b f'(x)dx=f(b)-f(a)$$

SO, $$\boxed{\int_{-45^\circ}^{+45^\circ}\frac{\tan^2(x)}{1+2030^x}dx=\frac{4-\pi}{4}}$$

04, Calculate integral $I=\int_{-\infty}^{+\infty}\frac{\sin(x)}{x}dx$

Answer

They give $$I=\underbrace{\int_{-\infty}^{+\infty}\frac{\sin(x)}{x}dx}_{is\ an\ even\ function}$$

$$\Rightarrow I(t)=2\int_0^{+\infty}\frac{\sin(x)e^{-tx}}{x}dx$$

$$\frac{\partial}{\partial t}I(t)=2\int_0^{+\infty}\frac{\partial}{\partial t}\left(\frac{\sin(x)e^{-tx}}{x}\right)dx$$

$$\Rightarrow I'(t)=-2\int_0^{+\infty}\frac{x\sin(x)e^{-tx}}{x}dx$$

$$=-2\int_0^{+\infty}e^{-tx}\sin(x)dx$$

$$\Rightarrow I'(t) = -2\int_0^{+\infty} \frac{x\sin(x)e^{-tx}}{x}dx$$

$$= -2\int_0^{+\infty} e^{-tx}\sin(x)dx$$

$$= -2\left[\frac{t\sin(x)+\cos(x)}{t^2+1}e^{-tx}\right]_0^{+\infty}$$

$$= -2\left(\frac{1}{t^2+1}\right)$$

$$\Rightarrow I(t) = -2\arctan(t) + C$$

$if: t = +\infty \Rightarrow I(+\infty) = 0 = -2\arctan(+\infty) + C \Rightarrow C = \pi$

$if: t = 0 \Rightarrow I(0) = D = 0 + C$

$$\Rightarrow I = \pi$$

SO, $\boxed{\int_{-\infty}^{+\infty} \frac{\sin(x)}{x}dx = \pi}$

05, Calculate integral $J = \int_0^1 \frac{x^n - 1}{\log(x)}dx \quad , n \geq 0$

Answer

They give $J = \int_0^1 \frac{x^n - 1}{\log(x)}dx \quad , n \geq 0$

$$\Rightarrow J(t) = \int_0^1 \frac{x^t - 1}{\log(x)}dx$$

$$\Rightarrow J'(t) = \int_0^1 \frac{\partial}{\partial t}\left(\frac{x^t - 1}{\log(x)}\right)dx$$

$$= \int_0^1 \left(\frac{x^t\log(x)}{\log(x)}\right)dx = \int_0^1 x^t dx = \frac{1}{t+1}$$

$$\Rightarrow J(t) = \log(t+1) + C$$

$if: J(0) = 0 = \log(0+1) + C \Rightarrow C = 0$

$if: J(n) = J = \log(n+1)$

SO, $\boxed{\int_0^1 \frac{x^n - 1}{\log(x)}dx = \log(n+1)}$

06, Calculate integral $K=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{a+\cos(x)}dx$

Answer

They give $K=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{n+\cos(x)}dx$

$$=\frac{2}{2\pi}\int_0^{\pi}\frac{1}{n+\cos(x)}dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\frac{1}{n+\cos(x)}dx \quad ,Use:\begin{cases} f(2\pi-x)=f(x) \\ \int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx \end{cases}$$

$$let: t=\tan\left(\frac{x}{2}\right)\Rightarrow dt=2\left(1+\tan^2\left(\frac{x}{2}\right)\right)dx \quad ,if\ : x\in(0,\pi)\Rightarrow t\in(0,\infty)$$

$$\Leftrightarrow dx=\frac{2}{1+t^2}dt \quad ,Note:\cos(x)=\frac{1-t^2}{1+t^2}$$

$$\Rightarrow K=\frac{1}{\pi}\int_0^{\infty}\left(\frac{1}{n+\frac{1-t^2}{1+t^2}}\right)\left(\frac{2}{1+t^2}\right)dt$$

$$=\frac{2}{\pi}\int_0^{\infty}\frac{1}{n(1+t^2)+1-t^2}dt$$

$$=\frac{2}{\pi(n-1)}\int_0^{\infty}\frac{1}{\left(\frac{n+1}{n-1}\right)+t^2}dt$$

$$=\frac{2}{\pi(n-1)}.\sqrt{\frac{n-1}{n+1}}\left[\arctan\left(t\sqrt{\frac{n-1}{n+1}}\right)\right]_0^{\infty}$$

$$=\frac{2}{\pi}.\frac{1}{\sqrt{n^2-1}}.[\arctan(\infty)-\arctan(0)]$$

$$=\frac{1}{\sqrt{n^2-1}}$$

SO, $\boxed{\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{n+\cos(x)}dx=\frac{1}{\sqrt{n^2-1}}}$

07, Calculate integral $I = \int_0^{+\infty} \frac{1}{x^4 + x^2 + 1} dx$

Answer

They give $\quad I = \int_0^{+\infty} \frac{1}{x^4 + x^2 + 1} dx \quad (*)$

$let: x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt, If: x \in (0, +\infty) \Rightarrow t \in (+\infty, 0)$

$$\Rightarrow I = -\int_0^{+\infty} \frac{1}{\frac{1}{t^4} + \frac{1}{t^2} + 1} \times \frac{1}{t^2} dx = \int_0^{+\infty} \frac{t^2}{t^4 + t^2 + 1} dx \qquad (**)$$

$Take: (*) + (**) They\ have:\ 2I = \int_0^{+\infty} \frac{t^2 + 1}{t^4 + t^2 + 1} dx \quad, Note: \int_a^b f(x)dx = \int_a^b f(t)dt$

$$= \int_0^{+\infty} \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right) + 3} dt = \int_0^{+\infty} \frac{\left(t - \frac{1}{t}\right)'}{\left(t - \frac{1}{t}\right)^2 + 3} dt$$

$$= \frac{1}{\sqrt{3}} \left[\arctan\left(\frac{t - t^{-1}}{\sqrt{3}} \right) \right]_0^{+\infty} = \frac{\pi}{\sqrt{3}}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{3}}$$

SO, $\boxed{\int_0^{+\infty} \frac{1}{x^4 + x^2 + 1} dx = \frac{\pi}{2\sqrt{3}}}$

008, Calculate integral $J = \int_0^{\frac{\pi}{2}} \log(\sin(x)) dx$

Answer

They give $\quad J = \int_0^{\frac{\pi}{2}} \log(\sin(x)) dx \quad (*)$

$$= \int_0^{\frac{\pi}{2}} \log\left(\sin(\frac{\pi}{2} - x) \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log(\cos(x)) dx \quad (**)$$

$Take: (*) + (**) They\ have: 2J = \int_0^{\frac{\pi}{2}} \log(\sin(x)\cos(x)) dx$

$$=\int_0^{\frac{\pi}{2}}\log\left(\frac{1}{2}\sin(2x)\right)dx=\int_0^{\frac{\pi}{2}}\left(\log\left(\sin(2x)\right)-\log(2)\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\left(\sin(2x)\right)dx-\frac{\pi}{2}\log(2)$$

$$2J+\frac{\pi}{2}\log(2)=\int_0^{\frac{\pi}{2}}\log\left(\sin(2x)\right)dx \quad ,Take:\begin{cases} let:t=2x\Rightarrow dx=\frac{1}{2}dt \\ if\ :x\in\left(0,\frac{\pi}{2}\right)\Rightarrow t\in\left(0,\pi\right)\end{cases}$$

$$\Rightarrow 2J+\frac{\pi}{2}=\frac{1}{2}\int_0^{\pi}\log\left(\sin(t)\right)dt$$

$$=\frac{1}{2}.2\int_0^{\frac{\pi}{2}}\log\left(\sin(t)\right)dt \quad ,Take:\begin{cases} f\left(2a-t\right)=f\left(t\right) \\ \int_0^{2a}f\left(t\right)dt=2\int_0^{a}f\left(t\right)dt\end{cases}$$

$$2J+\frac{\pi}{2}\log(2)=J\Rightarrow J=-\frac{\pi}{2}\log(2)$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx=\int_0^{\frac{\pi}{2}}\log\left(\cos(x)\right)dx=-\frac{\pi}{2}\log(2)}$$

09, Calculate integral $K=\int_0^{+\infty}\frac{\log(x^2+1)}{(x^2+1)}dx$

Answer

They give $K=\int_0^{+\infty}\frac{\log(x^2+1)}{(x^2+1)}dx$

$$let:x=\tan(u)\Rightarrow dx=\frac{1}{\cos^2(u)}du\ ,if\ \ :x\in\left(0,+\infty\right)\Rightarrow t\in\left(0,\frac{\pi}{2}\right)$$

$$K=\int_0^{\frac{\pi}{2}}\frac{\log\left(\tan^2(u)+1\right)}{\left(\tan^2(u)+1\right)}.\frac{1}{\cos^2(x)}du$$

$$=\int_0^{\frac{\pi}{2}}\log\left(\frac{1}{\cos^2(u)}\right)du=\int_0^{\frac{\pi}{2}}\log\left(\cos^{-2}(u)\right)du$$

$$=-2\int_0^{\frac{\pi}{2}}\log\left(\cos(u)\right)du=-2\left(-\frac{\pi}{2}\log(2)\right)=\pi\log(2)$$

SO, $$\boxed{\int_0^{+\infty}\frac{\log(x^2+1)}{(x^2+1)}dx=\pi\log(2)}$$

010, Calculate integral $I=\int_0^1 \frac{\log(x+1)}{(x^2+1)}dx$

Answer

They give $I=\int_0^1 \frac{\log(x+1)}{(x^2+1)}du$

$$let: x=\tan(u) \Rightarrow dx=\left(\tan^2(u)+1\right)du \text{ ,if } : x\in(0,1) \Rightarrow \in\left(0,\frac{\pi}{4}\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{\log(\tan(u)+1)}{(\tan^2(u)+1)}.\left(\tan^2(u)+1\right)dx=\int_0^{\frac{\pi}{4}}\log(\tan(u)+1)du$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\tan\left(\frac{\pi}{4}-u\right)+1\right)du \quad ,Use: \int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\frac{1-\tan(u)}{1+\tan(u)}+1\right)du=\int_0^{\frac{\pi}{4}}\log\left(\frac{1-\tan(u)+1+\tan(u)}{1+\tan(u)}\right)du$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan(u)}\right)du=\int_0^{\frac{\pi}{4}}\log(2)du-\int_0^{\frac{\pi}{4}}\log(\tan(u)+1)du$$

$$\Leftrightarrow I=\frac{\pi}{4}\log(2)-J \Rightarrow J=\frac{\pi}{8}\log(2)$$

SO, $\boxed{\int_0^1 \frac{\log(x+1)}{(x^2+1)}dx=\frac{\pi}{8}\log(2)}$

011, Calculate integral $I=\int_0^{\frac{\pi}{2}} \frac{x}{\tan(x)}dx$

Answer

They give $I=\int_0^{\frac{\pi}{2}} \frac{x}{\tan(x)}dx$

$$=\int_0^{\frac{\pi}{2}} \frac{\arctan(\tan(x))}{\tan(x)}dx$$

$$\Rightarrow I(a)=\int_0^{\frac{\pi}{2}} \frac{\arctan(a\tan(x))}{\tan(x)}dx$$

$$\Rightarrow I'(a)=\int_0^{\frac{\pi}{2}} \frac{\arctan(a\tan(x))}{\tan(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\tan(x)}{\tan(x)\left[1+\left(a\tan(x)\right)^2\right]}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\left(1+\tan^2(x)\right)\left[1+\left(a\tan(x)\right)^2\right]}d\left(\tan(x)\right)$$

$$=\int_0^{\frac{\pi}{2}}\left(\frac{\frac{a^2}{a^2-1}}{1+\left(a\tan(x)\right)^2}-\frac{\frac{1}{a^2-1}}{1+\tan^2(x)}\right)d\left(\tan(x)\right)$$

$$=\frac{1}{a^2+1}\left[a\left(\arctan\left(a\tan(x)\right)\right)-\arctan\left(\tan(x)\right)\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{a^2+1}\left(a\frac{\pi}{2}-\frac{\pi}{2}\right)=\frac{\pi}{2}\times\frac{1}{a+1}$$

$$\Rightarrow I(a)=\frac{\pi}{2}\log(a+1)+c\quad,\begin{cases} if: a=0\Rightarrow K(0)=0=C\\ if: a=1\Rightarrow K(1)=K=\frac{\pi}{2}\log(2)\end{cases}$$

$$SO,\ \boxed{\int_0^{\frac{\pi}{2}}\frac{x}{\tan(x)}dx=\frac{\pi}{2}\log(2)}$$

012, Calculate integral $J=\int_0^{2\pi}\left(|\sin(x)|+|\cos(x)|\right)dx$

Answer

They give $J=\int_0^{2\pi}\left(|\sin(x)|+|\cos(x)|\right)dx$

$$=\int_0^{2\pi}\left(|\sin(x)|\right)dx+\int_0^{2\pi}\left(|\cos(x)|\right)dx$$

$Mathod:1$

$$for:\int_0^{2\pi}|\sin(x)|dx=\int_0^{\pi}\sin(x)dx-\int_{\pi}^{2\pi}\sin(x)dx$$

$$=-[\cos(x)]_0^{\pi}+[\cos(x)]_{\pi}^{2\pi}=4$$

$$for:\int_0^{2\pi}|\cos(x)|dx=\int_0^{\frac{\pi}{2}}\cos(x)dx-\int_{\frac{\pi}{2}}^{\pi}\cos(x)dx-\int_{\pi}^{\frac{3\pi}{2}}\cos(x)dx+\int_{\frac{3\pi}{2}}^{2\pi}\cos(x)dx$$

$$=[\sin(x)]_0^{\frac{\pi}{2}}-[\sin(x)]_{\frac{\pi}{2}}^{\pi}-[\sin n(x)]_{\pi}^{\frac{3\pi}{2}}+[\sin(x)]_{\frac{3\pi}{2}}^{2\pi}=4$$

$$\Rightarrow J=4+4=8$$

Mathod : 2

$$for: \int_0^{2\pi} |\sin(x)|dx = 2\int_0^{\pi} |\sin(x)|dx$$

$$= 2\int_0^{\pi} \sin(x)dx = 2\times 2 = 4$$

$$for: \int_0^{2\pi} |\cos(x)|dx = 2\int_0^{\pi} |\cos(x)|dx$$

$$= 2\int_0^{\pi} \cos(x)dx = 2\int_0^{\frac{\pi}{2}} \cos(x)dx - 2\int_{\frac{\pi}{2}}^{\pi} \cos(x)dx = 4$$

$$\Rightarrow J = 4+4 = 8 \qquad Note: \begin{cases} \int_0^{n\pi} |\sin(x)|\,dx = n\int_0^{\pi} |\sin(x)|\,dx \\ \int_0^{n\pi} |\cos(x)|\,dx = n\int_0^{\pi} |\cos(x)|\,dx \end{cases}$$

$$SO, \quad \boxed{\int_0^{2\pi} \left(|\sin(x)| + |\cos(x)|\right)dx = 8}$$

013, Calculate integral $K = \int_{-1}^{+1} x\arcsin(x)\arccos(x)dx$

Answer

$$They\ give \quad K = \int_{-1}^{+1} x\arcsin(x)\arccos(x)dx$$

$$= \int_{-1}^{+1} (-x)\arcsin(-x)\arccos(-x)dx$$

$$Note: \arcsin(-x) = -\arcsin(x)\ , \arccos(-x) = \pi - \arccos(x)$$

$$\Rightarrow K = \int_{-1}^{+1} x\arcsin(x)\left(\pi - \arccos(x)\right)dx$$

$$= \pi\int_{-1}^{+1} x\arcsin(x)dx - \int_{-1}^{+1} x\arcsin(x)\arccos(x)dx$$

$$\Rightarrow K = \frac{\pi}{2}\int_{-1}^{+1} x\arcsin(x)dx = \pi\underbrace{\int_0^1 x\arcsin(x)dx}_{is\ an\ even\ function}$$

$$let: u = \arcsin(x) \Rightarrow du = \frac{1}{\sqrt{1-x^2}}dx\ , dv = x \Rightarrow v = \frac{x^2}{2}$$

$$\Rightarrow K = \frac{\pi}{2}\left[x^2\arcsin(x)\Big|_0^1 - \int_0^1 \frac{x^2}{\sqrt{1-x^2}}dx\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{2} + \int_0^1 \frac{1-x^2-1}{\sqrt{1-x^2}}dx\right]$$

$$=\frac{\pi}{2}\left[\frac{\pi}{2}+\int_0^1\sqrt{1-x^2}\,dx-\int_0^1\frac{1}{\sqrt{1-x^2}}\,dx\right]$$

$$=\frac{\pi}{2}\left[\frac{\pi}{2}+K'-\arcsin(x)\Big|_0^1\right]=\frac{\pi}{2}(K')\ \ (*)$$

$$for: K'=\int_0^1\sqrt{1-x^2}\,dx$$

$$Let: x=\sin(t)\Rightarrow dx=\cos(t)dt,\ If\ \ : x\in(0,1)\Rightarrow t\in\left(0,\frac{\pi}{2}\right)$$

$$=\frac{\pi}{2}\left[\frac{\pi}{2}+\int_0^1\frac{1-x^2-1}{\sqrt{1-x^2}}\,dx\right]$$

$$\Rightarrow K'=\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2(t)}\cos(t)dt$$

$$=\int_0^{\frac{\pi}{2}}\cos^2(t)dt=\frac{1}{2}\int_0^{\frac{\pi}{2}}\big(1+\cos(2t)\big)dt=\frac{\pi}{4}$$

$$Take\,(*)\ that\ have: K=\frac{\pi}{2}\times\frac{\pi}{4}=\frac{\pi^2}{8}$$

$$SO,\quad \boxed{\int_{-1}^{+1}x\arcsin(x)\arccos(x)dx=\frac{\pi^2}{8}}$$

014, Calculate integral $I=\int_0^{\frac{\pi}{4}}\frac{\log\left(\sqrt{1-\tan^2(x)}\right)}{\sqrt{1-\tan^2(x)}}\big(1+\tan^2(x)\big)dx$

Answer

$$They\ give\quad I=\int_0^{\frac{\pi}{4}}\frac{\log\left(\sqrt{1-\tan^2(x)}\right)}{\sqrt{1-\tan^2(x)}}\big(1+\tan^2(x)\big)dx$$

$$let: \tan(x)=\sin(t)\Rightarrow\big(1+\tan^2(x)\big)dx=\cos(t)dt, if\ \ : x\in\left(0,\frac{\pi}{4}\right)\Rightarrow t\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\log\left(\sqrt{1-\sin^2(t)}\right)}{\sqrt{1-\sin^2(t)}}\cos(t)dt=\int_0^{\frac{\pi}{2}}\log\big(\cos(t)\big)dt=-\frac{\pi}{2}\log(2)$$

$$SO,\quad \boxed{\int_0^{\frac{\pi}{4}}\frac{\log\left(\sqrt{1-\tan^2(x)}\right)}{\sqrt{1-\tan^2(x)}}\big(1+\tan^2(x)\big)=-\frac{\pi}{2}\log(2)}$$

015, Calculate integral $J=\int_0^1 (1-x)^5\,dx$

Answer

They give $J=\int_0^1 (1-x)^5\,dx$

$$\Rightarrow J=\int_0^1 \sum_{k=0}^{5} C_5^k\,(1)^{5-k}(-x)^k dx \quad ,Because:(a+b)^n=\sum_{k=0}^{n} C_n^k\,(a)^{n-k}(b)^k$$

$$=\sum_{k=0}^{5} C_5^k\,(-1)^k\int_0^1 (x)^k dx=\sum_{k=0}^{5} C_5^k\,(-1)^k\left.\frac{x^{k+1}}{k+1}\right|_0^1$$

$$=\sum_{k=0}^{5} C_5^k\,(-1)^k\frac{x^{k+1}}{k+1}=\frac{1}{6}$$

SO, $\boxed{\int_0^1 (x-1)^5\,dx=\frac{1}{6}}$

016, Calculate integral $K=\int_0^{+\infty}\frac{x\tan^{-1}(x)}{x^4+x^2+1}dx$

They give $K=\int_0^{+\infty}\frac{x\tan^{-1}(x)}{x^4+x^2+1}dx \quad (*)$

$$let: x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt\,,if\ :x\in(0,+\infty)\Rightarrow t\in(+\infty,0)$$

$$\Rightarrow K=-\int_{+\infty}^{0}\frac{\frac{1}{t}\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t^4}+\frac{1}{t^2}+1}\times\frac{1}{t^2}dt=\int_0^{+\infty}\frac{t\tan^{-1}\left(\frac{1}{t}\right)}{t^4+t^2+1}dt \quad (**)$$

$$Take:(*)+(**)\ That\ have:2K=\int_0^{+\infty}\frac{t\left[\tan^{-1}(t)+\tan^{-1}\left(\frac{1}{t}\right)\right]}{t^4+t^2+1}dt$$

$$By:\tan^{-1}(t)+\tan^{-1}\left(\frac{1}{t}\right)=\frac{\pi}{2}\ \ That\ \ K=\frac{\pi}{4}\int_0^{+\infty}\frac{t}{t^4+t^2+1}dt=\frac{\pi}{8}\int_0^{+\infty}\frac{1}{t^4+t^2+1}d\left(t^2\right)$$

$$=\frac{\pi}{4\sqrt{3}}\arctan\left.\left(\frac{2}{\sqrt{3}}t^2+\frac{1}{2}\right)\right|_0^{+\infty}=\frac{\pi^2}{12\sqrt{3}}$$

SO, $\boxed{\int_0^{+\infty}\frac{x\tan^{-1}(x)}{x^4+x^2+1}dx=\frac{\pi^2}{12\sqrt{3}}}$

017, Calculate integral $I=\int_2^4 \frac{\binom{x}{1}\binom{x}{3}\binom{x}{5}}{\binom{x}{2}\binom{x}{4}\binom{x}{6}}dx$

Answer

They give $I=\int_2^4 \frac{\binom{x}{1}\binom{x}{3}\binom{x}{5}}{\binom{x}{2}\binom{x}{4}\binom{x}{6}}dx$

$$=\int_2^4 \frac{\frac{x!}{(x-1)!.1!}}{\frac{x!}{(x-2)!.2!}}.\frac{\frac{x!}{(x-3)!.3!}}{\frac{x!}{(x-4)!.4!}}.\frac{\frac{x!}{(x-5)!.5!}}{\frac{x!}{(x-6)!.6!}}dx$$

$$=\frac{2!.4!.6!}{3!.5!}\int_2^4 \frac{x}{x(x-1)}.\frac{x(x-1)(x-2)}{x(x-1)(x-2)(x-3)}.\frac{x(x-1)(x-2)(x-3)(x-4)}{x(x-1)(x-2)(x-3)(x-4)(x-5)}dx$$

$$=48\int_2^4 \frac{1}{(x-1)(x-3)(x-5)}dx$$

$let: t=x-3 \Rightarrow dt=dx, \left(x-1=t+2, x-5=t-2\right)$

$if\ : x\in\left(2,4\right)\Rightarrow t\in\left(-1,1\right)$

$$\Rightarrow I=48\int_{-1}^1 \frac{1}{t(t+2)(t-2)}dx$$

$$=48\int_{-1}^1 \frac{1}{t(t^2-4)}dx$$

$by: f(-t)=-f(t)\ that\ f(x)\ an\,odd\ function\,on\,a\,space\ \left[-1,1\right]$

$$\Rightarrow I=0$$

SO, $\boxed{\int_2^4 \frac{\binom{x}{1}\binom{x}{3}\binom{x}{5}}{\binom{x}{2}\binom{x}{4}\binom{x}{6}}dx=0}$

$Note: C\left(n,r\right)=C_n^r=\binom{n}{r}=\frac{n!}{(n-r)!.r!}\quad ,n>r\wedge r\ge 0$

018, Calculate integral $J=\int_0^1 \frac{\log(x)}{x^2-1}dx$

Answer

They give $J=\int_0^1 \frac{\log(x)}{x^2-1}dx$

$$we\ have: \frac{1}{1-x^2}=\sum_{k=0}^{\infty} x^{2k} \Rightarrow \frac{1}{x^2-1}=-\sum_{k=0}^{\infty} x^{2k}$$

$$\Rightarrow J=-\sum_{k=0}^{\infty}\int_0^1 x^{2k}\log(x)dx$$

$$Let: u=\log(x)\Rightarrow du=\frac{1}{x}dx\ ,dv=x^{2k}dx \Leftrightarrow v=\frac{x^{2k+1}}{2k+1}$$

$$\Rightarrow I=-\sum_{k=0}^{\infty}\left[\underbrace{\frac{x^{2k+1}}{2k+1}\log(x)\Big|_0^1}_{0}-\frac{1}{2k+1}\int_0^1\frac{x^{2k+1}}{x}dx\right]=\sum_{k=0}^{\infty}\left[\frac{1}{2k+1}\times\frac{x^{2k+1}}{2k+1}\Big|_0^1\right]$$

$$=\sum_{k=0}^{\infty}\left[\frac{1}{(2k+1)^2}\right]=\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)$$

$$=\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)-\frac{1}{4}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)$$

$$=\frac{3}{4}\zeta(2)=\frac{3}{4}.\frac{\pi^2}{6}=\frac{\pi^2}{8}$$

SO, $\boxed{\int_0^1\frac{\log(x)}{x^2-1}dx=\frac{\pi^2}{8}}$

019, Calculate integral $K=\int_0^{\pi}\frac{\sin^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx$

Answer

$$They\ give\ K=\int_0^{\pi}\frac{\sin^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx$$

$$=2\int_0^{\frac{\pi}{2}}\frac{\sin^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx\ \ (*)\quad ,Because:\begin{cases} f(2a-t)=f(t)\\ \int_0^{2a} f(t)dt=2\int_0^{a} f(t)dt\end{cases}$$

$$=2\int_0^{\frac{\pi}{2}}\frac{\sin^{2030}\left(\frac{\pi}{2}-x\right)}{\sin^{2030}\left(\frac{\pi}{2}-x\right)+\cos^{2030}\left(\frac{\pi}{2}-x\right)}dx \quad ,Use:\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$$

$$=2\int_0^{\frac{\pi}{2}}\frac{\cos^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx \quad (**)$$

$$Take:(*)+(**)\ That\ have:2K=2\int_0^{\frac{\pi}{2}}\frac{\sin^{2030}(x)+\cos^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx$$

$$\Rightarrow K=x\Big|_0^{\pi/2}=\frac{\pi}{2}$$

SO, $$\boxed{\int_0^{\pi}\frac{\sin^{2030}(x)}{\sin^{2030}(x)+\cos^{2030}(x)}dx=\frac{\pi}{2}}$$

020, Calculate integral $$I=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(x)}{e^{\frac{1}{x}}+1}dx$$

Answer

They give $$I=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(x)}{e^{\frac{1}{x}}+1}dx \quad (*)$$

$$=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(-x)}{e^{-\frac{1}{x}}+1}dx \quad ,Use:\int_{-a}^{+a} f(x)dx=\int_{-a}^{+a} f(-x)dx$$

$$I=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{e^{\frac{1}{x}}\cos(x)}{e^{\frac{1}{x}}+1}dx \quad (**)$$

$$Take:(*)+(**)\ that\ have:\ 2I=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{e^{\frac{1}{x}}\cos(x)}{e^{\frac{1}{x}}+1}dx+\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(x)}{e^{\frac{1}{x}}+1}dx$$

$$=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos(x)dx=\left[\sin(x)\right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}=2$$

$$\Rightarrow I=1$$

SO, $$\boxed{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos(x)}{e^{\frac{1}{x}}+1}dx=1}$$

021, Calculate integral $J=\int_0^{\frac{\pi}{2}}\sqrt{\sin(x)-\sin^2(x)}dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\sqrt{\sin(x)-\sin^2(x)}dx$

$$=\int_0^{\frac{\pi}{2}}\sqrt{\sin(x)}\sqrt{1-\sin(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin(x)}}{\sqrt{1+\sin(x)}}\cos(x)dx$$

$$Let: t=\sin(x)\Rightarrow dt=\cos(x)dx, if: x\in\left(0,\frac{\pi}{2}\right)\Rightarrow t\in(0,1)$$

$$\Rightarrow J=\int_0^1\frac{\sqrt{t}}{\sqrt{1+t}}dt$$

$$Let: y=\sqrt{\frac{t}{t+1}}\Leftrightarrow t=\frac{y^2}{1-y^2}\Rightarrow dt=d\left(\frac{y^2}{1-y^2}\right), if: t\in(0,1)\Rightarrow y\in\left(0,\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow J=\int_0^{\frac{\sqrt{2}}{2}}y\,d\left(\frac{y^2}{1-y^2}\right)$$

$$Let: u=y\Rightarrow du=dy\;, dv=d\left(\frac{y^2}{1-y^2}\right)\Rightarrow v=\frac{y^2}{1-y^2}$$

$$\Rightarrow J=y.\frac{y^2}{1-y^2}\Bigg|_0^{\frac{\sqrt{2}}{2}}-\int_0^{\frac{\sqrt{2}}{2}}\frac{y^2}{1-y^2}dy$$

$$=\frac{\sqrt{2}}{2}+\int_0^{\frac{\sqrt{2}}{2}}\frac{1-y^2+1}{1-y^2}dy$$

$$=\frac{\sqrt{2}}{2}+\left[y-\tanh^{-1}(y)\right]_0^{\frac{\sqrt{2}}{2}}$$

$$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-\frac{1}{2}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

$$=\sqrt{2}-\log\left(\sqrt{2}+1\right)$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\sqrt{\sin(x)-\sin^2(x)}dx=\sqrt{2}-\log\left(\sqrt{2}+1\right)}$

022, Calculate integral $K=\int_0^{\frac{\pi}{4}} e^x\left(\tan^2(x)+\log\left(\cos(x)\right)\right)dx$

Answer

$$\text{They give}\quad K=\int_0^{\frac{\pi}{4}} e^x\left(\tan^2(x)+\log\left(\cos(x)\right)\right)dx$$

$$=\int_0^{\frac{\pi}{4}} e^x\left(\sec^2(x)-1+\log\left(\cos(x)\right)\right)dx$$

$$=-\int_0^{\frac{\pi}{4}} e^x dx+\int_0^{\frac{\pi}{4}} e^x\sec^2(x)dx+\int_0^{\frac{\pi}{4}} e^x\log(\cos(x))dx$$

$$=1-e^{\frac{\pi}{4}}+V'+V''$$

$$For: K'=\int_0^{\frac{\pi}{4}} e^x\sec^2(x)dx$$

$$Let:\begin{cases} u=e^x\Rightarrow du=e^x dx \\ dv=\sec^2(x)dx\Rightarrow v=\tan(x)\end{cases}$$

$$\Rightarrow K'=\left[e^x\tan(x)\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} e^x\tan(x)dx$$

$$For: K''=\int_0^{\frac{\pi}{4}} e^x\log(\cos(x))dx$$

$$Let:\begin{cases} u=\log\left(\cos(x)\right)\Rightarrow du=-\tan(x)dx \\ dv=e^x dx\Rightarrow v=e^x\end{cases}$$

$$\Rightarrow K''=\left[e^x\log\left(\cos(x)\right)\right]_0^{\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^x\tan(x)dx$$

$$\Rightarrow K\ =1-e^{\frac{\pi}{4}}+\left(\left[e^x\tan(x)\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} e^x\tan(x)dx\right)+\left(\left[e^x\log\left(\cos(x)\right)\right]_0^{\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^x\tan(x)dx\right)$$

$$=1-e^{\frac{\pi}{4}}+e^{\frac{\pi}{4}}+e^{\frac{\pi}{4}}\log\left(\frac{\sqrt{2}}{2}\right)$$

$$=1-\frac{1}{2}e^{\frac{\pi}{4}}\log(2)$$

$$SO,\quad \boxed{\int_0^{\frac{\pi}{4}} e^x\left(\tan^2(x)+\log\left(\cos(x)\right)\right)dx=1-\frac{1}{2}e^{\frac{\pi}{4}}\log(2)}$$

023, Calculate integral $I=\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)+\cos(x)}dx$

Answer

They give $I=\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)+\cos(x)}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx \quad ,Use:\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)+\cos(x)}dx-\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)+\cos(x)}dx$$

$$I=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)+\cos(x)}dx-I$$

$$\Rightarrow I=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)+\cos(x)}dx$$

Let : $u=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+u^2}du$, If $:x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(0,1)$

And : $\cos(x)=\frac{1-u^2}{1+u^2}$, $\sin(x)=\frac{2u}{1+u^2}$

$$\Rightarrow I=\frac{\pi}{4}\int_0^1\frac{1}{\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}\times\frac{2}{1+u^2}du$$

$$=\frac{\pi}{2}\int_0^1\frac{1}{1+2u-u^2}du$$

$$=\frac{\pi}{2}\times\left(\frac{1}{2\sqrt{2}}\log\left(\frac{\sqrt{2}+u}{\sqrt{2}-u}\right)\right)\Bigg|_0^1$$

$$=\frac{\pi\sqrt{2}}{8}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)+\cos(x)}dx=\frac{\pi\sqrt{2}}{4}\log\left(\sqrt{2}+1\right)}$

024, Calculate integral $J=\int_0^{\pi}\frac{x}{\cos^2(x)+9\sin^2(x)}dx$

Answer

They give $J=\int_0^{\pi}\frac{x}{\cos^2(x)+9\sin^2(x)}dx$

$$=\int_0^{\pi}\frac{\pi-x}{\cos^2(\pi-x)+9\sin^2(\pi-x)}dx \quad ,Use:\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$\Rightarrow J=\frac{\pi}{2}\int_0^{\pi}\frac{1}{\cos^2(x)+9\sin^2(x)}dx$$

$$Let:u=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+u^2}du \ \ ,If\ :x\in(0,\pi)\Rightarrow u\in(0,+\infty)$$

$$\Rightarrow J=\frac{\pi}{2}\int_0^{+\infty}\frac{1}{\left(\frac{1-u^2}{1+u^2}\right)^2+9\left(\frac{2u}{1+u^2}\right)^2}\times\frac{2}{1+u^2}du$$

$$=\pi\int_0^{+\infty}\frac{1+u^2}{\left(u-u^{-1}\right)^2+36}du$$

$$Let:v=u-u^{-1}\Rightarrow dv=(1+u^{-2})du \ ,If\ :u\in(0,+\infty)\Rightarrow v\in(-\infty,+\infty)$$

$$=\pi\int_{-\infty}^{+\infty}\frac{v}{v^2+36}dv=\frac{\pi}{6}\tan^{-1}\left(\frac{v}{6}\right)\Bigg|_{-\infty}^{+\infty}=\frac{\pi^2}{6}$$

SO, $\boxed{\int_0^{\pi}\frac{x}{\cos^2(x)+9\sin^2(x)}dx=\frac{\pi^2}{6}}$

025, Calculate integral $K=\int_{-1}^{0}\frac{x^2+x}{\left(e^x+x+1\right)^2}dx$

Answer

They give $K=\int_0^{+\infty}\frac{x^2+x}{\left(e^x+x+1\right)^2}dx$

$$=\int_0^{+\infty}\frac{(x+1)e^{-x}}{\left(1+(x+1)e^{-x}\right)^2}xe^{-x}dx \qquad ,\begin{cases}let:u=(x+1)e^{-x}\Rightarrow du=-xe^{-x}dx\\ if\ \ :x\in(0,+\infty)\Rightarrow u\in(1,0)\end{cases}$$

$$\Rightarrow K=-\int_1^0\frac{u}{(1+u)^2}du=\int_0^1\frac{(1+u)-1}{(1+u)^2}du=\log(2)-\frac{1}{2}$$

SO, $\boxed{\int_0^{+\infty}\frac{x^2+x}{\left(e^x+x+1\right)^2}dx=\log(2)-\frac{1}{2}}$

026, Calculate integral $I=\int_0^1 \frac{W\left(\frac{x\log(3)}{e^{-x\log(3)}}\right)}{e^{-x\log(3)}}dx$

Answer

They give $$I=\int_0^1 \frac{W\left(\frac{x\log(3)}{e^{-x\log(3)}}\right)}{e^{-x\log(3)}}dx$$

$$=\int_0^1 W\left(x\log(3)e^{x\log(3)}\right)e^{x\log(3)}dx$$

$$=\int_0^1 x\log(3)e^{x\log(3)}dx=\log(3)\int_0^1 x3^x dx$$

$$Let: u=x\Rightarrow du=dx \;\; And \;\; dv=3^x dx\Rightarrow v=\int 3^x dx=\frac{3^x}{\log(3)}$$

$$\Rightarrow I=\left.\frac{x3^x}{\log(3)}\right|_0^1-\int_0^1\frac{3^x}{\log(3)}dx=\frac{3\log(3)-2}{\log^2(3)}$$

SO, $$\boxed{\int_0^1 \frac{W\left(\frac{x\log(3)}{e^{-x\log(3)}}\right)}{e^{-x\log(3)}}dx=\frac{3\log(3)-2}{\log^2(3)}}$$

027, Calculate integral $J=\int_0^1\frac{1}{\sqrt[3]{(1+x)^2(1-x)}}dx$

Answer

They give $$J=\int_0^1\frac{1}{\sqrt[3]{(1+x)^2(1-x)}}dx$$

$$=\int_0^1\frac{1}{(1+x)\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}}dx \quad ,Let:\begin{cases} Let: y^3=\left(\frac{1-x}{1+x}\right)\Rightarrow 3y^2dy=-\frac{2}{(1+x)^2}dx \\ If: x\in(0,1)\Rightarrow y\in(1,0) \\ By: y^3=\left(\frac{1-x}{1+x}\right)\Rightarrow\frac{1}{1+x}=\frac{2}{1+y^3}\end{cases}$$

$$\Rightarrow J=-3\int_1^0\frac{y^2}{(1+y^3)y}dx=3\int_0^1\frac{y}{(1+y)(y^2-y+1)}dx$$

$$=\int_0^1\frac{(1+y)^2-(y^2-y+1)}{(1+y)(y^2-y+1)}dx=\int_0^1\frac{y+1}{y^2-y+1}dx-\int_0^1\frac{1}{y+1}dx$$

$$Take: J_1=\int_0^1 \frac{1}{y+1}dx=\log(2)$$

$$Take: J_2=\frac{1}{2}\int_0^1 \frac{2y-1+3}{y^2-y+1}dx=\frac{1}{2}\int_0^1 \frac{2y-1}{y^2-y+1}dx+\frac{3}{2}\int_0^1 \frac{1}{y^2-y+1}dx$$

$$for: \frac{1}{2}\int_0^1 \frac{2y-1}{y^2-y+1}dx=\frac{1}{2}\log\left(y^2-y+1\right)\Big|_0^1=0$$

$$for: \frac{3}{2}\int_0^1 \frac{1}{y^2-y+1}dx=\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\Bigg|_0^1=\frac{\pi}{\sqrt{3}}$$

$$\Rightarrow J=\frac{\pi}{\sqrt{3}}-\log(2)$$

SO, $$\boxed{\int_0^1 \frac{1}{\sqrt[3]{(1+x)^2(1-x)}}dx=\frac{\pi}{\sqrt{3}}-\log(2)}$$

028, Calculate integral $K=\int_0^{+\infty}\frac{x^{n-1}}{e^x-1}dx$

Answer

They give $$K=\int_0^{+\infty}\frac{x^{n-1}}{e^x-1}dx$$

$$=\int_0^{+\infty}\frac{x^{n-1}e^{-x}}{1-e^{-x}}dx=\int_0^{+\infty}\left(x^{n-1}e^{-x}\sum_{i=0}^{\infty}e^{-xi}\right)dx \quad ,Take: \frac{1}{1-a}=\sum_{i=0}^{\infty}a^i$$

$$=\sum_{i=0}^{\infty}\int_0^{+\infty}x^{n-1}e^{-x(i+1)}dx$$

$$Let: y=(i+1)x\Rightarrow dx=\frac{1}{i+1}dy, If \ : x\in(0,+\infty)\Rightarrow y\in(0,+\infty)$$

$$\Rightarrow K=\sum_{i=0}^{\infty}\int_0^{+\infty}\left(\frac{y}{i+1}\right)^{n-1}\frac{e^{-y}}{(i+1)}dy$$

$$=\sum_{i=0}^{\infty}\frac{1}{(i+1)^n}\int_0^{+\infty}y^{n-1}e^{-y}dy$$

$$=\sum_{i=0}^{\infty}\frac{\Gamma(n)}{(i+1)^n}=\zeta(n)\Gamma(n)=\zeta(n)\Gamma(n)$$

SO, $$\boxed{\int_0^{+\infty}\frac{x^{n-1}}{e^x-1}dx=\zeta(n)\Gamma(n)}$$

029, Calculate integral $I=\int_0^{\pi}\frac{1}{(1+\sin(x))^2}dx$

Answer

They give $I=\int_0^{\pi}\frac{1}{(1+\sin(x))^2}dx$

$$Let: y=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+y^2}dy\ ,\ If\ :x\in(0,\pi)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow I=\int_0^{\infty}\frac{1}{\left(1+\frac{2y}{1+y^2}\right)^2}\times\frac{2}{1+y^2}dy=2\int_0^{\infty}\frac{1+y^2}{(1+y)^4}dy$$

$$Let: u=1+y\Leftrightarrow y=u-1\Rightarrow dy=du\ ,If\ :y\in(0,\infty)\Rightarrow u\in(1,\infty)$$

$$\Rightarrow I=2\int_1^{\infty}\frac{1+(u-1)^2}{(u)^4}du=2\int_1^{\infty}\left(\frac{1}{u^2}-\frac{2}{u^3}+\frac{2}{u^4}\right)du=\frac{4}{3}$$

SO, $\boxed{\int_0^{\pi}\frac{1}{(1+\sin(x))^2}dx=\frac{4}{3}}$

030, Calculate integral $J=\int_0^1\log(\Gamma(x))dx$

Answer

They give $J=\int_0^1\log(\Gamma(x))dx \quad (1)$

$$=\int_0^1\log(\Gamma(1-x))dx\quad(2)\quad ,Use:\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$Take\ (1)+(2)\ That\ have\ :2J=\int_0^1\log(\Gamma(x))dx+\int_0^1\log(\Gamma(1-x))dx$$

$$\Rightarrow J=\frac{1}{2}\int_0^1\log(\Gamma(x)\Gamma(1-x))dx=\frac{1}{2}\int_0^1\log\left(\frac{\pi}{\sin(\pi x)}\right)dx$$

$$=\frac{1}{2}\left(\int_0^1\log(\pi)\,dx-\int_0^1\log(\sin(\pi x))\,dx\right)$$

$$=\frac{1}{2}(\log(\pi)-J')\qquad(3)$$

$$For: J'=\int_0^1\log(\sin(\pi x))\,dx\quad ,\begin{cases}Let: t=\pi x\Rightarrow dx=\frac{1}{\pi}dt\\ If\ :x\in(0,1)\Rightarrow t\in(0,\pi)\end{cases}$$

$$\Rightarrow J' = \frac{1}{\pi}\int_0^{\pi}\log\left(\sin(t)\right)dt \quad ,Take:\begin{cases} f(2a-x)=f(x) \\ \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \end{cases}$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\log\left(\sin(t)\right)dt = \frac{2}{\pi}\left(-\frac{\pi}{2}\log(2)\right) = -\log(2)$$

$$Take: (3)\ That\ J = \frac{1}{2}\left(\log(\pi)+\log(2)\right) = \frac{\log(2\pi)}{2}$$

$$SO, \quad \boxed{\int_0^1 \log\left(\Gamma(x)\right)dx = \frac{\log(2\pi)}{2}}$$

$Note: \oplus\ \Gamma(n)\Gamma(n+1) = \dfrac{\pi}{\sin(n)} = \pi\csc(n) \qquad \oplus\ \log(m)+\log(n) = \log(mn)$

031, Calculate integral $K = \int_0^1 \Gamma\left(1+\frac{x}{2}\right)\Gamma\left(1-\frac{x}{2}\right)dx$

Answer

$$They\ give \quad K = \int_0^1 \Gamma\left(1+\frac{x}{2}\right)\Gamma\left(1-\frac{x}{2}\right)dx$$

$$= \int_0^1 \frac{x}{2}\Gamma\left(\frac{x}{2}\right)\Gamma\left(1-\frac{x}{2}\right)dx = \int_0^1 \frac{x}{2}\times\pi\csc\left(\frac{x\pi}{2}\right)dx$$

$$Let: t = \frac{x\pi}{2} \Rightarrow dx = \frac{2}{\pi}dt, If: x\in(0,1) \Rightarrow t\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow K = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} t\csc(t)dt$$

$$Let: u = t \Rightarrow du = dt\ \ And\ \ dv = \csc(t)dx \Rightarrow v = -\log\left(\csc(t)+\cot(t)\right)$$

$$\Rightarrow K = \frac{2}{\pi}\left[\underbrace{-t\log\left(\csc(t)+\cot(t)\right)\Big|_0^{\frac{\pi}{2}}}_{0} + \int_0^{\frac{\pi}{2}}\log\left(\csc(t)+\cot(t)\right)dt\right]$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\log\left(\frac{1+\cos(t)}{\sin(t)}\right)dt = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\log\left(\cot\left(\frac{t}{2}\right)\right)\frac{dt}{2}$$

$$Let: y = \frac{t}{2} \Rightarrow dy = \frac{dt}{2}, If: t\in\left(0,\frac{\pi}{2}\right) \Rightarrow y\in\left(0,\frac{\pi}{4}\right)$$

$$\Rightarrow K = \frac{4}{\pi}\int_0^{\frac{\pi}{4}}\log\left(\cot(y)\right)dy = \frac{4}{\pi}G$$

$$SO, \quad \boxed{\int_0^1 \Gamma\left(1+\frac{x}{2}\right)\Gamma\left(1-\frac{x}{2}\right)dx = \frac{4}{\pi}G}$$

032, Calculate integral $J=\int_0^1 \frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}}dx$

Answer

They give $J=\int_0^1 \frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}}dx$

$$=-\int_0^1 \frac{1}{(x+x^{-1})\sqrt{x^2+x^{-2}}}\times\left(1-x^{-2}\right)dx$$

$$=-\int_0^1 \frac{1}{(x+x^{-1})\sqrt{(x+x^{-1})^2-2}}\times\left(1-x^{-2}\right)dx$$

$$let:\sqrt{2}\sec(t)=x+x^{-1}\Rightarrow\sqrt{2}\sec(t)\tan(t)dt=(1-x^{-2})dx \quad ,if:t\in(0,1)\Rightarrow y\in\left(\frac{\pi}{2},\frac{\pi}{4}\right)$$

$$\Rightarrow J=-\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\frac{\sqrt{2}\sec(t)\tan(t)dt}{\sqrt{2}\sec(t)\sqrt{2\sec^2(t)-2}}=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\sqrt{2}\sec(t)\tan(t)dt}{\sqrt{2}.\sqrt{2}\sec(t)\tan(t)}=\frac{\pi}{4\sqrt{2}}$$

SO, $$\boxed{\int_0^1 \frac{(1-x^2)}{(1+x^2)\sqrt{1+x^4}}dx=\frac{\pi}{4\sqrt{2}}}$$

033, Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{\cos(x)\sqrt{\cos(x)}-\sin(x)\sqrt{\sin(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\frac{\cos(x)\sqrt{\cos(x)}-\sin(x)\sqrt{\sin(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)\sqrt{\cos\left(\frac{\pi}{2}-x\right)}-\sin\left(\frac{\pi}{2}-x\right)\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sin\left(\frac{\pi}{2}-x\right)\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\cos\left(\frac{\pi}{2}-x\right)\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(x)\sqrt{\sin(x)}-\cos(x)\sqrt{\cos(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx$$

$$=-\int_0^{\frac{\pi}{2}}\frac{\cos(x)\sqrt{\cos(x)}-\sin(x)\sqrt{\sin(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx$$

$$\Leftrightarrow K=-K\Rightarrow K=0$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{\cos(x)\sqrt{\cos(x)}-\sin(x)\sqrt{\sin(x)}}{\sin(x)\sqrt{\cos(x)}+\cos(x)\sqrt{\sin(x)}}dx=0}$$

034, Calculate integra $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^5(x)}dx$

Answer

They give $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^5(x)}dx$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^3(x)}.\sec^2(x)dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^3(x)}.d\left(\tan(x)\right)$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(\frac{\sin^2(x)+\cos^2(x)}{\sin(x)\cos(x)}\right)^3 d\left(\tan(x)\right)=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(\tan(x)+\frac{1}{\tan(x)}\right)^3 d\left(\tan(x)\right)$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left[\tan^3(x)+3\tan(x)+\frac{3}{\tan(x)}+\frac{1}{\tan^3(x)}\right]d\left(\tan(x)\right)$$

$$=\left[\frac{\tan^4(x)}{4}+\frac{3\tan^2(x)}{2}+3\log\left(\tan(x)\right)-\frac{1}{2\tan^2(x)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{16}{3}+3\log\sqrt{3}$$

SO, $\boxed{\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin^3(x)\cos^5(x)}dx=\frac{16}{3}+3\log\sqrt{3}}$

035 Calculate integral $J=\sum_{n=0}^{\infty}\int_{2n}^{2n+1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx-\sum_{n=1}^{\infty}\int_{2n}^{2n-1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx$

Answer

They give $J=\sum_{n=0}^{\infty}\int_{2n}^{2n+1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx-\sum_{n=1}^{\infty}\int_{2n}^{2n-1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx$

We have $\left(\frac{\sin(x)}{x}\right)'=\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)$

$$\Rightarrow J=\sum_{n=0}^{\infty}\int_{2n}^{2n+1}\left(\frac{\sin(x)}{x}\right)'dx+\sum_{n=1}^{\infty}\int_{2n-1}^{2n}\left(\frac{\sin(x)}{x}\right)'dx$$

$$=\left(\int_0^1\left(\frac{\sin(x)}{x}\right)'dx+\int_2^3\left(\frac{\sin(x)}{x}\right)'dx+...\right)+\left(\int_1^2\left(\frac{\sin(x)}{x}\right)'dx+\int_3^4\left(\frac{\sin(x)}{x}\right)'dx+...\right)$$

$$=\int_0^1\left(\frac{\sin(x)}{x}\right)'dx+\int_1^2\left(\frac{\sin(x)}{x}\right)'dx+\int_2^3\left(\frac{\sin(x)}{x}\right)'dx+\int_3^4\left(\frac{\sin(x)}{x}\right)'dx+...$$

$$=\int_0^{\infty}\left(\frac{\sin(x)}{x}\right)'dx=\lim_{x\to\infty}\frac{\sin(x)}{x}-\lim_{x\to 0}\frac{\sin(x)}{x}=-1$$

SO, $\boxed{\sum_{n=0}^{\infty}\int_{2n}^{2n+1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx-\sum_{n=1}^{\infty}\int_{2n}^{2n-1}\left(\frac{\cos(x)}{x}-\frac{\sin(x)}{x^2}\right)dx=-1}$

036 Calculate integral $K=\int_0^1 \frac{1}{1+x+x^2+x^3+\dots}dx$

Answer

They give $K=\int_0^1 \frac{1}{1+x+x^2+x^3+\dots}dx$

$$=\int_0^1 \frac{1}{1+\frac{x}{1-x}}dx$$

$$=\int_0^1 (1-x)dx=\frac{1}{2}$$

SO, $\boxed{\int_0^1 \frac{1}{1+x+x^2+x^3+\dots}dx=\frac{1}{2}}$

Note : $u_1+u_2+u_3+\dots+u_n=\frac{u_1}{1-q}$, $|q|<1$

037 Calculate integral $I=\int_{-1}^0 \frac{1}{\sqrt{e^x}+\sqrt{e^{3x}}+\sqrt{e^{5x}}+\sqrt{e^{7x}}+\dots}dx$

Answer

They give $I=\int_{-1}^0 \frac{1}{\sqrt{e^x}+\sqrt{e^{3x}}+\sqrt{e^{5x}}+\sqrt{e^{7x}}+\dots}dx$

$$=\int_{-1}^0 \frac{e^{-\frac{1}{2}x}}{1+e^x+e^{2x}+e^{3x}+\dots}dx$$

$$=\int_{-1}^0 \frac{e^{-\frac{1}{2}x}}{\frac{1}{1-e^x}}dx$$

$$=\int_{-1}^0 \left(e^{-\frac{1}{2}x}-e^{\frac{1}{2}x}\right)dx$$

$$=-2\left(e^{-\frac{1}{2}x}+e^{\frac{1}{2}x}\right)\Big|_{-1}^0$$

$$=\frac{2(\sqrt{e}-e)}{e}$$

SO, $\boxed{\int_{-1}^0 \frac{1}{\sqrt{e^x}+\sqrt{e^{3x}}+\sqrt{e^{5x}}+\sqrt{e^{7x}}+\dots}dx=\frac{2(\sqrt{e}-e)}{e}}$

038 Calculate integral $J=\int_0^1 x^x dx$

Answer

They give $J=\int_0^1 x^x dx$

$$=\int_0^1 e^{x\log(x)}dx \quad ,By: e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\Rightarrow e^{x\log(x)}=\sum_{n=0}^{\infty}\frac{(x\log(x))^n}{n!}$$

$$\Rightarrow J=\sum_{n=0}^{\infty}\int_0^1\frac{(x\log(x))^n}{n!}dx=\sum_{n=0}^{\infty}\frac{1}{n!}\int_0^1 x^n\log^n(x)dx$$

$$Let: t=-\log(x)\Leftrightarrow x=e^{-t}\Rightarrow dx=-e^{-t}dt, If\ :x\in(0,1)\Rightarrow t\in(\infty,0)$$

$$\Rightarrow J=-\sum_{n=0}^{\infty}\frac{1}{n!}\int_{\infty}^{0}e^{-tn}(-t)^n e^{-t}dt=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\int_0^{\infty}e^{-t(n+1)}(t)^n dt$$

$$Let: u=t(n+1)\Leftrightarrow \frac{u}{n+1}=t\Rightarrow\frac{du}{n+1}=dt,\ If\ :t\in(0,\infty)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow J=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\int_0^{\infty}e^{-u}\left(\frac{u}{n+1}\right)^n\times\frac{du}{n+1}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^{n+1}n!}\int_0^{\infty}u^n e^{-u}du$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^{n+1}n!}\Gamma(n+1)\quad ,Note:\Gamma(n+1)=n!$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^{n+1}}=1-\frac{1}{2^2}+\frac{1}{3^3}-\frac{1}{4^4}+\ldots$$

SO, $$\boxed{\int_0^1 x^x dx=1-\frac{1}{2^2}+\frac{1}{3^3}-\frac{1}{4^4}+\ldots}$$

039 Calculate integral $K=\int_0^{+\infty}\frac{nx^{n-1}}{(x^2+1)^{\frac{n+2}{2}}}dx \quad ,(n>0)$

Answer

They give $K=\int_0^{+\infty}\frac{nx^{n-1}}{(x^2+1)^{\frac{n+2}{2}}}dx \quad ,(n>0)$

$$Let: x=\tan(y)\Rightarrow dx=\sec^2(y)dy\ ,If\ :x\in(0,+\infty)\Rightarrow y\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow K=\int_0^{+\infty}\frac{n\tan^{n-1}(y)}{(1+\tan^2(y))^{\frac{n+2}{2}}}.\sec^2(y)dy=n\int_0^{+\infty}\frac{\tan^{n-1}(y)}{\sec^n(y)}dy$$

$$=\frac{n}{2}\int_0^{+\infty}2\sin^{n-1}(y)\cos(y)dy=\frac{n}{2}\int_0^{+\infty}2\sin^{2\left(\frac{n}{2}\right)-1}(y)\cos^{2\times 1-1}(y)dy$$

$$=\frac{n}{2}B\left(\frac{n}{2},1\right)=\frac{n}{2}\times\frac{\Gamma\left(\frac{n}{2}\right)\Gamma(1)}{\Gamma\left(\frac{n}{2}+1\right)}\quad ,Note:\Gamma\left(n+1\right)=n\Gamma\left(n\right)$$

$$=\frac{n}{2}\times\frac{\Gamma\left(\frac{n}{2}\right)}{\frac{n}{2}\Gamma\left(\frac{n}{2}\right)}=1$$

SO, $$\boxed{\int_0^{+\infty}\frac{nx^{n-1}}{(x^2+1)^{\frac{n+2}{2}}}dx=1}$$

040 Calculate integral $\quad I=\int_0^{\infty}\left\lfloor ne^{-x}\right\rfloor dx\ ,n\in\mathbb{N}$

Answer

They give $\quad I=\int_0^{\infty}\left\lfloor ne^{-x}\right\rfloor dx\ ,n\in\mathbb{N}$

Let $:t=ne^{-x}\Rightarrow dt=-ne^{-x}dx\ ,If\ :x\in\left(0,\infty\right)\Rightarrow t\in\left(n,0\right)$

$$\Rightarrow I=-\int_n^0\frac{\lfloor y\rfloor}{y}dx=\int_0^n\frac{\lfloor y\rfloor}{y}dx$$

$$=\int_0^1\frac{\lfloor y\rfloor}{y}dx+\int_1^2\frac{\lfloor y\rfloor}{y}dx+\int_2^3\frac{\lfloor y\rfloor}{y}dx...+\int_{n-1}^n\frac{\lfloor y\rfloor}{y}dx$$

$$=\int_0^1\frac{0}{y}dx+\int_1^2\frac{1}{y}dx+\int_2^3\frac{2}{y}dx...+\int_{n-1}^n\frac{n-1}{y}dx$$

$$=\left(\log(2)-\log(1)\right)+2\left(\log(3)-\log(2)\right)+...+(n-1)\left(\log(n)-\log(n-1)\right)$$

$$=-\log(2)-\log(3)-\log(4)-...-\log(n-1)+(n-1)\log(n)$$

$$=\log\left(\frac{1}{(n-1)!}\right)+\log(n^{n-1})$$

$$=\log\left(\frac{n^{n-1}}{(n-1)!}\right)$$

SO, $$\boxed{\int_0^{\infty}\left\lfloor ne^{-x}\right\rfloor dx=\log\left(\frac{n^{n-1}}{(n-1)!}\right)}$$

041 Calculate integral $J=\int_1^2 (x+1)^2 e^{\frac{x^2-1}{x}}dx$

Answer

$$\text{They give} \qquad J=\int_1^2 (x+1)^2 e^{\frac{x^2-1}{x}}dx$$

$$=\int_1^2\left(2xe^{x-\frac{1}{x}}+x^2(1+\frac{1}{x^2})e^{x-\frac{1}{x}}\right)dx$$

$$=\int_1^2\left(x^2e^{x-\frac{1}{x}}\right)'dx=4e^{3/2}-1$$

$$SO, \quad \boxed{\int_1^2 (x+1)^2 e^{\frac{x^2-1}{x}}dx=4e^{3/2}-1}$$

042 Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx$

Answer

$$\text{They give} \qquad K=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx \quad (1)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos^2(x)}{\sin(x)+\cos(x)}dx \qquad (2)$$

$$Take:(1)+(2)\,That\,have:2K=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)+\cos^2(x)}{\sin(x)+\cos(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)+\cos(x)}dx=\int_0^{\frac{\pi}{2}}\sec\left(\frac{\pi}{4}-x\right)dx$$

$$=\log\left(\sec\left(\frac{\pi}{4}-x\right)+\tan\left(\frac{\pi}{4}-x\right)\right)\Bigg|_0^{\frac{\pi}{2}}=-\frac{\sqrt{2}}{2}\log\left(\sqrt{2}+1\right)$$

$$SO, \quad \boxed{\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx=-\frac{\sqrt{2}}{2}\log\left(\sqrt{2}+1\right)}$$

043 Calculate integral $I=\int_1^2 x^x\left(1+\log(x)\right)dx$

Answer

They give $I=\int_1^2 x^x\left(1+\log(x)\right)dx$

$$=\int_1^2 e^{x\log(x)}\left(1+\log(x)\right)dx$$

$$=\int_1^2 e^{x\log(x)}d\left(x\log(x)\right)$$

$$=e^{x\log(x)}\Big|_1^2=3$$

SO, $\boxed{\int_1^2 x^x\left(1+\log(x)\right)dx=3}$

044 Calculate integral $J=\int_0^{\frac{\pi}{2}}\sqrt{\tan(x)}dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\sqrt{\tan(x)}dx \quad (1)$

$$=\int_0^{\frac{\pi}{2}}\sqrt{\tan\left(\frac{\pi}{2}-x\right)}dx \quad (2)$$

$$=\int_0^{\frac{\pi}{2}}\sqrt{\cot(x)}dx \quad (2)$$

Take $(1)+(2)$ That have: $2J=\int_0^{\frac{\pi}{2}}\left(\sqrt{\tan(x)}+\sqrt{\cot(x)}\right)dx$

$$=\sqrt{2}\int_0^{\frac{\pi}{2}}\left(\frac{\sin(x)+\cos(x)}{\sqrt{1-\left(\sin(x)-\cos(x)\right)^2}}\right)dx$$

$$=\sqrt{2}\int_0^{\frac{\pi}{2}}\left(\frac{1}{\sqrt{1-\left(\sin(x)-\cos(x)\right)^2}}\right)d\left(\sin(x)-\cos(x)\right)$$

$$=\sqrt{2}\arcsin\left(\sin(x)-\cos(x)\right)\Big|_0^{\frac{\pi}{2}}$$

$$\Rightarrow J=\frac{\pi\sqrt{2}}{2}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\sqrt{\tan(x)}dx=\frac{\pi\sqrt{2}}{2}}$

045 Calculate integral $K=\int_0^1 (-1)^x e^{\frac{\pi}{2}i}dx \quad ,\left(i^2=-1\right)$

Answer

They give $K=\int_0^1 (-1)^x e^{\frac{\pi}{2}i}dx \quad ,\left(i^2=-1\right)$

$$=i\int_0^1 e^{i\pi x}dx \qquad ,\left(e^{\frac{\pi}{2}i}=i,(-1)^x=e^{i\pi x}\right)$$

$$=i\times\frac{1}{i\pi}e^{i\pi x}\Big|_0^1=\frac{2}{\pi}$$

SO, $\boxed{\int_0^1 (-1)^x dx=\frac{2}{\pi}}$

046 Calculate integral $I=\int_1^0 \frac{\log(1-x^2)}{(1+x^2)}dx$

Answer

They give $I=\int_1^0 \frac{\log(1-x^2)}{(1+x^2)}dx$

$Let: x=\tan(u)\Rightarrow dx=\left(1+\tan^2(u)\right)du, If \; : x\in(1,0)\Rightarrow u\in\left(\frac{\pi}{4},0\right)$

$$\Rightarrow I=\int_{\frac{\pi}{4}}^0 \frac{\log\left(1-\tan^2(u)\right)}{\left(1+\tan^2(u)\right)}\times\left(1+\tan^2(u)\right)du$$

$$=2\int_0^{\frac{\pi}{4}}\log\left(\cos(u)\right)du-\int_0^{\frac{\pi}{4}}\log\left(\cos(2u)\right)du$$

$$=2\underbrace{\int_0^{\frac{\pi}{4}}\log\left(\cos(u)\right)du}_{I_1}-\frac{1}{2}\underbrace{\int_0^{\frac{\pi}{4}}\log\left(\cos(2u)\right)d(2u)}_{I_2}$$

$For: I_1=\int_0^{\frac{\pi}{4}}\log\left(\cos(u)\right)du-\frac{\pi}{4}\log(2)+\frac{1}{2}G$

$For: I_2=\underbrace{\int_0^{\frac{\pi}{4}}\log\left(\cos(2u)\right)d(2u)}_{Let:t=2u}=\int_0^{\frac{\pi}{2}}\log\left(\cos(t)\right)dt=-\frac{\pi}{2}\log(2)$

$$\Rightarrow I=2\left(-\frac{\pi}{4}\log(2)+\frac{1}{2}G\right)-\frac{1}{2}\left(-\frac{\pi}{2}\log(2)\right)$$

$$=G-\frac{\pi}{4}\log(2)$$

SO, $\boxed{\int_1^0 \frac{\log(1-x^2)}{(1+x^2)}dx=G-\frac{\pi}{4}\log(2)}$

047 Calculate integral $J=\int_0^{\frac{\pi}{4}}\log\left(\cos(x)\right)dx$

Answer

They give $J=\int_0^{\frac{\pi}{4}}\log\left(\cos(x)\right)dx$

Take $K=\int_0^{\frac{\pi}{4}}\log\left(\sin(x)\right)dx$

That $J+K=\int_0^{\frac{\pi}{4}}\left[\log\left(\sin(2x)\right)-\log(2)\right]dx$

$$=-\frac{\pi}{4}\log(2)+\frac{1}{2}\int_0^{\frac{\pi}{4}}\log\left(\sin(2x)\right)d(2x)$$

$$=-\frac{\pi}{4}\log(2)+\frac{1}{2}\left(-\frac{\pi}{2}\log(2)\right)=-\frac{\pi}{2}\log(2)$$

That $J-K=\int_0^{\frac{\pi}{4}}\log\left(\cot(x)\right)dx=-\int_0^{\frac{\pi}{4}}\log\left(\tan(x)\right)dx$

$Let: y=\tan(x)\Rightarrow dx=\frac{1}{1+y^2}dy\ ,\ if: x\in\left(0,\frac{\pi}{4}\right)\Rightarrow y\in(0,1)$

$$\Rightarrow J-K=-\int_0^1\frac{\log(y)}{1+y^2}dy=-\int_0^1\sum_{n=0}^{\infty}(-1)^n y^{2n}\log(y)dy$$

$$=-\sum_{n=0}^{\infty}(-1)^n\int_0^1 y^{2n}\log(y)dy$$

$Let: u=\log(y)\Rightarrow dt=\frac{1}{y}dy\quad And\quad dv=y^{2n}dy\Rightarrow v=\frac{y^{2n+1}}{2n+1}$

$$=-\sum_{n=0}^{\infty}(-1)^n\left(\underbrace{\frac{\log(y)y^{2n+1}}{2n+1}\Bigg|_0^1}_{0}-\int_0^1\frac{y^{2n}}{2n+1}dy\right)$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}=G$$

Take $(J+K)+(J-K)=-\frac{\pi}{2}\log(2)+G$ *but*: $(J+K)-(J-K)=-\frac{\pi}{2}\log(2)-G$

$$\Rightarrow J=\frac{1}{2}\left(G-\frac{\pi}{2}\log(2)\right)\qquad\Rightarrow K=-\frac{1}{2}\left(G+\frac{\pi}{2}\log(2)\right)$$

SO, $\boxed{\int_0^{\frac{\pi}{4}}\log\left(\cos(x)\right)dx=\frac{1}{2}\left(G-\frac{\pi}{2}\log(2)\right)}$ *And* $\boxed{\int_0^{\frac{\pi}{4}}\log\left(\sin(x)\right)dx=-\frac{1}{2}\left(G+\frac{\pi}{2}\log(2)\right)}$

048 Calculate integral $$K=\int_0^1 \frac{\tan^{-1}(x)}{(1+x^2)^2}dx$$

Answer

They give $$K=\int_0^1 \frac{\tan^{-1}(x)}{(1+x^2)^2}dx$$

$Let: x=\tan(u) \Rightarrow dx=(1+\tan^2(u))du\ , If\ : x\in(0,1)\Rightarrow u\in\left(0,\frac{\pi}{4}\right)$

$$\Rightarrow K=\int_0^{\frac{\pi}{4}} \frac{u}{(1+\tan^2(u))^2}(1+\tan^2(u))du$$

$$=\int_0^{\frac{\pi}{4}} \frac{u}{(1+\tan^2(u))}du=\int_0^{\frac{\pi}{4}} u\cos^2(u)du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}} u(1+\cos(2u))du=\frac{1}{2}\int_0^{\frac{\pi}{4}} udu+\frac{1}{2}\int_0^{\frac{\pi}{4}} u\cos(2u)du$$

$$=\frac{1}{4}u^2\Big|_0^{\frac{\pi}{4}}+\frac{1}{2}\left[\frac{1}{2}u\sin(2u)+\frac{1}{4}\cos(2u)\right]\Bigg|_0^{\frac{\pi}{4}}=\frac{\pi^2}{64}+\frac{1}{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)$$

SO, $$\boxed{\int_0^1 \frac{\tan^{-1}(x)}{(1+x^2)^2}dx=\frac{\pi^2}{64}+\frac{\pi}{16}-\frac{1}{8}}$$

049 Calculate integral $$I=\int_0^1 \frac{\log(1/x^2)}{(1+x^2)^2}dx$$

Answer

They give $$I=\int_0^1 \frac{\log(1/x^2)}{(1+x^2)^2}dx$$

$Let: y=-\log(x) \Leftrightarrow x=e^{-y} \Rightarrow dx=-e^{y}dy\ ,\ If\ \ : x\in(0,1)\Rightarrow y\in(\infty,0)$

$$\Rightarrow I=-2\int_\infty^0 \frac{ye^{-y}}{(1+e^{-2y})^2}dy=2\int_0^\infty y\times\frac{e^{-y}}{(1+e^{-2y})^2}dy$$

$$=2\int_0^\infty y\left(\sum_{n=0}^{\infty}(-1)^n(n+1)e^{-(2n+1)y}\right)dy=2\sum_{n=0}^{\infty}(-1)^n(n+1)\int_0^\infty ye^{-(2n+1)y}dy$$

$$=2\sum_{n=0}^{\infty}(-1)^n(n+1)\frac{\Gamma(2)}{(2n+1)^2}=2\sum_{n=0}^{\infty}(-1)^n\frac{(n+1)}{(2n+1)^2}$$

$$=\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}+\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\right)=\left(\frac{\pi}{4}+G\right)$$

SO, $$\boxed{\int_0^1 \frac{\log(1/x^2)}{(1+x^2)^2}dx=\left(\frac{\pi}{4}+G\right)}$$

050 Calculate integral $J=\int_0^{\infty}\frac{1}{1+x+x^2+x^3+x^4+x^5}dx$

Answer

They give $\quad J=\int_0^{\infty}\frac{1}{1+x+x^2+x^3+x^4+x^5}dx \quad (1)$

$$Let: x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt \quad ,If: x\in(0,\infty)\Rightarrow t\in(\infty,0)$$

$$\Rightarrow J=-\int_{\infty}^{0}\frac{1}{1+\frac{1}{t}+\frac{1}{t^2}+\frac{1}{t^3}+\frac{1}{t^4}+\frac{1}{t^5}}\times\frac{1}{t^2}dt \quad (x=t)$$

$$=\int_0^{\infty}\frac{x^3}{1+x+x^2+x^3+x^4+x^5}dx \quad (2)$$

$$Take\ (1)+(2)\ We\,have: 2J=\int_0^{\infty}\frac{x^3+1}{1+x+x^2+x^3+x^4+x^5}dx$$

$$=\int_0^{\infty}\frac{(x^3+1)}{(1+x^3)(1+x+x^2)}dx$$

$$=\int_0^{\infty}\frac{1}{(1+x+x^2)}dx$$

$$=\int_0^{\infty}\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}dx$$

$$Let:\left(x+\frac{1}{2}\right)=\frac{\sqrt{3}}{2}\tan(u)\Rightarrow dx=\frac{\sqrt{3}}{2}\left(1+\tan^2(u)\right)du \ ,If: x\in(0,\infty)\Rightarrow u\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$$

$$\Rightarrow 2J=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\frac{\sqrt{3}}{2}\left(1+\tan^2(u)\right)}{\frac{3}{4}\tan^2(u)+\frac{3}{4}}du$$

$$=\frac{2\sqrt{3}}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\left(1+\tan^2(u)\right)}{\tan^2(u)+1}du$$

$$\Rightarrow J=\frac{\sqrt{3}}{3}\left(\frac{\pi}{2}-\frac{\pi}{6}\right)=\frac{\pi}{3\sqrt{3}}$$

SO, $\boxed{\int_0^{\infty}\frac{1}{1+x+x^2+x^3+x^4+x^5}dx=\frac{\pi}{3\sqrt{3}}}$

051 Calculate integral $K=\int_0^1 x\left\lfloor \frac{1}{x}\right\rfloor dx$

Answer

They give $K=\int_0^1 x\left\lfloor \frac{1}{x}\right\rfloor dx$

$Let: u=\frac{1}{x}\Rightarrow dx=-\frac{1}{u^2}du \;\; , If: x\in(0,1)\Rightarrow u\in(\infty,1)$

$$\Rightarrow K=-\int_\infty^1 \frac{\lfloor u\rfloor}{u^3}du$$

$$=\lim_{n\to\infty}\int_1^n \frac{\lfloor u\rfloor}{u^3}du \qquad Note: \forall k\in\mathbb{Z}, x\in\mathbb{R}\;: k\le x\le k+1\Rightarrow \lfloor x\rfloor=k$$

$$=\lim_{n\to\infty}\left(\int_1^2\frac{\lfloor u\rfloor}{u^3}du+\int_2^3\frac{\lfloor u\rfloor}{u^3}du+\int_3^4\frac{\lfloor u\rfloor}{u^3}du+\ldots+\int_{n-1}^n\frac{\lfloor u\rfloor}{u^3}du\right)$$

$$=\lim_{n\to\infty}\left(\int_1^2\frac{1}{u^3}du+\int_2^3\frac{2}{u^3}du+\int_3^4\frac{3}{u^3}du+\ldots+\int_{n-1}^n\frac{n-1}{u^3}du\right)$$

$$=-\frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{u^2}\Big|_1^2+\frac{2}{u^2}\Big|_2^3+\frac{3}{u^2}\Big|_3^4+\ldots+\frac{n-1}{u^2}\Big|_{n-1}^n\right)$$

$$=-\frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{2^2}-\frac{1}{1^2}+\frac{2}{3^2}-\frac{2}{2^2}+\frac{3}{4^2}-\frac{3}{3^2}+\ldots+\frac{n-1}{n^2}-\frac{n-1}{(n-1)^2}\right)$$

$$=-\frac{1}{2}\lim_{n\to\infty}\left(-\frac{1}{1^2}-\frac{1}{2^2}-\frac{1}{3^2}-\frac{1}{4^2}-\ldots-\frac{1}{(n-1)^2}-\frac{1}{n^2}+\frac{1}{n}\right)$$

$$=\frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{(n-1)^2}+\frac{1}{n^2}-\frac{1}{n}\right)$$

$$=\frac{1}{2}\zeta(2)-\frac{1}{2}\lim_{n\to\infty}\frac{1}{n}$$

$$=\frac{1}{2}\times\frac{\pi^2}{6}-0$$

$$=\frac{\pi^2}{12}$$

SO, $\boxed{\int_0^1 x\left\lfloor \frac{1}{x}\right\rfloor dx=\frac{\pi^2}{12}}$

052 Calculate integral $I=\int_0^1 \sin\left(\sqrt{-\log(x)}\right)dx$

Answer

They give $I=\int_0^1 \sin\left(\sqrt{-\log(x)}\right)dx$

$Let: u=\sqrt{-\log(x)} \Rightarrow x=e^{-u^2} \Rightarrow dx=-2ue^{-u^2}du \quad , If: x\in(0,1) \Rightarrow u\in(\infty,0)$

$$\Rightarrow I=-2\int_\infty^0 u\sin(u)e^{-u^2}du$$
$$=2\int_0^\infty u\sin(u)e^{-u^2}du$$
$$=\underbrace{-\sin(u)e^{-u^2}\Big|_0^\infty}_{0}+\int_0^\infty \cos(u)e^{-u^2}du$$
$$=\int_0^\infty \cos(u)e^{-u^2}du$$
$$\Rightarrow I(a)=\int_0^\infty \cos(au)e^{-u^2}du$$
$$\frac{\partial}{\partial a}I(a)=\int_0^\infty \frac{\partial}{\partial a}\cos(au)e^{-u^2}du$$
$$\Rightarrow I'(a)=\frac{a}{2}\int_0^\infty \sin(au)(-2u)e^{-u^2}du$$
$$=\frac{a}{2}\left[\underbrace{\sin(u)e^{-u^2}\Big|_0^\infty}_{0}-\int_0^\infty \cos(au)e^{-u^2}du\right]$$
$$=-\frac{a}{2}\left(\int_0^\infty \cos(au)e^{-u^2}du\right)$$
$$\Leftrightarrow I'(a)=-\frac{a}{2}I(a) \Leftrightarrow I'(a)+\frac{a}{2}I(a)=0 \Rightarrow I(a)=Ce^{-\frac{a^2}{4}}$$

$If: a=1 \Rightarrow I(1)=I=Ce^{-\frac{1}{4}}$

$If: a=0 \Rightarrow I(0)=\int_0^\infty e^{-u^2}du=\frac{\sqrt{\pi}}{2}$

$$\Rightarrow I=\frac{\sqrt{\pi}}{2}e^{-\frac{1}{4}}$$

SO, $\boxed{\int_0^1 \sin\left(\sqrt{-\log(x)}\right)dx=\frac{\sqrt{\pi}}{2}e^{-\frac{1}{4}}}$

053 Calculate integral $J=\int_0^1 \frac{\log(x)}{1+x}dx$

Answer

They give $J=\int_0^1 \frac{\log(x)}{1+x}dx$

$$=\int_0^1 \sum_{n=0}^{\infty}(-1)^n x^n \log(x)dx \qquad , Because: \frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$$

$$=\sum_{n=0}^{\infty}(-1)^n \int_0^1 x^n \log(x)dx$$

$$Let: u=\log(x)\Rightarrow du=\frac{1}{x}dx\ , dv=x^n dx\Rightarrow v=\frac{x^{n+1}}{n+1}$$

$$\Rightarrow J=\sum_{n=0}^{\infty}(-1)^n\left[\underbrace{\frac{x^{n+1}\log(x)}{n+1}\Bigg|_0^1}_{0}-\int_0^1 \frac{x^{n+1}}{n+1}\times\frac{1}{x}dx\right]$$

$$=-\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left[\int_0^1 x^n dx\right]=-\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left[\frac{x^{n+1}}{n+1}\Bigg|_0^1\right]$$

$$=-\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^2}=-\left(\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\dots\right)$$

$$=-\left[\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\dots\right)-\frac{2}{2^2}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\dots\right)\right]$$

$$=-\frac{1}{2}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\dots\right)$$

$$=-\frac{1}{2}\zeta(2)=-\frac{1}{2}\times\frac{\pi^2}{6}=-\frac{\pi^2}{12}$$

SO, $\boxed{\int_0^1 \frac{\log(x)}{1+x}dx=-\frac{\pi^2}{12}}$

$$OR: J=-\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^2}=-\eta(2)=-(1-2^{1-2})\zeta(2)=-\frac{1}{2}\times\frac{\pi^2}{6}=-\frac{\pi^2}{12}$$

$$That: \eta(2)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^2}=\frac{\pi^2}{12}$$

054 Calculate integral $K=\int_1^e \frac{\log\left[\Gamma\left(1-\log(x)\right)\right]}{x}dx$

Answer

They give $K=\int_1^e \frac{\log\left[\Gamma\left(1-\log(x)\right)\right]}{x}dx$

$$Let: y=\log(x)\Rightarrow dy=\frac{1}{x}dx\ ,If\ : x\in(1,e)\Rightarrow y\in(0,1)$$

$$\Rightarrow K=\int_0^1 \log\left(\Gamma(1-y)\right)dy\quad (1)$$

$$=\int_0^1 \log\left[\Gamma\left(1-(1-y)\right)\right]dy$$

$$=\int_0^1 \log\left(\Gamma(y)\right)dy\qquad (2)$$

$$Take\,(1)+(2)\Leftrightarrow 2K=\int_0^1 \log\left(\Gamma(1-y)\right)dy+\int_0^1 \log\left(\Gamma(y)\right)dy$$

$$=\int_0^1 \log\left(\Gamma(y)\Gamma(1-y)\right)dy$$

$$=\int_0^1 \log\left(\frac{\pi}{\sin(\pi y)}\right)dy$$

$$=\int_0^1 \log(\pi)dy-\int_0^1 \log\left(\sin(\pi y)\right)dy$$

$$=\log(\pi)-\int_0^1 \log\left(\sin(\pi y)\right)dy\quad (3)$$

$$Take: K'=\int_0^1 \log\left(\sin(\pi y)\right)dy$$

$$Let: u=\pi y\Rightarrow \frac{du}{\pi}=dy\ ,If\ : y\in(0,1)\Rightarrow u\in(0,\pi)$$

$$\Rightarrow K'=\frac{1}{\pi}\int_0^{\pi} \log\left(\sin(u)\right)du$$

$$=\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \log\left(\sin(u)\right)du$$

$$=\frac{2}{\pi}\left(-\frac{\pi}{2}\log(2)\right)=-\log(2)$$

$$Take\ (3):\ \Leftrightarrow 2K=\log(\pi)+\log(2)$$

$$\Rightarrow K=\log\sqrt{2\pi}$$

SO, $\boxed{\int_1^e \frac{\log\left[\Gamma\left(1-\log(x)\right)\right]}{x}dx=\log\sqrt{2\pi}}$

055 Calculate integral $I=\int_0^1 \frac{1}{x\sqrt{1-x^2}}\log\left(\frac{2+x}{2-x}\right)dx$

Answer

They give $I=\int_0^1 \frac{1}{x\sqrt{1-x^2}}\log\left(\frac{2+x}{2-x}\right)dx$

$$\Rightarrow I(a)=\int_0^1 \frac{1}{x\sqrt{1-x^2}}\log\left(\frac{a+x}{a-x}\right)dx$$

$$\Rightarrow I'(a)=\int_0^1 \frac{1}{x\sqrt{1-x^2}}\left(\frac{1}{a+x}-\frac{1}{a-x}\right)dx$$

$$=\int_0^1 \frac{1}{x\sqrt{1-x^2}}\left(\frac{-2x}{(a+x)(a-x)}\right)dx=-2\int_0^1 \frac{1}{\left(a^2-x^2\right)\sqrt{1-x^2}}dx$$

Let $: x=\sin(y)\Rightarrow dx=\cos(y)dy$, *If* $: x\in(0,1)\Rightarrow y\in\left(0,\frac{\pi}{2}\right)$

$$=-2\int_0^{\frac{\pi}{2}} \frac{\cos(y)}{\left(a^2-\sin^2(y)\right)\sqrt{1-\sin^2(y)}}dy=-2\int_0^{\frac{\pi}{2}} \frac{1}{\left(a^2-\sin^2(y)\right)}dy$$

$$=-2\int_0^{\frac{\pi}{2}} \frac{\sec^2(y)}{\left(a^2\left(1+\tan^2(y)\right)-\tan^2(y)\right)}dy=\frac{-2}{a^2-1}\int_0^{\frac{\pi}{2}} \frac{1}{\left(\frac{a^2}{a^2-1}+\tan^2(y)\right)}d(\tan y)$$

$$=\frac{-2}{1-a^2}\left[\frac{\sqrt{1-a^2}}{a}\tan^{-1}\left(\frac{\sqrt{1-a^2}}{a}\tan(y)\right)\right]_0^{\frac{\pi}{2}}=\frac{-2}{(1-a^2)}\times\frac{\sqrt{(1-a^2)}}{a}\times\frac{\pi}{2}$$

$$=\frac{-\pi}{a\sqrt{1-a^2}}$$

$$\Leftrightarrow \int I'(a)da=\int \frac{-\pi}{a\sqrt{1-a^2}}da\Rightarrow I(a)=-\pi\sec^{-1}(a)+c$$

If $: a=2\Rightarrow I(2)=I=-\pi\sec^{-1}(2)+c=-\frac{\pi^2}{3}+c$

If $: a=\infty\Rightarrow I(\infty)=0=-\pi\sec^{-1}(\infty)+c\Rightarrow c=\frac{\pi^2}{2}$

$$\Rightarrow I=-\frac{\pi^2}{3}+\frac{\pi^2}{2}=\frac{\pi^2}{6}$$

SO, $\boxed{\int_0^1 \frac{1}{x\sqrt{1-x^2}}\log\left(\frac{2+x}{2-x}\right)dx=\frac{\pi^2}{6}}$

056 Calculate integral $J=\int_0^{\infty}\frac{\sqrt{x}}{(x+9)^2}dx$

Answer

They give $J=\int_0^{\infty}\frac{\sqrt{x}}{(x+9)^2}dx$

$Let: x=9y\Rightarrow dx=9dy, if: x\in(0,\infty)\Rightarrow y\in(0,\infty)$

$$\Rightarrow J=\frac{1}{81}\int_0^{\infty}\frac{\sqrt{9y}}{(y+1)^2}9dy=\frac{1}{3}\int_0^{\infty}\frac{\sqrt{y}}{(y+1)^2}dy$$

$$=\frac{1}{3}\int_0^{\infty}\frac{y^{\frac{3}{2}-1}}{(y+1)^{\frac{3}{2}+\frac{1}{2}}}dy=\frac{1}{3}B\left(\frac{3}{2},\frac{1}{2}\right)$$

$$=\frac{1}{3}\times\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(2)}=\frac{1}{6}\Gamma^2\left(\frac{1}{2}\right)=\frac{\pi}{6}\qquad,\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

SO, $\boxed{\int_0^{\infty}\frac{\sqrt{x}}{(x+9)^2}dx=\frac{\pi}{6}}$

057 Calculate integral $K=\int_0^1\frac{1}{\sqrt{1-x^3}}dx$

Answer

They give $K=\int_0^1\frac{1}{\sqrt{1-x^3}}dx$, $\begin{cases} Let: x=y^{\frac{1}{3}}\Rightarrow dx=\frac{1}{3}y^{-\frac{2}{3}}dy \\ if: x\in(0,1)\Rightarrow y\in(0,1)\end{cases}$

$$\Rightarrow K=\frac{1}{3}\int_0^1\frac{y^{-\frac{2}{3}}}{\sqrt{1-y}}dy=\frac{1}{3}\int_0^1 y^{\frac{1}{3}-1}(1-y)^{\frac{1}{2}-1}dy$$

$$=\frac{1}{3}B\left(\frac{1}{3},\frac{1}{2}\right)=\frac{1}{3}\times\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{3}\right)}=\frac{\sqrt{\pi}}{3}\times\frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{6}\right)}$$

SO, $\boxed{\int_0^1\frac{1}{\sqrt{1-x^3}}dx=\frac{\sqrt{\pi}}{3}\times\frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{6}\right)}}$

058 Calculate integral $I=\int_0^1 \frac{\pi-4\tan^{-1}(x)}{1-x^2}dx$

Answer

They give
$$I=\int_0^1 \frac{\pi-4\tan^{-1}(x)}{1-x^2}dx$$
$$=4\int_0^1 \frac{\tan^{-1}(1)-\tan^{-1}(x)}{1-x^2}dx$$
$$=4\int_0^1 \frac{\tan^{-1}\left(\frac{1-x}{1+x}\right)}{1-x^2}dx$$

$$Let: y=\frac{1-x}{1+x} \Leftrightarrow x=\frac{1-y}{1+y} \Rightarrow dx=-\frac{2}{(1+y)^2}dy\ , if: x\in(0,1)\Rightarrow y\in(1,0)$$

$$\Rightarrow I=-4\int_1^0 \frac{\tan^{-1}(y)}{1-\left(\frac{1-y}{1+y}\right)^2}\times\frac{2}{(1+y)^2}dy$$
$$=8\int_0^1 \frac{\tan^{-1}(y)}{(1+y)^2-(1-y)^2}dy$$
$$=2\int_0^1 \frac{\tan^{-1}(y)}{y}dy \qquad By: \tan^{-1}(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)}$$
$$=2\int_0^1 \frac{1}{y}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)}dy$$
$$=2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\int_0^1 \frac{y^{2n+1}}{y}dy$$
$$=2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\,y^{2n+1}\Big|_0^1$$
$$=2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}=2G$$

SO,
$$\boxed{\int_0^1 \frac{\pi-4\tan^{-1}(x)}{1-x^2}dx=2G}$$

Note: $\tan^{-1}(x)-\tan^{-1}(y)=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

059 Calculate integral $J=\int_0^1 \frac{x^p\log(x)}{x-1}dx$

Answer

They give
$$J=\int_0^1 \frac{x^p\log(x)}{x-1}dx$$
$$=-\sum_{n=0}^{\infty}\int_0^1 x^n x^p\log(x)dx \qquad ,\text{Note}: \frac{1}{x-1}=-\frac{1}{1-x}=-\sum_{n=0}^{\infty}x^n$$
$$=-\sum_{n=0}^{\infty}\int_0^1 x^{n+p}\log(x)dx \qquad ,(\text{Use partial integral})$$
$$=-\sum_{n=0}^{\infty}\left[\underbrace{\frac{x^{n+p+1}\log(x)}{n+p+1}\Bigg|_0^1}_{0}-\int_0^1\frac{x^{n+p+1}}{n+p+1}\times\frac{1}{x}dx\right]$$
$$=\frac{1}{n+p+1}\int_0^1 x^{n+p}dx=\sum_{n=0}^{\infty}\frac{1}{(n+p+1)^2}$$

SO, $\boxed{\int_0^1\frac{x^p\log(x)}{x-1}dx=\frac{1}{(p+1)^2}+\frac{1}{(p+2)^2}+\frac{1}{(p+3)^2}+\dots}$

060 Calculate integral $K=\int_0^1\frac{1}{\sqrt{x(1-x)}}dx$

Answer

They give
$$K=\int_0^1\frac{1}{\sqrt{x(1-x)}}dx$$
$$=\int_0^1 x^{-\frac{1}{2}}(1-x)^{-\frac{1}{2}}dx=\int_0^1 x^{\frac{1}{2}-1}(1-x)^{\frac{1}{2}-1}dx$$
$$=B\left(\frac{1}{2},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\right)}$$
$$=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(1)}=\pi$$

SO, $\boxed{\int_0^1\frac{1}{\sqrt{x(1-x)}}dx=\pi}$

061 Calculate integral $I=\int_0^n \frac{\log(x)}{x^2+n^2}dx$

Answer

They give $I=\int_0^n \frac{\log(x)}{x^2+n^2}dx$

$Let: x=nt \Rightarrow dx=ndt$, $If: x\in(0,n)\Rightarrow t\in(0,1)$

$$\Rightarrow I=n\int_0^1 \frac{\log(nt)}{(nt)^2+n^2}dt$$

$$=\int_0^1 \frac{\log(nt)}{t^2+1}dt$$

$$=\int_0^1 \frac{\log(n)}{t^2+1}dt+\int_0^1 \frac{\log(t)}{t^2+1}dt$$

$$=\log(n)\tan^{-1}(t)\Big|_0^1+\int_0^1\sum_{m=0}^{\infty}(-1)^m t^{2m}\log(t)dt$$

$$=\frac{\pi\log(n)}{4}+\sum_{m=0}^{\infty}\left((-1)^m\int_0^1 t^{2m}\log(t)dt\right)$$

$$=\frac{\pi\log(n)}{4}+\sum_{m=0}^{\infty}(-1)^m\left[\underbrace{\frac{t^{2m+1}\log(t)}{2m+1}\Bigg|_0^1}_{0}-\int_0^1\frac{t^{2m+1}}{2m+1}\times\frac{1}{t}dt\right]$$

$$=\frac{\pi\log(n)}{4}-\sum_{m=0}^{\infty}\left[(-1)^m\left(\int_0^1\frac{t^{2m}}{2m+1}dt\right)\right]$$

$$=\frac{\pi\log(n)}{4}-\sum_{m=0}^{\infty}\left[(-1)^m\left(\frac{t^{2m+1}}{(2m+1)^2}\Bigg|_0^1\right)\right]$$

$$=\frac{\pi\log(n)}{4}-\sum_{m=0}^{\infty}\frac{(-1)^m}{(2m+1)^2}$$

$$=\frac{\pi\log(n)}{4}-G$$

SO, $$\boxed{\int_0^n \frac{\log(x)}{x^2+n^2}dx=\frac{\pi\log(n)}{4}-G}$$

062 Calculate integral $J=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1+\sin(x)}}dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1+\sin(x)}}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1+\sin\left(\frac{\pi}{2}-x\right)}}dx=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1+\cos(x)}}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{2\cos^2\left(\frac{x}{2}\right)}}dx=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}}\sec\left(\frac{x}{2}\right)dx$$

$$=\frac{2}{\sqrt{2}}\left[\log\left|\sec\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)\right|\right]_0^{\frac{\pi}{2}}=\sqrt{2}\log\left(\sqrt{2}+1\right)$$

SO, $\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1+\sin(x)}}dx=\sqrt{2}\log\left(\sqrt{2}+1\right)$

063 Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{x}{\tan(x)}dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\frac{x}{\tan(x)}dx$

$Let: u=x\Rightarrow du=dx$ *, and* $dv=\frac{1}{\tan(x)}dx\Rightarrow v=\log(\sin x)$

$$\Rightarrow J=\left.\frac{x}{\tan(x)}\right|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx$$

$$=0-\left(-\frac{\pi}{2}\log(2)\right)$$

$$=\frac{\pi}{2}\log(2)$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{x}{\tan(x)}dx=\frac{\pi}{2}\log(2)}$

064 Calculate integral $K=\int_0^{\infty}\frac{\tan^{-1}(2x)}{\sqrt[3]{x^2}}dx$

Answer

They give $\quad K=\int_0^{\infty}\frac{\tan^{-1}(2x)}{\sqrt[3]{x^2}}dx$

$$Let: u=\tan^{-1}(2x)\Rightarrow du=\frac{2}{1+(2x)^2}dx \;\; And \;\; dv=\int\frac{1}{\sqrt[3]{x^2}}dx\Rightarrow v=-2x^{-\frac{1}{2}}$$

$$\Rightarrow K=-2x^{-\frac{1}{2}}\tan^{-1}(2x)\Big|_0^{\infty}+\int_0^{\infty}\frac{4}{\left(1+4x^2\right)\sqrt{x}}dx$$

$$=\int_0^{\infty}\frac{4}{\left(1+4x^2\right)\sqrt{x}}dx$$

$$Let: y=\sqrt{x}\Rightarrow dx=2ydy\ ,If\ : x\in(0,\infty)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow K=\int_0^{\infty}\frac{4}{\left(1+4y^4\right)y}\times 2ydy$$

$$=8\int_0^{\infty}\frac{1}{\left(1+4y^4\right)}dy$$

$$Let: t=4y^4\Rightarrow y=\frac{1}{\sqrt{2}}t^{\frac{1}{4}}\Rightarrow dy=\frac{1}{4\sqrt{2}}t^{\frac{1}{4}-1}dt\ ,\ If\ : y\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow K=\frac{8}{4\sqrt{2}}\int_0^{\infty}\frac{t^{\frac{1}{4}-1}}{(1+t)^{\frac{1}{4}+\frac{3}{4}}}dy$$

$$=\sqrt{2}B\left(\frac{1}{4},\frac{3}{4}\right)=\sqrt{2}\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}+\frac{3}{4}\right)}$$

$$=\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)=\frac{\pi\sqrt{2}}{\sin\left(\frac{\pi}{4}\right)}=2\pi$$

SO, $\quad\boxed{\int_0^{\infty}\frac{\tan^{-1}(2x)}{\sqrt[3]{x^2}}dx=2\pi}$

065 Calculate integral $I=\int_0^{2\pi}\lfloor 2023\sin(x)\rfloor dx$

Answer

They give $I=\int_0^{2\pi}\lfloor 2023\sin(x)\rfloor dx$

$$By:\begin{cases}0\le x\le\dfrac{\pi}{2}\Rightarrow \sin(0+x)=\sin(x)\\ \dfrac{\pi}{2}\le x\le\pi\Rightarrow \sin\left(\dfrac{\pi}{2}+x\right)=\cos(x)\\ \pi\le x\le\dfrac{3\pi}{2}\Rightarrow \sin(\pi+x)=-\sin(x)\\ \dfrac{3\pi}{2}\le x\le 2\pi\Rightarrow \sin\left(\dfrac{3\pi}{2}+x\right)=-\cos(x)\end{cases}$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\lfloor 2023\sin(x)\rfloor dx+\int_0^{\frac{\pi}{2}}\lfloor 2023\cos(x)\rfloor dx+\int_0^{\frac{\pi}{2}}\lfloor -2023\sin(x)\rfloor dx+\int_0^{\frac{\pi}{2}}\lfloor -2023\cos(x)\rfloor dx$$

$$=\int_0^{\frac{\pi}{2}}\lfloor 2023\sin(x)\rfloor dx+\int_0^{\frac{\pi}{2}}\lfloor 2023\cos(x)\rfloor dx-\int_0^{\frac{\pi}{2}}\left(\lfloor 2023\sin(x)\rfloor+1\right)dx-\int_0^{\frac{\pi}{2}}\left(\lfloor 2023\cos(x)\rfloor+1\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\lfloor 2023\sin(x)\rfloor dx-\int_0^{\frac{\pi}{2}}\lfloor 2023\sin(x)\rfloor dx+\int_0^{\frac{\pi}{2}}\lfloor 2023\cos(x)\rfloor dx-\int_0^{\frac{\pi}{2}}\lfloor 2023\cos(x)\rfloor dx-\int_0^{\frac{\pi}{2}}2dx$$

$$=-\int_0^{\frac{\pi}{2}}2dx=-\pi$$

SO, $\boxed{\int_0^{2\pi}\lfloor 2023\sin(x)\rfloor dx=-\pi}$

066 Calculate integral $J=\int_1^e\frac{\log^2(x)}{x^3}dx$

Answer

They give $J=\int_1^e\frac{\log^2(x)}{x^3}dx$

$$=\int_1^e x^{-3}\log^2(x)dx$$

$Let: y=\log(x)\Rightarrow x=e^y\Rightarrow dx=e^ydy, if\ :x\in(1,e)\Rightarrow y\in(0,1)$

$$\Rightarrow J=\int_0^1 e^{-3y}y^2e^ydy=\int_0^1 e^{-2y}y^2dy$$

$$=\left(-\frac{1}{2}y^2-\frac{1}{2}y-\frac{1}{4}\right)e^{-2y}\Big|_0^1=\frac{1}{4}-\frac{5}{4}e^{-2}$$

SO, $\boxed{\int_1^e\frac{\log^2(x)}{x^3}dx=\frac{e^{-2}}{4}\left(e^2-5\right)}$

067 Calculate integral $K=\int_0^1 \lfloor x \rfloor^{-1} dx$

Answer

They give $K=\int_0^1 \lfloor x \rfloor^{-1} dx$

$$=\int_{\frac{1}{2}}^{1} 1^{-1} dx+\int_{\frac{1}{3}}^{\frac{1}{2}} 2^{-1} dx+\int_{\frac{1}{4}}^{\frac{1}{3}} 3^{-1} dx+\int_{\frac{1}{5}}^{\frac{1}{4}} 4^{-1} dx+\ldots$$

$$=\frac{1}{1}\left(1-\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{3}\right)+\frac{1}{3}\left(\frac{1}{3}-\frac{1}{4}\right)+\frac{1}{4}\left(\frac{1}{4}-\frac{1}{5}\right)+\ldots$$

$$=1-\frac{1}{1\times 2}+\frac{1}{2^2}-\frac{1}{2\times 3}+\frac{1}{3^2}-\frac{1}{3\times 4}+\frac{1}{4^2}-\frac{1}{4\times 5}+\ldots$$

$$=\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)-\lim_{n\to\infty}\left(\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}\right)$$

$$=\zeta(2)-\lim_{n\to\infty}\left(1-\frac{1}{(n+1)}\right)=\frac{\pi^2}{6}-1$$

SO, $\boxed{\int_0^1 \lfloor x \rfloor^{-1} dx=\frac{\pi^2}{6}-1}$

068 Calculate integral $I=\int_0^1 \frac{x^{-1}}{\sqrt{x^{-1}-1}} dx$

Answer

They give $I=\int_0^1 \frac{x^{-1}}{\sqrt{x^{-1}-1}} dx$

$$=\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx=\int_0^1 x^{-\frac{1}{2}}(1-x)^{-\frac{1}{2}} dx$$

$$=\int_0^1 x^{\frac{1}{2}-1}(1-x)^{\frac{1}{2}-1} dx=B\left(\frac{1}{2},\frac{1}{2}\right)$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\right)}=\Gamma^2\left(\frac{1}{2}\right)=\pi$$

SO, $\boxed{\int_0^1 \frac{x^{-1}}{\sqrt{x^{-1}-1}} dx=\pi}$

069 Calculate integral $J=\int_{e}^{\infty}x^{1-\log(x)}dx$

Answer

They give
$$\begin{aligned}J&=\int_{e}^{\infty}x^{1-\log(x)}dx\\&=\int_{e}^{\infty}e^{(1-\log(x))\log(x)}dx\end{aligned}$$

$$Let: t=\log(x)\Leftrightarrow x=e^{t}\Rightarrow dx=e^{t}dt\,,if\ :x\in(e,\infty)\Rightarrow t\in(1,\infty)$$

$$\begin{aligned}\Rightarrow J&=\int_{1}^{\infty}e^{(1-t)t}e^{t}dt\\&=e\int_{1}^{\infty}e^{-\left(t^{2}-2t+1\right)}dt\\&=e\int_{1}^{\infty}e^{-(t-1)^{2}}dt\end{aligned}$$

$$Let: u=t-1\Rightarrow du=dt\,,if\ :x\in(e,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=e\int_{0}^{\infty}e^{-u^{2}}du=\frac{e\sqrt{\pi}}{2}$$

SO, $\boxed{\int_{e}^{\infty}x^{1-\log(x)}dx=\frac{e\sqrt{\pi}}{2}}$

070 Calculate integral $K=\int_{0}^{1}x^{\log(x)-1}\log(x)dx$

Answer

They give
$$\begin{aligned}K&=\int_{1}^{e}x^{\log(x)-1}\log(x)dx\\&=\int_{1}^{e}x^{\log(x)}\frac{\log(x)}{x}dx=\int_{1}^{e}e^{\log\left(x^{\log(x)}\right)}\frac{\log(x)}{x}dx\\&=\int_{1}^{e}e^{\log(x)\times\log(x)}\frac{\log(x)}{x}dx=\int_{1}^{e}e^{\log^{2}(x)}\frac{\log(x)}{x}dx\\&=\frac{1}{2}\int_{1}^{e}e^{\log^{2}(x)}\frac{2\log(x)}{x}dx\quad ,By:\left(\frac{2\log(x)}{x}dx=d\left(\log^{2}(x)\right)\right)\end{aligned}$$

$$\begin{aligned}\Rightarrow K&=\frac{1}{2}\int_{1}^{e}e^{\log^{2}(x)}d\left(\log^{2}(x)\right)\\&=\frac{1}{2}e^{\log^{2}(x)}\Big|_{1}^{e}=\frac{e-1}{2}\end{aligned}$$

SO, $\boxed{\int_{1}^{e}x^{\log(x)-1}\log(x)dx=\frac{e-1}{2}}$

071 Calculate integral $I=\int_0^1 \frac{\log(x)}{\sqrt{x}(x-1)}dx$

Answer

They give $I=\int_0^1 \frac{\log(x)}{\sqrt{x}(x-1)}dx$

$Let: u=\sqrt{x}\Rightarrow x=u^2\Rightarrow dx=2udu, if: x\in(0,1)\Rightarrow u\in(0,1)$

$$\Rightarrow I=2\int_0^1 \frac{\log(u^2)}{u(u^2-1)}udu=-4\int_0^1\frac{\log(u)}{1-u^2}du \quad ,By:\frac{1}{1-u^2}=\sum_{n=0}^{\infty}u^{2n}$$

$$\Rightarrow I=-4\int_0^1\sum_{n=0}^{\infty}u^{2n}\log(u)du=-4\sum_{n=0}^{\infty}\int_0^1 u^{2n}\log(u)du$$

$$=-4\sum_{n=0}^{\infty}\left[\underbrace{\frac{u^{2n+1}\log(u)}{2n+1}\bigg|_0^1}_{0}-\int_0^1\frac{u^{2n+1}}{2n+1}\times\frac{1}{u}du\right]=4\sum_{n=0}^{\infty}\int_0^1\frac{u^{2n}}{2n+1}du$$

$$=4\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=4\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)$$

$$=4\left[\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)-\frac{1}{2^2}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)\right]$$

$$=4\left(\zeta(2)-\frac{1}{4}\zeta(2)\right)=\frac{\pi^2}{2}$$

SO, $\boxed{\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx=\frac{\pi^2}{2}}$

072 Calculate integral $J=\int_0^n\frac{\log(x+1)}{x}dx$

Answer

They give $J=\int_0^n\frac{\log(x+1)}{x}dx$

$Let:\ x=-y\ \Rightarrow dx=-dy, if: x\in(0,n)\Rightarrow y\in(0,-n)$

$$=-\int_0^{-n}\frac{\log(1-y)}{-y}dx=\int_0^{-n}\frac{\log(1-y)}{y}dx\ \ dx$$

$$=-Li_2(-n) \quad ,\text{Note}: Li_2(n)=\int_0^n\frac{\log(-x+1)}{-x}$$

SO, $\boxed{\int_0^n\frac{\log(x+1)}{x}dx=-Li_2(-n)}$

073 Calculate integral $K=\int_0^{\pi} x\sin^4(x)dx$

Answer

They give $$K=\int_0^{\pi} x\sin^4(x)dx$$

$$=\int_0^{\pi}(\pi-x)\sin^4(\pi-x)dx$$

$$=\pi\int_0^{\pi}\sin^4(x)dx-\int_0^{\pi}x\sin^4(x)dx$$

$$\Leftrightarrow 2K=\frac{\pi}{4}\int_0^{\pi}\left(1-\cos(2x)\right)^2 dx$$

$$\Rightarrow K=\frac{\pi}{8}\int_0^{\pi}\left(1-2\cos(2x)+\cos^2(2x)\right)dx$$

$$=\frac{\pi}{8}\int_0^{\pi}\left(1-2\cos(2x)+\frac{1}{2}\left(1+\cos(4x)\right)\right)dx$$

$$=\frac{\pi}{8}\left(\frac{3x}{2}-\sin(x)+\frac{1}{8}\sin(4x)\right)\Bigg|_0^{\pi}=\frac{3\pi^2}{16}$$

SO, $$\boxed{\int_0^{\pi}x\sin^4(x)dx=\frac{3\pi^2}{16}}$$

074 Calculate integral $I=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sin^5(x)+\cos^5(x)}dx$

Answer

They give $$I=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sin^5(x)+\cos^5(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)+\cos(x)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx$$

$$=2\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{\sin^5(x)+\cos^5(x)}dx$$

$$=2\int_0^{\frac{\pi}{2}}\frac{\tan^2(x)+1}{\tan^5(x)+1}\sec^2(x)dx$$

$$Let: u=\tan(x)\Rightarrow du=\sec^2(x)dx, if: x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow I=2\int_0^{\infty}\frac{u^2+1}{u^5+1}du$$

$$= 2\int_0^{\infty}\frac{u^2}{u^5+1}du + 2\int_0^{\infty}\frac{1}{u^5+1}du$$

$$Let: y = u^5 \Rightarrow u = y^{\frac{1}{5}} \Rightarrow du = \frac{1}{5}y^{\frac{1}{5}-1}dy, if: x \in (0,\infty) \Rightarrow u \in (0,\infty)$$

$$\Rightarrow I = \frac{2}{5}\int_0^{\infty}\frac{y^{\frac{3}{5}-1}}{y+1}dy + \frac{2}{5}\int_0^{\infty}\frac{y^{\frac{1}{5}-1}}{y+1}du$$

$$= \frac{2}{5}\left[\Gamma\left(\frac{3}{5}\right)\Gamma\left(1-\frac{3}{5}\right)+\Gamma\left(\frac{1}{5}\right)\Gamma\left(1-\frac{1}{5}\right)\right]$$

$$= \frac{2\pi}{5}\left(\frac{1}{\sin\left(\frac{3\pi}{5}\right)}+\frac{1}{\sin\left(\frac{\pi}{5}\right)}\right) = \frac{2\pi}{5}\left(\frac{1}{\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}}+\frac{1}{\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}}\right)$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sin^5(x)+\cos^5(x)}dx = \frac{2\sqrt{2}\pi}{5}\left(\sqrt{1-\frac{1}{\sqrt{5}}}+\sqrt{1+\frac{1}{\sqrt{5}}}\right)}$$

075 Calculate integral $J = \int_0^{e^e} x^{2e-\log(xe)}dx$

Answer

They give $$J = \int_0^{e^e} x^{2e-\log(xe)}dx$$

$$= \int_0^{e^e} e^{(2e-\log(x))\log(x)}\frac{1}{x}dx = \int_0^{e^e} e^{-\log^2(x)+2e\log(x)}\frac{1}{x}dx$$

$$= e^{e^2}\int_0^{e^e} e^{-e^2+2e\log(x)-\log^2(x)}\frac{1}{x}dx = e^{e^2}\int_0^{e^e} e^{-(e-\log(x))^2}\frac{1}{x}dx$$

$$Let: u = e-\log(x) \Rightarrow du = -\frac{1}{x}dx, if: x \in (0,e^e) \Rightarrow u \in (\infty,0)$$

$$\Rightarrow = -e^{e^2}\int_{\infty}^{0} e^{-u^2}du$$

$$= e^{e^2}\int_0^{\infty} e^{-u^2}du$$

$$= \frac{\sqrt{\pi}}{2}e^{e^2}$$

SO, $$\boxed{\int_0^{e^e} x^{2e-\log(xe)}dx = \frac{\sqrt{\pi}}{2}e^{e^2}}$$

076 Calculate integral $I=\int_1^2 \frac{\log(x+1)-\log(2)}{(x^2-1)}dx$

Answer

They give $$I=\int_1^2 \frac{\log(x+1)-\log(2)}{(x^2-1)}dx$$

$$=-\int_1^2 \frac{\log\left(\frac{2}{x+1}\right)}{\left(\frac{x-1}{x+1}\right)(x+1)^2}dx=-\int_1^2 \frac{\log\left(1-\frac{x-1}{x+1}\right)}{\left(\frac{x-1}{x+1}\right)(x+1)^2}dx$$

$$Let: u=\frac{x-1}{x+1}\Rightarrow \frac{1}{2}du=\frac{1}{(x+1)^2}dx, if: x\in(1,2)\Rightarrow u\in\left(0,\frac{1}{3}\right)$$

$$=-\frac{1}{2}\int_0^{\frac{1}{3}} \frac{\log(1-u)}{u}du=\frac{1}{2}Li_2\left(\frac{1}{3}\right)$$

SO, $$\boxed{\int_1^2 \frac{\log(x+1)-\log(2)}{(x^2-1)}dx=\frac{1}{2}Li_2\left(\frac{1}{3}\right)}$$

077 Calculate integral $J=\int_{\frac{1}{2025}}^{2025} \frac{x^2+1}{x^2+x^{2025}}dx$

Answer

They give $$J=\int_{\frac{1}{2025}}^{2025} \frac{x^2+1}{x^2+x^{2025}}dx$$

$$=\int_{\frac{1}{2025}}^{2025} \left(\frac{x^2+x^{2025}+1-x^{2025}}{x^2+x^{2025}}\right)dx$$

$$=\int_{\frac{1}{2025}}^{2025} \left(1+\frac{1-x^{2025}}{x^2+x^{2025}}\right)dx$$

$$=2025-\frac{1}{2025}+\int_{\frac{1}{2025}}^{2025} \left(\frac{1-x^{2025}}{x^2+x^{2025}}\right)dx$$

$$Take: J'=\int_{\frac{1}{2025}}^{2025} \left(\frac{1-x^{2025}}{x^2+x^{2025}}\right)dx$$

$$Let: x=\frac{1}{u}\Rightarrow dx=-\frac{1}{u^2}du, if: x\in\left(2025,\frac{1}{2025}\right)\Rightarrow u\in\left(\frac{1}{2025},2025\right)$$

$$\Rightarrow J' = -\int_{2025}^{\frac{1}{2025}} \left(\frac{1 - \frac{1}{x^{2025}}}{\frac{1}{x^2} + \frac{1}{x^{2025}}} \right) \times \frac{1}{u^2} du$$

$$= \int_{\frac{1}{2025}}^{2025} \left(\frac{x^{2025} - 1}{x^2 + x^{2025}} \right) du$$

$$= -\int_{\frac{1}{2025}}^{2025} \left(\frac{1 - x^{2025}}{x^2 + x^{2025}} \right) du = -J'$$

$$\Leftrightarrow 2J' = 0 \Rightarrow J' = 0$$

That $J = 2025 - \frac{1}{2025} + 0 = \frac{2024 \times 2026}{2025}$

SO, $\boxed{\int_{\frac{1}{2025}}^{2025} \frac{x^2 + 1}{x^2 + x^{2025}} dx = \frac{2024 \times 2026}{2025}}$

078 Calculate integral $K = \int_1^{\infty} \frac{1}{x^n (x^2 + 1)} dx$

Answer

They give $K = \int_1^{\infty} \frac{1}{x^n (x^2 + 1)} dx$

Let $: x = \frac{1}{u} \Rightarrow dx = -\frac{1}{u^2} du, if : x \in (1, \infty) \Rightarrow u \in (1, 0)$

$$\Rightarrow K = -\int_1^0 \frac{1}{\frac{1}{u^n}\left(\frac{1}{u^2} + 1 \right)} \times \frac{1}{u^2} du$$

$$= \int_0^1 \frac{u^n}{(u^2 + 1)} du$$

$$= \sum_{m=0}^{\infty} (-1)^m \int_0^1 u^n \times u^{2m} du$$

$$= \sum_{m=0}^{\infty} (-1)^m \int_0^1 u^{2m+n} du$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m + n + 1)}$$

SO, $\boxed{\int_1^{\infty} \frac{1}{x^n (x^2 + 1)} dx = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m + n + 1)}}$

079 Calculate integral $I=\int_0^1 \frac{x^{\pi}-x^2}{x\log(x)}dx$

Answer

They give $$I=\int_0^1 \frac{x^{\pi}-x^2}{x\log(x)}dx$$

$$=\int_0^1 \frac{x^{\pi-1}-x}{\log(x)}dx=\int_0^1 \frac{x^{\pi-1}-1-x+1}{\log(x)}dx$$

$$=\int_0^1 \frac{x^{\pi-1}-1}{\log(x)}dx-\int_0^1 \frac{x^1-1}{\log(x)}dx=\log\big((\pi-1)+1\big)-\log(1+1)$$

$$=\log\left(\frac{\pi}{2}\right) \qquad , \textit{Note}: \int_0^1 \frac{x^n-1}{\log(x)}dx=\log(n+1)$$

SO, $$\boxed{\int_0^1 \frac{x^{\pi}-x^2}{x\log(x)}dx=\log\left(\frac{\pi}{2}\right)}$$

080 Calculate integral $J=\int_{-\infty}^{+\infty} \frac{\arctan^2(x)}{x^2}dx$

Answer

They give $$J=\int_{-\infty}^{+\infty} \frac{\arctan^2(x)}{x^2}dx$$

$$=2\int_0^{+\infty} \frac{\arctan^2(x)}{x^2}dx$$

$$Let: x=\tan(y)\Rightarrow dx=\sec^2(y)dy \;, if: x\in(0,+\infty)\Rightarrow u\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow J=2\int_0^{\frac{\pi}{2}} \frac{y^2\sec^2(y)}{\tan^2(y)}dy=2\int_0^{\frac{\pi}{2}} y^2\csc^2(y)dy$$

$$=2\left[y^2\cot(y)\Big|_0^{\frac{\pi}{2}}+2\int_0^{\frac{\pi}{2}} y\cot(y)dy\right] \quad (\text{Take } \textit{partial integral})$$

$$=2\left[0+2\int_0^{\frac{\pi}{2}} y\cot(y)dy\right]=4\int_0^{\frac{\pi}{2}} y\cot(y)dy$$

$$=4\left[y\log\big(\sin(y)\big)\Big|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\log\big(\sin(y)\big)dy\right] \quad (\text{Take } \textit{partial integral})$$

$$=4\left[0-\int_0^{\frac{\pi}{2}}\log\big(\sin(y)\big)dy\right]=2\pi\log(2)$$

SO, $$\boxed{\int_{-\infty}^{+\infty} \frac{\arctan^2(x)}{x^2}dx=2\pi\log(2)}$$

081 Calculate integral $K=\int_0^{2\pi}\left(\sin(x)+\cos(x)\right)^{11}dx$

Answer

They give $K=\int_0^{2\pi}\left(\sin(x)+\cos(x)\right)^{11}dx$

By contact : $\int_0^{2a}f(x)dx=\int_0^{a}f(x)dx+\int_0^{a}f(2a-x)dx$

$$\Rightarrow K=\underbrace{\int_0^{\pi}\left(\sin(x)+\cos(x)\right)^{11}dx}_{K_1}+\int_0^{\pi}\left(\sin(2\pi-x)+\cos(2\pi-x)\right)^{11}dx$$

$$=K_1+\int_0^{\pi}\left(-\sin(x)+\cos(x)\right)^{11}dx$$

By contact : $\int_0^{a}f(x)dx=\int_0^{a}f(a-x)dx$

$$\Rightarrow K=K_1+\int_0^{\pi}\left(-\sin(\pi-x)+\cos(\pi-x)\right)^{11}dx$$

$$=K_1+\int_0^{\pi}\left(-\sin(x)-\cos(x)\right)^{11}dx$$

$$=K_1-\underbrace{\int_0^{\pi}\left(\sin(x)+\cos(x)\right)^{11}dx}_{K_1}$$

$$=0$$

SO, $\boxed{\int_0^{2\pi}\left(\sin(x)+\cos(x)\right)^{11}dx=0}$

082 Calculate integral $I=\int_0^{\pi}\sqrt[x]{\log(x)}dx$

Answer

They give $I=\int_0^{\pi}\sqrt[\log(x)]{x}dx$

$$=\int_0^{\pi}x^{\frac{1}{\log(x)}}dx \qquad ,\ Note: e^{a\log(b)}=b^a$$

$$=\int_0^{\pi}e^{\log\left(x^{\frac{1}{\log(x)}}\right)}dx$$

$$=\int_0^{\pi}e^{\frac{1}{\log(x)}\times\log(x)}dx$$

$$=\int_0^{\pi}edx=\pi e$$

SO, $\boxed{\int_0^{1}\sqrt[\log(x)]{x}dx=\pi e}$

083 Calculate integral $J=\int_0^1\sqrt{\frac{1}{x}\log\left(\frac{1}{x}\right)}dx$

Answer

They give $J=\int_0^1\sqrt{\frac{1}{x}\log\left(\frac{1}{x}\right)}dx$

$Let:t=\log\left(\frac{1}{x}\right)\Leftrightarrow x=e^{-t}\Rightarrow dx=-e^{-t}dt\,,if\ :x\in(0,1)\Rightarrow t\in(\infty,0)$

$$\Rightarrow J=-\int_\infty^0\sqrt{te^t}\,e^{-t}dt=\int_0^\infty\sqrt{te^t}\,e^{-t}dt=\int_0^\infty t^{\frac{1}{2}}e^{-\frac{1}{2}t}dt$$

$Let:\frac{1}{2}t=y\Leftrightarrow dt=2dy\,,if\ :t\in(0,\infty)\Rightarrow y\in(0,\infty)$

$$\Rightarrow J=2\int_0^\infty(2y)^{\frac{1}{2}}e^{-y}dy=2\sqrt{2}\int_0^\infty y^{(\frac{1}{2}+1)-1}e^{-y}dy$$

$$=2\sqrt{2}\Gamma\left(1+\frac{1}{2}\right)=2\sqrt{2}\,\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$=\sqrt{2}\sqrt{\pi}=\sqrt{2\pi}$$

SO, $\boxed{\int_0^1\sqrt{\frac{1}{x}\log\left(\frac{1}{x}\right)}dx=\sqrt{2\pi}}$

084 Calculate integral $K=\int_0^{+\infty}e^{-\lfloor x\rfloor}dx$

Answer

They give $K=\int_0^{+\infty}e^{-\lfloor x\rfloor}dx$

$$=\int_0^1 e^{-\lfloor x\rfloor}dx+\int_1^2 e^{-\lfloor x\rfloor}dx+\int_2^3 e^{-\lfloor x\rfloor}dx+\int_3^4 e^{-\lfloor x\rfloor}dx+\ldots$$

$$=\int_0^1 e^{-0}dx+\int_1^2 e^{-1}dx+\int_2^3 e^{-2}dx+\int_3^4 e^{-3}dx+\ldots$$

$$=1+e^{-1}+e^{-2}+e^{-3}+\ldots$$

$$=1+\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}+\frac{1}{e^4}+\ldots$$

$$=\frac{1}{1-\frac{1}{e}}=\frac{e}{e-1}$$

SO, $\boxed{\int_0^{+\infty}e^{-\lfloor x\rfloor}dx=\frac{e}{e-1}}$

085 Calculate integral $I = \int_1^2 \frac{\log(x)}{x^2 - 2x + 2} dx$

Answer

They give $$I = \int_1^2 \frac{\log(x)}{x^2 - 2x + 2} dx$$

$$= \int_1^2 \frac{\log(x)}{(x-1)^2 + 1} dx$$

$$Let : x - 1 = \tan(y) \Rightarrow dx = \left(\tan^2(y) + 1\right) dy \text{ ,} if : x \in (1, 2) \Rightarrow u \in \left(0, \frac{\pi}{4}\right)$$

$$= \int_0^{\frac{\pi}{4}} \frac{\log\left(1 + \tan(y)\right)}{\tan^2(y) + 1} \times \left(\tan^2(y) + 1\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(1 + \tan(y)\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(1 + \tan\left(\frac{\pi}{4} - y\right)\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(1 + \frac{\tan\left(\frac{\pi}{4}\right) - \tan(y)}{\tan\left(\frac{\pi}{4}\right) + \tan(y)}\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(1 + \frac{1 - \tan(y)}{1 + \tan(y)}\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(\frac{1 + \tan(y) + 1 - \tan(y)}{1 + \tan(y)}\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1 + \tan(y)}\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \left(\log(2) - \log\left(1 + \tan(y)\right)\right) dy$$

$$= \int_0^{\frac{\pi}{4}} \log\left(2\right) dy - \int_1^{\frac{\pi}{4}} \log\left(1 + \tan(y)\right) dy$$

$$\Leftrightarrow I = \frac{\pi}{4} \log(2) - I \Rightarrow I = \frac{\pi}{8} \log(2)$$

SO, $$\boxed{\int_1^2 \frac{\log(x)}{x^2 - 2x + 2} dx = \frac{\pi}{8} \log(2)}$$

086 Calculate integral $J=\int_{-\infty}^{+\infty}\frac{1}{1+x^4}dx$

Answer

They give $J=\int_{-\infty}^{+\infty}\frac{1}{1+x^4}dx$

$$=2\int_{0}^{+\infty}\frac{1}{1+x^4}dx$$

$Let: x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt\ ,if: x\in(0,+\infty)\Rightarrow t\in(+\infty,0)$

$$\Rightarrow J=-2\int_{+\infty}^{0}\frac{1}{1+\frac{1}{t^4}}\left(\frac{1}{t^2}\right)dt=2\int_{0}^{+\infty}\frac{1}{t^2+t^{-2}}dt$$

$$=\int_{0}^{+\infty}\frac{1-t^{-2}+1+t^{-2}}{t^2+t^{-2}}dt$$

$$=\int_{0}^{+\infty}\frac{1-t^{-2}}{t^2+t^{-2}}dt+\int_{0}^{+\infty}\frac{1+t^{-2}}{t^2+t^{-2}}dt$$

$$=\int_{0}^{+\infty}\frac{d(t+t^{-1})}{(t+t^{-1})^2-2}+\int_{0}^{+\infty}\frac{d(t-t^{-1})}{(t-t^{-1})^2+2}$$

$$=\frac{1}{2\sqrt{2}}\log\left(\frac{t+t^{-1}-\sqrt{2}}{t+t^{-1}+\sqrt{2}}\right)\Bigg|_{0}^{+\infty}+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t+t^{-1}}{\sqrt{2}}\right)\Bigg|_{0}^{+\infty}$$

$$=\frac{1}{2\sqrt{2}}\log\left(\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right)\Bigg|_{0}^{+\infty}+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2+1}{t\sqrt{2}}\right)\Bigg|_{0}^{+\infty}$$

$$=\frac{1}{2\sqrt{2}}\left[\log\left(\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right)-\log\left(\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right)\right]+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2+1}{t\sqrt{2}}\right)\Bigg|_{0}^{+\infty}$$

$$=\frac{1}{2\sqrt{2}}(0-0)+\frac{1}{\sqrt{2}}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$$

$$=\frac{1}{\sqrt{2}}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$$

SO, $\boxed{\int_{-\infty}^{+\infty}\frac{1}{1+x^4}dx=\frac{1}{\sqrt{2}}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]}$

087 Calculate integral $K=\int_{-1}^{+1}\frac{e^{x}-1}{e^{x}+1}dx$

Answer

They give
$$K=\int_{-1}^{+1}\frac{e^{x}-1}{e^{x}+1}dx$$
$$=\int_{-1}^{+1}\frac{e^{-x}-1}{e^{-x}+1}dx \quad ,Use \quad \int_{a}^{b}f(x)dx=\int_{a}^{b}f(a+b-x)dx$$
$$=\int_{-1}^{+1}\frac{e^{-x}-1}{e^{-x}+1}\times\frac{e^{x}}{e^{x}}dx$$
$$=\int_{-1}^{+1}\frac{1-e^{x}}{e^{-x}+1}dx$$
$$=-\int_{-1}^{+1}\frac{e^{x}-1}{e^{-x}+1}dx$$
$$=-K$$
$$\Rightarrow K=0$$

SO, $\boxed{\int_{-1}^{+1}\frac{e^{x}-1}{e^{x}+1}dx=0}$

088 Calculate integral $I=\int_{-1}^{+1}\log\left(\frac{1-x}{1+x}\right)dx$

Answer

They give
$$I=\int_{-1}^{+1}\log\left(\frac{1-x}{1+x}\right)dx$$
$$=\int_{-1}^{+1}\log\left(\frac{1-(-x)}{1+(-x)}\right)dx \quad ,Use \quad \int_{a}^{b}f(x)dx=\int_{a}^{b}f(a+b-x)dx$$
$$=\int_{-1}^{+1}\log\left(\frac{1+x}{1-x}\right)dx$$
$$=\int_{-1}^{+1}\log\left(\frac{1-x}{1+x}\right)^{-1}dx$$
$$=-\int_{-1}^{+1}\log\left(\frac{1-x}{1+x}\right)dx$$
$$=-I$$
$$\Rightarrow I=0$$

SO, $\boxed{\int_{-1}^{+1}\log\left(\frac{1-x}{1+x}\right)dx=0}$

089 Calculate integral $J=\int_1^{\infty}\frac{\left(\frac{x-1}{x+1}\right)}{(x+1)(x-1)}\log^2\left(\frac{x+1}{x-1}\right)dx$

Answer

They give $$J=\int_1^{\infty}\frac{\left(\frac{x-1}{x+1}\right)}{(x+1)(x-1)}\log^2\left(\sqrt{\frac{x+1}{x-1}}\right)dx$$

$$=\int_1^{\infty}\frac{1}{(x+1)^2}\log^2\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}dx$$

$$=\int_1^{\infty}\frac{1}{(x+1)^2}\left(-\frac{1}{2}\right)^2\log^2\left(\frac{x-1}{x+1}\right)dx$$

$$=\frac{1}{4}\int_1^{\infty}\frac{1}{(x+1)^2}\log^2\left(\frac{x-1}{x+1}\right)dx$$

$$Let:t=\frac{x-1}{x+1}\Rightarrow dt=\frac{\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}}{(x+1)^2}dx\Leftrightarrow\frac{dt}{2}=\frac{1}{(x+1)^2}dx\ ,if\ :x\in(1,\infty)\Rightarrow t\in(0,1)$$

$$\Rightarrow J=\frac{1}{2\times 4}\int_0^1\log^2(t)dt$$

$$=\frac{1}{2\times 4}\left[t\log^2(t)\Big|_0^1-2\int_0^1\log(t)dt\right]$$

$$=\frac{1}{2\times 4}\left[0-2\int_0^1\log(t)dt\right]$$

$$=-\frac{1}{4}\left[\int_0^1\log(t)dt\right]$$

$$=-\frac{1}{4}\left[\left(t\log(t)\Big|_0^1-\int_0^1 dt\right)\right]$$

$$=-\frac{1}{4}\left(0-x\Big|_0^1\right)$$

$$=-\frac{1}{4}(-1+0)=\frac{1}{4}$$

SO, $$\boxed{\int_1^{\infty}\frac{\left(\frac{x-1}{x+1}\right)}{(x+1)(x-1)}\log^2\left(\sqrt{\frac{x+1}{x-1}}\right)dx=\frac{1}{4}}$$

090 Calculate integral $K=\int_0^{\infty}\frac{t^n}{e^x-1}dx$

Answer

They give $$K=\int_0^{\infty}\frac{x^n}{e^x-1}dx$$

$$=\int_0^{\infty}\frac{x^ne^{-x}}{1-e^{-x}}dx$$

$$=\sum_{m=0}^{\infty}\int_0^{\infty}x^ne^{-(m+1)x}dx$$

$$Let: u=(m+1)x\Rightarrow\frac{du}{(m+1)}=dx\ ,if\ :x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow K=\sum_{m=0}^{\infty}\int_0^{\infty}\left(\frac{u}{m+1}\right)^n e^{-u}\frac{du}{(m+1)}=\sum_{m=0}^{\infty}\frac{1}{(m+1)^{n+1}}\int_0^{\infty}u^ne^{-u}du$$

$$=\sum_{m=0}^{\infty}\frac{\Gamma(n+1)}{(m+1)^{n+1}}=\Gamma(n+1)\sum_{m=0}^{\infty}\frac{1}{(m+1)^{n+1}}$$

$$=\Gamma(n+1)\zeta(n+1)$$

SO, $$\boxed{\int_0^{\infty}\frac{x^n}{e^x-1}dx=\Gamma(n+1)\zeta(n+1)}$$

091 Calculate integral $I=\int_0^{e-1}\frac{x}{(x+1)\log(x+1)}dx$

Answer

They give $$I=\int_0^{e-1}\frac{x}{(x+1)\log(x+1)}dx$$

$$Let: u=\log(x+1)\Leftrightarrow x=e^u-1\Rightarrow dx=e^udu\ ,if\ :x\in(0,e-1)\Rightarrow u\in(0,1)$$

$$\Rightarrow I=\int_0^1\frac{(e^u-1)e^u}{ue^u}du\qquad ,But: e^u-1=u+\frac{u^2}{2!}+\frac{u^3}{3!}+\dots=\sum_{n=1}^{\infty}\frac{u^n}{n!}$$

$$\Rightarrow I=\sum_{n=1}^{\infty}\frac{1}{n!}\int_0^1\frac{u^n}{u}du=\sum_{n=1}^{\infty}\frac{1}{n!}\times\left.\frac{u^n}{n}\right|_0^1$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)$$

SO, $$\boxed{\int_0^{e-1}\frac{x}{\log(x+1)}dx=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)}$$

092 Calculate integral $J=\int_{1}^{2024}\lfloor \log_{43}(x)\rfloor dx$

Answer

They give $J=\int_{1}^{2024}\lfloor \log_{43}(x)\rfloor dx$

$By:\lfloor \log_{43}(43)\rfloor=1,\lfloor \log_{43}(43^2)\rfloor=2\ and\ \ n\le\lfloor n\rfloor\le n+1\Rightarrow\lfloor n\rfloor=n$

$Exampl:\lfloor 1.5\rfloor=1,\lfloor -1.5\rfloor=-2$

$$\Rightarrow J=\int_{1}^{43}\lfloor \log_{43}(x)\rfloor dx+\int_{43}^{43^2}\lfloor \log_{43}(x)\rfloor dx+\int_{43^2}^{2024}\lfloor \log_{43}(x)\rfloor dx$$

$$But:\lfloor \log_{43}(x)\rfloor=\begin{cases}0 & When\ 1\le x\le 43\\ 1 & When\ 43\le x\le 43^2\\ 2 & When\ 43^2\le x\le 2024\end{cases}$$

$$=\int_{1}^{43}0dx+\int_{43}^{43^2}1dx+\int_{43^2}^{2024}2dx$$

$$=43^2-43+2(2024-43^2)$$

$$=2156$$

SO, $\boxed{\int_{1}^{2024}\lfloor \log_{43}(x)\rfloor dx=2156}$

093 Calculate integral $K=\int_{-\infty}^{0}\frac{\log(x+1)-\log(x)}{(x+1)x}dx$

Answer

They give $K=\int_{-\infty}^{1}\frac{\log(x)-\log(x+1)}{x(x+1)}dx$

$$=-\int_{-\infty}^{1}\frac{\log\left(\frac{x+1}{x}\right)}{x^2\left(\frac{x+1}{x}\right)}dx=\int_{1}^{-\infty}\frac{\log\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)}dx$$

$$Let:u=\log\left(1+\frac{1}{x}\right)\Rightarrow du=x^2\left(1+\frac{1}{x}\right),if\ :x\in(1,-\infty)\Rightarrow u\in(\log(2),0)$$

$$K=\int_{\log(2)}^{0}udu=\frac{u^2}{2}\Bigg|_{\log(2)}^{0}=-\frac{1}{2}\log^2(2)$$

SO, $\boxed{\int_{1}^{\infty}\frac{\log(x)-\log(x+1)}{x(x+1)}dx=-\frac{1}{2}\log^2(2)}$

094 Calculate integral $I=\int_0^1 \frac{\log^2(x)}{x^2-1}dx$

Answer

They give $$I=\int_0^1 \frac{\log^2(x)}{x^2-1}dx$$

$$=-\int_0^1 \frac{\log^2(x)}{1-x^2}dx=-\sum_{n=0}^{\infty}\int_0^1 x^{2n}\log^2(x)dx$$

$$=-\sum_{n=0}^{\infty}\left[\underbrace{\frac{x^{2n+1}\log^2(x)}{(2n+1)}\Bigg|_0^1}_{0}-2\int_0^1 \frac{x^{2n+1}\log(x)}{(2n+1)x}dx\right] \quad (\textit{Use partial integral})$$

$$=2\sum_{n=0}^{\infty}\left[\int_0^1 \frac{x^{2n}\log(x)}{(2n+1)}dx\right]$$

$$=2\sum_{n=0}^{\infty}\left[\underbrace{\frac{x^{2n+1}\log^2(x)}{(2n+1)^2}\Bigg|_0^1}_{0}-\int_0^1 \frac{x^{2n+1}}{(2n+1)^2x}dx\right] \quad (\textit{Use partial integral})$$

$$=-2\sum_{n=0}^{\infty}\left[\int_0^1 \frac{x^{2n}}{(2n+1)^2}dx\right]=-2\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(2n+1)^3}\Bigg|_0^1\right]$$

$$=-2\sum_{n=0}^{\infty}\left(\frac{1}{(2n+1)^3}\right)=-2\left(1+\frac{1}{3^3}+\frac{1}{5^3}+\frac{1}{7^3}\dots\right)$$

$$=-2\left[\left(1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\frac{1}{5^3}\dots\right)-\frac{1}{2^3}\left(1+\frac{1}{2^3}+\frac{1}{4^3}+\frac{1}{6^3}+\frac{1}{8^3}\dots\right)\right]$$

$$=-2\left(\zeta(3)-\frac{1}{8}\zeta(3)\right)=-\frac{7\zeta(3)}{4}$$

SO, $$\boxed{\int_0^1 \frac{\log^2(x)}{x^2-1}dx=-\frac{7\zeta(3)}{4}}$$

095 Calculate integral $J=\int_0^1 \sqrt{1-x^{\pi}}\,dx$

Answer

They give $$J=\int_0^1 \sqrt{1-x^{\pi}}\,dx$$

$$Let: u=x^{\pi} \Leftrightarrow x=u^{\frac{1}{\pi}} \Rightarrow dx=\frac{1}{\pi}u^{\frac{1}{\pi}-1}du, if: x\in(0,1)\Rightarrow u\in(0,1)$$

$$\Rightarrow J=\frac{1}{\pi}\int_0^1 u^{\frac{1}{\pi}-1}\sqrt{1-u}\,du=\frac{1}{\pi}\int_0^1 u^{\frac{1}{\pi}-1}(1-u)^{\frac{3}{2}-1}du$$

$$=\frac{1}{\pi}B\left(\frac{1}{\pi},\frac{3}{2}\right)=\frac{1}{\pi}\frac{\Gamma\left(\frac{1}{\pi}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{\pi}+\frac{3}{2}\right)}$$

$$=\frac{\sqrt{\pi}}{(\pi+2)}\times\frac{\Gamma\left(\frac{1}{\pi}\right)}{\Gamma\left(\frac{1}{\pi}+\frac{1}{2}\right)}$$

SO, $$\boxed{\int_0^1\sqrt{1-x^\pi}\,dx=\frac{\sqrt{\pi}}{(\pi+2)}\times\frac{\Gamma(1/\pi)}{\Gamma((\pi+2)/2\pi)}}$$

096 Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+1}{\sin(x)+\cos(x)+1}dx$

Answer

They give $$K=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+1}{\sin(x)+\cos(x)+1}dx$$

$$=\int_0^{\frac{\pi}{2}}\left(1-\frac{\cos(x)}{\sin(x)+\cos(x)+1}\right)dx$$

$$=2\pi-\int_0^{\frac{\pi}{2}}\left(\frac{\cos(x)}{\sin(x)+\cos(x)+1}\right)dx$$

$$Let: u=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+u^2}du\,, if\ : x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(0,1)$$

$$\Rightarrow K=\frac{\pi}{2}-\int_0^1\left(\frac{\frac{1-u^2}{1+u^2}}{\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}+1}\right)du=\frac{\pi}{2}-\frac{1}{2}\int_0^1\left(\frac{1-u^2}{1+u}\right)du$$

$$=\frac{\pi}{2}-\frac{1}{2}\int_0^1(1-u)\,du=\frac{2\pi-1}{4}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{\sin(x)+1}{\sin(x)+\cos(x)+1}dx=\frac{2\pi-1}{4}}$$

097 Calculate integral $I=\int_0^9 \frac{x+\frac{x+\dots}{1+\dots}}{1+\frac{x+\dots}{1+\dots}}dx$

Answer

They give $I=\int_0^9 \frac{x+\frac{x+\dots}{1+\dots}}{1+\frac{x+\dots}{1+\dots}}dx$

$$Let: u=\frac{x+\frac{x+\dots}{1+\dots}}{1+\frac{x+\dots}{1+\dots}}=\frac{x+u}{1+u}\Rightarrow x=u^2\Rightarrow dx=2udu, if: x\in(0,9)\Rightarrow u\in(0,3)$$

$$\Rightarrow I=2\int_0^3 u^2dx=18$$

SO, $\boxed{I=18}$

098 Calculate integra $J=\int_0^\infty \frac{x\log(x)}{x^4+1}dx$

Answer

They give $J=\int_0^\infty \frac{x\log(x)}{x^4+1}dx$

$$=\int_0^1 \frac{x\log(x)}{x^4+1}dx+\underbrace{\int_1^\infty \frac{x\log(x)}{x^4+1}dx}_{J'}\quad (*)$$

$$Take: J'=\int_1^\infty \frac{x\log(x)}{x^4+1}dx$$

$$Let: x=\frac{1}{u}\Rightarrow dx=-\frac{1}{u^2}du, if: x\in(1,\infty)\Rightarrow u\in(1,0)$$

$$\Rightarrow J'=-\int_1^0 \frac{u^{-1}\log(u^{-1})}{u^{-4}+1}u^{-2}du=-\int_0^1 \frac{u\log(u)}{u^4+1}du$$

$$=-\int_0^1 \frac{x\log(x)}{x^4+1}dx\quad ,\left(\int_a^b f(x)dx=\int_a^b f(u)du\right)$$

$$Take: (*)\ That\quad J=\int_0^1 \frac{x\log(x)}{x^4+1}dx-\int_0^1 \frac{x\log(x)}{x^4+1}dx=0$$

SO, $\boxed{\int_0^\infty \frac{x\log(x)}{x^4+1}dx=0}$

099 Calculate integral $K=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^x}dx$

Answer

They give $$K=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^x}dx \qquad (*)$$

$$=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(-x)}{1+e^{-x}}dx=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^{-x}}dx$$

$$=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{e^x\sin^2(x)}{1+e^x}dx \qquad (**)$$

Take $(*)+(**)$ That : $$2K=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^x}dx+\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{e^x\sin^2(x)}{1+e^x}dx$$

$$=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{(1+e^x)\sin^2(x)}{(1+e^x)}dx=\underbrace{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\sin^2(x)dx}_{\text{is an even funcyion}}$$

$$=2\int_0^{\frac{\pi}{2}}\sin^2(x)dx=\int_0^{\frac{\pi}{2}}\left(1-\cos(2x)\right)dx=\frac{\pi}{2}$$

$$\Rightarrow K=\frac{\pi}{4}$$

SO, $$\boxed{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\sin^2(x)}{1+e^x}dx=\frac{\pi}{4}}$$

100 Calculate integral $I=\int_0^{\frac{\pi}{2}}\frac{x+\sin(x)}{1+\cos(x)}dx$

Answer

They give $$I=\int_0^{\frac{\pi}{2}}\frac{x+\sin(x)}{1+\cos(x)}dx$$

We have : $$\begin{cases}\sin(x)=2\sin\left(x/2\right)\cos\left(x/2\right)\\ 1+\cos(x)=2\cos^2\left(x/2\right)\end{cases}$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{x+2\sin\left(x/2\right)\cos\left(x/2\right)}{2\cos^2\left(x/2\right)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{x}{2\cos^2\left(x/2\right)}dx+\int_0^{\frac{\pi}{2}}\frac{\sin\left(x/2\right)}{\cos\left(x/2\right)}dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} x\sec^2(x/2)dx + \int_0^{\frac{\pi}{2}} \frac{\sin(x/2)}{\cos(x/2)}dx$$

$$= \int_0^{\frac{\pi}{2}} (x/2)\sec^2(x/2)dx + \int_0^{\frac{\pi}{2}} \tan(x/2)dx$$

$$Let : u = x/2 \Rightarrow du = 2dx \text{ , } if : x \in \left(0, \frac{\pi}{2}\right) \Rightarrow u \in \left(0, \frac{\pi}{4}\right)$$

$$= 2\int_0^{\frac{\pi}{4}} u\sec^2(u)du + 2\int_0^{\frac{\pi}{4}} \tan(u)du$$

$$= 2\int_0^{\frac{\pi}{4}} \left[u\sec^2(u) + \tan(u)\right]du$$

$$= 2\int_0^{\frac{\pi}{4}} \left[u\left(\tan(u)\right)' + u'\tan(u)\right]du$$

$$= 2\int_0^{\frac{\pi}{4}} \left(u\tan(u)\right)'dt = 2\left(u\tan(u)\right)\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{2}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}} \frac{x+\sin(x)}{1+\cos(x)}dx = \frac{\pi}{2}}$$

OR: $$I = \underbrace{2\int_0^{\frac{\pi}{4}} u\sec^2(u)du}_{Use\ partial\ integral} + 2\int_0^{\frac{\pi}{4}} \tan(u)du$$

$$= 2\left(u\tan(u)\right)\Big|_0^{\frac{\pi}{4}} \underbrace{-2\int_0^{\frac{\pi}{4}} \tan(u)du + 2\int_0^{\frac{\pi}{4}} \tan(u)du}_{0} = \frac{\pi}{2}$$

101 Calculate integral $J = \int_0^1 \frac{\sqrt{x(1-x)}}{1+x}dx$

Answer

They give $$J = \int_0^1 \frac{\sqrt{x(1-x)}}{1+x}dx$$

$$Let : x = \sin^2(u) \Rightarrow dx = 2\sin(u)\cos(u)du \text{ , } if : x \in (0,1) \Rightarrow u \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow J = 2\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin^2(u)\left(1-\sin^2(u)\right)}}{1+\sin^2(u)} \times \sin(u)\cos(u)du$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{\sin^2(u)\cos^2(u)}{1+\sin^2(u)}du$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\cos^2(u) - \frac{\cos^2(u)}{1+\sin^2(u)}\right)du$$

$$=\int_0^{\frac{\pi}{2}} 2\cos^2(u)du-2\int_0^{\frac{\pi}{2}}\frac{1}{\sec^2(u)+\tan^2(u)}du$$

$$=\int_0^{\frac{\pi}{2}}(1+\cos(2u))du-2\int_0^{\frac{\pi}{2}}\frac{\sec^2(u)}{\left(1+2\tan^2(u)\right)\left(1+\tan^2(u)\right)}du$$

$$=\left(u+\sin(u)\cos(u)\right)\Big|_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}\frac{1}{\left(1+2\tan^2(u)\right)\left(1+\tan^2(u)\right)}d\left(\tan(u)\right)$$

$$=\frac{\pi}{2}-2\int_0^{\frac{\pi}{2}}\left[\frac{2}{1+2\tan^2(u)}-\frac{1}{1+\tan^2(u)}\right]d\left(\tan(u)\right)$$

$$=\frac{\pi}{2}-2\left(\frac{2}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan(u)\right)-\tan^{-1}\left(\tan(u)\right)\right)\Bigg|_0^{\frac{\pi}{2}}\frac{3\pi}{2}-\sqrt{2}\pi$$

SO, $\boxed{\int_0^1\frac{\sqrt{x(1-x)}}{1+x}dx=\left(\frac{3}{2}-\sqrt{2}\right)\pi}$

102 Calculate integral $K=\int\frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)\cos(x)}}dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sqrt{\sin(2x)}}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sqrt{1-\left(1-2\sin(x)\cos(x)\right)}}dx=\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sqrt{1-\left(\sin(x)-\cos(x)\right)^2}}dx$$

$Let: u=\sin(x)-\cos(x)\Rightarrow du=\left(\sin(x)+\cos(x)\right)dx, if\ :x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(-1,1)$

$$\Rightarrow K=\underbrace{\int_{-1}^1\frac{1}{\sqrt{1-u^2}}du}_{is\,an\,even\ funtion}=2\int_0^1\frac{1}{\sqrt{1-u^2}}du$$

$Let: u=\sin(t)\Rightarrow du=\cos(t)dt, if\ :u\in(0,1)\Rightarrow t\in\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow K=2\int_0^{\frac{\pi}{2}}\frac{\cos(t)}{\sqrt{1-\sin^2(t)}}dt=2\int_0^{\frac{\pi}{2}}\frac{\cos(t)}{|\cos(t)|}dt=2\int_0^{\frac{\pi}{2}}\frac{\cos(t)}{\cos(x)}dt=\pi$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{\sin(x)+\cos(x)}{\sqrt{\sin(2x)}}dx=\pi}$

103 Calculate integral $I=\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx$

Answer

They give $I=\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx \quad (1)$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos(\pi/2-x)}{\left(1+\sqrt{\sin(\pi-2x)}\right)^2}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx \quad (2)$$

Take : (1) + (2) That : $2I=\int_0^{\frac{\pi}{2}}\frac{\cos(x)+\sin(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos(x)+\sin(x)}{\left(1+\sqrt{1-\left(\sin(x)+\cos(x)\right)^2}\right)^2}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\left(1+\sqrt{1-\left(\sin(x)-\cos(x)\right)^2}\right)^2}d\left(\sin(x)-\cos(x)\right)$$

Let : $\sin(x)-\cos(x)=\sin(u)\Rightarrow d\left(\cos(x)-\sin(x)\right)=\cos(u)du$, if : $x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$$\Rightarrow 2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos(u)}{\left(1+\sqrt{1-\sin^2(u)}\right)^2}du$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos(u)}{\left(1+\cos(u)\right)^2}du$$

Let : $y=\tan(u/2)\Rightarrow du=\frac{2}{1+y^2}dy$, if : $u\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(0,1)$, $\cos(u)=\frac{1-y^2}{1+y^2}$

$$\Rightarrow I=\int_0^1\frac{\frac{1-y^2}{1+y^2}}{\left(1+\frac{1-y^2}{1+y^2}\right)^2}\times\frac{2}{1+y^2}dy=\frac{1}{2}\int_0^1(1-y^2)dy=\frac{1}{3}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{\cos(x)}{\left(1+\sqrt{\sin(2x)}\right)^2}dx=\frac{1}{3}}$

104 Calculate integral $J=\int_{-1}^{+1}\frac{1}{\sqrt{1+x}+\sqrt{1-x}+2}dx$

Answer

They give $J=\int_{-1}^{+1}\frac{1}{\sqrt{1+x}+\sqrt{1-x}+2}dx$

$Let: x=\cos(2u)\Rightarrow dx=-2\sin(2u)du \quad ,if: x\in(-1,1)\Rightarrow u\in\left(\frac{\pi}{2},0\right)$

$$\Rightarrow J=\int_{\frac{\pi}{2}}^{0}\frac{-2\sin(2u)}{\sqrt{1+\cos(2u)}+\sqrt{1-\cos(2u)}+2}du$$

$$=2\int_{0}^{\frac{\pi}{2}}\frac{\sin(2u)}{\sqrt{2}\cos(u)+\sqrt{2}\sin(u)+2}du$$

$$=\int_{0}^{\frac{\pi}{2}}\frac{\sin(2u)}{\frac{\sqrt{2}}{2}\cos(u)+\frac{\sqrt{2}}{2}\sin(u)+1}du$$

$$=\int_{0}^{\frac{\pi}{2}}\frac{\sin(2u)}{\cos\left(\frac{\pi}{4}-u\right)+1}du$$

$Let: t=\frac{\pi}{4}-u\Rightarrow du=-dt, u=\frac{\pi}{4}-t,\ if: u\in\left(\frac{\pi}{2},0\right)\Rightarrow t\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

$$\Rightarrow J=-\int_{+\frac{\pi}{4}}^{-\frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{2}-2t\right)}{\cos(t)+1}dt=\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}\frac{\cos(2t)}{\cos(t)+1}dt$$

$$=2\int_{0}^{\frac{\pi}{4}}\frac{\cos(2t)}{\cos(t)+1}dt=2\int_{0}^{\frac{\pi}{4}}\frac{\left(1-2\sin^2(t)\right)\left(1-\cos(t)\right)}{\sin^2(t)}dt$$

$$=2\int_{0}^{\frac{\pi}{4}}\left(\csc^2(t)-\csc(t)\cot(t)+2\cos(t)-2\right)dt$$

$$=2\left(-\sin(t)+\csc(t)+2\sin(t)-2t\right)\Big|_{0}^{\pi/4}$$

$$=4\sqrt{2}-2-\pi$$

SO, $\boxed{\int_{-1}^{+1}\frac{1}{\sqrt{1+x}+\sqrt{1-x}+2}dx=4\sqrt{2}-2-\pi}$

105 Calculate integral $K=\int_0^{2\pi}\frac{1}{e^{\sin(x)}+1}dx$

Answer

They give $K=\int_0^{2\pi}\frac{1}{e^{\sin(x)}+1}dx \quad (*)$

$$=\int_0^{2\pi}\frac{1}{e^{\sin(2\pi-x)}+1}dx=\int_0^{2\pi}\frac{1}{e^{-\sin(x)}+1}dx$$

$$=\int_0^{2\pi}\frac{e^{\sin(x)}}{e^{\sin(x)}+1}dx \quad (**)$$

Take $(*)+(**)$ They have : $2K=\int_0^{2\pi}\frac{1}{e^{\sin(x)}+1}dx+\int_0^{2\pi}\frac{e^{\sin(x)}}{e^{\sin(x)}+1}dx$

$$=\int_0^{2\pi}\frac{e^{\sin(x)}+1}{e^{\sin(x)}+1}dx$$

$$=2\pi$$

$$\Rightarrow K=\pi$$

SO, $\boxed{\int_0^{2\pi}\frac{1}{e^{\sin(x)}+1}dx=\pi}$

106 Calculate integral $I=\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}dx$

Answer

They give $I=\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}dx$

$$=\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+(-x)}-\sqrt[3]{1-(-x)}}dx$$

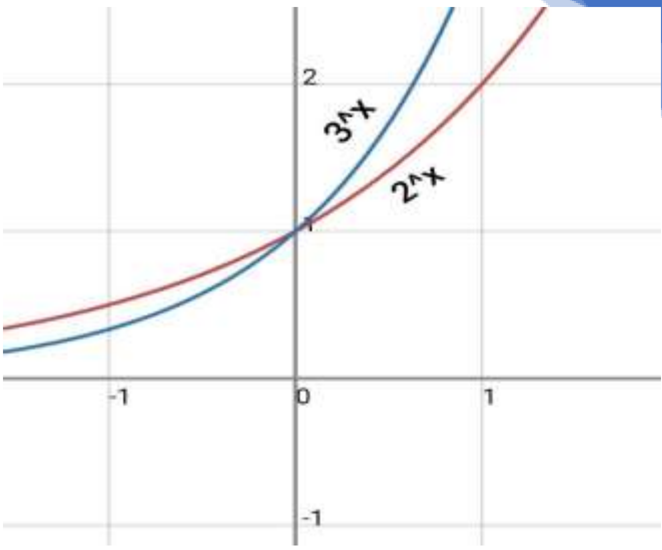
$$=\int_{-1}^{+1}\frac{1}{\sqrt[3]{1-x}-\sqrt[3]{1+x}}dx$$

$$=-\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}dx$$

$$=-I\Rightarrow I=0$$

SO, $\boxed{\int_{-1}^{+1}\frac{1}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}dx=0}$

106 Calculate integral $J=\int_{-1}^{1}\left|3^x-2^x\right|dx$



Answer

They give $J=\int_{-1}^{1}\left|3^x-2^x\right|dx$

$$\Rightarrow J=\int_{-1}^{0}\left|3^x-2^x\right|dx+\int_{0}^{1}\left|3^x-2^x\right|dx$$

$$=-\int_{-1}^{0}\left(3^x-2^x\right)dx+\int_{0}^{1}\left(3^x-2^x\right)dx$$

$$=\left(\frac{2^x}{\log(2)}-\frac{3^x}{\log(3)}\right)\Bigg|_{-1}^{0}+\left(\frac{3^x}{\log(3)}-\frac{2^x}{\log(2)}\right)\Bigg|_{0}^{1}$$

$$=\left(\frac{1}{\log(2)}-\frac{1}{\log(3)}\right)-\left(\frac{2^{-1}}{\log(2)}-\frac{3^{-1}}{\log(3)}\right)+\left(\frac{3}{\log(3)}-\frac{2}{\log(2)}\right)-\left(\frac{1}{\log(3)}-\frac{1}{\log(2)}\right)$$

$$=\left(\frac{3^{-1}+1}{\log(3)}\right)-\left(\frac{2^{-1}}{\log(2)}\right)$$

SO, $\boxed{\int_{-1}^{1}\left|3^x-2^x\right|dx=\left(\frac{3^{-1}+1}{\log(3)}\right)-\left(\frac{2^{-1}}{\log(2)}\right)}$

108 Calculate integral $K=\int_{0}^{1013\pi}\left|\sin(1013x)\right|dx$

Answer

They give $K=\int_{0}^{1013\pi}\left|\sin(1013x)\right|dx$

Let : $u=1013x\Rightarrow dx=\frac{1}{1013}du$, if : $x\in\left(0,1013\pi\right)\Rightarrow u\in\left(0,1013^2\pi\right)$

$$\Rightarrow K=\frac{1}{1013}\int_{0}^{1013^2\pi}\left|\sin(u)\right|du$$

$$=\frac{1013^2}{1013}\int_{0}^{\pi}\left|\sin(u)\right|du \quad , \text{Note}: \int_{0}^{n\pi}\left|\sin(x)\right|dx=n\int_{0}^{\pi}\left|\sin(x)\right|dx$$

$$=1013\int_{0}^{\pi}\sin(u)du$$

$$=-1013\left(\cos(u)\right)\Big|_{0}^{\pi}$$

$$=2026$$

SO, $\boxed{\int_{0}^{1013\pi}\left|\sin(1013x)\right|dx=2026}$

109 Calculate integral $I=\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1-x}{1+x}\right)dx$

Answer

They give $I=\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1-x}{1+x}\right)dx$

$$=\int_{-\pi}^{\pi}\cos(-x)\log\left(\frac{1-(-x)}{1+(-x)}\right)dx=\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1+x}{1-x}\right)dx$$

$$=-\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1-x}{1+x}\right)dx=-I$$

$$\Rightarrow I=0$$

SO, $\boxed{\int_{-\pi}^{\pi}\cos(x)\log\left(\frac{1-x}{1+x}\right)dx=0}$

111 Calculate integral $K=\int_{0}^{\infty}\frac{x^2}{x^4+1}dx$

Answer

They give $K=\int_{0}^{\infty}\frac{x^2}{x^4+1}dx \quad (*)$

$Let: x=1/u\Rightarrow dx=-1/u^2du, if: x\in(0,\infty)\Rightarrow u\in(\infty,0)$

$$\Rightarrow K=-\int_{\infty}^{0}\frac{1/u^2}{1/u^4+1}\times 1/u^2du=\int_{0}^{\infty}\frac{1}{u^4+1}du$$

$$=\int_{0}^{\infty}\frac{1}{x^4+1}dx \quad (**)$$

$Take:(*)+(**) That: 2K=\int_{0}^{\infty}\frac{x^2+1}{x^4+1}dx$

$$=\int_{0}^{\infty}\frac{1+x^{-2}}{x^2+x^{-2}}dx=\int_{0}^{\infty}\frac{1+x^{-2}}{(x-x^{-1})^2+2}dx$$

$$=\int_{0}^{\infty}\frac{d(x-x^{-1})}{(x-x^{-1})^2+2}=\frac{1}{\sqrt{2}}\arctan\left(\frac{x-x^{-1}}{\sqrt{2}}\right)\Big|_{0}^{\infty}$$

$$=\frac{1}{\sqrt{2}}\left[\lim_{x\to\infty}\arctan\left(\frac{x^2-1}{\sqrt{2}x}\right)-\lim_{x\to 0}\arctan\left(\frac{x^2-1}{\sqrt{2}x}\right)\right]$$

$$=\frac{1}{\sqrt{2}}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

$$\Rightarrow K=\frac{\pi}{2\sqrt{2}}$$

SO, $\boxed{\int_{0}^{\infty}\frac{x^2}{x^4+1}dx=\frac{\pi}{2\sqrt{2}}}$

112 Calculate integral $I=\int_0^{\infty}\frac{x^2}{(x^4+1)^2}dx$

Answer

They give $I=\int_0^{\infty}\frac{x^2}{(x^4+1)^2}dx$

$$=\frac{1}{4}\int_0^{\infty}\frac{4x^3}{x(x^4+1)^2}dx$$

$$Let: u=\frac{1}{x}\Rightarrow du=-\frac{1}{x^2}dx, v=\int\frac{4x^3}{(x^4+1)^2}dx=-\frac{1}{(x^4+1)}$$

$$=\frac{1}{4}\left[-\frac{1}{x(x^4+1)}\Big|_0^{\infty}-\int_0^{\infty}\frac{1}{x^2(x^4+1)}dx\right]$$

$$=\frac{1}{4}\left[-\frac{1}{x(x^4+1)}\Big|_0^{\infty}-\int_0^{\infty}\frac{(x^4+1)-x^4}{x^2(x^4+1)}dx\right]$$

$$=\frac{1}{4}\left[-\frac{1}{x(x^4+1)}\Big|_0^{\infty}-\int_0^{\infty}\frac{1}{x^2}dx+\int_0^{\infty}\frac{x^2}{x^4+1}dx\right]$$

$$=\frac{1}{4}\left[-\frac{1}{x(x^4+1)}\Big|_0^{\infty}+\frac{1}{x}\Big|_0^{\infty}+\frac{\pi}{2\sqrt{2}}\right]$$

$$=\frac{1}{4}\left[-\frac{1}{x(x^4+1)}\Big|_0^{\infty}+\frac{x^4+1}{x(x^4+1)}\Big|_0^{\infty}+\frac{\pi}{2\sqrt{2}}\right]$$

$$=\frac{1}{4}\left[\frac{-1+x^4+1}{x(x^4+1)}\Big|_0^{\infty}+\frac{\pi}{2\sqrt{2}}\right]$$

$$=\frac{1}{4}\left[\frac{x^3}{x^4+1}\Big|_0^{\infty}+\frac{\pi}{2\sqrt{2}}\right]$$

$$=\frac{1}{4}\left[(0-0)+\frac{\pi}{2\sqrt{2}}\right]$$

$$=\frac{\pi}{8\sqrt{2}}$$

$$SO,\ \boxed{\int_0^{\infty}\frac{x^2}{(x^4+1)^2}dx=\frac{\pi}{8\sqrt{2}}}$$

113 Calculate integral $J=\int_0^1 \frac{x^4(x^2-1)}{(2x^3+1)^3}dx$

Answer

They give $J=\int_0^1 \frac{x^4(x^2-1)}{(2x^3+1)^3}dx$

$$=\int_0^1 \frac{x^4(x^2-1)}{x^6(2x+x^{-2})^3}dx=\int_0^1 \frac{(1-x^{-3})}{(2x+x^{-2})^3}dx$$

$$=\frac{1}{2}\int_0^1 \frac{(2-2x^{-3})}{(2x+x^{-2})^3}dx=\frac{1}{2}\int_0^1 \frac{d(2x+x^{-2})}{(2x+x^{-2})^3}$$

$$=-\frac{1}{4}\times\frac{1}{(2x+x^{-2})^2}\Big|_0^1=-\frac{1}{36}$$

SO, $\boxed{\int_0^1 \frac{x^4(x^2-1)}{(2x^3+1)^3}dx=-\frac{1}{36}}$

114 Calculate integral $K=\int_1^{\sqrt[4]{2}} \frac{x^8-1}{x(x^8+1)}dx$

Answer

They give $K=\int_1^{\sqrt[4]{2}} \frac{x^8-1}{x(x^8+1)}dx$

$$=\int_1^{\sqrt[4]{2}} \frac{x^8-1}{x(x^8+1)}\times\frac{x^{-5}}{x^{-5}}dx$$

$$=\int_1^{\sqrt[4]{2}} \frac{x^3-x^{-5}}{x^4+x^{-4}}dx$$

$$=\frac{1}{4}\int_1^{\sqrt[4]{2}} \frac{4x^3-4x^{-5}}{x^4+x^{-4}}dx$$

$Let: u=x^4+x^{-4}\Rightarrow du=4x^3-4x^{-5}dx, if: x\in\left(1,\sqrt[4]{2}\right)\Rightarrow u\in\left(2,\frac{5}{2}\right)$

$$\Rightarrow K=\frac{1}{4}\int_2^{\frac{5}{2}}\frac{1}{u}du=\frac{1}{4}\log(u)\Big|_2^{\frac{5}{2}}$$

$$=\frac{1}{4}\log\left(\frac{5}{4}\right)$$

SO, $\boxed{\int_1^{\sqrt[4]{2}} \frac{x^8-1}{x(x^8+1)}dx=\frac{1}{4}\log\left(\frac{5}{4}\right)}$

115 Calculate integral $I=\int_{-1}^{+1}\frac{(x+\sqrt{x^2+1})^3}{\sqrt{x^2+1}}dx$

Answer

They give $I=\int_{-1}^{+1}\frac{(x+\sqrt{x^2+1})^3}{\sqrt{x^2+1}}dx$

$$=\int_{-1}^{+1}(x+\sqrt{x^2+1})^2\frac{(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}dx$$

$$=\int_{-1}^{+1}(x+\sqrt{x^2+1})^2\left(1+\frac{x}{\sqrt{x^2+1}}\right)dx$$

$$=\int_{-1}^{+1}(x+\sqrt{x^2+1})^2d(x+\sqrt{x^2+1})$$

$$=\frac{1}{3}\times(x+\sqrt{x^2+1})^3\Big|_{-1}^{+1}=\frac{14}{3}$$

SO, $\boxed{\int_{-1}^{+1}\frac{(x+\sqrt{x^2+1})^3}{\sqrt{x^2+1}}dx=\frac{14}{3}}$

116 Calculate integral $J=\int_{e^{-1}}^{e^3}\frac{\sqrt{1+\log(x)}}{x\log(x)}dx$

Answer

They give $J=\int_{e^{-1}}^{e^3}\frac{\sqrt{1+\log(x)}}{x\log(x)}dx$

$$Let: u^2=1+\log(x)\Leftrightarrow\log(x)=u^2-1\Rightarrow 2udu=\frac{1}{x}dx, if: x\in\left(e^{-1},e^3\right)\Rightarrow u\in\left(0,2\right)$$

$$\Rightarrow J=2\int_0^2\frac{u^2}{u^2-1}du$$

$$=2\int_0^2\left(1-\frac{1}{u^2-1}\right)du$$

$$=2\left(u-\frac{1}{2}\log\left|\frac{u-1}{u+1}\right|\right)\Big|_0^2=4-\log(3)$$

SO, $\boxed{\int_{e^{-1}}^{e^3}\frac{\sqrt{1+\log(x)}}{x\log(x)}dx=4-\log(3)}$

117 Calculate integral $K=\int_0^{\pi}\sqrt{\dfrac{1+\cos(2x)}{3}}dx$

Answer

They give $$K=\int_0^{\pi}\sqrt{\frac{1+\cos(2x)}{3}}dx$$

$$=\frac{1}{\sqrt{3}}\int_0^{\pi}\sqrt{2\cos^2(x)}dx=\frac{\sqrt{2}}{\sqrt{3}}\int_0^{\pi}|\cos(x)|dx$$

$$=\frac{\sqrt{6}}{3}\left(\int_0^{\frac{\pi}{2}}\cos(x)dx-\int_{\frac{\pi}{2}}^{\pi}\cos(x)dx\right)$$

$$=\frac{2\sqrt{6}}{3}$$

SO, $$\boxed{\int_0^{\pi}\sqrt{\frac{1+\cos(2x)}{3}}dx=\frac{2\sqrt{6}}{3}}$$

118 Calculate integral $I=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\sqrt{1+\sin(2x)}dx$

Answer

They give $$I=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\sqrt{1+\sin(2x)}dx$$

$$=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\sqrt{(\sin(x)+\cos(x))^2}dx$$

$$=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}|\sin(x)+\cos(x)|dx=\sqrt{2}\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\left|\sin\left(\frac{\pi}{4}+x\right)\right|dx$$

$$Let: u=\frac{\pi}{4}+x\Rightarrow du=dx, if\ : x\in\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)\Rightarrow u\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$\Rightarrow I=\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}|\sin(u)|du=2\sqrt{2}\int_0^{\frac{\pi}{2}}|\sin(u)|du$$

$$=2\sqrt{2}\times\frac{1}{2}\int_0^{\pi}|\sin(u)|du=\sqrt{2}\int_0^{\pi}\sin(u)du=2\sqrt{2}$$

SO, $$\boxed{\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\sqrt{1+\sin(2x)}dx=2\sqrt{2}}$$

119 Calculate integral $J=\int_0^{\infty}\frac{1}{(1+x^{\phi})^{\phi}}dx \quad ,\phi=\frac{1+\sqrt{5}}{2}$

Answer

They give $J=\int_0^{\infty}\frac{1}{(1+x^{\phi})^{\phi}}dx$

$$Let: u=x^{\phi}\Leftrightarrow x=u^{\frac{1}{\phi}}\Rightarrow dx=\frac{1}{\phi}u^{\frac{1}{\phi}-1}du, if: x\in(0,\infty)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow J=\frac{1}{\phi}\int_0^{\infty}\frac{u^{\frac{1}{\phi}-1}}{(1+u)^{\phi}}du \quad , By: \phi=\frac{1+\sqrt{5}}{2}\Rightarrow\frac{1}{\phi}=\phi-1$$

$$\Rightarrow J=\frac{1}{\phi}\int_0^{\infty}\frac{u^{(\phi-1)-1}}{(1+u)^{(\phi-1)+1}}du=\frac{1}{\phi}B(\phi-1,1)$$

$$=\frac{1}{\phi}\times\frac{\Gamma(\phi-1)\Gamma(1)}{\Gamma(\phi-1+1)}=\frac{1}{\phi}\times\frac{(\phi-2)!}{(\phi-1)!}$$

$$=\frac{1}{\phi(\phi-1)}$$

$$=\frac{(\phi-1)}{(\phi-1)}=1 \qquad , Because: \left(\frac{1}{\phi}=\phi-1\right)$$

SO, $\boxed{\int_0^{\infty}\frac{1}{(1+x^{\phi})^{\phi}}dx=1}$

120 Calculate integral $K=\int_0^1\frac{x^e-x^{\pi}}{\log(x)}dx$

Answer

They give $K=\int_0^1\frac{x^e-x^{\pi}}{\log(x)}dx$

$$=\int_0^1\frac{x^e-1}{\log(x)}dx-\int_0^1\frac{x^{\pi}-1}{\log(x)}dx \quad , Tkae: \int_0^1\frac{x^n-1}{\log(x)}dx=\log(n+1)$$

$$=\log(e+1)-\log(\pi+1)$$

$$=\log\left(\frac{e+1}{\pi+1}\right)$$

SO, $\boxed{\int_0^1\frac{x^e-x^{\pi}}{\log(x)}dx=\log\left(\frac{e+1}{\pi+1}\right)}$

121 Calculate integral $I=\int_0^1 \left(x\log(x)\right)^n dx$

Answer

They give $I=\int_0^1 \left(x\log(x)\right)^n dx$

$$Let: u=-\log(x) \Rightarrow x=e^{-u} \Rightarrow dx=-e^{-u}du, if: x\in(0,1) \Rightarrow u\in(\infty,0)$$

$$\Rightarrow I=-\int_\infty^0 e^{-nu}\left(-u\right)^n e^{-u}du=(-1)^n\int_0^\infty u^n e^{-(n+1)u}du$$

$$Let: t=(n+1)u \Leftrightarrow u=\frac{1}{n+1}t \Rightarrow du=\frac{1}{n+1}dt, if: u\in(0,\infty) \Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=(-1)^n\int_0^\infty\left(\frac{t}{n+1}\right)^n\frac{e^{-t}}{n+1}dt=\frac{(-1)^n}{(n+1)^{n+1}}\int_0^\infty t^n e^{-t}dt$$

$$=\frac{(-1)^n\Gamma(n+1)}{(n+1)^{(n+1)}}=\frac{(-1)^n n!}{(n+1)^{(n+1)}}$$

SO, $\boxed{\int_0^1\left(x\log(x)\right)^n dx=\frac{(-1)^n n!}{(n+1)^{(n+1)}}}$

122 Calculate integral $J=\int_0^1\frac{\log(x+1)\log(x)}{x}dx$

Answer

They give $J=\int_0^1\frac{\log(x+1)\log(x)}{x}dx$

$$=\int_0^1\frac{\log(x)}{x}\sum_{n=0}^\infty\frac{(-1)^n x^{n+1}}{n+1}dx$$

$$=\sum_{n=0}^\infty\frac{(-1)^n}{n+1}\int_0^1 x^n\log(x)dx$$

$$=\sum_{n=0}^\infty\frac{(-1)^n}{n+1}\left(\underbrace{\frac{x^{n+1}\log(x)}{n+1}\bigg|_0^1}_{0}-\int_0^1\frac{x^n}{n+1}dx\right)\quad(Use\ partial\ integral)$$

$$=-\sum_{n=0}^\infty\frac{(-1)^n}{(n+1)^3}=-\left(1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+\ldots\right)$$

$$=-\frac{3}{4}\zeta(3)$$

SO, $\boxed{\int_0^1\frac{\log(x+1)\log(x)}{x}dx=-\frac{3}{4}\zeta(3)}$

123 Calculate integral $K=\int_0^{\infty}\left(x-\frac{x^2}{2}+\frac{x^5}{2\times4}-\frac{x^7}{2\times4\times6}+\ldots\right)\left(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\times4^2}+\frac{x^6}{2^2\times4^2\times6^2}+\ldots\right)dx$

Answer

They give $\quad K=\int_0^{\infty}\left(x-\frac{x^2}{2}+\frac{x^5}{2\times4}-\frac{x^7}{2\times4\times6}+\ldots\right)\left(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\times4^2}+\frac{x^6}{2^2\times4^2\times6^2}+\ldots\right)dx$

$$\text{We have}\begin{cases} x-\frac{x^2}{2}+\frac{x^5}{2\times4}-\frac{x^7}{2\times4\times6}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2^n\times n!}=x\sum_{n=0}^{\infty}\frac{\left(-\frac{x^2}{2}\right)^n}{n!}=xe^{-\frac{x^2}{2}} \\ 1+\frac{x^2}{2^2}+\frac{x^4}{2^2\times4^2}+\frac{x^6}{2^2\times4^2\times6^2}+\ldots=\sum_{m=0}^{\infty}\frac{x^{2m}}{2^{2m}\times(m!)^2}\end{cases}$$

$$\Rightarrow K=\int_0^{\infty}\left(xe^{-\frac{x^2}{2}}\right)\left(\sum_{m=0}^{\infty}\frac{(x^2)^m}{2^{2m}\times(m!)^2}\right)dx$$

$$=\int_0^{\infty}\left(e^{-\frac{x^2}{2}}\sum_{m=0}^{\infty}\frac{(x^2)^m}{2^{2m}\times(m!)^2}\right)xdx$$

$$Let: u=\frac{x^2}{2}\Rightarrow du=xdx, if\ : x\in(0,\infty)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow K=\int_0^{\infty}\left(e^{-u}\sum_{m=0}^{\infty}\frac{(2u)^m}{2^{2m}\times(m!)^2}\right)du$$

$$=\sum_{m=0}^{\infty}\left(\frac{2^m}{2^{2m}\times(m!)^2}\int_0^{\infty}e^{-u}u^m du\right)$$

$$=\sum_{m=0}^{\infty}\frac{m!}{2^m\times(m!)^2}$$

$$=\sum_{m=0}^{\infty}\frac{1}{2^m\times m!}$$

$$=\sum_{m=0}^{\infty}\frac{\left(\frac{1}{2}\right)^m}{m!}=e^{1/2}$$

SO, $\boxed{\int_0^{\infty}\left(x-\frac{x^2}{2}+\frac{x^5}{2\times4}-\frac{x^7}{2\times4\times6}+\ldots\right)\left(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\times4^2}+\frac{x^6}{2^2\times4^2\times6^2}+\ldots\right)dx=\sqrt{e}}$

124 Calculate integral $I=\int_0^1 \log\left((x-1)!\right)dx$

Answer

They give $I=\int_0^1 \log\left((x-1)!\right)dx$

$$=\int_0^1 \log\left(\Gamma(x)\right)dx \quad (1)$$

$$=\int_0^1 \log\left(\Gamma(x-1)\right)dx \quad (2)$$

Take $(1)+(2)$ That have: $2I=\int_0^1 \log\left(\Gamma(x)\Gamma(1-x)\right)dx$

$$\Rightarrow 2I=\int_0^1 \log\left(\frac{\pi}{\sin(\pi x)}\right)dx$$

$$=\int_0^1 \left[\log\left(\pi\right)-\log\left(\sin(\pi x)\right)\right]dx$$

$$=\int_0^1 \log\left(\pi\right)dx-\int_0^1 \log\left(\sin(\pi x)\right)dx$$

$$=\log(\pi)-I' \quad (3)$$

For: $I'=\int_0^1 \log\left(\sin(\pi x)\right)dx$

Let: $t=\pi x\Rightarrow dx=\frac{1}{\pi}dt$,If : $x\in\left(0,1\right)\Rightarrow t\in\left(0,\pi\right)$

$$\Rightarrow I'=\frac{1}{\pi}\int_0^{\pi} \log\left(\sin(t)\right)dt \quad ,Take:\begin{cases}\int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx\\ f(2a-x)=f(x)\end{cases}$$

$$=\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \log\left(\sin(t)\right)dt$$

$$=\frac{2}{\pi}\left(-\frac{\pi}{2}\log(2)\right)$$

$$=-\log(2)$$

Take (3) That have: $\Rightarrow 2I=\log(\pi)-\left[-\log(2)\right]$

$$=\log(2\pi)$$

$$\Leftrightarrow I=\frac{\log(2\pi)}{2}$$

SO, $\boxed{\int_0^1 \log\left((x-1)!\right)dx=\frac{\log(2\pi)}{2}}$

125 Calculate integral $J=\int_{-\infty}^{+\infty}\Gamma(1+ix)\Gamma(1-ix)dx$

Answer

They give $J=\int_{-\infty}^{\infty}\Gamma(1+ix)\Gamma(1-ix)dx$

$$=\int_{-\infty}^{\infty}ix\Gamma(ix)\Gamma(1-ix)dx=\int_{-\infty}^{\infty}ix\frac{\pi}{\sin(i\pi x)}dx$$

$$=\int_{-\infty}^{\infty}\frac{ix\pi}{\frac{e^{i(i\pi x)}-e^{-i(i\pi x)}}{2i}}dx=-\int_{-\infty}^{\infty}\frac{2\pi x}{e^{-\pi x}-e^{\pi x}}dx$$

$$=2\pi\int_{-\infty}^{\infty}\frac{x}{e^{\pi x}-e^{-\pi x}}dx=4\pi\int_{0}^{\infty}\frac{xe^{-\pi x}}{1-e^{-2\pi x}}dx$$

Let : $u=\pi x\Rightarrow du=\pi dx$, if : $x\in(-\infty,\infty)\Rightarrow u\in(-\infty,\infty)$

$$\Rightarrow K=\frac{4}{\pi}\int_0^{\infty}\frac{ue^{-u}}{1-e^{-2u}}du$$

$$=\frac{4}{\pi}\int_0^{\infty}ue^{-u}\sum_{n=0}^{\infty}e^{-2un}du$$

$$=\frac{4}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}ue^{-(2n+1)u}du$$

Let : $t=(2n+1)u\Rightarrow du=\frac{dt}{(2n+1)}$, if : $u\in(0,\infty)\Rightarrow t\in(0,\infty)$

$$=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\int_0^{+\infty}te^{-t}dt$$

$$=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\Gamma(2)}{(2n+1)^2}$$

$$=\frac{4}{\pi}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)$$

$$=\frac{4}{\pi}\left[\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\dots\right)-\frac{1}{4}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\dots\right)\right]$$

$$=\frac{4}{\pi}\left(\frac{\pi^2}{6}-\frac{1}{4}\times\frac{\pi^2}{6}\right)=\frac{\pi}{2}$$

SO, $\boxed{\int_{-\infty}^{\infty}\Gamma(1+ix)\Gamma(1-ix)dx=\frac{\pi}{2}}$

126 Calculate integral $K=\int_0^{\infty}\log\left(1+\frac{1}{x^2}+\frac{1}{x^4}\right)dx$

Answer

They give $\quad K=\int_0^{\infty}\log\left(1+\frac{1}{x^2}+\frac{1}{x^4}\right)dx$

$Let: x=1/u \Rightarrow dx=-1/u^2du, if: x\in(0,\infty)\Rightarrow u\in(\infty,0)$

$$\Rightarrow K=-\int_{\infty}^{0}\frac{\log(1+u^2+u^4)}{u^2}du=\int_0^{\infty}\frac{\log(1+u^2+u^4)}{u^2}du$$

$$=-\frac{\log(1+u^2+u^4)}{u}\Bigg|_0^{\infty}+\int_0^{\infty}\frac{2u+4u^3}{u(1+u^2+u^4)}du$$

$$=-\left[\lim_{x\to\infty}\frac{\log(1+u^2+u^4)}{u}-\lim_{x\to 0}\frac{\log(1+u^2+u^4)}{u}\right]+\int_0^{\infty}\frac{2+4u^2}{1+u^2+u^4}du$$

$$=-[0-0]+\int_0^{\infty}\frac{2+4u^2}{1+u^2+u^4}du$$

$$=\int_0^{\infty}\frac{2+4u^2}{1+u^2+u^4}du \quad (1)$$

$Let: u=1/t\Rightarrow du=-1/t^2dt, if: u\in(0,\infty)\Rightarrow t\in(\infty,0)$

$$\Rightarrow K=-\int_{\infty}^{0}\frac{2+4(1/t^2)}{1+1/t^2+1/t^4}(1/t^2)dt$$

$$=\int_0^{\infty}\frac{2t^2+4}{1+t^2+t^4}dt=\int_0^{\infty}\frac{2x^2+4}{1+x^2+x^4}dx \quad (2)$$

$$Take:(1)+(2)\,That\; 2K=6\int_0^{\infty}\frac{1+x^2}{1+x^2+x^4}dx$$

$$\Rightarrow K=3\int_0^{\infty}\frac{(1+1/x^2)}{(x-1/x)^2+3}dx$$

$$=3\int_0^{\infty}\frac{(1+1/x)'}{(x-1/x)^2+3}dx$$

$$=\sqrt{3}\arctan\left(\frac{x^2-1}{\sqrt{3}x}\right)\Bigg|_0^{\infty}=\sqrt{3}\pi$$

SO, $\boxed{\int_0^{\infty}\log\left(1+\frac{1}{x^2}+\frac{1}{x^4}\right)dx=\sqrt{3}\pi}$

127 Calculate integral $I=\int_0^{\infty}\frac{x\sqrt{x}}{e^{2x}-1}dx$

Answer

They give $$I=\int_0^{\infty}\frac{x\sqrt{x}}{e^{2x}-1}dx$$

$$=\int_0^{\infty}\frac{x^{\frac{3}{2}}e^{-2x}}{1-e^{-2x}}dx=\int_0^{\infty}\left(x^{\frac{3}{2}}e^{-2x}\sum_{n=0}^{\infty}e^{-2xn}\right)dx$$

$$=\sum_{n=0}^{\infty}\left(\int_0^{\infty}x^{\frac{3}{2}}e^{-2x}e^{-2xn}dx\right)=\sum_{n=0}^{\infty}\left(\int_0^{\infty}x^{\frac{3}{2}}e^{-2(n+1)x}dx\right)$$

$$Let: t=2(n+1)x\Leftrightarrow x=\frac{t}{2(n+1)}\Rightarrow dx=\frac{1}{2(n+1)}dt, if: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\sum_{n=0}^{\infty}\left(\int_0^{\infty}\left(\frac{t}{2(n+1)}\right)^{\frac{3}{2}}\frac{e^{-t}}{2(n+1)}dt\right)$$

$$=\sum_{n=0}^{\infty}\left(\frac{1}{\left(2(n+1)\right)^{\frac{5}{2}}}\int_0^{\infty}t^{\frac{3}{2}}e^{-t}dx\right)$$

$$=\frac{1}{4\sqrt{2}}\sum_{n=0}^{\infty}\left(\frac{1}{(n+1)^{\frac{5}{2}}}\int_0^{\infty}t^{\left(1+\frac{3}{2}\right)-1}e^{-t}dx\right)$$

$$=\frac{1}{4\sqrt{2}}\zeta\left(\frac{5}{2}\right)\Gamma\left(1+\frac{3}{2}\right)$$

$$=\frac{1}{4\sqrt{2}}\times\frac{3}{2}\zeta\left(\frac{5}{2}\right)\Gamma\left(1+\frac{1}{2}\right)$$

$$=\frac{3}{8\sqrt{2}}\times\frac{1}{2}\zeta\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)$$

$$=\frac{3}{16}\times\sqrt{\frac{\pi}{2}}\zeta\left(\frac{5}{2}\right)$$

SO, $$\boxed{\int_0^{\infty}\frac{x\sqrt{x}}{e^{2x}-1}dx=\frac{3}{16}\times\sqrt{\frac{\pi}{2}}\zeta\left(\frac{5}{2}\right)}$$

128 Calculate integral $J=\int_0^1 \frac{x^8+1}{x^{10}+1}dx$

Answer

They give $$J=\int_0^1 \frac{x^8+1}{x^{10}+1}dx \qquad (1)$$

$Let: x=1/u \Rightarrow dx=-1/u^2du, if: x\in(0,1)\Rightarrow u\in(\infty,1)$

$$\Rightarrow J=\int_\infty^1 \frac{1/u^8+1}{1/u^{10}+1}(-1/u^2)du=\int_1^\infty \frac{u^8+1}{u^{10}+1}du \qquad (2)$$

$Take:(1)+(2)\ That\ 2J=\int_0^\infty \frac{u^8+1}{u^{10}+1}du$

$$\Rightarrow J=\frac{1}{2}\int_0^\infty \frac{u^8+1}{u^{10}+1}du$$

$Let: t=u^{10}\Leftrightarrow u=t^{\frac{1}{10}}\Rightarrow du=\frac{1}{10}t^{\frac{1}{10}-1}dt, if: u\in(0,\infty)\Rightarrow t\in(0,\infty)$

$$=\frac{1}{20}\int_0^\infty \frac{t^{\frac{8}{10}}+1}{t+1}\times t^{\frac{1}{10}-1}dt=\frac{1}{20}\left(\int_0^\infty \frac{t^{\frac{9}{10}-1}}{t+1}dt+\int_0^\infty \frac{t^{\frac{1}{10}-1}}{t+1}dt\right)$$

$$=\frac{1}{20}\left(\int_0^\infty \frac{t^{\frac{9}{10}-1}}{(t+1)^{\frac{9}{10}+\frac{1}{10}}}dt+\int_0^\infty \frac{t^{\frac{1}{10}-1}}{(t+1)^{\frac{1}{10}+\frac{9}{10}}}dt\right)$$

$$=\frac{1}{20}\left(B\left(\frac{9}{10},\frac{1}{10}\right)+B\left(\frac{1}{10},\frac{9}{10}\right)\right)$$

$$=\frac{1}{20}\left(\frac{\Gamma\left(\frac{9}{10}\right)\Gamma\left(\frac{1}{10}\right)}{\Gamma\left(\frac{9}{10}+\frac{1}{10}\right)}+\frac{\Gamma\left(\frac{1}{10}\right)\Gamma\left(\frac{9}{10}\right)}{\Gamma\left(\frac{1}{10}+\frac{9}{10}\right)}\right)$$

$$=\frac{\Gamma\left(\frac{1}{10}\right)\Gamma\left(1-\frac{1}{10}\right)}{10}$$

$$=\frac{\pi}{10\sin\left(\frac{\pi}{10}\right)}=\frac{(\sqrt{5}+1)\pi}{10}$$

SO, $$\boxed{\int_0^1 \frac{x^8+1}{x^{10}+1}dx=\frac{(\sqrt{5}+1)\pi}{10}}$$

129 Calculate integral $K=\int_{-\infty}^{+\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx$

Answer

$$\text{They give}\quad K=\int_{-\infty}^{\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx$$

$$=\underbrace{\int_{-\infty}^{0}e^{-(x-x^{-1})^2}(x+x^{-2})dx}_{K'}+\int_{0}^{\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx$$

$$=K'+\int_{0}^{+\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx\quad(*)$$

$$For: K'=\int_{-\infty}^{0}e^{-(x-x^{-1})^2}(x+x^{-2})dx$$

$$Let: x=-\frac{1}{u}\Rightarrow dx=\frac{1}{u^2}du, if\ : x\in(-\infty,0)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow K'=\int_{0}^{\infty}e^{-(u-u^{-1})^2}(-\frac{1}{u}+u^2)\frac{1}{u^2}du$$

$$=\int_{0}^{\infty}e^{-(u-u^{-1})^2}(-\frac{1}{u^3}+1)du$$

$$=\int_{0}^{\infty}e^{-(u-u^{-1})^2}(1-u^{-3})du$$

$$Take\ (*) That: K=\int_{0}^{\infty}e^{-(x-x^{-1})^2}(1-x^{-3}+x+x^{-2})dx$$

$$=\int_{0}^{\infty}e^{-(x-x^{-1})^2}\left(x-x^{-1}+1\right)\left(1+x^{-2}\right)dx$$

$$Let: u=x-x^{-1}\Rightarrow du=(1+x^{-2})du, if\ : x\in(0,\infty)\Rightarrow u\in(-\infty,\infty)$$

$$=\int_{-\infty}^{\infty}e^{-u^2}\left(u+1\right)du$$

$$=\int_{-\infty}^{\infty}ue^{-u^2}du+\int_{-\infty}^{\infty}e^{-u^2}du$$

$$=\frac{1}{2}\int_{-\infty}^{\infty}e^{-u^2}d(u^2)+2\int_{0}^{\infty}e^{-u^2}du$$

$$=-\frac{1}{2}e^{-u^2}\Big|_{-\infty}^{\infty}+2\times\frac{\sqrt{\pi}}{2}$$

$$=\sqrt{\pi}$$

$$SO,\quad \boxed{\int_{-\infty}^{\infty}e^{-(x-x^{-1})^2}(x+x^{-2})dx=\sqrt{\pi}}$$

130 Calculate integral $I=\int_0^{\infty}\frac{x\log(x)}{(x^2+1)^2}dx$

Answer

They give $$I=\int_0^{\infty}\frac{x\log(x)}{(x^2+1)^2}dx$$

$$Let: x=\tan(u)\Rightarrow dx=\sec^2(u)du, if : x\in(0,\infty)\Rightarrow u\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\tan(u)\log(\tan(u))}{(\tan^2(u)+1)^2}\times\sec^2(u)du$$

$$=\int_0^{\frac{\pi}{2}}\frac{\tan(u)\log(\tan(u))}{\sec^4(u)}\times\sec^2(u)du$$

$$=\int_0^{\frac{\pi}{2}}\frac{\tan(u)\log(\tan(u))}{\sec^2(u)}du$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(u)}{\cos(x)}.\cos^2(u)\log(\tan(u))du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}2\sin(u)\cos(u)\log(\tan(u))du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin(2u)\log(\tan(u))du$$

$$Use: \int_0^{2a}f(x)dx=\int_0^{a}\left(f(x)+f(2a-x)\right)dx$$

$$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{4}}\left(\sin(2u)\log(\tan(u))+\sin 2\left(\frac{\pi}{2}-u\right)\log\left(\tan\left(\frac{\pi}{2}-u\right)\right)\right)du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\left(\sin(2u)\log(\tan(u))+\sin(\pi-2u)\log(\cot(u))\right)du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\left(\sin(2u)\log(\tan(u))+\sin(2u)\log(\tan(u)^{-1}\right)du$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\left(\sin(2u)\log(\tan(u))-\sin(2u)\log(\tan(u)\right)du$$

$$=0$$

SO, $$\boxed{\int_0^{\infty}\frac{x\log(x)}{(x^2+1)^2}dx=0}$$

131 Calculate integral $I=\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{1}{(x^2+1)}dx$

Answer

They give $$I=\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{1}{(x^2+1)}dx$$

$$Let: x=\tan(u)\Rightarrow dx=\left(1+\tan^2(u)\right)du, if: x\in(0,\infty)\Rightarrow u\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left(\tan(u)+\frac{1}{\tan(u)}\right)\frac{1}{(\tan^2(u)+1)}\left(1+\tan^2(u)\right)du$$

$$=\int_0^{\frac{\pi}{2}}\log\left(\frac{\tan^2(u)+1}{\tan(u)}\right)du$$

$$=\int_0^{\frac{\pi}{2}}\log\left(\tan^2(u)+1\right)du-\int_0^{\frac{\pi}{2}}\log\left(\tan(u)\right)du$$

$$=-2\int_0^{\frac{\pi}{2}}\log\left(\cos(u)\right)du-0$$

$$=-2\left(-\frac{\pi}{2}\log(2)\right)=\pi\log(2)$$

SO, $$\boxed{\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{1}{(x^2+1)}dx=\pi\log(2)}$$

132 Calculate integral $J=\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x)-\sin(x)}dx$

Answer

They give $$J=\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x)-\sin(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\cos(\pi/2-x)-\sin(\pi/2-x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)-\cos(x)}dx$$

$$=-\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x)-\sin(x)}dx$$

$$\Leftrightarrow J=-J\Rightarrow J=0$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x)-\sin(x)}dx=0}$$

133 Calculate integral $K=\int_0^{\frac{\pi}{4}}\frac{1}{\cos(x)+\sin(x)}dx$

Answer

They give $K=\int_0^{45^\circ}\frac{1}{\cos(x)+\sin(x)}dx$

$$=\frac{1}{\sqrt{2}}\int_0^{45^\circ}\frac{1}{\cos(x)\cos(45^\circ)+\sin(x)\sin(45^\circ)}dx$$

$$=\frac{1}{\sqrt{2}}\int_0^{45^\circ}\frac{1}{\cos(x-45^\circ)}dx$$

$$=\frac{1}{\sqrt{2}}\log\left|\sec(x-45^\circ)+\tan(x-45^\circ)\right|\Big|_0^{45^\circ}$$

$$=\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$$

SO, $\boxed{\int_0^{45^\circ}\frac{1}{\cos(x)+\sin(x)}dx=\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)}$

134 Calculate integral $I=\int_0^{90^\circ}\frac{1}{5\cos^2(x)+4\sin^2(x)-3}dx$

Answer

They give $I=\int_0^{90^\circ}\frac{1}{5\cos^2(x)+4\sin^2(x)-3}dx$

$$=\int_0^{90^\circ}\frac{1}{5\cos^2(x)+4\sin^2(x)-3}\times\frac{\sec^2(x)}{\sec^2(x)}dx$$

$$=\int_0^{90^\circ}\frac{\sec^2(x)}{5+4\tan^2(x)-3\sec^2(x)}dx$$

$$=\int_0^{90^\circ}\frac{1}{5+4\tan^2(x)-3\left(1+\tan^2(x)\right)}d(\tan(x))$$

$$=\int_0^{90^\circ}\frac{1}{2+\tan^2(x)}d(\tan(x))=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)\Bigg|_0^{90^\circ}$$

$$=\frac{1}{\sqrt{2}}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{2\sqrt{2}}$$

SO, $\boxed{\int_0^{90^\circ}\frac{1}{5\cos^2(x)+4\sin^2(x)-3}dx=\frac{\pi}{2\sqrt{2}}}$

135 Calculate integral $J=\int_0^1 x^{-x}dx$

Answer

They give $$J=\int_0^1 x^{-x}dx$$
$$=\int_0^1 e^{-x\log(x)}dx$$

$$By: e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\Rightarrow e^{-x\log(x)}=\sum_{n=0}^{\infty}\frac{(-x\log(x))^n}{n!}$$

$$\Rightarrow J=\sum_{n=0}^{\infty}\int_0^1\frac{(-x\log(x))^n}{n!}dx$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\int_0^1 x^n\log^n(x)dx$$

$$Let: t=-\log(x)\Leftrightarrow x=e^{-t}\Rightarrow dx=-e^{-t}dt\ ,\ If\ : x\in(0,1)\Rightarrow t\in(\infty,0)$$

$$\Rightarrow J=-\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{n!}\int_{\infty}^0 e^{-tn}(-t)^n e^{-t}dt\right)$$
$$=\sum_{n=0}^{\infty}\left(\frac{(-1)^n(-1)^n}{n!}\int_0^{\infty}e^{-t(n+1)}(t)^n dt\right)$$

$$Let: u=t(n+1)\Leftrightarrow\frac{u}{n+1}=t\Rightarrow\frac{du}{n+1}=dt\ ,\ If\ : t\in(0,\infty)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow J=\sum_{n=0}^{\infty}\left(\frac{1}{n!}\int_0^{\infty}e^{-u}\frac{u^n}{(n+1)^n}\times\frac{du}{n+1}\right)$$
$$=\sum_{n=0}^{\infty}\left(\frac{1}{(n+1)^{n+1}\Gamma(n+1)}\int_0^{\infty}u^n e^{-u}du\right)$$
$$=\sum_{n=0}^{\infty}\frac{1}{(n+1)^{n+1}\Gamma(n+1)}\Gamma(n+1)$$
$$=\sum_{n=0}^{\infty}\frac{1}{(n+1)^{n+1}}$$
$$=1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\dots$$

SO, $$\boxed{\int_0^1 x^x dx=\sum_{n=1}^{\infty}\frac{1}{n^n}}$$

136 Calculate integral $K=\int_0^{90^\circ}\sin^3(2x)\cos(x)dx$

Answer

They give $K=\int_0^{90^\circ}\sin^3(2x)\cos(x)dx$

$$=\int_0^{90^\circ}\left(2\sin(x)\cos(x)\right)^3\cos(x)dx$$

$$=8\int_0^{90^\circ}\sin^3(x)\cos^4(x)dx$$

$$=8\int_0^{90^\circ}\sin^{2(2)-1}(x)\cos^{2\left(\frac{5}{2}\right)-1}(x)dx$$

$$=4B\left(2,\frac{5}{2}\right)=\frac{4\Gamma(2)\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(2+\frac{5}{2}\right)}$$

$$=\frac{4\Gamma\left(\frac{5}{2}\right)}{\frac{7}{2}\times\frac{5}{2}\Gamma\left(\frac{5}{2}\right)}=\frac{16}{35}$$

SO, $\boxed{\int_0^{90^\circ}\sin^3(2x)\cos(x)dx=\frac{16}{35}}$

137 Calculate integral $I=\int_0^1\frac{1}{\sqrt{x}(1+\sqrt{x})\sqrt{1-x}}dx$

Answer

They give $I=\int_0^1\frac{1}{\sqrt{x}(1+\sqrt{x})\sqrt{1-x}}dx$

$$=2\int_0^1\frac{1}{(1+\sqrt{x})\sqrt{(1-\sqrt{x})(1+\sqrt{x})}}d(\sqrt{x})=2\int_0^1\frac{1}{(1+\sqrt{x})^2\sqrt{\frac{2}{(1+\sqrt{x})}-1}}d(\sqrt{x})$$

$$=2\int_0^1\frac{1}{(1+\sqrt{x})^2\sqrt{\frac{2}{(1+\sqrt{x})}-1}}d(1+\sqrt{x})=-\int_0^1\frac{1}{\sqrt{\frac{2}{(1+\sqrt{x})}-1}}d\left(\frac{2}{1+\sqrt{x}}\right)$$

$$=-2\sqrt{\frac{2}{(1+\sqrt{x})}-1}\,\Bigg|_0^1=2$$

SO, $\boxed{\int_0^1\frac{1}{\sqrt{x}(1+\sqrt{x})\sqrt{1-x}}dx=2}$

138 Calculate integral $J=\int_0^{\infty}\frac{e^{-2x}\sin(3x)}{x}dx$

Answer

$$\text{They give} \qquad J=\int_0^{\infty}\frac{e^{-2x}\sin(3x)}{x}dx$$

$$\Rightarrow J(a)=\int_0^{\infty}\frac{e^{-ax}\sin(3x)}{x}dx$$

$$\Rightarrow J'(a)=-\int_0^{\infty}\frac{xe^{-ax}\sin(3x)}{x}dx$$

$$=-\int_0^{\infty}e^{-ax}\sin(3x)dx$$

$$=-\left[\left(-\frac{1}{3}e^{-ax}\cos(3x)-\frac{a}{9}e^{-ax}\sin(3x)\right)\Bigg|_0^{\infty}-\frac{a^2}{9}\int_0^{\infty}e^{-ax}\sin(3x)dx\right]$$

$$=\left(\frac{1}{3}e^{-ax}\cos(3x)+\frac{a}{9}e^{-ax}\sin(3x)\right)\Bigg|_0^{\infty}+\frac{a^2}{9}J'(a)$$

$$\Rightarrow J'(a)-\frac{a^2}{9}J'(a)=-\frac{1}{3}$$

$$\Rightarrow J'(a)=-\frac{9}{3}\left(\frac{1}{9-a^2}\right)=\frac{3}{a^2-9}$$

$$\Rightarrow J(a)=\int\frac{3}{a^2-9}da$$

$$=\frac{1}{2}\log\left|\frac{a-3}{a+3}\right|+C$$

$$If: a=2 \Rightarrow J(2)=J=\frac{1}{2}\log\left|\frac{1}{5}\right|+C$$

$$If: a=\infty \Rightarrow J(\infty)=0=\lim_{a\to\infty}\left(\frac{1}{2}\log\left|\frac{a-3}{a+3}\right|+C\right)$$

$$\Leftrightarrow 0=0+C\Rightarrow C=0$$

$$That: J=-\frac{1}{2}\log(5)+0=-\frac{1}{2}\log(5)$$

$$SO,\quad \boxed{\int_0^{\infty}\frac{e^{-2x}\sin(3x)}{x}dx=-\frac{1}{2}\log(5)}$$

139 Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{\left[\cos(x)\left(1+\frac{\sin(x)}{\cos(x)}\right)\right]^2}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{(1+\tan(x))^2\cos^2(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{(1+\tan(x))^2}d(\tan(x))$$

$Let: u=\tan(x)\Rightarrow du=d(\tan(x))\ ,if\ :x\in\left(0,\frac{\pi}{2}\right)\Rightarrow u\in(0,\infty)$

$$\Rightarrow K=\int_0^{\infty}\frac{\sqrt[3]{u}}{(1+u)^2}du=\int_0^{\infty}\frac{u^{\frac{4}{3}-1}}{(1+u)^{\frac{4}{3}+\frac{2}{3}}}du$$

$$=B\left(\frac{4}{3},\frac{2}{3}\right)=\frac{\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{4}{3}+\frac{2}{3}\right)}$$

$$=\frac{\Gamma\left(1+\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)}{\Gamma(2)}$$

$$=\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\Gamma\left(1-\frac{1}{3}\right)$$

$$=\frac{1}{3}\times\frac{\pi}{\sin\left(\frac{\pi}{3}\right)}=\frac{2\pi}{3\sqrt{3}}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx=\frac{2\pi}{3\sqrt{3}}}$

140 Calculate integral $I=\int_{-1}^{1} x^{\frac{x}{\log(x)}}dx$

Answer

They give $I=\int_{-1}^{1} x^{\frac{x}{\log(x)}}dx$

$$=\int_{-1}^{1} e^{\log\left(x^{\frac{x}{\log(x)}}\right)}dx=\int_{-1}^{1} e^{\frac{x}{\log(x)}\log(x)}dx$$

$$=\int_{-1}^{1} e^{x}dx=\frac{(e-1)(e+1)}{e}$$

SO, $\boxed{\int_{-1}^{1} x^{\frac{x}{\log(x)}}dx=\frac{(e-1)(e+1)}{e}}$

141 Calculate integral $J=\int_{-1}^{1} x^{\frac{1}{\log(2x)}}dx$

Answer

They give $J=\int_{0}^{1} \frac{x^{\frac{1}{\log(2x)}-1}}{\log^2(2x)}dx$

$$=\int_{0}^{1} \frac{(2x)^{\frac{1}{\log(2x)}}\times(2)^{-\frac{1}{\log(2x)}}}{x\log^2(2x)}dx$$

$$=\int_{0}^{1} \frac{e\times(2)^{-\frac{1}{\log(2x)}}}{x\log^2(2x)}dx$$

$$=e\int_{0}^{1} \frac{(2)^{-\frac{1}{\log(2x)}}}{x\log^2(2x)}dx$$

$$Let: u=-\frac{1}{\log(2x)}\Rightarrow du=\frac{1}{x\log^2(2x)}dx, if: x\in(0,1)\Rightarrow u\in\left(0,-\frac{1}{\log(2)}\right)$$

$$\Rightarrow J=e\int_{0}^{1} 2^u du=\frac{2^u e}{\log(2)}\Bigg|_{0}^{-\frac{1}{\log(2)}}$$

$$=\frac{1-e}{\log(2)}$$

SO, $\boxed{\int_{0}^{1} \frac{x^{\frac{1}{\log(2x)}-1}}{\log^2(2x)}dx=\frac{1-e}{\log(2)}}$

142 Calculate integral $K=\int_e^{\pi}\sqrt{x-e}\sqrt{\pi-x}dx$

Answer

They give $$K=\int_e^{\pi}\sqrt{x-e}\sqrt{\pi-x}dx$$
$$=\int_e^{\pi}(\pi-x)\sqrt{\frac{x-e}{\pi-x}}dx$$

$$Let: u=\frac{x-e}{\pi-x}\Leftrightarrow x=\frac{\pi u+e}{u+1}\Rightarrow dx=\frac{\pi+e}{(u+1)^2}du, if: x\in(e,\pi)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow K=\int_0^{\infty}\left(\pi-\frac{\pi u+e}{u+1}\right)\sqrt{u}\times\frac{\pi-e}{(u+1)^2}du$$
$$=(\pi-e)^2\int_0^{\infty}\frac{u^{\frac{1}{2}}}{(u+1)^3}du=(\pi-e)^2\int_0^{\infty}\frac{u^{\frac{3}{2}-1}}{(u+1)^{\frac{3}{2}+\frac{3}{2}}}du$$
$$=(\pi-e)^2B\left(\frac{3}{2},\frac{3}{2}\right)=\frac{(\pi-e)^2\Gamma^2\left(\frac{3}{2}\right)}{\Gamma(3)}$$
$$=\frac{\pi(\pi-e)^2}{8}$$

SO, $$\boxed{\int_e^{\pi}\sqrt{x-e}\sqrt{\pi-x}dx=\frac{\pi(\pi-e)^2}{8}}$$

143 Calculate integral $I=\int_{\pi}^{e}x^{\frac{1-2x}{x}}\log(x/e)dx$

Answer

They give $$I=\int_{\pi}^{e}x^{\frac{1-2x}{x}}\log(x/e)dx$$
$$=\int_{\pi}^{e}x^{\frac{1}{x}-2}\log(x/e)dx=\int_e^{\pi}x^{\frac{1}{x}}\frac{\log(e/x)}{x^2}dx$$
$$=\int_e^{\pi}e^{\frac{\log(x)}{x}}\frac{\log(e/x)}{x^2}dx=\int_e^{\pi}e^{\frac{\log(x)}{x}}d\left(\frac{\log(x)}{x}\right)$$
$$=e^{\frac{\log(x)}{x}}\Bigg|_e^{\pi}=\sqrt[\pi]{\pi}-\sqrt[e]{e}$$

SO, $$\boxed{\int_{\pi}^{e}x^{\frac{1-2x}{x}}\log(x/e)dx=\sqrt[\pi]{\pi}-\sqrt[e]{e}}$$

144 Calculate integral $J=\int_0^{\frac{\pi}{2}}\sin^4(x)\cos^5(x)dx$

Answer

They give $$J=\int_0^{\frac{\pi}{2}}\sin^4(x)\cos^5(x)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin^{2(5/2)-1}(x)\cos^{2(3)-1}(x)dx=\frac{1}{2}B(5/2,3)$$

$$=\frac{1}{2}\cdot\frac{\Gamma(5/2)\Gamma(3)}{\Gamma(3+5/2)}=\frac{1}{2}\cdot\frac{2\Gamma(5/2)}{(2+5/2)(1+5/2)(5/2)\Gamma(5/2)}=\frac{8}{315}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\sin^4(x)\cos^5(x)dx=\frac{8}{315}}$$

145 Calculate integral $K=\int_0^{\pi}\frac{1}{1+e^{\tan(x)}}dx$

Answer

They give $$K=\int_0^{\pi}\frac{1}{1+e^{\tan(x)}}dx\ (1)$$

$$=\int_0^{\pi}\frac{1}{1+e^{\tan(\pi-x)}}dx=\int_0^{\pi}\frac{1}{1+e^{-\tan(x)}}dx$$

$$=\int_0^{\pi}\frac{e^{\tan(x)}}{1+e^{\tan(x)}}dx\ \ (2)$$

$$Take\,(1)+(2)That:2K=\int_0^{\pi}\frac{e^{\tan(x)}}{1+e^{\tan(x)}}dx+\int_0^{\pi}\frac{1}{1+e^{\tan(x)}}dx=\int_0^{\pi}\frac{1+e^{\tan(x)}}{1+e^{\tan(x)}}dx=\pi$$

$$\Rightarrow K=\frac{\pi}{2}$$

SO, $$\boxed{\int_0^{\pi}\frac{1}{1+e^{\tan(x)}}dx=\frac{\pi}{2}}$$

146 Calculate integral $I=\int_0^{1}\frac{\sin^{-1}(x)}{x}dx$

Answer

They give $$I=\int_0^{1}\frac{\sin^{-1}(x)}{x}dx$$

$$Let:x=\sin(u)\Rightarrow dx=\cos(u)du\ ,if\ :x\in(0,1)\Rightarrow u\in\left(0,\frac{\pi}{2}\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{-1}(\sin(u))}{\sin(u)}\times\cos(u)du=\int_0^{\frac{\pi}{2}}\frac{u}{\tan(u)}du=\frac{\pi}{2}\log(2)$$

SO, $$\boxed{\int_0^{1}\frac{\sin^{-1}(x)}{x}dx=\frac{\pi}{2}\log(2)}$$

147 Calculate integral $J=\int_{e^{-1}}^{e^{2}}\left|\frac{\log(x)}{x}\right|dx$

Answer

They give $$J=\int_{e^{-1}}^{e^{2}}\left|\frac{\log(x)}{x}\right|dx$$

$$By: x=1\Rightarrow\frac{\log(x)}{x}=0, e^{-1}<x<1\Rightarrow\frac{\log(x)}{x}<0\ ,1<x<e^{2}\Rightarrow\frac{\log(x)}{x}>0$$

$$That: \ J=\int_{e^{-1}}^{1}\left|\frac{\log(x)}{x}\right|dx+\int_{1}^{e^{2}}\left|\frac{\log(x)}{x}\right|dx$$

$$=-\int_{e^{-1}}^{1}\frac{\log(x)}{x}dx+\int_{1}^{e^{2}}\frac{\log(x)}{x}dx$$

$$=-\frac{1}{2}\left(\log^{2}(x)\right)\Big|_{e^{-1}}^{1}+\frac{1}{2}\left(\log^{2}(x)\right)\Big|_{1}^{e^{2}}$$

$$=-\frac{1}{2}\left(0-(-1)^{2}\right)+\frac{1}{2}(4-0)=\frac{5}{2}$$

$$SO, \ \boxed{\int_{e^{-1}}^{e^{2}}\left|\frac{\log(x)}{x}\right|dx=\frac{5}{2}}$$

148 Calculate integral $K=\int_{0}^{2}(1-x)\log(x)dx$

Answer

They give $$K=\int_{0}^{2}\left|(1-x)\log(x)\right|dx$$

$$=\int_{0}^{2}\left|(x-1)\log(x)\right|dx$$

$$By: x=1\Rightarrow\left|(x-1)\log(x)\right|=0\ ;0<x<1\Rightarrow\left|(x-1)\log(x)\right|>0;1<x<2\Rightarrow\left|(x-1)\log(x)\right|>0$$

$$\Rightarrow K=\int_{0}^{2}(x-1)\log(x)dx$$

$$=\left(\frac{x^{2}}{2}-x\right)\log(x)\Big|_{0}^{2}-\int_{0}^{2}\left(\frac{x^{2}}{2}-x\right).\frac{1}{x}dx \quad (Use\ partial\ integral)$$

$$=\lim_{x\to2}\left(\frac{x^{2}}{2}-x\right)\log(x)-\lim_{x\to0^{+}}\left(\frac{x^{2}}{2}-x\right)\log(x)-\int_{0}^{2}\left(\frac{x}{2}-1\right)dx$$

$$=0\times\log(2)-\lim_{x\to0^{+}}\left(\frac{x^{2}-2x}{2}\right)\log(x)-\int_{0}^{2}\left(\frac{x}{2}-1\right)dx$$

$$=-\lim_{x\to0^{+}}\frac{(\log(x))}{\left(\frac{2}{x^{2}-2x}\right)}+\left(x-\frac{x^{2}}{4}\right)\Big|_{0}^{2}$$

$$= -\lim_{x\to 0^+} \frac{d\left(\log(x)\right)}{d\left(\dfrac{2}{x^2-2x}\right)} + 1 = 1 + \lim_{x\to 0^+} \frac{\dfrac{1}{x}}{\dfrac{4(x-1)}{(x^2-2x)^2}}$$

$$= 1 + \lim_{x\to 0^+} \frac{(x^2-2x)^2}{4x(x-1)} = 1 + \lim_{x\to 0^+} \frac{x^2(x-2)^2}{4x(x-1)} = 1$$

SO, $\boxed{\int_0^2 \left|(1-x)\log(x)\right| dx = 1}$

149 Calculate integral $I = \int_1^e \frac{x-1}{x^2 - \log(x^x)} dx$

Answer

They give $I = \int_1^e \frac{x-1}{x^2-\log(x^x)} dx$

$$= \int_1^e \frac{x-1}{x(x-\log(x))} dx = \int_1^e \frac{1-1/x}{x-\log(x)} dx$$

$$= \int_1^e \frac{d\left(x-\log(x)\right)}{\left(x-\log(x)\right)} = \log\left|x-\log(x)\right|\Big|_1^e$$

$$= \log\left|e-\log(e)\right| - \log\left|1-\log(1)\right|$$

$$= \log(e-1)$$

SO, $\boxed{\int_1^e \frac{x-1}{x^2-\log(x^x)} dx = \log(e-1)}$

150 Calculate integral $J = \int_{-1}^1 \sqrt{\frac{1-x}{1+x}} dx$

Answer

They give $J = \int_{-1}^1 \sqrt{\frac{1-x}{1+x}} dx$

$$= \int_{-1}^1 \frac{1-x}{\sqrt{1-x^2}} dx$$

$$= \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} dx - \underbrace{\int_{-1}^1 \frac{x}{\sqrt{1-x^2}} dx}_{\text{is an odd function}}$$

$$= \int_{-1}^1 (\arcsin(x))' dx = 2\arcsin(1) = \pi$$

SO, $\boxed{\int_{-1}^1 \sqrt{\frac{1-x}{1+x}} dx = \pi}$

151 Calculate integral $J=\int_1^2\sqrt{\frac{x-1}{2-x}}dx$

Answer

They give $K=\int_1^2\sqrt{\frac{x-1}{2-x}}dx$

$Let: u=\sqrt{2-x} \Leftrightarrow x=2-u^2 \Rightarrow dx=-2udu \ ,if: x\in(1,2)\Rightarrow u\in(1,0)$

$$\Rightarrow K=-2\int_1^0\frac{\sqrt{1-u^2}}{u}\times udu=2\int_0^1\sqrt{1-u^2}du$$

$Let: u=\sin(t)\Rightarrow du=\cos(t)dt \ ,if: u\in(0,1)\Rightarrow t\in\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow K=2\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2(t)}\cos(t)dt=2\int_0^{\frac{\pi}{2}}|\cos(t)|\cos(t)dt$$

$$=2\int_0^{\frac{\pi}{2}}\cos^2(t)dt=\frac{\pi}{2}$$

SO, $\boxed{\int_1^2\sqrt{\frac{x-1}{2-x}}dx=\frac{\pi}{2}}$

152 Calculate integral $I=\int_{-1}^1\sqrt{\frac{x+1}{x-1}}dx$

Answer

They give $I=\int_{-1}^1\sqrt{\frac{x+1}{x-1}}dx \quad (*)$

$$=\int_{-1}^1\sqrt{\frac{-x+1}{-x-1}}dx$$

$$=\int_{-1}^1\sqrt{\frac{x-1}{x+1}}dx \quad (**)$$

$Take: (**)+(*) They\ have: 2I=\int_{-1}^1\sqrt{\frac{x+1}{x-1}}dx+\int_{-1}^1\sqrt{\frac{x-1}{x+1}}dx=\int_{-1}^1\frac{x+1+x-1}{\sqrt{x^2-1}}dx$

$$=\underbrace{\int_{-1}^1\frac{2x}{\sqrt{x^2-1}}dx}_{is\,an\,odd\ function}$$

$$\Rightarrow I=0$$

SO, $\boxed{\int_{-1}^1\sqrt{\frac{x+1}{x-1}}dx=0}$

153 Calculate integral $J=\int_{-\infty}^{\infty}\frac{1}{x^{12}+1}dx$

Answer

They give $$J=\underbrace{\int_{-\infty}^{\infty}\frac{1}{x^{12}+1}dx}_{is\,an\,even\ function}$$

$$=2\int_{0}^{\infty}\frac{1}{x^{12}+1}dx$$

$$Let: t=x^{12}\Leftrightarrow x=t^{\frac{1}{12}}\Rightarrow dx=\frac{1}{12}t^{\frac{1}{12}-1}dt\,,if\ :x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=2\times\frac{1}{12}\int_{0}^{\infty}\frac{t^{\frac{1}{12}-1}}{(1+t)}dx$$

$$=\frac{1}{6}\int_{0}^{\infty}\frac{t^{\frac{1}{12}-1}}{(1+t)^{\frac{1}{12}+\frac{11}{12}}}dx=\frac{1}{6}.B\left(\frac{1}{12},\frac{11}{12}\right)$$

$$=\frac{1}{6}.\frac{\Gamma\left(\frac{1}{12}\right)\Gamma\left(\frac{11}{12}\right)}{\Gamma\left(\frac{1}{12}+\frac{11}{12}\right)}$$

$$=\frac{1}{6}.\frac{\Gamma\left(\frac{1}{12}\right)\Gamma\left(1-\frac{1}{12}\right)}{\Gamma(1)}$$

$$=\frac{1}{6}.\frac{\pi}{\sin\left(\frac{\pi}{12}\right)}=\frac{1}{6}.\frac{\pi}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$=\frac{\pi(\sqrt{6}+\sqrt{2})}{6}$$

SO, $$\boxed{\int_{-\infty}^{\infty}\frac{1}{x^{12}+1}dx=\frac{\pi(\sqrt{6}+\sqrt{2})}{6}}$$

Noet : $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}$

Because : $\sin\left(\frac{\pi}{12}\right)=\sqrt{\frac{1}{2}\left(1-\cos(\frac{\pi}{6})\right)}=\sqrt{\frac{1}{2}\left(\frac{2-\sqrt{3}}{2}\right)}=\sqrt{\frac{1}{2}\left(\frac{4-2\sqrt{3}}{4}\right)}=\frac{\sqrt{3}-1}{2\sqrt{2}}$

154 Calculate integral $K=\int_0^{45^\circ}\tan(x)\log\left(\tan(x)\right)dx$

Answer

They give $K=\int_0^{45^\circ}\tan(x)\log\left(\tan(x)\right)dx$

$Let:u=\tan(x)\Leftrightarrow x=\tan^{-1}(t)\Rightarrow dx=\frac{1}{1+t^2}dt\ ,if\ :x\in\left(0,45^\circ\right)\Rightarrow t\in(0,1)$

$$\Rightarrow K=\int_0^1\frac{t\log(t)}{1+t^2}dt$$

$Let:u=\log(t)\Rightarrow du=\frac{1}{t}dt\ ,v=\int\frac{t}{1+t^2}dt=\frac{1}{2}\log(1+t^2)$

$$\Rightarrow K=\underbrace{\frac{1}{2}.\log(t)\log(t^2+1)\Big|_0^1}_{0}-\frac{1}{2}\int_0^1\frac{\log(1+t^2)}{t}dt$$

$$=-\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^1 x^{2n-1}dt$$

$$=-\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\times\frac{x^{2n}}{2n}\Big|_0^1$$

$$=-\frac{1}{4}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=-\frac{1}{4}\eta(2)$$

$$=-\frac{1}{4}\times\frac{\pi^2}{12}=-\frac{\pi^2}{48}$$

SO, $\boxed{\int_0^{45^\circ}\tan(x)\log\left(\tan(x)\right)dx=-\frac{\pi^2}{48}}$

Why $:\log(t)\log(t^2+1)\Big|_0^1=0$

Because $:\lim_{x\to1}\log(t)\log(t^2+1)=\log(1)\log(2)=0$

$$\lim_{x\to0}\log(t)\log(t^2+1)=\lim_{x\to0}\frac{d\left(\log(t^2+1)\right)}{d\left(\frac{1}{\log(t)}\right)}$$

$$=-\lim_{x\to0}\frac{\frac{2t}{t^2+1}}{t\log^2(t)}=-2\lim_{x\to0}\frac{t^2\log^2(t)}{t^2+1}=0$$

155 Calculate integral $I=\int_1^2 \lfloor x^2-x \rfloor dx$

Answer

They give $$I=\int_1^2 \lfloor x^2-x \rfloor dx$$

$$Let: t=x^2-x \Rightarrow dx=\frac{1}{\sqrt{4t+1}}dt, if: x\in(1,2)\Rightarrow t\in(0,2)$$

$$\Rightarrow I=\int_0^2 \frac{\lfloor t \rfloor}{\sqrt{4t+1}}dt=\int_0^1 \frac{\lfloor t \rfloor}{\sqrt{4t+1}}dt+\int_1^2 \frac{\lfloor t \rfloor}{\sqrt{4t+1}}dt$$

$$By: \forall t\in[0,1] \vee 0\le t\le 1 \Rightarrow \lfloor t \rfloor=0 \text{ and } \forall t\in[1,2] \vee 1\le t\le 2 \Rightarrow \lfloor t \rfloor=1$$

$$\Rightarrow I=\int_0^1 \frac{0}{\sqrt{4t+1}}dt+\int_1^2 \frac{1}{\sqrt{4t+1}}dt$$

$$=\int_1^2 \frac{1}{\sqrt{4t+1}}dt$$

$$=\frac{1}{2}(3-\sqrt{5})$$

SO, $$\boxed{\int_1^2 \lfloor x^2-x \rfloor dx=\frac{1}{2}(3-\sqrt{5})}$$

156 Calculate integral $J=\int_0^{\log(2)} \frac{\lfloor e^x \rfloor}{\lfloor e^x-1 \rfloor}dx$

Answer

They give $$J=\int_{\log(2)}^{\log(3)} \frac{\lfloor e^x \rfloor}{\lfloor e^x-1 \rfloor}dx$$

$$Let: t=e^x-1 \Rightarrow dx=\frac{1}{t+1}dt, if: x\in(\log(2),\log(3))\Rightarrow t\in(1,2)$$

$$\Rightarrow J=\int_1^2 \frac{\lfloor t+1 \rfloor}{\lfloor t \rfloor}\times\frac{1}{t+1}dt$$

$$if: x\in[1,2]\Rightarrow \lfloor t \rfloor=1, \lfloor t+1 \rfloor=2 \ \vee 1\le t\le 2 \ \Rightarrow \lfloor t \rfloor=1, 2\le t+1\le 3 \ \Rightarrow \lfloor t+1 \rfloor=2$$

$$\Rightarrow J=\int_1^2 \frac{2}{1}\times\frac{1}{t+1}dx=2\log\left(\frac{3}{2}\right)$$

SO, $$\boxed{\int_{\log(2)}^{\log(3)} \frac{\lfloor e^x \rfloor}{\lfloor e^x-1 \rfloor}dx=2\log\left(\frac{3}{2}\right)}$$

157 Calculate integral $K=\int_0^2 \sin\left(\lfloor x+1\rfloor\right)dx$

Answer

They give $$K=\int_0^2 \sin\left(\lfloor x+1\rfloor\right)dx$$

$$=\int_0^2 \sin\left(\lfloor x\rfloor+1\right)dx \quad ,Because:\lfloor x+m\rfloor=\lfloor x\rfloor+m \ \ ,m\in\mathbb{Z}$$

$$=\int_0^1 \sin\left(\lfloor x\rfloor+1\right)dx+\int_1^2 \sin\left(\lfloor x\rfloor+1\right)dx$$

$$=\int_0^1 \sin\left(0+1\right)dx+\int_1^2 \sin\left(1+1\right)dx$$

$$=\sin(1)+\sin(2)=2\sin\left(\frac{3}{2}\right)\cos\left(\frac{1}{2}\right)$$

SO, $$\boxed{\int_0^2 \sin\left(\lfloor x+1\rfloor\right)dx=2\sin\left(\frac{3}{2}\right)\cos\left(\frac{1}{2}\right)}$$

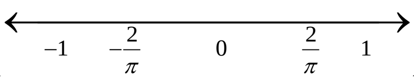
158 Calculate integral $I=\int_{-2}^{2}\dfrac{\sin(x)}{\left\lfloor \dfrac{x}{\pi}\right\rfloor+2}dx$

Answer

They give $$I=\int_{-2}^{2}\frac{\sin(x)}{\left\lfloor \dfrac{x}{\pi}\right\rfloor+2}dx$$

$$Let: t=\frac{x}{\pi}\Rightarrow dx=\pi dt, if\ : x\in(-2,2)\Rightarrow t\in\left(-\frac{2}{\pi},\frac{2}{\pi}\right)$$

$$\Rightarrow I=\pi\int_{-\frac{2}{\pi}}^{\frac{2}{\pi}}\frac{\sin(\pi t)}{\lfloor t\rfloor+2}dt=\pi\int_{-\frac{2}{\pi}}^{0}\frac{\sin(\pi t)}{\lfloor t\rfloor+2}dt+\pi\int_{0}^{\frac{2}{\pi}}\frac{\sin(\pi t)}{\lfloor t\rfloor+2}dt$$

By: 

$$\Rightarrow -\frac{2}{\pi}\le t\le 0\Rightarrow \lfloor t\rfloor=-1, 0\le t\le\frac{2}{\pi}\Rightarrow\lfloor t\rfloor=0$$

$$\Rightarrow I=\pi\int_{-\frac{2}{\pi}}^{0}\frac{\sin(\pi t)}{-1+2}dt+\pi\int_{0}^{\frac{2}{\pi}}\frac{\sin(\pi t)}{0+2}dt$$

$$=\pi\int_{-\frac{2}{\pi}}^{0}\sin(\pi t)dt+\frac{\pi}{2}\int_{0}^{\frac{2}{\pi}}\sin(\pi t)dt=\frac{1}{2}\left(\cos(2)-1\right)$$

SO, $$\boxed{\int_{-2}^{2}\frac{\sin(x)}{\lfloor x/\pi\rfloor+2}dx=\frac{1}{2}\left(\cos(2)-1\right)}$$

159 Calculate integral $J=\int_0^{\infty}\lfloor x\rfloor e^{-x}dx$

Answer

They give $J=\int_0^{\infty}\lfloor x\rfloor e^{-x}dx$

$$=\lim_{n\to\infty}\int_0^{n}\lfloor x\rfloor e^{-x}dx$$

$$=\lim_{n\to\infty}\left(\int_0^1\lfloor x\rfloor e^{-x}dx+\int_1^2\lfloor x\rfloor e^{-x}dx+\int_2^3\lfloor x\rfloor e^{-x}dx+\ldots+\int_{n-1}^{n}\lfloor x\rfloor e^{-x}dx\right)$$

$$=\lim_{n\to\infty}\left(\int_0^1 0.e^{-x}dx+\int_1^2 1.e^{-x}dx+\int_2^3 2.e^{-x}dx\ldots+\int_{n-1}^{n}(n-1)e^{-x}dx\right)$$

$$=\lim_{n\to\infty}\left(-e^{-x}\Big|_0^1-2e^{-x}\Big|_1^2-3e^{-x}\Big|_2^3-\ldots-(n-1)e^{-x}\Big|_{n-1}^{n}\right)$$

$$=\lim_{n\to\infty}\left(-\left(e^{-2}-e^{-1}\right)-\left(2e^{-2}-2e^{-3}\right)-\ldots-\left((n-1)e^{-n}-(n-1)e^{-n+1}\right)\right)$$

$$=\lim_{n\to\infty}\left(e^{-1}+e^{-2}+e^{-3}\ldots+e^{-n+1}+e^{-n}-ne^{-n}\right)$$

$$=\lim_{n\to\infty}\left(e^{-1}\frac{e^{-n}-1}{e^{-1}-1}-ne^{-n}\right)=\lim_{n\to\infty}\left(\frac{e^{-n}-1}{e(e^{-1}-1)}-ne^{-n}\right)$$

$$=\lim_{n\to\infty}\left(\frac{e^{-n}-1-(1-e)ne^{-n}}{1-e}\right)\qquad,\left(\lim_{n\to\infty}(ne^{-n})=0\right)$$

$$=\frac{0-1-0}{1-e}=\frac{1}{e-1}$$

SO, $\boxed{\int_0^{\infty}\lfloor x\rfloor e^{-x}dx=\frac{1}{e-1}}$

160 Calculate integral $K=\int_0^{\frac{\pi}{2}}\lfloor 1+\sin(x)\rfloor dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\lfloor 1+\sin(x)\rfloor dx$

$$=\int_0^{\frac{\pi}{2}}\left(\lfloor\sin(x)\rfloor+1\right)dx\quad ,By:0\le\sin(x)\le1\Rightarrow\lfloor\sin(x)\rfloor=0$$

$$\Rightarrow K=\int_0^{\frac{\pi}{2}}(0+1)dx=\frac{\pi}{2}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\lfloor 1+\sin(x)\rfloor dx=\frac{\pi}{2}}$

161 Calculate integral $I=\int_{\sqrt{2}}^{\sqrt{3}}\frac{\lfloor x^2\rfloor}{\lfloor x^2-1\rfloor}dx$

Answer

They give $I=\int_{\sqrt{2}}^{\sqrt{3}}\frac{\lfloor x^2\rfloor}{\lfloor x^2-1\rfloor}dx$

$$Let: t=x^2-1\Rightarrow dx=\frac{1}{2\sqrt{t+1}}dt, if: x\in\left(\sqrt{2},\sqrt{3}\right)\Rightarrow t\in(1,2)$$

$$\Rightarrow I=\int_1^2\frac{\lfloor t+1\rfloor}{\lfloor t\rfloor}\times\frac{1}{2\sqrt{t+1}}dt$$

$$=\int_1^2\frac{2}{1}\times\frac{1}{2\sqrt{t+1}}dt=\int_1^2\frac{1}{\sqrt{t+1}}dt$$

$$=2\sqrt{t+1}\Big|_1^2=\frac{2}{\sqrt{3}-\sqrt{2}}$$

SO, $\boxed{\int_{\sqrt{2}}^{\sqrt{3}}\frac{\lfloor x^2\rfloor}{\lfloor x^2-1\rfloor}dx=\frac{2}{\sqrt{3}-\sqrt{2}}}$

162 Calculate integral $J=\int_0^{\frac{\pi}{2}}\lfloor\cos(2x)\rfloor dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\lfloor\cos(2x)\rfloor dx$

$$Let: t=\cos(2x)\Rightarrow dx=-\frac{1}{2\sqrt{1-t^2}}dt, if: x\in\left(0,\frac{\pi}{2}\right)\Rightarrow t\in(1,-1)$$

$$\Rightarrow J=-\int_1^{-1}\frac{\lfloor t\rfloor}{2\sqrt{1-t^2}}dx$$

$$=\int_{-1}^0\frac{\lfloor t\rfloor}{2\sqrt{1-t^2}}dx+\int_0^1\frac{\lfloor t\rfloor}{2\sqrt{1-t^2}}dx=\int_{-1}^0\frac{-1}{2\sqrt{1-t^2}}dx+\int_0^1\frac{0}{2\sqrt{1-t^2}}dx$$

$$=-\frac{1}{2}\sin^{-1}(t)\Big|_{-1}^0=-\frac{\pi}{4}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\lfloor\cos(2x)\rfloor dx=-\frac{\pi}{4}}$

163 Calculate integral $K=\int_{-2}^{2}\frac{\left\lfloor \frac{x}{\pi}\right\rfloor}{\left\lfloor \frac{x}{\pi}\right\rfloor+\frac{1}{2}}dx$

Answer

They give $$K=\int_{-2}^{2}\frac{\left\lfloor \frac{x}{\pi}\right\rfloor}{\left\lfloor \frac{x}{\pi}\right\rfloor+\frac{1}{2}}dx$$

$$Let:t=x/\pi\Rightarrow dx=\pi dt,if\ :x\in(-2,2)\Rightarrow t\in\left(-\frac{2}{\pi},\frac{2}{\pi}\right)$$

$$\Rightarrow K=\pi\int_{-\frac{2}{\pi}}^{\frac{2}{\pi}}\frac{\lfloor t\rfloor}{\lfloor t\rfloor+\frac{1}{2}}dx=\pi\int_{-\frac{2}{\pi}}^{0}\frac{\lfloor t\rfloor}{\lfloor t\rfloor+\frac{1}{2}}dt+\pi\int_{0}^{\frac{2}{\pi}}\frac{\lfloor t\rfloor}{\lfloor t\rfloor+\frac{1}{2}}dt$$

$$=\pi\int_{-\frac{2}{\pi}}^{0}\frac{-1}{-1+\frac{1}{2}}dt+\pi\int_{0}^{\frac{2}{\pi}}\frac{0}{0+\frac{1}{2}}dt=\pi\int_{-\frac{2}{\pi}}^{0}2dt=4$$

SO, $$\boxed{\int_{-2}^{2}\frac{\left\lfloor \frac{x}{\pi}\right\rfloor}{\left\lfloor \frac{x}{\pi}\right\rfloor+\frac{1}{2}}dx=4}$$

164 Calculate integral $I=\int_{0}^{\log(3)}\left\lfloor e^{x}+1\right\rfloor dx$

Answer

They give $$I=\int_{0}^{\log(3)}\left\lfloor e^{x}+1\right\rfloor dx$$

$$=\int_{0}^{\log(3)}\left(1+\left\lfloor e^{x}\right\rfloor\right)dx=\log(3)+\int_{0}^{\log(3)}\left\lfloor e^{x}\right\rfloor dx$$

$$Let:t=e^{x}\Rightarrow dx=\frac{1}{t}dt,if\ :x\in(0,\log(3))\Rightarrow t\in(1,3)$$

$$=\log(3)+\int_{1}^{3}\frac{\lfloor t\rfloor}{t}dt=\log(3)+\int_{1}^{2}\frac{\lfloor t\rfloor}{t}dt+\int_{2}^{3}\frac{\lfloor t\rfloor}{t}dt$$

$$=\log(3)+\int_{1}^{2}\frac{1}{t}dt+\int_{2}^{3}\frac{2}{t}dt=\log\left(\frac{27}{2}\right)$$

SO, $$\boxed{\int_{0}^{\log(3)}\left\lfloor e^{x}+1\right\rfloor dx=\log\left(\frac{27}{2}\right)}$$

165 Calculate integral $K=\int_e^{e^2}\lfloor \log(x)\rfloor^{\lfloor \log(x)+1\rfloor}dx$

Answer

They give $K=\int_e^{e^2}\lfloor \log(x)\rfloor^{\lfloor \log(x)+1\rfloor}dx$

$Let: t=\log(x)\Rightarrow dx=e^t dt, if: x\in\left(e,e^2\right)\Rightarrow t\in(1,2)$

$$\Rightarrow K=\int_1^2\lfloor t\rfloor^{\lfloor t+1\rfloor}e^t dt$$

$\forall t\in[1,2]\Rightarrow\lfloor t\rfloor=1,\lfloor t+1\rfloor=2$

$$\Rightarrow K=\int_1^2 1^2 e^t dt=e(e-1)$$

SO, $\boxed{\int_e^{e^2}\lfloor \log(x)\rfloor^{\lfloor \log(x)+1\rfloor}dx=e(e-1)}$

166 Calculate integral $I=\int_{e-1}^{e^2-1}\lfloor \log(x+1)\rfloor dx$

Answer

They give $I=\int_{e-1}^{e^2-1}\lfloor \log(x+1)\rfloor dx$

$Let: t=\log(x+1)\Rightarrow dx=e^t dt, if: x\in\left(e-1,e^2-1\right)\Rightarrow t\in(1,2)$

$$\Rightarrow I=\int_1^2\lfloor t\rfloor e^t dt=e(e-1)$$

SO, $\boxed{\int_{e-1}^{e^2-1}\lfloor \log(x+1)\rfloor dx=e(e-1)}$

167 Calculate integral $J=\int_0^4\frac{|x-1|}{|x-2|+|x-3|}dx$

Answer

They give $J=\int_0^4\frac{|x-1|}{|x-2|+|x-3|}dx$

$$=\int_0^1\frac{|x-1|}{|x-2|+|x-3|}dx+\int_1^2\frac{|x-1|}{|x-2|+|x-3|}dx+\int_2^3\frac{|x-1|}{|x-2|+|x-3|}dx+\int_3^4\frac{|x-1|}{|x-2|+|x-3|}dx$$

$By:\forall x(0,1)\Rightarrow|x-1|=-(x-1),|x-2|=-(x-2),|x-3|=-(x-3)$

$\forall x(1,2)\Rightarrow|x-1|=(x-1),|x-2|=-(x-2),|x-3|=-(x-3)$

$\forall x(2,3)\Rightarrow|x-1|=(x-1),|x-2|=(x-2),|x-3|=-(x-3)$

$\forall x(3,4)\Rightarrow|x-1|=(x-1),|x-2|=(x-2),|x-3|=(x-3)$

$$\Rightarrow J=\int_0^1\frac{-(x-1)}{-(x-2)-(x-3)}dx+\int_1^2\frac{(x-1)}{-(x-2)-(x-3)}dx+\int_2^3\frac{(x-1)}{(x-2)-(x-3)}dx+\int_3^4\frac{(x-1)}{(x-2)-(x-3)}dx$$

$$=\int_0^1\frac{x-1}{2x-5}dx+\int_1^2\frac{x-1}{-2x+5}dx+\int_2^3\frac{x-1}{1}dx+\int_3^4\frac{x-1}{2x-5}dx$$

$$=\left(\frac{1}{2}+\frac{3}{4}\log\left(\frac{3}{5}\right)\right)+\left(-\frac{1}{2}+\frac{3}{4}\log(3)\right)+\left(\frac{3}{2}\right)+\left(\frac{1}{2}+\frac{3}{4}\log(3)\right)$$

$$=2+\frac{3}{4}\log\left(\frac{27}{5}\right)$$

SO, $\boxed{\int_0^4\frac{|x-1|}{|x-2|+|x-3|}dx=2+\frac{3}{4}\log\left(\frac{27}{5}\right)}$

168 Calculate integral $K=\int_0^{\frac{\pi}{2}}\log\left(9\cos^2(x)+\cos(x)\right)dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\log\left(9\cos^2(x)+\sin^2(x)\right)dx$

$$\Rightarrow K(a)=\int_0^{\frac{\pi}{2}}\log\left(a^2\cos^2(x)+\sin^2(x)\right)dx$$

$$\Rightarrow K'(a)=\int_0^{\frac{\pi}{2}}\frac{2a\cos^2(x)}{a^2\cos^2(x)+\sin^2(x)}dx$$

$$=2a\int_0^{\frac{\pi}{2}}\frac{1}{a^2+\tan^2(x)}dx=2a\int_0^{\frac{\pi}{2}}\frac{\sec^2(x)}{\left(a^2+\tan^2(x)\right)\left(1+\tan^2(x)\right)}dx$$

$$=\frac{2a}{a^2-1}\int_0^{\frac{\pi}{2}}\frac{\left(a^2+\tan^2(x)\right)-\left(1+\tan^2(x)\right)}{\left(a^2+\tan^2(x)\right)\left(1+\tan^2(x)\right)}d\left(\tan(x)\right)$$

$$=\frac{2a}{a^2-1}\int_0^{\frac{\pi}{2}}\left[\frac{1}{\left(1+\tan^2(x)\right)}-\frac{1}{\left(a^2+\tan^2(x)\right)}\right]d\left(\tan(x)\right)$$

$$=\frac{2a}{a^2-1}\left[\tan^{-1}\left(\tan(x)\right)-\frac{1}{a}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{a}}\right)\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$=\frac{2a}{a^2-1}\left(\frac{\pi}{2}-\frac{1}{a}\times\frac{\pi}{2}\right)=\frac{2\pi a(a-1)}{a(a-1)(a+1)}=\frac{\pi}{a+1}$$

$$\Rightarrow K(a)=\int\frac{\pi}{a+1}da=\pi\log(a+1)+C$$

$$If: a=3 \Rightarrow K(3)=K=\pi\log(4)+C \quad (*)$$
$$If: a=1 \Rightarrow K(1)=0=\pi\log(2)+C \Rightarrow C=-\pi\log(2)$$
$$Take: K=\pi\log(4)-\pi\log(2)=\pi\log(2)$$

$$SO, \quad \boxed{\int_0^{\frac{\pi}{2}} \log\left(9\cos^2(x)+\sin^2(x)\right)dx=\pi\log(2)}$$

169 Calculate integral $I=\int_0^{\pi}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx$

Answer

They give $\quad I=\int_0^{3}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx$

$$=\int_0^{1}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx+\int_1^{2}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx+\int_2^{3}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx$$

$$By:\begin{cases}\forall x(0,1)\Leftrightarrow 0<x<1\Rightarrow\lfloor x\rfloor=0\\ \forall x(1,2)\Leftrightarrow 1<x<2\Rightarrow\lfloor x\rfloor=1\\ \forall x(2,3)\Leftrightarrow 2<x<3\Rightarrow\lfloor x\rfloor=2\end{cases}$$

$$\Rightarrow I=\int_0^{1}\frac{1}{x^2+1+0(0-2x)}dx+\int_1^{2}\frac{1}{x^2+1+1(1-2x)}dx+\int_2^{3}\frac{1}{x^2+1+2(2-2x)}dx$$

$$=\int_0^{1}\frac{1}{x^2+1}dx+\int_1^{2}\frac{1}{(x-1)^2+1}dx+\int_2^{3}\frac{1}{(x-2)^2+1}dx$$

$$=\int_0^{1}\frac{1}{x^2+1}dx+\int_1^{2}\frac{1}{(x-1)^2+1}d(x-1)+\int_2^{3}\frac{1}{(x-2)^2+1}d(x-2)$$

$$=\tan^{-1}(x)\Big|_0^1+\tan^{-1}(x-1)\Big|_1^2+\tan^{-1}(x-2)\Big|_2^3$$

$$=\tan^{-1}(1)-\tan^{-1}(0)+\tan^{-1}(2-1)+\tan^{-1}(1-1)+\tan^{-1}(3-2)+\tan^{-1}(2-2)$$

$$=\tan^{-1}(1)-\tan^{-1}(0)+\tan^{-1}(1)+\tan^{-1}(0)+\tan^{-1}(1)+\tan^{-1}(0)$$

$$=\left(\frac{\pi}{4}-0\right)+\left(\frac{\pi}{4}-0\right)+\left(\frac{\pi}{4}-0\right)$$

$$=\frac{3\pi}{4}$$

$$SO, \quad \boxed{\int_0^{3}\frac{1}{x^2+1+\lfloor x\rfloor\left(\lfloor x\rfloor-2x\right)}dx=\frac{3\pi}{4}}$$

170 Calculate integral $J=\int_{-\infty}^{\infty}\frac{x^2+1}{x^8+1}dx$

Answer

They give $$J=\underbrace{\int_{-\infty}^{\infty}\frac{x^2+1}{x^8+1}dx}_{\text{is an even function}}$$

$$=2\int_0^{\infty}\frac{x^2+1}{x^8+1}dx$$

$$Let: x=t^{\frac{1}{8}}\Rightarrow dx=\frac{1}{8}t^{\frac{1}{8}-1}dt, if: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=2\int_0^{\infty}\frac{t^{\frac{2}{8}}+1}{t+1}\times\frac{1}{8}t^{\frac{1}{8}-1}dt=\frac{1}{4}\int_0^{\infty}\left(\frac{t^{\frac{3}{8}-1}}{t+1}+\frac{t^{\frac{1}{8}-1}+1}{t+1}\right)dt$$

$$=\frac{1}{4}\int_0^{\infty}\left(\frac{t^{\frac{3}{8}-1}}{(t+1)^{\frac{3}{8}+\frac{5}{8}}}+\frac{t^{\frac{1}{8}-1}+1}{(t+1)^{\frac{1}{8}+\frac{7}{8}}}\right)dt=\frac{1}{4}\left[B\left(\frac{3}{8},\frac{5}{8}\right)+B\left(\frac{1}{8},\frac{7}{8}\right)\right]$$

$$=\frac{1}{4}\left[\frac{\Gamma\left(\frac{3}{8}\right)\Gamma\left(\frac{5}{8}\right)}{\Gamma\left(\frac{3}{8}+\frac{5}{8}\right)}+\frac{\Gamma\left(\frac{1}{8}\right)\Gamma\left(\frac{7}{8}\right)}{\Gamma\left(\frac{1}{8}+\frac{7}{8}\right)}\right]$$

$$=\frac{1}{4}\left[\Gamma\left(\frac{3}{8}\right)\Gamma\left(1-\frac{3}{8}\right)+\Gamma\left(\frac{1}{8}\right)\Gamma\left(1-\frac{1}{8}\right)\right]$$

$$=\frac{1}{4}\left[\frac{\pi}{\sin\left(\frac{3\pi}{8}\right)}+\frac{\pi}{\sin\left(\frac{\pi}{8}\right)}\right]=\frac{\pi}{4}\left[\frac{1}{\cos\left(\frac{\pi}{8}\right)}+\frac{1}{\sin\left(\frac{\pi}{8}\right)}\right]$$

$$=\frac{\pi}{4}\left[\frac{1}{\cos\left(\frac{\pi}{8}\right)}+\frac{1}{\sin\left(\frac{\pi}{8}\right)}\right]=\frac{\pi}{4}\left[\frac{1}{\frac{\sqrt{2+\sqrt{2}}}{2}}+\frac{1}{\frac{\sqrt{2-\sqrt{2}}}{2}}\right]$$

$$=\frac{\pi}{2}\left(\sqrt{2+\sqrt{2}}\right)$$

SO, $$\boxed{\int_{-\infty}^{\infty}\frac{x^2+1}{x^8+1}dx=\frac{\pi}{2}\left(\sqrt{2+\sqrt{2}}\right)}$$

171 Calculate integral $K=\int_{-1}^{1}x\sqrt{x^2}dx$

Answer

They give $K=\int_{-1}^{1}x\sqrt{x^2}dx$

$$=\int_{-1}^{1}x|x|dx=\int_{-1}^{1}(-x)|-x|dx$$

$$=-\int_{-1}^{1}x|x|dx$$

$$\Leftrightarrow K=-K\Rightarrow K=0$$

SO, $\boxed{\int_{-1}^{1}x|x|dx=0}$

172 Calculate integral $I=\int_{0}^{\infty}\frac{\sqrt{x}}{(x+1)^2}dx$

Answer

They give $I=\int_{0}^{\infty}\frac{\sqrt{x}}{(x+1)^2}dx$

$$=\int_{0}^{\infty}\frac{x^{(1+\frac{1}{2})-1}}{(x+1)^{(1+\frac{1}{2})+\frac{1}{2}}}dx$$

$$=B\left(1+\frac{1}{2},\frac{1}{2}\right)=\frac{\Gamma\left(1+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(1+\frac{1}{2}+\frac{1}{2}\right)}$$

$$=\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2}$$

SO, $\boxed{\int_{0}^{\infty}\frac{\sqrt{x}}{(x+1)^2}dx=\frac{\sqrt{\pi}}{2}}$

173 Calculate integral $J=\int_{0}^{1}x^2(x-1)^3dx$

Answer

They give $J=\int_{0}^{1}x^2(x-1)^3dx$

$$=-\int_{0}^{1}x^{3-1}(1-x)^{4-1}dx$$

$$=-B(3,4)=-\frac{\Gamma(3)\Gamma(4)}{\Gamma(3+4)}$$

$$=-\frac{2!\times 3!}{6!}=-\frac{1}{60}$$

SO, $\boxed{\int_{0}^{1}x^2(x-1)^3dx=-\frac{1}{60}}$

174 Calculate integral $K=\int_0^{\frac{\pi}{2}}\left(x\cos(x)+1\right)e^{\sin(x)}dx$

Answer

They give $K=\int_0^{\frac{\pi}{2}}\left(x\cos(x)+1\right)e^{\sin(x)}dx$

$$=\int_0^{\frac{\pi}{2}}\left[x\left(\sin(x)\right)'e^{\sin(x)}+x'e^{\sin(x)}\right]dx$$

$$=\int_0^{\frac{\pi}{2}}\left[x\left(e^{\sin(x)}\right)'+x'e^{\sin(x)}\right]dx \quad ,Use: u'v+v'u=(uv)',(e^u)'=u'e^u$$

$$=\int_0^{\frac{\pi}{2}}\left(xe^{\sin(x)}\right)'dx \quad ,Use:\begin{cases}\int_a^b f'(x)dx=f(b)-f(a)\\ \int f'(x)dx=f(x)+C\end{cases}$$

$$=xe^{\sin(x)}\Big|_0^{\frac{\pi}{2}}=\frac{\pi}{2}e$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\left(x\cos(x)+1\right)e^{\sin(x)}dx=\frac{\pi}{2}e}$

175 Calculate integral $I=\int_0^{\frac{\pi}{2}}\left(x\sin(x)-1\right)e^{\cos(x)}dx$

Answer

They give $I=\int_0^{\frac{\pi}{2}}\left(x\sin(x)-1\right)e^{\cos(x)}dx$

$$=-\int_0^{\frac{\pi}{2}}\left(1-x\sin(x)\right)e^{\cos(x)}dx$$

$$=-\int_0^{\frac{\pi}{2}}\left[x'e^{\cos(x)}-x\left(\cos(x)\right)'e^{\cos(x)}\right]dx$$

$$=-\int_0^{\frac{\pi}{2}}\left[x'e^{\cos(x)}-x\left(e^{\cos(x)}\right)'\right]dx$$

$$=-\int_0^{\frac{\pi}{2}}\left(xe^{\cos(x)}\right)'dx$$

$$=-xe^{\cos(x)}\Big|_0^{\frac{\pi}{2}}=-\frac{\pi}{2}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\left(x\sin(x)-1\right)e^{\cos(x)}dx=-\frac{\pi}{2}}$

176 Calculate integral $J=\int_0^{\infty}\frac{x}{e^{\pi x}-1}dx$

Answer

They give $J=\int_0^{\infty}\frac{x}{e^{\pi x}-1}dx$

$$Let: t=\pi x\Rightarrow dx=\frac{1}{\pi}dt, if: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=\frac{1}{\pi^2}\int_0^{\infty}\frac{t}{e^t-1}dt$$

$$=\frac{1}{\pi^2}\int_0^{\infty}\frac{te^{-t}}{(e^t-1)e^{-t}}dt$$

$$=\frac{1}{\pi^2}\int_0^{\infty}\frac{te^{-t}}{1-e^{-t}}dt$$

$$=\frac{1}{\pi^2}\int_0^{\infty}\left(te^{-t}\sum_{n=0}^{\infty}e^{-nt}\right)dt$$

$$=\frac{1}{\pi^2}\sum_{n=0}^{\infty}\left[\int_0^{\infty}te^{-(n+1)t}dt\right]$$

$$Let: u=(n+1)t\Rightarrow dt=\frac{1}{n+1}du, if: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=\frac{1}{\pi^2}\sum_{n=0}^{\infty}\left[\int_0^{\infty}\left(\frac{u}{n+1}\right)e^{-u}\left(\frac{1}{n+1}\right)du\right]$$

$$=\frac{1}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(n+1)^2}\left[\int_0^{\infty}ue^{-u}du\right]$$

$$=\frac{1}{\pi^2}\sum_{n=0}^{\infty}\frac{\Gamma(2)}{(n+1)^2}=\frac{1}{\pi^2}\zeta(2)$$

$$=\frac{1}{\pi^2}\times\frac{\pi^2}{6}=\frac{1}{6}$$

SO, $\boxed{\int_0^{\infty}\frac{x}{e^{\pi x}-1}dx=\frac{1}{6}}$

$$Note: J=\frac{1}{\pi^2}\int_0^{\infty}\frac{t}{e^t-1}dt=\frac{1}{\pi^2}\int_0^{\infty}\frac{t^{2-1}}{e^t-1}dt$$

$$=\frac{1}{\pi^2}\zeta(2)\Gamma(2)=\frac{1}{\pi^2}\times\frac{\pi^2}{6}\times 1!=\frac{1}{6}$$

177 Calculate integral $K=\int_{-2026}^{2026} x^{2026}\cot^{-1}(2026x)dx$

Answer

They give
$$K=\int_{-2026}^{2026} x^{2026}\cot^{-1}\left(2026x\right)dx$$
$$=\int_{-2026}^{2026} (-x)^{2026}\cot^{-1}\left(-2026x\right)dx$$
$$=\int_{-2026}^{2026} x^{2026}\left[\pi-\cot^{-1}\left(2026x\right)\right]dx$$
$$=\pi\underbrace{\int_{-2026}^{2026} x^{2026}}_{is\,an\,even\ function}dx-K$$
$$\Rightarrow K=\pi\int_{0}^{2026} x^{2026}dx=\frac{\pi\times 2026^{2027}}{2027}$$

SO, $\boxed{\int_{-2026}^{2026} x^{2026}\cot^{-1}\left(2026x\right)dx=\frac{\pi\times 2026^{2027}}{2027}}$

178 Calculate integral $I=\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor x^2-4x+4\rfloor+\lfloor x^2\rfloor}dx$

Answer

They give
$$I=\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor x^2-4x+4\rfloor+\lfloor x^2\rfloor}dx$$
$$=\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor (2-x)^2\rfloor+\lfloor x^2\rfloor}dx \qquad (1)$$
$$=\int_0^2\frac{\lfloor (2-x)^2\rfloor}{\lfloor (x)^2\rfloor+\lfloor (2-x)^2\rfloor}dx \qquad (2)$$

$$Take:(1)+(2)That:2I=\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor (2-x)^2\rfloor+\lfloor x^2\rfloor}dx+\int_0^2\frac{\lfloor (2-x)^2\rfloor}{\lfloor (2-x)^2\rfloor+\lfloor (x)^2\rfloor}dx$$
$$\Rightarrow I=\frac{1}{2}\int_0^2\frac{\lfloor (2-x)^2\rfloor+\lfloor (x)^2\rfloor}{\lfloor (2-x)^2\rfloor+\lfloor (x)^2\rfloor}dx=1$$

SO, $\boxed{\int_0^2\frac{\lfloor x^2\rfloor}{\lfloor x^2-4x+4\rfloor+\lfloor x^2\rfloor}dx=1}$

179 Calculate integral $J=\int_0^{45}\lfloor 45x\rfloor dx$

Answer

They give $J=\int_0^{45}\lfloor 45x\rfloor dx$

$$Let:t=45x\Rightarrow dx=\frac{1}{45}dt, if\ :x\in(0,45)\Rightarrow t\in(0,2025)$$

$$\Rightarrow J=\frac{1}{45}\int_0^{2025}\lfloor t\rfloor dt=\frac{1}{45}\left[\int_0^1\lfloor t\rfloor dt+\int_1^2\lfloor t\rfloor dt+\int_2^3\lfloor t\rfloor dt+...+\int_{2024}^{2025}\lfloor t\rfloor dt\right]$$

$$=\frac{1}{45}\left[\int_0^1 0dt+\int_1^2 1dt+\int_2^3 2dt+...+\int_{2024}^{2025}2024dt\right]$$

$$=\frac{1}{45}(1+2+3+...+2025)$$

$$=\frac{1}{45}\times\frac{2025\times2026}{2}=25325$$

SO, $\boxed{\int_0^{45}\lfloor 45x\rfloor dx=25325}$

180 Calculate integral $K=\int_0^{\infty}\frac{(2-x)^{2023}}{(2+x)^{2025}}dx$

Answer

They give $K=\int_0^{\infty}\frac{(2-x)^{2024}}{(2+x)^{2026}}dx$

$$=-\frac{1}{4}\int_0^{\infty}\left(\frac{2-x}{2+x}\right)^{2024}\times\frac{-4}{(2+x)^2}dx$$

$$Let:t=\frac{2-x}{2+x}\Rightarrow dt=\frac{\begin{bmatrix}-1&2\\1&2\end{bmatrix}}{(2+x)^2}dx=\frac{-4}{(2+x)^2}dx, if\ :x\in(0,\infty)\Rightarrow t\in(1,-1)$$

$$\Rightarrow K=-\frac{1}{4}\int_1^{-1}(t)^{2024}dt=\frac{1}{4}\underbrace{\int_{-1}^{1}(t)^{2024}dt}_{is\,an\,even\ funaction}$$

$$=\frac{1}{2}\int_0^1(t)^{2024}dt=\frac{1}{4050}$$

SO, $\boxed{\int_0^{\infty}\frac{(2-x)^{2024}}{(2+x)^{2026}}dx=\frac{1}{4050}}$

181 Calculate integral $I=\int_0^{\infty}\frac{x^6+1}{x^{12}+1}dx$

Answer

They give $I=\int_0^{\infty}\frac{x^6+1}{x^{12}+1}dx$

$$Let: x=t^{\frac{1}{12}}\Rightarrow dx=\frac{1}{12}t^{\frac{1}{12}-1}dt, if: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=\int_0^{\infty}\frac{t^{\frac{6}{12}}+1}{t+1}\times\frac{1}{12}t^{\frac{1}{12}-1}dt=\frac{1}{12}\int_0^{\infty}\left(\frac{t^{\frac{7}{12}-1}}{t+1}+\frac{t^{\frac{1}{12}-1}+1}{t+1}\right)dt$$

$$=\frac{1}{12}\int_0^{\infty}\left(\frac{t^{\frac{7}{12}-1}}{(t+1)^{\frac{7}{12}+\frac{5}{12}}}+\frac{t^{\frac{1}{12}-1}+1}{(t+1)^{\frac{1}{12}+\frac{11}{12}}}\right)dt$$

$$=\frac{1}{12}\left[B\left(\frac{7}{12},\frac{5}{2}\right)+B\left(\frac{1}{12},\frac{11}{12}\right)\right]$$

$$=\frac{1}{12}\left[\frac{\Gamma\left(\frac{7}{12}\right)\Gamma\left(\frac{5}{12}\right)}{\Gamma\left(\frac{7}{12}+\frac{5}{12}\right)}+\frac{\Gamma\left(\frac{1}{12}\right)\Gamma\left(\frac{11}{12}\right)}{\Gamma\left(\frac{1}{12}+\frac{11}{12}\right)}\right]$$

$$=\frac{1}{12}\left[\Gamma\left(\frac{7}{12}\right)\Gamma\left(1-\frac{7}{12}\right)+\Gamma\left(\frac{1}{12}\right)\Gamma\left(1-\frac{1}{12}\right)\right]$$

$$=\frac{1}{12}\left[\frac{\pi}{\sin\left(\frac{7\pi}{12}\right)}+\frac{\pi}{\sin\left(\frac{\pi}{12}\right)}\right]=\frac{\pi}{12}\left[\frac{1}{\cos\left(\frac{\pi}{12}\right)}+\frac{1}{\sin\left(\frac{\pi}{12}\right)}\right]$$

$$=\frac{\pi}{12}\left[\frac{1}{\frac{\sqrt{2+\sqrt{3}}}{2}}+\frac{1}{\frac{\sqrt{2-\sqrt{3}}}{2}}\right]=\frac{\pi}{\sqrt{6}}$$

SO, $\boxed{\int_0^{\infty}\frac{x^6+1}{x^{12}+1}dx=\frac{\pi}{\sqrt{6}}}$

Note: $\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{2+\sqrt{3}}}{2}, \sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{2-\sqrt{3}}}{2}$

182 Calculate integral $J=\int_0^{\frac{\pi}{2}}\frac{x\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\frac{x\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-x\right)\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx$$

$$=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\frac{\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx-\int_0^{\frac{\pi}{2}}\frac{x\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx$$

$$=\frac{\pi}{2}\int_0^{\frac{\pi}{2}}\frac{\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx-J$$

$$\Rightarrow J=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\tan(x)}{1+\tan^4(x)}\times\frac{1}{\cos^2(x)}dx$$

$$=\frac{\pi}{4}\int_0^{\frac{\pi}{2}}\frac{\tan(x)}{1+\tan^4(x)}d\left(\tan(x)\right)$$

$$=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{2\tan(x)}{1+\tan^4(x)}d\left(\tan(x)\right)$$

$$=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{1}{1+\tan^2(x)}d\left(\tan^2(x)\right)$$

$$=\frac{\pi}{8}\left(\arctan\left(\tan^2(x)\right)\right)\Big|_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{8}\left(\arctan(\infty)-\arctan(0)\right)$$

$$=\frac{\pi}{8}\left(\frac{\pi}{2}-0\right)=\frac{\pi^2}{16}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\frac{x\sin(x)\cos(x)}{\sin^4(x)+\cos^4(x)}dx=\frac{\pi^2}{16}}$

183 Calculate integral $K = \int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1+\sin(x)\cos(x)}dx$

Answer

They give $\quad K = \int_0^{\frac{\pi}{2}} \frac{\sin^2(x)}{1+\sin(x)\cos(x)}dx \quad (*)$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^2(x)}{1+\sin(x)\cos(x)}dx \quad (**)$$

$Take\,(*)+(**)\,That\,have: 2K = \int_0^{\frac{\pi}{2}} \frac{\sin^2(x)}{1+\sin(x)\cos(x)}dx + \int_0^{\frac{\pi}{2}} \frac{\cos^2(x)}{1+\sin(x)\cos(x)}dx$

$$\Rightarrow K = \int_0^{\frac{\pi}{2}} \frac{1}{2+2\sin(x)\cos(x)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2+\sin(2x)}dx$$

$Let: t = 2x \Rightarrow dx = \frac{1}{2}dt, if\ : x \in \left(0, \frac{\pi}{2}\right) \Rightarrow t \in (0, \pi)$

$$\Rightarrow K = \int_0^{\pi} \frac{1}{2+\sin(t)}dt$$

$Let: y = \tan\left(\frac{t}{2}\right) \Rightarrow dt = \frac{2}{1+y^2}dy, if\ : x \in (0, \pi) \Rightarrow t \in (0, \infty), \sin(t) = \frac{2y}{1+y^2}$

$$\Rightarrow K = \int_0^{\infty} \frac{1}{2+\frac{2y}{1+y^2}} \times \frac{2}{1+y^2}dy = \int_0^{\infty} \frac{1}{y^2+t+1}dy$$

$$= \int_0^{\infty} \frac{1}{(2y+1)^2+3}dy = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)\Bigg|_0^{\infty}$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}$$

SO, $\boxed{\int_0^{\frac{\pi}{2}} \frac{\sin^2(x)}{1+\sin(x)\cos(x)}dx = \frac{\pi}{3\sqrt{3}}}$

184 Calculate integral $I=\int_0^{\infty}\frac{\log(x)}{x^2-x+1}dx$

Answer

They give $I=\int_0^{\infty}\frac{\log(x)}{x^2-x+1}dx$

$$Let: x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt, If: x\in(0,\infty)\Rightarrow t\in(\infty,0)$$

$$\Rightarrow I=\int_{\infty}^{0}\frac{\log(1/t)}{\left(\frac{1}{t}\right)^2-\left(\frac{1}{t}\right)+1}\left(-\frac{1}{t^2}\right)dt$$

$$=-\int_0^{\infty}\frac{\log(t)}{t^2-t+1}dt=-I \quad , Note: \int_0^a f(x)dx=\int_0^a f(t)dt$$

$$\Leftrightarrow I=-I\Rightarrow I=0$$

SO, $\boxed{\int_0^{\infty}\frac{\log(x)}{x^2-x+1}dx=0}$

185 Calculate integral $J=\int_0^1\frac{1}{\sqrt{-\log(x)}}dx$

Answer

They give $J=\int_0^1\frac{1}{\sqrt{-\log(x)}}dx$

$$Let: t=-\log(x)\Rightarrow dx=-e^{-t}dt, If: x\in(0,1)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=-\int_{\infty}^{0}\frac{e^{-t}}{\sqrt{t}}dt$$

$$=\int_0^{\infty}t^{-\frac{1}{2}}e^{-t}dt$$

$$=\int_0^{\infty}t^{\frac{1}{2}-1}e^{-t}dt$$

$$=\Gamma\left(\frac{1}{2}\right)=\left(-\frac{1}{2}\right)!$$

$$=\sqrt{\pi}$$

SO, $\boxed{\int_0^1\frac{1}{\sqrt{-\log(x)}}dx=\sqrt{\pi}}$

186 Calculate integral $K=\int_0^1\left(\log\left(\frac{1}{x}\right)\right)^{n-1}dx$

Answer

They give $$K=\int_0^1\left(\log\left(\frac{1}{x}\right)\right)^{n-1}dx$$

$$Let: t=\log(1/x)\Leftrightarrow x=e^{-t}\Rightarrow dx=-e^{-t}dt\,, If: x\in(0,1)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow K=-\int_\infty^0 t^{n-1}e^{-t}dt$$

$$=\int_0^\infty t^{n-1}e^{-t}dt$$

$$=\Gamma(n)=(n-1)!$$

SO, $$\boxed{\int_0^1\left(\log\left(\frac{1}{x}\right)\right)^{n-1}dx=\Gamma(n)}$$

187 Calculate integral $I=\int_0^{\pi/2}\tan^n(x)dx$

Answer

They give $$I=\int_0^{\pi/2}\tan^n(x)dx$$

$$=\int_0^{\pi/2}\sin^n(x)\cos^{-n}(x)dx$$

$$=\int_0^{\pi/2}\sin^{2\left(\frac{1+n}{2}\right)-1}(x)\cos^{2\left(\frac{1-n}{2}\right)-1}(x)dx$$

$$=\frac{1}{2}B\left(\frac{1+n}{2},\frac{1-n}{2}\right)$$

$$=\frac{1}{2}\times\frac{\Gamma\left(\frac{1+n}{2}\right)\Gamma\left(\frac{1-n}{2}\right)}{\left(\frac{1+n}{2}+\frac{1-n}{2}\right)}$$

$$=\frac{1}{2}\times\Gamma\left(\frac{1+n}{2}\right)\Gamma\left(1-\frac{1+n}{2}\right)$$

$$=\frac{\pi}{2}\csc\left(\frac{1+n}{2}\right)$$

SO, $$\boxed{\int_0^{\pi/2}\tan^n(x)dx=\frac{\pi}{2}\csc\left(\frac{1+n}{2}\right)}$$

188 Calculate integral $J=\int_0^1 \frac{e^x-1}{x}dx$

Answer

$Mathod:1$

$$They\ give \qquad J=\int_0^1 \frac{e^x-1}{x}dx$$

$$By: e^x=\sum_{n=0}^{\infty}\left(\frac{x^n}{n!}\right) \Rightarrow e^x-1=\sum_{n=1}^{\infty}\left(\frac{x^n}{n!}\right)$$

$$\Rightarrow J=\int_0^1\left(\frac{1}{x}\sum_{n=1}^{\infty}\left(\frac{x^n}{n!}\right)dx\right)=\sum_{n=1}^{\infty}\left(\frac{1}{n!}\int_0^1 x^{n-1}dx\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{n!}\right)\frac{x^n}{n}\Bigg|_0^1=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)$$

$$SO, \qquad \boxed{\int_0^1 \frac{e^x-1}{x}dx=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)}$$

$Mathod:2$

$$They\ give \qquad J=\int_0^1 \frac{e^x-1}{x}dx$$

$$\Rightarrow J(a)=\int_0^1 \frac{e^{ax}-1}{x}dx$$

$$\Rightarrow J'(a)=\int_0^1 e^{ax}dx=\frac{e^{ax}}{a}\Bigg|_0^1$$

$$=\frac{e^a-1}{a}$$

$$\Rightarrow J(a)=\int \frac{e^a-1}{a}da=\sum_{n=1}^{\infty}\left(\frac{a^n}{n.n!}\right)+C$$

$$If\ : a=0 \Rightarrow J(0)=0=\sum_{n=1}^{\infty}\left(\frac{1}{n!}\right)\frac{0^n}{n}+C \Rightarrow C=0$$

$$If\ : a=1 \Rightarrow J(1)=J=\sum_{n=1}^{\infty}\left(\frac{1}{n!}\right)\frac{1^n}{n}+C \quad ,But: C=0$$

$$That:\ J=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)$$

$$SO, \qquad \boxed{\int_0^1 \frac{e^x-1}{x}dx=\sum_{n=1}^{\infty}\left(\frac{1}{n.n!}\right)}$$

189 Calculate integral $K=\int_0^{\infty} x^{-\log(x)}\log(x^x)dx$

Answer

They give $K=\int_0^{\infty} x^{-\log(x)}\log(x^x)dx$

$$=\int_0^{\infty} x^{-\log(x)+1}\log(x)dx$$

Let : $t=\log(x)\Rightarrow dx=e^t dt$, *If* : $x\in(0\infty)\Rightarrow t\in(-\infty,\infty)$

$$\Rightarrow K=\int_{-\infty}^{\infty}(e^t)^{-t+1}t.e^t dt=e\int_{-\infty}^{\infty}e^{-(t-1)^2}tdt$$

$$=e\int_{-\infty}^{\infty}e^{-(t-1)^2}[(t-1)+1]dt=e\int_{-\infty}^{\infty}(t-1)e^{-(t-1)^2}dt+e\int_{-\infty}^{\infty}e^{-(t-1)^2}dt$$

$$=\frac{e}{2}\int_{-\infty}^{\infty}e^{-(t-1)^2}d\left((t-1)^2\right)+e\int_{-\infty}^{\infty}e^{-(t-1)^2}d(t-1)$$

$$=\frac{e}{2}e^{-(t-1)^2}\Big|_{-\infty}^{\infty}+e\sqrt{\pi}=\frac{e}{2}(0-0)+e\sqrt{\pi}=e\sqrt{\pi}$$

SO, $\boxed{\int_0^{\infty} x^{-\log(x)}\log(x^x)dx=e\sqrt{\pi}}$

190 Calculate integral $I=\int_0^{\infty}\frac{x^4}{(1+x^3)^2}dx$

Answer

They give $I=\int_0^{\infty}\frac{x^4}{(1+x^3)^2}dx$

Let : $t=x^3\Rightarrow dx=\frac{1}{3}t^{1/3-1}dt$, *If* : $x\in(0\infty)\Rightarrow t\in(0,\infty)$

$$\Rightarrow I=\frac{1}{3}\int_0^{\infty}\frac{t^{\frac{4}{3}}}{(1+t)^2}\times t^{\frac{1}{3}-1}dt=\frac{1}{3}\int_0^{\infty}\frac{t^{\frac{4}{3}}}{(1+t)^2}\times t^{\frac{1}{3}-1}dt$$

$$=\frac{1}{3}\int_0^{\infty}\frac{t^{\frac{5}{3}-1}}{(1+t)^{\frac{5}{3}+\frac{1}{3}}}dt=\frac{1}{3}B\left(\frac{5}{3},\frac{1}{3}\right)$$

$$=\frac{1}{3}\times\frac{\Gamma\left(\frac{5}{3}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma(2)}=\frac{1}{3}\times\frac{2}{3}\Gamma\left(1-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)$$

$$=\frac{2\pi}{9}\csc\left(\frac{\pi}{3}\right)=\frac{4\pi}{9\sqrt{3}}$$

SO, $\boxed{\int_0^{\infty}\frac{x^4}{(1+x^3)^2}dx=\frac{4\pi}{9\sqrt{3}}}$

191 Calculate integral $J=\int_{-1}^{1}(1-x^2)^n\,dx$

Answer

They give $$J=\underbrace{\int_{-1}^{1}(1-x^2)^n\,dx}_{\text{is an even function}}$$

$$=2\int_0^1(1-x^2)^n\,dx \qquad ,\begin{cases} Let: t=x^2 \Rightarrow dx=\frac{1}{2}t^{\frac{1}{2}-1}dt \\ If: x\in(0,1)\Rightarrow t\in(0,1)\end{cases}$$

$$\Rightarrow J=2\times\frac{1}{2}\int_0^1(1-t)^n t^{\frac{1}{2}-1}dt=\int_0^1 t^{\frac{1}{2}-1}(1-t)^{(n+1)-1}dt$$

$$=B\left(\frac{1}{2},n+1\right)=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma(n+1)}{\Gamma\left(\frac{1}{2}+n+1\right)}$$

$$=\frac{\sqrt{\pi}.n!}{\left(\frac{1}{2}+n\right)\Gamma\left(\frac{1}{2}+n\right)}=\frac{2\sqrt{\pi}.n!}{(2n+1)\Gamma\left(n+\frac{1}{2}\right)}$$

$$=\frac{2\sqrt{\pi}.n!\Gamma(n)}{(2n+1)\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)}=\frac{2\sqrt{\pi}.n!(n-1)!}{(2n+1)2^{1-2n}\sqrt{\pi}\,\Gamma(2n)}$$

$$=\frac{n!(n-1)!}{(2n+1)2^{-2n}\,\Gamma(2n)}=\frac{2^{2n}(2n)n!(n-1)!}{(2n+1)(2n)(2n-1)!}$$

$$=\frac{2^{2n+1}n!(n)!}{(2n+1)(2n)!}=\frac{2^{2n+1}}{(2n+1)\frac{(2n)!}{n!(n)!}}$$

$$=\frac{2^{2n+1}}{(2n+1)\frac{(2n)!}{n!(n)!}}=\frac{2^{2n+1}}{(2n+1)\binom{2n}{n}} \quad ,\text{Note}:\binom{r}{n}=C_r^n=C(n,r)$$

SO, $$\boxed{\int_{-1}^{1}(1-x^2)^n\,dx=\frac{2^{2n+1}}{(2n+1)\binom{2n}{n}}}$$

192 Calculate integral $K=\int_0^4 \binom{x}{5}^{-1} dx$

Answer

They give $\quad K=\int_0^4 \binom{x}{5}^{-1} dx$

By : $\binom{x}{5}=\dfrac{x!}{(x-5).5!} \Rightarrow \binom{x}{5}^{-1}=\dfrac{(x-5).5!}{x!}=\dfrac{5!}{x(x-1)(x-2)(x-3)(x-4)}$

That $\quad K=\int_0^4 \dfrac{5!}{x(x-1)(x-2)(x-3)(x-4)}dx$

Let : $t=x-2 \Rightarrow dx=dt$, If : $x\in(0,4)\Rightarrow t\in(-2,2)$

That: $x=t+2, x-1=t+1, x-3=t-1, x-4=t-2$

$$\Rightarrow K=\int_{-2}^{2}\frac{5!}{(t+2)(t+1)(t)(t-1)(t-2)}dt$$

$$=\underbrace{\int_{-2}^{2}\frac{5!}{(t)(t^2-1)(t^2-4)}}_{is\,an\,odd\ function}dt=0$$

SO, $\boxed{\int_0^4 \binom{x}{5}^{-1} dx=0}$

193 Calculate integral $I=\int_0^{45^\circ}\arcsin\left(\dfrac{2x}{1+x^2}\right)dx$

Answer

They give $\quad I=\int_0^1 \arcsin\left(\dfrac{2x}{1+x^2}\right)dx$

Let : $x=\tan(t)\Rightarrow dx=\sec^2(t)dt$, If : $x\in(0,1)\Rightarrow t\in(0,45^\circ)$

$$\Rightarrow I=\int_0^{45^\circ}\arcsin\left(\frac{2\tan(t)}{1+\tan^2(t)}\right)\sec^2(t)dt=\int_0^{45^\circ}\arcsin\left(\sin(2t)\right)\sec^2(t)dt$$

$$=\int_0^{45^\circ}2t\sec^2(t)dt=2\left(t.\tan(t)+\log\left|\cos(t)\right|\right)\Big|_0^{45^\circ}$$

$$=\frac{\pi}{2}-\log(2)$$

SO, $\boxed{\int_0^1 \arcsin\left(\dfrac{2x}{1+x^2}\right)dx=\dfrac{\pi}{2}-\log(2)}$

194 Calculate integral $J=\int_0^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx$

Answer

They give $J=\int_0^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx$

$$=\int_0^{\frac{1}{2}} \frac{\sin(\pi x)}{1+e^{2x-1}}dx+\int_{\frac{1}{2}}^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx \quad (*)$$

For : $J'=\int_{\frac{1}{2}}^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx \quad ,\begin{cases} Let: x=1-t \Rightarrow dx=-dt \\ If : x\in\left(\frac{1}{2},1\right)\Rightarrow t\in\left(\frac{1}{2},0\right)\end{cases}$

$$\Rightarrow J'=-\int_{\frac{1}{2}}^0 \frac{\sin\left(\pi(1-t)\right)}{1+e^{2(1-t)-1}}dt=\int_0^{\frac{1}{2}} \frac{\sin(t)}{1+e^{-(2t-1)}}dt=\int_0^{\frac{1}{2}} \frac{e^{(2t-1)}\sin(t)}{1+e^{(2t-1)}}dt$$

Take :(*) That $J=\int_0^{\frac{1}{2}} \frac{\sin(\pi x)}{1+e^{2x-1}}dx+\int_0^{\frac{1}{2}} \frac{e^{(2x-1)}\sin(x)}{1+e^{(2x-1)}}dx$

$$=\int_0^{\frac{1}{2}} \sin(\pi x)dx=\frac{1}{\pi}$$

SO, $\boxed{\int_0^1 \frac{\sin(\pi x)}{1+e^{2x-1}}dx=\frac{1}{\pi}}$

195 Calculate integral $K=\int_{-\pi}^{2\pi}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)dx$

Answer

They give $K=\int_{-\pi}^{2\pi}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)dx$

$$=\int_{-\pi}^{0}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)dx+\int_{0}^{2\pi}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)dx$$

$$=\int_{-\pi}^{0}\left(-\frac{\pi}{2}\right)dx+\int_{0}^{2\pi}\left(\frac{\pi}{2}\right)dx=\frac{\pi^2}{2}$$

SO, $\boxed{\int_{-\pi}^{2\pi}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)=\frac{\pi^2}{2}}$

Note : $\tan^{-1}(x)+\cot^{-1}(x)=\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)=\begin{cases}\frac{\pi}{2} & ,x>0 \\ -\frac{\pi}{2} & ,x<0\end{cases}$

196 Calculate integral $I=\int_0^{\infty}\frac{1}{1+x^n}dx$

Answer

They give $I=\int_0^{\infty}\frac{1}{1+x^n}dx$

$$Let : x=t^{\frac{1}{n}} \Rightarrow dx=\frac{1}{n}t^{\frac{1}{n}-1}dt \,, If : x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\frac{1}{n}\int_0^{\infty}\frac{t^{\frac{1}{n}-1}}{(1+t)}dt=\frac{1}{n}\int_0^{\infty}\frac{t^{\frac{1}{n}-1}}{(1+t)^{\frac{1}{n}+\frac{n-1}{n}}}dt$$

$$=\frac{1}{n}B\left(\frac{1}{n},\frac{n-1}{n}\right)=\frac{1}{n}\times\pi\csc\left(\frac{\pi}{n}\right)$$

SO, $\boxed{\int_0^{\infty}\frac{1}{1+x^n}dx=\frac{\pi}{n}\csc\left(\frac{\pi}{n}\right)}$

197 Calculate integral $J=\int_0^{\frac{\pi}{4}}\log\left(\cot(x)-1\right)dx$

Answer

They give $J=\int_0^{\frac{\pi}{4}}\log\left(\cot(x)-1\right)dx$

$$=\int_0^{\frac{\pi}{4}}\log\left(\cot\left(\frac{\pi}{4}-x\right)-1\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\frac{\cot(x)+1}{\cot(x)-1}-1\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{\cot(x)-1}\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\log(2)dx-\int_0^{\frac{\pi}{4}}\log\left(\cot(x)-1\right)dx$$

$$\Leftrightarrow J=\frac{\pi}{4}\log(2)-J$$

$$\Rightarrow J=\frac{\pi}{8}\log(2)$$

SO, $\boxed{\int_0^{\frac{\pi}{4}}\log\left(\cot(x)-1\right)dx=\frac{\pi}{8}\log(2)}$

198 Calculate integral $K=\int_0^{\infty}\left(\dfrac{\log(x)}{(1+x)}\right)dx$

Answer

They give
$$K=\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx$$
$$=\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx+\int_1^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx \quad (*)$$

$$For:\ K'=\int_1^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx$$

$$Let: x=\frac{1}{u}\Rightarrow dx=-\frac{1}{u^2}du\,, If\ : x\in(1,\infty)\Rightarrow u\in(1,0)$$

$$That:\ K'=-\int_1^{0}\left(\frac{-\log(u)}{1+\frac{1}{u}}\right)^2\frac{1}{u^2}du=\int_0^{1}\left(\frac{\log(u)}{1+u}\right)^2 du$$

$$Take:(*)They\, have: K=2\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx=2\int_0^{1}\frac{\log^2(x)}{(1+x)^2}dx$$

$$By:\frac{-1}{(1+x)^2}=\sum_{n=0}^{\infty}(-1)^n nx^{n-1}\Rightarrow\frac{1}{(1+x)^2}=\sum_{n=0}^{\infty}(-1)^{n-1}nx^{n-1}$$

$$\Rightarrow K=2\int_0^{1}\log^2(x)\left(\sum_{n=0}^{\infty}(-1)^{n-1}nx^{n-1}\right)dx$$
$$=2\sum_{n=0}^{\infty}n.(-1)^{n-1}\underbrace{\int_0^{1}x^{n-1}\log^2(x)dx}_{Use\ partial\ integral}$$
$$=4\sum_{n=0}^{\infty}n.(-1)^{n-1}\left(\frac{1}{n^3}\right)$$
$$=4\sum_{n=0}^{\infty}\left(\frac{(-1)^{n-1}}{n^2}\right)=4\eta(2)$$
$$=4\frac{\pi^2}{12}=\frac{\pi^2}{3}\qquad ,Note:\eta(s)=(1-2^{1-s})\zeta(s)$$

$$SO,\quad \boxed{\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx=\frac{\pi^2}{3}}$$

199 Calculate integral $I=\int_0^1 \log(1+x)\log(1-x)dx$

Answer

They give $$I=\underbrace{\int_0^1 \log(1+x)\log(1-x)dx}_{is\, an\, even\ function}$$

$$=\frac{1}{2}\int_{-1}^1 \log(1+x)\log(1-x)dx$$

$Let: t=x+1 \Leftrightarrow 1-x=2-t \Rightarrow dx=dt\,, If: x\in(-1,1)\Rightarrow u\in(0,2)$

$$\Rightarrow I=\frac{1}{2}\int_0^2 \log(t)\log(2-t)dt=\frac{1}{2}\int_0^2 \log(t)\left[\log(2)+\log\left(1-\frac{t}{2}\right)\right]dt$$

$$=\frac{1}{2}\log(2)\int_0^2 \log(t)dt+\frac{1}{2}\int_0^2 \log(t)\log\left(1-\frac{t}{2}\right)dt$$

$$=\log(2)\big(\log(2)-1\big)+I' \quad (*)$$

$$For: I'=\frac{1}{2}\int_0^2 \log(t)\log\left(1-\frac{t}{2}\right)dt \qquad , \begin{cases} Let: u=\dfrac{t}{2}\Rightarrow du=\dfrac{1}{2}dt \\ If: t\in(0,2)\Rightarrow u\in(0,1)\end{cases}$$

$$\Rightarrow I'=\int_0^1 \log(2u)\log(1-u)du=\int_0^1 [\log(2)+\log(u)]\log(1-u)du$$

$$=\log(2)\underbrace{\int_0^1 \log(1-u)du}_{0}+\int_0^1 \log(u)\log(1-u)du$$

$$=\int_0^1 \log(u)\left(-\sum_{n=1}^{\infty}\frac{u^n}{n}\right)du=-\sum_{n=1}^{\infty}\frac{1}{n}\int_0^1 u^n\log(u)du$$

$$=-\sum_{n=1}^{\infty}\frac{1}{n}\left(\underbrace{\frac{u^{n+1}\log(u)}{n+1}\bigg|_0^1}_{0}-\int_0^1 \frac{u^{n+1}}{n+1}\times\frac{1}{u}du\right)$$

$$=\sum_{n=1}^{\infty}\frac{1}{n(n+1)^2}=\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{n+1}\right)}_{1}-\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)$$

$$=1-\left[\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)-1\right]=2-\frac{\pi^2}{6}$$

Take (*) *That* $: I=\log(2)[\log(2)-1]+2-\dfrac{\pi^2}{6}$

SO, $$\boxed{\int_0^1 \log(1+x)\log(1-x)dx=\log(2)[\log(2)-1]+2-\frac{\pi^2}{6}}$$

200 Calculate integral $J=\int_0^{\infty}\frac{\sqrt{x}}{e^{\sqrt{x}}}dx$

Answer

They give $J=\int_0^{\infty}\frac{\sqrt{x}}{e^{\sqrt{x}}}dx$

$Let: x=t^2 \Rightarrow dx=2tdt\, , If: x\in(0,\infty)\Rightarrow u\in(0,\infty)$

$$\Rightarrow J=\int_0^{\infty}\frac{t}{e^t}\times 2tdt=2\int_0^{\infty}t^2e^{-t}dt=2.2!=4$$

SO, $\boxed{\int_0^{\infty}\frac{\sqrt{x}}{e^{\sqrt{x}}}dx=4}$

201 Calculate integral $K=\int_0^{\infty}\frac{x}{e^x+e^{-x}}dx$

Answer

They give $K=\int_0^{\infty}\frac{x}{e^x+e^{-x}}dx$

$$=\int_0^{\infty}\frac{e^{-x}x}{e^{-x}(e^x+e^{-x})}dx$$

$$=\int_0^{\infty}\frac{xe^{-x}}{1+e^{-2x}}dx$$

$$=\int_0^{\infty}xe^{-x}\sum_{n=0}^{\infty}(-1)^n e^{-2nx}dx$$

$$=\sum_{n=0}^{\infty}(-1)^n\int_0^{\infty}xe^{-x}e^{-2nx}dx$$

$$=\sum_{n=0}^{\infty}(-1)^n\underbrace{\int_0^{\infty}xe^{-(2n+1)x}}_{Use\ partial\ integral}dx$$

$$=\sum_{n=0}^{\infty}(-1)^n\times\frac{1}{(2n+1)^2}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}=G$$

SO, $\boxed{\int_0^{\infty}\frac{x}{e^x+e^{-x}}dx=G}$

202 Calculate integral $I=\int_0^{\frac{\pi}{4}}\frac{x}{(\sin(x)+\cos(x))\cos(x)}dx$

Answer

They give $I=\int_0^{\frac{\pi}{4}}\frac{x}{(\sin(x)+\cos(x))\cos(x)}dx$

$$=\int_0^{\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-x\right)}{\left(\sin\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{\pi}{4}-x\right)\right)\cos\left(\frac{\pi}{4}-x\right)}dx$$

$$=\int_0^{\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-x\right)}{\left(\frac{1}{\sqrt{2}}\cos(x)-\frac{1}{\sqrt{2}}\sin(x)+\frac{1}{\sqrt{2}}\cos(x)+\frac{1}{\sqrt{2}}\sin(x)\right)\left(\frac{1}{\sqrt{2}}\cos(x)+\frac{1}{\sqrt{2}}\sin(x)\right)}dx$$

$$=\int_0^{\frac{\pi}{4}}\frac{\left(\frac{\pi}{4}-x\right)}{(\cos(x)+\sin(x))\cos(x)}dx$$

$$=\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{1}{(\cos(x)+\sin(x))\cos(x)}dx-\int_0^{\frac{\pi}{4}}\frac{x}{(\sin(x)+\cos(x))\cos(x)}dx$$

$$\Rightarrow I=\frac{\pi}{8}\int_0^{\frac{\pi}{4}}\frac{1}{\tan(x)+1}\times\frac{1}{\cos^2(x)}dx$$

$$=\frac{\pi}{8}\int_0^{\frac{\pi}{4}}\frac{1}{\tan(x)+1}d\left(\tan(x)\right)$$

$$=\frac{\pi}{8}\log|\tan(x)+1|\Big|_0^{\frac{\pi}{4}}$$

$$=\frac{\pi}{8}\left(\log\left|\tan\left(\frac{\pi}{4}\right)+1\right|-\log|\tan(0)+1|\right)$$

$$=\frac{\pi}{8}\left(\log|1+1|-\log|1|\right)$$

$$=\frac{\pi}{8}\log(2)$$

SO, $\boxed{\int_0^{\frac{\pi}{4}}\frac{x}{(\sin(x)+\cos(x))\cos(x)}dx=\frac{\pi}{8}\log(2)}$

203 Calculate integral $J=\int_1^2 \frac{\log(x-1)}{x(2-x)}dx$

Answer

They give
$$J=\int_1^2 \frac{\log(x-1)}{x(2-x)}dx$$
$$=-\int_1^2 \frac{\log(x-1)}{x^2-2x}dx$$
$$=-\int_1^2 \frac{\log(x-1)}{1+(x-1)^2}dx$$

$Let: u=x-1 \Rightarrow du=dx, If: x\in(1,2)\Rightarrow u\in(0,1)$

$$\Rightarrow J=-\int_0^1 \frac{\log(u)}{1+u^2}du$$

$Mathod:1$

We have: $J=-\int_0^1 \frac{\log(u)}{1+u^2}du=-\int_0^1 \log(u)\sum_{n=0}^{\infty}(-1)^n u^{2n}du$

$$=-\sum_{n=0}^{\infty}(-1)^n \underbrace{\int_0^1 u^{2n}\log(u)du}_{Use\ partial\ integral}$$
$$=-\sum_{n=0}^{\infty}(-1)^n\left[\underbrace{\frac{u^{2n+1}\log(u)}{2n+1}\Bigg|_0^1}_{0}-\int_0^1 \frac{u^{2n+1}}{2n+1}\times\frac{1}{u}du\right]$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}=G$$

$Mathod:2$

$Let: u=\tan(y)\Rightarrow du=\sec^2(y)dy, If: u\in(0,1)\Rightarrow y\in(0,\frac{\pi}{4})$

$$\Rightarrow J=-\int_0^{\frac{\pi}{4}}\frac{\log(\tan(y))}{1+\tan^2(y)}\times\sec^2(y)dy=-\int_0^{\frac{\pi}{4}}\log(\tan(y))dy=G$$

$Mathod:3$

$Let: u=\cot(y)\Rightarrow du=-\csc^2(y)dy, If: u\in(0,1)\Rightarrow y\in(0,\frac{\pi}{4})$

$$\Rightarrow J=\int_0^{\frac{\pi}{4}}\frac{\log(\cot(y))}{1+\cot^2(y)}\times\csc^2(y)dy=\int_0^{\frac{\pi}{4}}\log(\cot(y))dy=G$$

SO, $\boxed{\int_1^2 \frac{\log(x-1)}{x(2-x)}dx=G}$

204 Calculate integral $K=\int_0^{\sqrt{2}}\lfloor x^2\rfloor dx$

Answer

They give $K=\int_0^{\sqrt{2}}\lfloor x^2\rfloor dx$

Let : $x=\sqrt{t}\Rightarrow dx=\frac{1}{2\sqrt{t}}dt$, If : $x\in(0,\sqrt{2})\Rightarrow t\in(0,2)$

$$\Rightarrow K=\frac{1}{2}\int_0^2\frac{\lfloor t\rfloor}{\sqrt{t}}dx=\frac{1}{2}\int_0^1\frac{\lfloor t\rfloor}{\sqrt{t}}dx+\frac{1}{2}\int_1^2\frac{\lfloor t\rfloor}{\sqrt{t}}dx$$

$$=\frac{1}{2}\int_0^1\frac{0}{\sqrt{t}}dx+\frac{1}{2}\int_1^2\frac{1}{\sqrt{t}}dx=\int_1^2\left(\sqrt{t}\right)'dx=\sqrt{2}-1$$

SO, $\boxed{\int_0^{\sqrt{2}}\lfloor x^2\rfloor dx=\sqrt{2}-1}$

205 Calculate integral $I=\int_0^{\sqrt{2}}\left(\lfloor x\rfloor\right)^2dx$

Answer

They give $I=\int_0^{\sqrt{2}}\left(\lfloor x\rfloor\right)^2dx$

$$=\int_0^1\left(\lfloor x\rfloor\right)^2dx+\int_1^{\sqrt{2}}\left(\lfloor x\rfloor\right)^2dx$$

$$=\int_0^1(0)^2dx+\int_1^{\sqrt{2}}(1)^2dx=\sqrt{2}-1$$

SO, $\boxed{\int_0^{\sqrt{2}}\left(\lfloor x\rfloor\right)^2dx=\sqrt{2}-1}$

206 Calculate integral $J=\int_0^{\infty}\frac{x}{1+x^3}dx$

Answer

They give $J=\int_0^{\infty}\frac{x}{1+x^3}dx$

Let : $x=t^{\frac{1}{3}}\Rightarrow dx=\frac{1}{3}t^{\frac{1}{3}-1}dt$, If : $x\in(0,\infty)\Rightarrow t\in(0,\infty)$

$$\Rightarrow J=\int_0^{\infty}\frac{t^{\frac{1}{3}}}{(1+t)}\times\frac{1}{3}t^{\frac{1}{3}-1}dt=\frac{1}{3}\int_0^{\infty}\frac{t^{\frac{2}{3}-1}}{(1+t)^{\frac{2}{3}+\frac{1}{3}}}dt$$

$$=\frac{1}{3}\left(\pi\csc\left(\frac{\pi}{3}\right)\right)=\frac{2\pi}{3\sqrt{3}}$$

SO, $\boxed{\int_0^{\infty}\frac{x}{1+x^3}dx=\frac{2\pi}{3\sqrt{3}}}$

207 Calculate integral $K=\int_0^{\frac{\pi}{2}}\sin^2(x)\log(\tan(x))dx$

Answer

They give $$K=\int_0^{\frac{\pi}{2}}\sin^2(x)\log(\tan(x))dx \qquad (*)$$

$$=\int_0^{\frac{\pi}{2}}\sin^2\left(\frac{\pi}{2}-x\right)\log\left(\tan\left(\frac{\pi}{2}-x\right)\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\cos^2(x)\log\left(\cot(x)\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\cos^2(x)\log\left(\left[\tan(x)\right]^{-1}\right)dx$$

$$=-\int_0^{\frac{\pi}{2}}\cos^2(x)\log\left(\tan(x)\right)dx \quad (**)$$

$$Take:(*)+(**)\,They\,have: 2K=\int_0^{\frac{\pi}{2}}\left(\sin^2(x)-\cos^2(x)\right)\log(\tan(x))dx$$

$$=-\int_0^{\frac{\pi}{2}}\left(\cos^2(x)-\sin^2(x)\right)\log(\tan(x))dx$$

$$=-\int_0^{\frac{\pi}{2}}\cos(2x)\log(\tan(x))dx$$

$$(Use\ partial\ integral) \qquad =-\underbrace{\frac{\sin(2x)\log(\tan)}{2}\Bigg|_0^{\frac{\pi}{2}}}_{0}+\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin(2x)\left(\frac{\sec^2(x)}{\tan(x)}\right)dx$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}2\sin(x)\cos(x)\left(\frac{\sec^2(x)}{\tan(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin(x)\cos(x)\left(\frac{1}{\frac{\sin(x)}{\cos(x)}\times\cos^2(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin(x)\cos(x)\left(\frac{1}{\sin(x)\cos(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{2}}(1)\,dx=\frac{\pi}{2}$$

$$\Rightarrow K=\frac{\pi}{4}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\sin^2(x)\log(\tan(x))dx=\frac{\pi}{4}}$$

208 Calculate integral $I=\int_0^{\frac{\pi}{2}}\log\left(\lfloor \sin(x)+1\rfloor\right)dx$

Answer

They give $$I=\int_0^{\frac{\pi}{2}}\log\left(\lfloor 1+\sin(x)\rfloor\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\log\left(1+\lfloor \sin(x)\rfloor\right)dx$$

$$By:\sin(x)\in\left(0,\frac{\pi}{2}\right)\Rightarrow 0<\sin(x)<1\Rightarrow\lfloor \sin(x)\rfloor=0$$

$$\Rightarrow K=\int_0^{\frac{\pi}{2}}\log(1+0)\,dx=0$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\log\left(\lfloor \sin(x)+1\rfloor\right)dx=0}$$

209 Calculate integral $I=\int_0^1\frac{x-1}{(x+1)^3}e^x dx$

Answer

They give $$I=\int_0^1\frac{x-1}{(x+1)^3}e^x dx$$

$$=\int_0^1\left(\frac{1}{(x+1)^2}-\frac{2}{(x+1)^3}\right)e^x dx$$

$$=\int_0^1\left(\frac{e^x}{(x+1)^2}-\frac{2e^x}{(x+1)^3}\right)dx$$

$$=\int_0^1\left[\frac{1}{(x+1)^2}(e^x)'-\left(\frac{1}{(x+1)^2}\right)'e^x\right]dx$$

$$=\int_0^1\left(\frac{e^x}{(x+1)^2}\right)'dx$$

$$=\left.\frac{e^x}{(x+1)^2}\right|_0^1$$

$$=\frac{e-4}{4}$$

SO, $$\boxed{\int_0^1\frac{x-1}{(x+1)^3}e^x dx=\frac{e-4}{4}}$$

210 Calculate integral $J=\int_0^{\frac{\pi}{2}}\left(1-\sin(x)+\sin^2(x)-\sin^3(x)+..\right)dx$

Answer

They give $J=\int_0^{\frac{\pi}{2}}\left(1-\sin(x)+\sin^2(x)-\sin^3(x)+...\right)dx$

$By: \forall x\in(0,1)$ *we have* $\dfrac{1}{1+x}=\sum_{n=0}^{\infty}(-x)^n=1-x+x^2-x^3+...$

$$\Rightarrow J=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1-\sin(x)}{\left(1+\sin(x)\right)\left(1-\sin(x)\right)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1-\sin(x)}{1-\sin^2(x)}dx=\int_0^{\frac{\pi}{2}}\frac{1-\sin(x)}{\cos^2(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{\cos^2(x)}-\frac{\sin(x)}{\cos^2(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\left[\left(\tan(x)\right)'-\left(\frac{1}{\cos(x)}\right)'\right]dx$$

$$=\left(\tan(x)-\frac{1}{\cos(x)}\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$=\lim_{x\to\frac{\pi}{2}}\left(\tan(x)-\frac{1}{\cos(x)}\right)-\lim_{x\to 0}\left(\tan(x)-\frac{1}{\cos(x)}\right)$$

$$=\underbrace{\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin(x)-1}{\cos(x)}\right)}_{Use\ Lopital}-\left(\tan(0)-\frac{1}{\cos(0)}\right)$$

$$=-\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos(x)}{\sin(x)}\right)-(0-1)$$

$$=-\frac{0}{1}-(0-1)$$

$$=-0-(0-1)=1$$

SO, $\boxed{\int_0^{\frac{\pi}{2}}\left(1-\sin(x)+\sin^2(x)-\sin^3(x)+...\right)dx=1}$

211 Calculate integral $K=\int_1^2 \frac{\sqrt{x-1}\tan^{-1}\left(\sqrt{x-1}\right)}{x}dx$

Answer

They give $$K=\int_1^2 \frac{\sqrt{x-1}\tan^{-1}\left(\sqrt{x-1}\right)}{x}dx$$

$Let: \sqrt{x-1}=\tan(u) \Leftrightarrow x=\sec^2(u) \Rightarrow dx=2\tan(u)\sec^2(u)du, If: x\in(1,2) \Rightarrow t\in(0,\frac{\pi}{4})$

$$\Rightarrow K=2\int_0^{\frac{\pi}{4}} \frac{\tan^2(u)\tan^{-1}\left(\tan(u)\right)\sec^2(u)}{\sec^2(u)}du$$

$$=\underbrace{2\int_0^{\frac{\pi}{4}} u\tan^2(u)du}_{Use\ partial\ integral}=2\left[u\left(\tan(u)-u\right)+\frac{u^2}{2}+\log\left(\cos(u)\right)\right]_0^{\frac{\pi}{4}}$$

$$=\frac{\pi}{2}-\frac{\pi^2}{16}-\log(2)$$

SO, $$\boxed{\int_1^2 \frac{\sqrt{x-1}\tan^{-1}\left(\sqrt{x-1}\right)}{x}dx=\frac{\pi}{2}-\frac{\pi^2}{16}-\log(2)}$$

212 Calculate integral $I=\int_0^{\frac{\pi}{2}} \frac{\sin(x)-\cos(x)}{e^x-\sin(x)}dx$

Answer

They give $$I=\int_0^{\frac{\pi}{2}} \frac{\sin(x)-\cos(x)}{e^x-\sin(x)}dx$$

$Let: f(x)=e^x-\sin(x) \Rightarrow f'(x)=e^x-\cos(x), If: x\in(0,\frac{\pi}{2}) \Rightarrow f(0)=1, f(\frac{\pi}{2})=e^{\frac{\pi}{2}}-1$

$That: f'(x)-f(x)=\sin(x)-\cos(x)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{f'(x)-f(x)}{f(x)}dx=\int_0^{\frac{\pi}{2}}\left(\frac{f'(x)}{f(x)}-1\right)dx$$

$$=\left(\log|f(x)|-x\right)\Big|_0^{\frac{\pi}{2}}=\left(\log\left|e^{\frac{\pi}{2}}-1\right|-\frac{\pi}{2}\right)-\left(\log|1|-0\right)$$

$$=-\frac{\pi}{2}+\log\left|e^{\frac{\pi}{2}}-1\right|$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}} \frac{\sin(x)-\cos(x)}{e^x-\sin(x)}dx=-\frac{\pi}{2}+\log\left|e^{\frac{\pi}{2}}-1\right|}$$

213 Calculate integral $J=\int_{-2}^{2}\left[x^{2025}\cos\left(\frac{x}{2026}\right)+\frac{1}{2}\right]\sqrt{4-x^2}\,dx$

Answer

They give $J=\int_{-2}^{2}\left[x^{2025}\cos\left(\frac{x}{2026}\right)+\frac{1}{2}\right]\sqrt{4-x^2}\,dx$

$$=\underbrace{\int_{-2}^{2}x^{2025}\cos\left(\frac{x}{2026}\right)\sqrt{4-x^2}\,dx}_{\text{is an odd function}}+\frac{1}{2}\underbrace{\int_{-2}^{2}\sqrt{4-x^2}\,dx}_{\text{is an even function}}$$

$$=0+\frac{1}{2}\times 2\int_{0}^{2}\sqrt{4-x^2}\,dx$$

$$=\int_{0}^{2}\sqrt{4-x^2}\,dx$$

Let $: x=2\sin(u)\Rightarrow dx=2\cos(u)du$, *If* $: x\in(0,2)\Rightarrow u\in\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow J=2\int_{0}^{\frac{\pi}{2}}\sqrt{4-4\sin^2(u)}\cos(u)du$$

$$=4\int_{0}^{\frac{\pi}{2}}\sqrt{1-\sin^2(u)}\cos(u)du$$

$$=4\int_{0}^{\frac{\pi}{2}}|\cos(u)|\cos(u)du$$

By $:\forall u\in\left(0,\frac{\pi}{2}\right)\Rightarrow|\cos(u)|=\cos(u)$

$$\Rightarrow J=4\int_{0}^{\frac{\pi}{2}}\cos^2(u)du$$

$$=2\int_{0}^{\frac{\pi}{2}}(1+\cos(2u))du$$

$$=2\left(u+\frac{1}{2}\sin(2u)\right)\Big|_{0}^{\frac{\pi}{2}}$$

$$=2\left(\frac{\pi}{2}-0\right)-2(0+0)$$

$$=\pi$$

SO, $\boxed{\int_{-2}^{2}\left[x^{2025}\cos\left(\frac{x}{2026}\right)+\frac{1}{2}\right]\sqrt{4-x^2}\,dx=\pi}$

214 Calculate integral $K=\int_0^{\infty}\frac{\sin^2(x)}{x^2(x^2+1)}dx$

Answer

They give $$K=\int_0^{\infty}\frac{\sin^2(x)}{x^2(x^2+1)}dx$$
$$=\int_0^{\infty}\frac{\left((x^2+1)-x^2\right)\sin^2(x)}{x^2(x^2+1)}dx$$
$$=\int_0^{\infty}\frac{\sin^2(x)}{x^2}dx-\int_0^{\infty}\frac{\sin^2(x)}{x^2+1}dx$$
$$=\frac{\pi}{2}-I \quad (*)$$

$$For: I=\int_0^{\infty}\frac{\sin^2(x)}{x^2+1}dx \;\;, Take: J=\int_0^{\infty}\frac{\cos^2(x)}{x^2+1}dx$$

$$Take: J+I=\int_0^{\infty}\frac{\sin^2(x)+\cos^2(x)}{x^2+1}dx$$
$$=\int_0^{\infty}\frac{1}{x^2+1}dx$$
$$=\tan^{-1}(x)\Big|_0^{\infty}=\frac{\pi}{2}$$

$$Take: J-I=\int_0^{\infty}\frac{\cos^2(x)-\sin^2(x)}{x^2+1}dx$$
$$=\int_0^{\infty}\frac{\cos(2x)}{x^2+1}dx=\frac{\pi}{2e^2} \qquad ,\left(Use: \int_0^{\infty}\frac{\cos(ax)}{x^2+1}dx=\frac{\pi}{2e^a}\right)$$

$$That: (J+I)-(J-I)=\frac{\pi}{2}-\frac{\pi}{2e^2}$$
$$2I=\frac{\pi}{2}-\frac{\pi}{2e^2}$$
$$\Rightarrow I=\frac{\pi}{4}-\frac{\pi e^{-2}}{4}$$

$$Take: (*) :\Rightarrow K=\frac{\pi}{2}-\frac{\pi}{4}+\frac{\pi e^{-2}}{4}=\frac{\pi}{4}\left(e^{-2}+1\right)$$

SO, $$\boxed{\int_0^{\infty}\frac{\sin^2(x)}{x^2(x^2+1)}dx=\frac{\pi}{4}\left(e^{-2}+1\right)}$$

215 Calculate integral $I=\int_1^{\infty}\left(\dfrac{\log(x)}{x}\right)^{n+m}dx$

Answer

They give $I=\int_1^{\infty}\left(\dfrac{\log(x)}{x}\right)^{n+m}dx$

$Let: u=\log(x)\Leftrightarrow x=e^u \Rightarrow dx=e^u du\,, If\ : x\in(1,\infty)\Rightarrow u\in\left(0,\infty\right)$

$$\Rightarrow I=\int_0^{\infty}\left(\frac{u}{e^u}\right)^{n+m} e^u du=\int_0^{\infty}u^{n+m}e^{-(n+m-1)u}du$$

$Let: y=(n+m-1)u\Rightarrow du=\dfrac{1}{n+m-1}dy\,, If\ : u\in(0,\infty)\Rightarrow y\in\left(0,\infty\right)$

$$\Rightarrow I=\int_0^{\infty}\left(\frac{y}{n+m-1}\right)^{n+m}\left(\frac{e^{-y}}{n+m-1}\right)dy$$

$$=\left(\frac{1}{n+m-1}\right)^{n+m+1}\int_0^{\infty}y^{n+m}e^{-y}dy$$

$$=\left(\frac{1}{n+m-1}\right)^{n+m+1}(n+m)!=\frac{(n+m)!}{\left(n+m-1\right)^{n+m+1}}$$

SO, $\boxed{\int_1^{\infty}\left(\dfrac{\log(x)}{x}\right)^{n+m}dx=\dfrac{(n+m)!}{\left(n+m-1\right)^{n+m+1}}}$

216 Calculate integral $J=\int_{-2}^{2}\dfrac{\lfloor x\rfloor}{|x+1|}dx$

Answer

They give $J=\int_{-2}^{2}\dfrac{\lfloor x\rfloor}{|x+1|}dx$

$$=\int_{-2}^{-1}\frac{\lfloor x\rfloor}{|x|+1}dx+\int_{-1}^{0}\frac{\lfloor x\rfloor}{|x|+1}dx+\int_{0}^{1}\frac{\lfloor x\rfloor}{|x|+1}dx+\int_{0}^{2}\frac{\lfloor x\rfloor}{|x|+1}dx$$

$$=\int_{-2}^{-1}\frac{-2}{-x+1}dx+\int_{-1}^{0}\frac{-1}{-x+1}dx+\int_{0}^{1}\frac{0}{x+1}dx+\int_{1}^{2}\frac{1}{x+1}dx$$

$$=2\log|x-1|\Big|_{-2}^{-1}+\log|x-1|\Big|_{-1}^{0}dx+0+\log|x+1|\Big|_{1}^{2}$$

$$=-\log(3)$$

SO, $\boxed{\int_{-2}^{2}\dfrac{\lfloor x\rfloor}{|x+1|}dx=\log\left(\dfrac{1}{3}\right)}$

217 Calculate integral $K=\int_{-2}^{2}\frac{\lceil x\rceil}{|x+1|}dx$

Answer

They give $$K=\int_{-2}^{2}\frac{\lceil x\rceil}{|x+1|}dx$$

$$=\int_{-2}^{-1}\frac{\lceil x\rceil}{|x|+1}dx+\int_{-1}^{0}\frac{\lceil x\rceil}{|x|+1}dx+\int_{0}^{1}\frac{\lceil x\rceil}{|x|+1}dx+\int_{0}^{2}\frac{\lceil x\rceil}{|x|+1}dx$$

$$=\int_{-2}^{-1}\frac{-1}{-x+1}dx+\int_{-1}^{0}\frac{0}{-x+1}dx+\int_{0}^{1}\frac{1}{x+1}dx+\int_{1}^{2}\frac{2}{x+1}dx$$

$$=\log|x-1|\Big|_{-2}^{-1}+0+\log|x+1|\Big|_{0}^{1}dx+\log|x+1|\Big|_{1}^{2}$$

$$=\log(2)-\log(3)+\log(2)-\log(1)+\log(3)-\log(2)$$

$$=\log(2)$$

SO, $$\boxed{\int_{-2}^{2}\frac{\lceil x\rceil}{|x+1|}dx=\log(2)}$$

218 Calculate integral $I=\int_{0}^{1}x\left(-\log(x)\right)^{3}dx$

Answer

They give $$I=\int_{0}^{1}x\left(-\log(x)\right)^{3}dx$$

$Let: u=-\log(x)\Leftrightarrow x=e^{-u}\Rightarrow dx=-e^{-u}du\,, If\ : x\in(0,1)\Rightarrow u\in(\infty,0)$

$$\Rightarrow I=-\int_{\infty}^{0}u^{3}e^{-u}\times e^{-u}du=\int_{0}^{\infty}u^{3}e^{-2u}du$$

$Let: y=2u\Rightarrow du=\frac{dy}{2}, If\ : u\in(0,\infty)\Rightarrow y\in(0,\infty)$

$$\Rightarrow I=\frac{1}{2}\int_{0}^{\infty}\left(\frac{y}{2}\right)^{3}e^{-y}dy$$

$$=\frac{1}{16}\int_{0}^{\infty}y^{3}e^{-y}dy$$

$$=\frac{1}{16}\times 3!=\frac{3}{8}$$

SO, $$\boxed{\int_{0}^{1}x\left(-\log(x)\right)^{3}dx=\frac{3}{8}}$$

219 Calculate integral $J=\int_0^{\pi}\log\left(\left|\tan(x)\right|\right)dx$

Answer

They give $J=\int_0^{\pi}\log\left(\left|\tan(x)\right|\right)dx$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\left|\tan(x)\right|\right)dx \quad ,\left(Use:\int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx\ ,\ f(2a-x)=f(x)\right)$$

$$=2\left[\int_0^{\frac{\pi}{2}}\log\left(\left|\sin(x)\right|\right)dx-\int_0^{\frac{\pi}{2}}\log\left(\left|\cos(x)\right|\right)dx\right]$$

$$=2\left[\int_0^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx-\int_0^{\frac{\pi}{2}}\log\left(\cos(x)\right)dx\right]$$

$$=2\left[\left(-\frac{\pi}{2}\log(2)\right)-\left(-\frac{\pi}{2}\log(2)\right)\right]=0$$

SO, $\boxed{\int_0^{\pi}\log\left(\left|\tan(x)\right|\right)dx=0}$

220 Calculate integral $K=\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx$

Answer

They give $K=\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\left|\sin(x)\right|\right)dx \quad ,\left(Take:\int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx\ ,\ f(2a-x)=f(x)\right)$$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx$$

$$=2\left(-\frac{\pi}{2}\log(2)\right)$$

$$=-\pi\log(2)$$

And : $\int_0^{\pi}\log\left(\left|\cos(x)\right|\right)dx=2\int_0^{\frac{\pi}{2}}\log\left(\left|\cos(x)\right|\right)dx$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\cos(x)\right)dx$$

$$=2\left(-\frac{\pi}{2}\log(2)\right)$$

$$=-\pi\log(2)$$

SO, $\boxed{\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx=\int_0^{\pi}\log\left(\left|\cos(x)\right|\right)dx=-\pi\log(2)}$

220 Calculate integral $K=\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx$

Answer

They give $K=\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\left|\sin(x)\right|\right)dx\ ,\left(Take:\int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx\ ,f(2a-x)=f(x)\right)$$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\sin(x)\right)dx$$

$$=2\left(-\frac{\pi}{2}\log(2)\right)=-\pi\log(2)$$

And : $\int_0^{\pi}\log\left(\left|\cos(x)\right|\right)dx=2\int_0^{\frac{\pi}{2}}\log\left(\left|\cos(x)\right|\right)dx$

$$=2\int_0^{\frac{\pi}{2}}\log\left(\cos(x)\right)dx$$

$$=2\left(-\frac{\pi}{2}\log(2)\right)=-\pi\log(2)$$

SO, $\boxed{\int_0^{\pi}\log\left(\left|\sin(x)\right|\right)dx=\int_0^{\pi}\log\left(\left|\cos(x)\right|\right)dx=-\pi\log(2)}$

221 Calculate integral $I=\int_1^{e}\left[\left(x/e\right)^x+\left(e/x\right)^x\right]\log(x)dx$

Answer

They give $I=\int_1^{e}\left[\left(x/e\right)^x+\left(e/x\right)^x\right]\log(x)dx$

$$=\int_1^{e}\left[\left(x/e\right)^x+\frac{1}{\left(x/e\right)^x}\right]\log(x)dx$$

Let : $u=\left(x/e\right)^x\Leftrightarrow\log(u)=x\log(x)-x\Rightarrow\frac{1}{u}du=\log(x)dx, If\ :x\in(1,e)\Rightarrow u\in(1/e,1)$

$$\Rightarrow I=\int_{\frac{1}{e}}^{1}\left(u+\frac{1}{u}\right)\frac{1}{u}du=\left(u-\frac{1}{u}\right)\Bigg|_{\frac{1}{e}}^{1}$$

$$=\frac{(e-1)(e+1)}{e}$$

SO, $\boxed{\int_1^{e}\left[\left(x/e\right)^x+\left(e/x\right)^x\right]\log(x)dx=\frac{(e-1)(e+1)}{e}}$

222 Calculate integral $J=\int_0^1 \frac{\log(x^2+1)}{x}dx$

Answer

They give $$J=\int_0^1 \frac{\log(x^2+1)}{x}dx$$

$$Take:\log(x+1)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n \Rightarrow \log(x^2+1)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^{2n}$$

$$\Rightarrow J=\int_0^1 \frac{1}{x}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^{2n}dx$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^1 x^{2n}dx$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\times\left.\frac{x^{2n+1}}{2n+1}\right|_0^1$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(2n+1)}$$

SO, $$\boxed{\int_0^1 \frac{\log(x^2+1)}{x}dx=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(2n+1)}}$$

223 Calculate integral $K=\int_0^{\pi}\frac{\log\left(1-\sin(x)\right)}{\sin(x)}dx$

Answer

They give $$K=\int_0^{\pi}\frac{\log\left(1-\sin(x)\right)}{\sin(x)}dx$$

$$K(a)=\int_0^{\pi}\frac{\log\left(1-\sin(a)\sin(x)\right)}{\sin(x)}dx$$

$$K'(a)=-\int_0^{\pi}\frac{\cos(a)\sin(x)}{\left(1-\sin(a)\sin(x)\right)\sin(x)}dx$$

$$=-\cos(a)\int_0^{\pi}\frac{1}{1-\sin(a)\sin(x)}dx$$

$$Let: y=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+y^2}dy, If: x\in(0,\pi)\Rightarrow y\in(0,\infty),\sin(x)=\frac{2y}{1+y^2}$$

$$\Rightarrow K'(a)=\cos(a)\int_0^{\infty}\frac{1}{1-\sin(a)\times\frac{2y}{1+y^2}}\times\frac{2}{1+y^2}dy$$

$$= -2\cos(a)\int_0^{\infty}\frac{1}{1+y^2-2y\sin(a)}dy$$

$$= -2\cos(a)\int_0^{\infty}\frac{1}{\cos^2(y)+(y-\sin(x))^2}dy$$

$$= -2\cos(a)\left[\frac{1}{\cos(a)}\tan^{-1}\left(\frac{y-\sin(a)}{\cos(a)}\right)\right]_0^{\infty}$$

$$= -2\left(\frac{\pi}{2}+a\right)$$

$$\Rightarrow K(a) = -2\int\left(\frac{\pi}{2}+a\right)da$$

$$= -(\pi a + a^2) + C$$

$$If\ : a = 0 \Rightarrow K(0) = 0 = 0 + C \Rightarrow C = 0$$

$$If\ : a = \frac{\pi}{2} \Rightarrow K\left(\frac{\pi}{2}\right) = K = -\left(\frac{\pi^2}{2}+\frac{\pi^2}{4}\right) + 0 = -\frac{3\pi^2}{4}$$

$$SO,\quad \boxed{\int_0^{\pi}\frac{\log\big(1-\sin(x)\big)}{\sin(x)}dx = -\frac{3\pi^2}{4}}$$

224 Calculate integral $I = \int_1^e (x-1)\log^2(x)dx$

Answer

$$They\ give \qquad I = \int_1^e (x-1)\log^2(x)dx$$

$$Let\ : u = \log(x) \Leftrightarrow x = e^u \Rightarrow dx = e^u du\ , If\ : x \in (1,e) \Rightarrow u \in (0,1)$$

$$\Rightarrow I = \int_0^1 u^2(e^u-1)e^u dx = \underbrace{\int_0^1 u^2 e^{2u}dx - \int_0^1 u^2 e^u dx}_{(Use\ partial\ integral)}$$

$$= \left(\frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{4}\right)e^{2u}\Bigg|_0^1 - \left(u^2-2u+2\right)e^{2u}\Big|_0^1$$

$$= \frac{e^2-4e+7}{4}$$

$$SO,\quad \boxed{\int_1^e (x-1)\log^2(x)dx = \frac{e^2-4e+7}{4}}$$

225 Calculate integral $J=\int_{\frac{\pi}{2}}^{\pi}\log^2\left(1+(e-1)\sin(x)\right)\sin(2x)dx$

Answer

They give $$J=\int_{\frac{\pi}{2}}^{\pi}\log^2\left(1+(e-1)\sin(x)\right)\sin(2x)dx$$

$$=2\int_{\frac{\pi}{2}}^{\pi}\log^2\left(1+(e-1)\sin(x)\right)\sin(x)\cos(x)dx$$

$$Let: y=1+(e-1)\sin(x)\Leftrightarrow \sin(x)=\frac{y-1}{e-1}\Rightarrow \cos(x)dx=\frac{1}{e-1}dy$$

$$If: u\in(\frac{\pi}{2},\pi)\Rightarrow x\in(e,1)$$

$$\Rightarrow J=2\int_e^1\left(\frac{y-1}{e-1}\right)\log^2(y)\frac{dy}{e-1}$$

$$=-\frac{2}{(e-1)^2}\int_1^e(y-1)\log^2(y)dy$$

$$=-\frac{2}{(e-1)^2}\times\frac{e^2-4e+7}{4}=-\frac{e^2-4e+7}{2(e-1)^2}$$

SO, $$\boxed{\int_{\frac{\pi}{2}}^{\pi}\log^2\left(1+(e-1)\sin(x)\right)\sin(2x)dx=-\frac{e^2-4e+7}{2(e-1)^2}}$$

226 Calculate integral $K=\int_0^1\frac{\log(x+1)}{x}dx$

Answer

They give $$K=\int_0^1\frac{\log(x+1)}{x}dx$$

$$=\int_0^1\frac{1}{x}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n dx$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^1\frac{1}{x}x^n dx$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^1 x^{n-1}dx$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\eta(2)=\frac{\pi^2}{12}$$

SO, $$\boxed{\int_0^1\frac{\log(x+1)}{x}dx=\frac{\pi^2}{12}}$$

227 Calculate integral $I=\int_{-1}^{1}\log\left(x+\sqrt{1+x^2}\right)dx$

Answer

They give $$I=\int_{-1}^{1}\log\left(x+\sqrt{1+x^2}\right)dx \quad (*)$$

$$=\int_{-1}^{1}\log\left(-x+\sqrt{1+(-x)^2}\right)dx \quad ,Use:\int_{-a}^{a}f(x)dx=\int_{-a}^{a}f(-x)dx$$

$$=\int_{-1}^{1}\log\left(\sqrt{1+x^2}-x\right)dx \quad (**)$$

$$Take:(*)+(**)That:2I=\int_{-1}^{1}\log\left(x+\sqrt{1+x^2}\right)dx+\int_{-1}^{1}\log\left(\sqrt{1+x^2}-x\right)dx$$

$$=\int_{-1}^{1}\log\left[\left(x+\sqrt{1+x^2}\right)\left(\sqrt{1+x^2}-x\right)\right]dx$$

$$=\int_{-1}^{1}\log\left[\left(1+x^2-x^2\right)\right]dx$$

$$=\int_{-1}^{1}\log(1)\,dx=0$$

$$\Rightarrow I=0$$

SO, $\boxed{\int_{-1}^{1}\log\left(x+\sqrt{1+x^2}\right)dx=0}$

228 Calculate integral $J=\int_{0}^{\infty}\frac{\tan^{-1}(ex)-\tan^{-1}(x)}{x}dx$

Answer

They give $$J=\int_{0}^{\infty}\frac{\tan^{-1}(ex)-\tan^{-1}(x)}{x}dx$$

$$\Rightarrow J(a)=\int_{0}^{\infty}\frac{\tan^{-1}(ax)-\tan^{-1}(x)}{x}dx$$

$$\Rightarrow J'(a)=\int_{0}^{\infty}\frac{1}{x}\left(\frac{x}{1+(ax)^2}\right)dx=\int_{0}^{\infty}\left(\frac{1}{1+(ax)^2}\right)dx=\frac{\pi}{2a}$$

$$\Rightarrow J(a)=\frac{\pi}{2}\log(a)+C$$

$$If:a=1\Rightarrow J(1)=\frac{\pi}{2}\log(0)+C=0\Rightarrow C=0$$

$$If:a=e\Rightarrow J(e)=J=\frac{\pi}{2}\log(e)+0=\frac{\pi}{2}$$

SO, $\boxed{\int_{0}^{\infty}\frac{\tan^{-1}(ex)-\tan^{-1}(x)}{x}dx=\frac{\pi}{2}}$

229 Calculate integral $K=\int_0^1 \frac{x^3(1+x^2)}{(1+x)^{10}}dx$

Answer

They give $$K=\int_0^1 \frac{x^3(1+x^2)}{(1+x)^{10}}dx$$

$$=\int_0^1 \frac{x^3+x^5}{(1+x)^{10}}dx=\int_0^1 \frac{x^{4-1}+x^{6-1}}{(1+x)^{4+6}}dx$$

$$=B(4,6)=\frac{\Gamma(4)\Gamma(6)}{\Gamma(10)}$$

$$=\frac{3!5!}{9!}=\frac{1}{504}$$

SO, $\int_0^1 \frac{x^3(1+x^2)}{(1+x)^{10}}dx=\frac{1}{504}$

230 Calculate integral $I=\int_0^{\pi}\sin^5(x)\left(1-\cos(x)\right)^3 dx$

Answer

They give $$I=\int_0^{\pi}\sin^5(x)\left(1-\cos(x)\right)^3 dx$$

Let $:x=y+\frac{\pi}{2}\Rightarrow dx=dy$, If $:x\in(0,\pi)\Rightarrow y\in(-\frac{\pi}{2},\frac{\pi}{2})$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^5(\frac{\pi}{2}+y)\left(1-\cos(\frac{\pi}{2}+y)\right)^3 dy=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^5(y)\left(1-\sin(y)\right)^3 dy$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^5(y)\left(1+3\sin(y)+3\sin^2(y)+\sin^3(y)\right)dy$$

$$=\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^5(y)\left(1+\sin^3(y)\right)dy}_{\text{is an even function}}+\underbrace{3\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^5(y)\left(\sin(y)+\sin^2(y)\right)dy}_{\text{is an odd function}}$$

$$=2\int_0^{\frac{\pi}{2}}\cos^5(y)\left(1+\sin^3(y)\right)dy+0$$

$$=2\int_0^{\frac{\pi}{2}}\cos^5(y)dy+2\int_0^{\frac{\pi}{2}}\cos^7(y)dy$$

$$=2\frac{4\times2}{5\times3\times1}+2\frac{6\times4\times2}{7\times5\times3\times1}=\frac{32}{21}$$

SO, $\int_0^{\pi}\sin^5(x)\left(1-\cos(x)\right)^3 dx=\frac{32}{21}$

231 Calculate integral $J=\int_0^1 \tan^{-1}\big(\sec(x)+\tan(x)\big)dx$

Answer

They give $J=\int_0^1 \tan^{-1}\big(\sec(x)+\tan(x)\big)dx$

$$=\int_0^1 \tan^{-1}\left(\frac{1+\sin(x)}{\cos(x)}\right)dx=\int_0^1 \tan^{-1}\left(\frac{1-\cos(\pi/2+x)}{\sin(\pi/2+x)}\right)dx$$

$$=\int_0^1 \tan^{-1}\left(\frac{2\sin^2(\pi/4+x/2)}{2\sin(\pi/4+x/2)\cos(\pi/4+x/2)}\right)dx$$

$$=\int_0^1 \tan^{-1}\big(\tan(\pi/4+x/2)\big)dx=\int_0^1\big(\pi/4+x/2\big)dx=3\pi/4$$

SO, $\int_0^1 \tan^{-1}\big(\sec(x)+\tan(x)\big)dx=\dfrac{3\pi}{4}$

232 Calculate integral $K=\int_0^{\pi}\frac{x\sin^2(x)}{1+\cos^2(x)}dx$

Answer

They give $K=\int_0^{\pi}\frac{x\sin^2(x)}{1+\cos^2(x)}dx$

$$=\int_0^{\pi}\frac{(\pi-x)\sin^2(\pi-x)}{1+\cos^2(\pi-x)}dx \qquad ,Use:\int_0^a f(x)dx=\int_0^a f(-x)dx$$

$$=\pi\int_0^{\pi}\frac{\sin^2(x)}{1+\cos^2(x)}dx-K$$

$$\Rightarrow K=\frac{\pi}{2}\int_0^{\pi}\frac{\sin^2(x)}{1+\cos^2(x)}dx$$

$$\Rightarrow K=\frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{1+\cos^2(x)}dx \qquad ,Use:\begin{cases}\int_0^{2a} f(x)dx=2\int_0^a f(x)dx\\ f(2a-x)=f(x)\end{cases}$$

$$=\pi\int_0^{\frac{\pi}{2}}\frac{\tan^2(x)}{2+\tan^2(x)}\times\frac{\big(1+\tan^2(x)\big)}{\big(1+\tan^2(x)\big)}dx$$

$$=\pi\int_0^{\frac{\pi}{2}}\frac{\tan^2(x)}{\big(2+\tan^2(x)\big)\big(1+\tan^2(x)\big)}\big(1+\tan^2(x)\big)dx$$

Let $:u=\tan(x)\Rightarrow du=\big(1+\tan^2(x)\big)dx, If :x\in(0,\frac{\pi}{2})\Rightarrow u\in(0,\infty)$

$$\Rightarrow K=\pi\int_0^{\infty}\frac{u^2}{\left(2+u^2\right)\left(1+u^2\right)}du$$

$$=\pi\int_0^{\infty}\left[\frac{2}{\left(2+u^2\right)}-\frac{1}{\left(1+u^2\right)}\right]du$$

$$=\pi\left(\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)-\tan^{-1}(u)\right)\Bigg|_0^{\infty}$$

$$=\pi\left(\frac{2}{\sqrt{2}}\times\frac{\pi}{2}-\frac{\pi}{2}\right)=\left(\frac{\sqrt{2}-1}{2}\right)\pi^2$$

SO, $\boxed{\int_0^{\pi}\frac{x\sin^2(x)}{1+\cos^2(x)}dx=\left(\frac{\sqrt{2}-1}{2}\right)\pi^2}$

233 Calculate integral $I=\int_1^{\infty}\frac{x-1}{x^4\log(x)}dx$

Answer

They give $K=\int_1^{\infty}\frac{x-1}{x^4\log(x)}dx$

Let $:u=\log(x)\Leftrightarrow x=e^u\Rightarrow x=e^u du$, *If* $:x\in(1,\infty)\Rightarrow u\in(0,\infty)$

$$\Rightarrow K=\int_0^{\infty}\frac{e^u-1}{ue^{4u}}\times e^u du=\int_0^{\infty}\frac{1-e^{-u}}{u}e^{-2u}du$$

$$\Rightarrow K(a)=\int_0^{\infty}\frac{1-e^{-u}}{u}e^{-au}du$$

$$\Rightarrow K'(a)=-\int_0^{\infty}\left(1-e^{-u}\right)e^{-au}du$$

$$=\int_0^{\infty}e^{-(a+1)u}du-\int_0^{\infty}e^{-au}du=-\frac{1}{a+1}e^{-(a+1)u}\Bigg|_0^{\infty}+\frac{1}{a}e^{-au}\Bigg|_0^{\infty}=\frac{1}{a+1}-\frac{1}{a}$$

$$\Rightarrow K(a)=\int\left(\frac{1}{a+1}-\frac{1}{a}\right)da=\log\left|\frac{a+1}{a}\right|+C$$

If $:a=2\Rightarrow K(2)=K=\log\left|\frac{2+1}{2}\right|+C$ *And* *If* $:a=\infty\Rightarrow K(\infty)=0=0+C\Rightarrow C=0$

That $:K=\log\left(\frac{3}{2}\right)$

SO, $\boxed{\int_1^{\infty}\frac{x-1}{x^4\log(x)}dx=\log\left(\frac{3}{2}\right)}$

234 Calculate integral $I=\int_0^{\infty}\frac{x}{x^8+2x^4+1}dx$

Answer

They give $$I=\int_0^{\infty}\frac{x}{x^8+2x^4+1}dx$$

$$=\frac{1}{2}\int_0^{\infty}\frac{1}{(x^4+1)^2}d(x^2)=\frac{1}{2}\int_0^{\infty}\frac{1}{\left((x^2)^2+1\right)^2}d(x^2)$$

$$Let: x^2=t^{\frac{1}{2}}\Rightarrow d(x^2)=\frac{1}{2}t^{\frac{1}{2}-1}dt\ ,\ If: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\frac{1}{2}\times\frac{1}{2}\int_0^{\infty}\frac{t^{\frac{1}{2}-1}}{(t+1)^2}d(t)=\frac{1}{4}\int_0^{\infty}\frac{t^{\frac{1}{2}-1}}{(t+1)^{\frac{1}{2}+\frac{3}{2}}}d(t)=\frac{1}{4}B\left(\frac{1}{2},\frac{3}{2}\right)$$

$$=\frac{1}{4}.\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{3}{2}\right)}=\frac{1}{4}.\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(1+\frac{1}{2}\right)}{\Gamma(2)}=\frac{1}{4}.\frac{\Gamma\left(\frac{1}{2}\right)\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{1!}$$

$$=\frac{1}{8}\Gamma^2\left(\frac{1}{2}\right)=\frac{1}{8}\left(\sqrt{\pi}\right)^2=\frac{\pi}{8}$$

SO, $\boxed{\int_0^{\infty}\frac{x}{x^8+2x^4+1}dx=\frac{\pi}{8}}$

235 Calculate integral $J=\int_0^{1}\frac{1}{1+\lfloor 1/x\rfloor}dx$

Answer

They give $$J=\int_0^{1}\frac{1}{1+\lfloor 1/x\rfloor}dx$$

$$Let: t=\frac{1}{x}\Rightarrow dx=-\frac{1}{t^2}dt\ ,\ If: x\in(0,1)\Rightarrow t\in(\infty,1)$$

$$\Rightarrow J=-\int_{\infty}^{1}\frac{1}{1+\lfloor t\rfloor}\times\frac{1}{t^2}dx=\int_1^{\infty}\frac{1}{1+\lfloor t\rfloor}\times\frac{1}{t^2}dx$$

$$=\sum_{n=1}^{\infty}\int_n^{n+1}\frac{1}{1+n}\times\frac{1}{t^2}dx=\sum_{n=1}^{\infty}\left(\frac{1}{1+n}\right)\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{n(1+n)}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{(1+n)^2}\right)$$

$$=\sum_{n=1}^{\infty}\left(1-\frac{1}{n+1}\right)-\left(\zeta(2)-1\right)=2-\frac{\pi^2}{6}$$

SO, $\boxed{\int_0^{1}\frac{1}{1+\lfloor 1/x\rfloor}dx=2-\frac{\pi^2}{6}}$

236 Calculate integral $K=\int_0^1 \frac{\sin\left(\sqrt[x]{\log(x)}\right)}{\log(x)}dx$

Answer

They give $$K=\int_0^1 \frac{\sin\left(\sqrt[x]{\log(x)}\right)}{\log(x)}dx$$

$$=\int_0^1 \frac{\sin\left(\log(x)\right)}{x\log(x)}dx$$

$$Let: t=-\log(x) \Leftrightarrow x=e^{-t} \Rightarrow dx=-e^{-t}dt \ , \ If: x\in(0,1) \Rightarrow t\in(\infty,0)$$

$$\Rightarrow K=-\int_\infty^0 \frac{\sin(t)}{(t)e^{-t}}\times e^{-t}dt=\int_0^\infty \frac{\sin(t)}{(t)}dt=\frac{\pi}{2}$$

$$SO, \ \int_0^1 \frac{\sin\left(\sqrt[x]{\log(x)}\right)}{\log(x)}dx=\frac{\pi}{2}$$

237 Calculate integral $I=\int_0^\infty \frac{\sin(x)}{x+x\cos^2(x)}dx$

Answer

They give $$I=\int_0^\infty \frac{\sin(x)}{x+x\cos^2(x)}dx$$

$$=\int_0^\infty \frac{\sin(x)}{x}\times\frac{1}{1+\cos^2(x)}dx \ , By: f(x)=\frac{1}{1+\cos^2(x)} \ and \ f(\pi\pm x)=f(x)$$

$$That \quad I=\int_0^\infty \frac{\sin(x)}{x}\times\frac{1}{1+\cos^2(x)}dx=\int_0^{\frac{\pi}{2}}\frac{1}{1+\cos^2(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sec^2(x)}{\left(1+\cos^2(x)\right)\sec^2(x)}dx=\int_0^{\frac{\pi}{2}}\frac{\sec^2(x)}{2+\tan^2(x)}dx$$

$$Let: \tan(x)=\sqrt{2}t \Rightarrow \sec^2(x)dx=\sqrt{2}dt \ , \ If: x\in(0,\frac{\pi}{2}) \Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\sqrt{2}\int_0^\infty \frac{1}{2+2t^2}dt=\frac{\sqrt{2}}{2}\tan^{-1}(t)\Big|_0^\infty=\frac{\pi}{2\sqrt{2}}$$

$$SO, \ \boxed{\int_0^\infty \frac{\sin(x)}{x+x\cos^2(x)}dx=\frac{\pi}{2\sqrt{2}}}$$

$$Note: \int_0^\infty \frac{\sin(x)}{x}f(x)dx=\int_0^{\frac{\pi}{2}}f(x)dx \ , If: f(x)=f(\pi\pm x)$$

238 Calculate integral $J=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx$

Answer

They give $$J=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx\ (*)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos^2(x)}{\sin(x)+\cos(x)}dx\ \ (**)$$

$$Take\,(*)+(**)\,That: 2J=\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx+\int_0^{\frac{\pi}{2}}\frac{\cos^2(x)}{\sin(x)+\cos(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{\sin(x)+\cos(x)}dx$$

$$Let: y=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+y^2}dy, If: x\in\left(0,\frac{\pi}{2}\right)\Rightarrow y\in(0,1)$$

$$But: \sin(x)=\frac{2y}{1+y^2}, \cos(x)=\frac{1-y^2}{1+y^2}$$

$$\Rightarrow J=\int_0^1\frac{1}{\frac{2y}{1+y^2}+\frac{1-y^2}{1+y^2}}\times\frac{2}{1+y^2}dy$$

$$=2\int_0^1\frac{1}{2-(y-1)^2}dy$$

$$=2\times\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{y-1}{\sqrt{2}}\right)\Bigg|_0^1$$

$$=\sqrt{2}\left(\sin^{-1}(0)-\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$$

$$=\sqrt{2}\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)=\frac{\pi}{2\sqrt{2}}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{\sin^2(x)}{\sin(x)+\cos(x)}dx=\frac{\pi}{2\sqrt{2}}}$$

239 Calculate integral $K=\int_0^{2022}\left(x^2-\lfloor x\rfloor\lceil x\rceil\right)dx$

Answer

They give $K=\int_0^{2022}\left(x^2-\lfloor x\rfloor\lceil x\rceil\right)dx$

$$=\int_0^{2022}\left(x^2\right)dx-\int_0^{2022}\lfloor x\rfloor\lceil x\rceil dx$$

$$=\frac{2022^3}{3}-\int_0^{2022}\lfloor x\rfloor\lceil x\rceil dx$$

$\forall k\in\mathbb{Z}$ We have: $k\le x<k+1\Rightarrow\lfloor x\rfloor=k$

$\forall k\in\mathbb{Z}$ We have: $k<x\le k+1\Rightarrow\lceil x\rceil=k+1$

That $K=\dfrac{2022^3}{3}-\sum_{k=0}^{2021}\int_k^{k+1}k(k+1)dx$

$$=\frac{2022^3}{3}-\lim_{n\to2021}\sum_{k=0}^{n}k(k+1)\int_k^{k+1}1dx$$

$$=\frac{2022^3}{3}-\lim_{n\to2021}\sum_{k=0}^{n}k(k+1)$$

$$=\frac{2022^3}{3}-\sum_{k=0}^{n}\left(k^2+k\right)$$

$$=\frac{2022^3}{3}-\lim_{n\to2021}\left(\frac{n\times(n+1)\times(2n+1)}{6}+\frac{n\times(n+1)}{2}\right)$$

$$=\lim_{n\to2021}\left(\frac{(n+1)^3}{3}-\frac{2n\times(n+1)\times(n+2)}{6}\right)$$

$$=\lim_{n\to2021}\left(\frac{(n+1)^3-n\times(n+1)\times(n+2)}{3}\right)$$

$$=\lim_{n\to2021}\left(\frac{(n+1)\left((n+1)^2-n\times(n+2)\right)}{3}\right)$$

$$=\lim_{n\to2021}\left(\frac{(n+1)\left(n^2+2n+1-n^2-2n\right)}{3}\right)$$

$$=\lim_{n\to2021}\left(\frac{(n+1)}{3}\right)=\frac{2022}{3}$$

SO, $\boxed{\int_0^{2022}\left(x^2-\lfloor x\rfloor\lceil x\rceil\right)dx=\frac{2022}{3}}$

240 Calculate integral $I=\int_0^{\pi} x\cos^4(x)\sin^5(x)dx$

Answer

They give $I=\int_0^{\pi} x\cos^4(x)\sin^5(x)dx$

$$=\int_0^{\pi}(\pi-x)\cos^4(\pi-x)\sin^5(\pi-x)dx$$

$$=\int_0^{\pi}(\pi-x)\cos^4(x)\sin^5(x)dx$$

$$=\pi\int_0^{\pi}\cos^4(x)\sin^5(x)dx-\underbrace{\int_0^{\pi}x\cos^4(x)\sin^5(x)dx}_{I}$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\cos^4(x)\sin^5(x)dx$$

$$=\frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}}\cos^4(x)\sin^5(x)dx \qquad \left(Take:\begin{cases}\int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx\\ f(2a-x)=f(x)\end{cases}\right)$$

$$=\pi\int_0^{\frac{\pi}{2}}\cos^4(x)\sin^5(x)dx$$

$$=\pi\int_0^{\frac{\pi}{2}}\cos^{2(\frac{5}{2})-1}(x)\sin^{2(3)-1}(x)dx$$

$$=\pi\int_0^{\frac{\pi}{2}}\cos^{2(\frac{5}{2})-1}(x)\sin^{2(3)-1}(x)dx$$

$$=\frac{\pi}{2}B\left(\frac{5}{2},3\right)=\frac{\pi}{2}\times\frac{\Gamma\left(\frac{5}{2}\right)\Gamma(3)}{\Gamma\left(3+\frac{5}{2}\right)}$$

$$=\frac{\pi\Gamma\left(\frac{5}{2}\right)}{\left(2+\frac{5}{2}\right)\left(1+\frac{5}{2}\right)\left(\frac{5}{2}\right)\Gamma\left(\frac{5}{2}\right)}$$

$$=\frac{8\pi}{315}$$

SO, $\boxed{\int_0^{\pi}x\cos^4(x)\sin^5(x)dx=\frac{8\pi}{315}}$

241 Calculate integral $J=\int_0^1 \frac{\log(x)}{\sqrt{x}(x-1)}dx$

Answer

They give $$J=\int_0^1 \frac{\log(x)}{\sqrt{x}(x-1)}dx$$

$Let: x=t^2 \Rightarrow dx=2tdt\ ,\ If\ : x\in(0,1)\Rightarrow t\in(0,1)$

$$\Rightarrow J=\int_0^1 \frac{\log(t^2)}{t(t^2-1)}\times 2tdt=-4\int_0^1 \frac{\log(t)}{(1-t^2)}dt$$

$$=-4\int_0^1 \log(t)\sum_{n=0}^{\infty} t^{2n}dt=-4\sum_{n=0}^{\infty}\int_0^1 t^{2n}\log(t)dt$$

$$=\underbrace{-4\sum_{n=0}^{\infty}\int_0^1 t^{2n}\log(t)dt}_{(Use\ partial\ integral)}$$

$$=-4\sum_{n=0}^{\infty}\left(\left.\frac{t^{2n+1}\log(t)}{2n+1}\right|_0^1-\int_0^1 \frac{t^{2n+1}}{2n+1}\times\frac{1}{t}dt\right)$$

$$=4\sum_{n=0}^{\infty}\left(\int_0^1 \frac{t^{2n}}{2n+1}dt\right)$$

$$=4\sum_{n=0}^{\infty}\left(\frac{\left.t^{2n+1}\right|_0^1}{(2n+1)^2}\right)$$

$$=4\sum_{n=0}^{\infty}\left(\frac{1}{(2n+1)^2}\right)$$

$$=4\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)$$

$$=4\left[\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)-\frac{1}{2^2}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)\right]$$

$$=4\left(\zeta(2)-\frac{1}{4}\zeta(2)\right)=3\zeta(2)$$

$$=\frac{\pi^2}{2}$$

SO, $\boxed{\int_0^1 \frac{\log(x)}{\sqrt{x}(x-1)}dx=\frac{\pi^2}{2}}$

242 Calculate integral $K=\int_0^{\infty}\frac{x^n}{e^x+1}dx$

Answer

They give $$K=\int_0^{\infty}\frac{x^n}{e^x+1}dx$$

$$=\int_0^{\infty}\frac{x^n e^{-x}}{1+e^{-x}}dx=\int_0^{\infty}x^n e^{-x}\sum_{m=0}^{\infty}(-1)^m e^{-mx}dx$$

$$=\sum_{m=0}^{\infty}(-1)^m\int_0^{\infty}x^n e^{-x}\times e^{-mx}dx=\sum_{m=0}^{\infty}(-1)^m\int_0^{\infty}x^n e^{-(m+1)x}dx$$

$Let: y=(m+1)x\Rightarrow dx=\frac{dy}{m+1}$, $If: x\in(0,\infty)\Rightarrow y\in(0,\infty)$

$$=\sum_{m=0}^{\infty}(-1)^m\int_0^{\infty}\left(\frac{y}{m+1}\right)^n e^{-y}\frac{dy}{m+1}=\sum_{m=0}^{\infty}\frac{(-1)^m}{(m+1)^{n+1}}\int_0^{\infty}(y)^n e^{-y}dy$$

$$=\Gamma(n+1)\sum_{m=0}^{\infty}\frac{(-1)^m}{(m+1)^{n+1}}=\Gamma(n+1)\left(1-\frac{1}{2^{n+1}}+\frac{1}{3^{n+1}}-\frac{1}{4^{n+1}}+\frac{1}{5^{n+1}}-\frac{1}{6^{n+1}}+\dots\right)$$

$$=\Gamma(n+1)\left[\left(1+\frac{1}{2^{n+1}}+\frac{1}{3^{n+1}}+\frac{1}{4^{n+1}}+\dots\right)-\frac{2}{2^{n+1}}\left(1+\frac{1}{2^{n+1}}+\frac{1}{3^{n+1}}+\frac{1}{4^{n+1}}+\dots\right)\right]$$

$$=\Gamma(n+1)\left[\zeta(n+1)-\frac{1}{2^n}\zeta(n+1)\right]=\frac{2^n-1}{2^n}\Gamma(n+1)\zeta(n+1)$$

SO, $$\boxed{\int_0^{\infty}\frac{x^n}{e^x+1}dx=\frac{2^n-1}{2^n}\Gamma(n+1)\zeta(n+1)}$$

243 Calculate integral $I=\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx$

Answer

They give $$I=\int_0^{\infty}\frac{\log(x)}{\sqrt{x}(x-1)}dx$$

$$=\int_0^{1}\frac{\log(x)}{\sqrt{x}(x-1)}dx+\underbrace{\int_1^{\infty}\frac{\log(x)}{\sqrt{x}(x-1)}dx}_{I_1}$$

$For: I_1=\int_1^{\infty}\frac{\log(x)}{\sqrt{x}(x-1)}dx$, $\begin{cases}Let: y=\frac{1}{x}\Rightarrow dx=-\frac{1}{y^2}dy\\ If: x\in(1,\infty)\Rightarrow y\in(1,0)\end{cases}$

$$That: I_1=-\int_1^0\frac{\log(1/y)}{\sqrt{1/y}\,(1/y-1)}\cdot\frac{1}{y^2}dy=\int_0^1\frac{\log(y)}{\sqrt{y}(y-1)}dy=\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx\ ,(f(x)=f(y))$$

$$That: I=\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx+\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx$$

$$=2\int_0^1\frac{\log(x)}{\sqrt{x}(x-1)}dx=2\left(\frac{\pi^2}{2}\right)=\pi^2$$

SO, $\boxed{\int_0^\infty\frac{\log(x)}{\sqrt{x}(x-1)}dx=\pi^2}$

244 Calculate integral $J=\int_1^{\int_1^{\int_1^{\cdots}(2x)dx}(2x)dx}(2x)dx$

Answer

They give
$$J=\int_1^{\int_1^{\int_1^{\cdots}(2x)dx}(2x)dx}(2x)dx$$

$$=\int_1^J(2x)dx=x^2\Big|_1^J=J^2-1$$

$$\Leftrightarrow J=J^2-1$$

$$\Leftrightarrow J^2-J-1=0\Rightarrow J=\frac{1\pm\sqrt{5}}{2}$$

SO, $\boxed{\int_1^{\int_1^{\int_1^{\cdots}(2x)dx}(2x)dx}(2x)dx=\frac{1\pm\sqrt{5}}{2}}$

245 Calculate integral $K=\int_0^\infty\frac{e^{-2x}\cos(x)-e^{-3x}}{x}dx$

Answer

They give
$$K=\int_0^\infty\frac{e^{-2x}\cos(x)-e^{-3x}}{x}dx$$

$$=\int_0^\infty\frac{\cos(x)-e^{-x}}{x}e^{-2x}dx$$

$$K(a)=\int_0^\infty\frac{\cos(x)-e^{-x}}{x}e^{-ax}dx$$

$$\frac{d}{da}(K(a))=-\int_0^\infty\left(\cos(x)-e^{-x}\right)e^{-ax}dx$$

$$=\underbrace{\int_0^\infty e^{-(a+1)x}dx}_{K_1}-\underbrace{\int_0^\infty\cos(x)e^{-ax}dx}_{K_2}$$

$$Take: K_2=\underbrace{\int_0^\infty\cos(x)e^{-ax}dx}_{(Use\ partial\ integral)}=\frac{1}{a^2+1}\left[e^{-ax}\left(\sin(x)-a\cos(x)\right)\right]_0^\infty=\frac{a}{a^2+1}$$

$$Take: K_1=\int_0^{\infty} e^{-(a+1)x}dx=-\frac{1}{a+1}e^{-(a+1)x}\Big|_0^{\infty}=\frac{1}{a+1}$$

$$That: \frac{d}{da}\left(K(a)\right)=\frac{1}{a+1}-\frac{a}{a^2+1}$$

$$\Rightarrow K(a)=\log(a+1)-\frac{1}{2}\log(a^2+1)+C=\log\left(\frac{a+1}{\sqrt{a^2+1}}\right)+C$$

$$If: a=\infty \Rightarrow K(\infty)=0=\lim_{a\to\infty}\left[\log\left(\frac{a+1}{\sqrt{a^2+1}}\right)+C\right]\Leftrightarrow 0=0+C\Rightarrow C=0$$

$$If: a=2\Rightarrow K(2)=I=\log\left(\frac{2+1}{\sqrt{2^2+1}}\right)+0=\log\left(\frac{3}{\sqrt{5}}\right)$$

$$SO,\ \boxed{\int_0^{\infty}\frac{e^{-2x}\cos(x)-e^{-3x}}{x}dx=\log\left(\frac{3}{\sqrt{5}}\right)}$$

246 Calculate integral $I=\int_0^{\infty}\left(\frac{\log^2(x)}{x(x+1)}\right)dx$

Answer

$$They\ give \qquad I=\int_1^{\infty}\left(\frac{\log^2(x)}{x(x+1)}\right)dx$$

$$Let: x=e^y \Rightarrow dx=e^y dy, If: x\in(1,\infty)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow I=\int_0^{\infty}\frac{y^2e^y}{e^y(e^y+1)}dy=\int_0^{\infty}\frac{y^2e^{-y}}{(1+e^{-y})}dy$$

$$By: \frac{1}{1+e^{-y}}=\sum_{n=0}^{\infty}(-1)^n e^{-ny}\Leftrightarrow\frac{1}{1+e^{-x}}=1+\sum_{n=1}^{\infty}(-1)^n e^{-ny}$$

$$\Rightarrow I=\int_0^{\infty}y^2e^{-y}\left(1+\sum_{n=1}^{\infty}(-1)^n e^{-ny}\right)dy$$

$$=\int_0^{\infty}y^2e^{-y}dy+\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}y^2e^{-(n+1)y}dy$$

$$=\Gamma(3)+\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}y^2e^{-(n+1)y}dy$$

$$Let: t=(n+1)y \Leftrightarrow y=\frac{t}{n+1} \Rightarrow dy=\frac{1}{n+1}dt, If: y\in(0,\infty) \Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\Gamma(3)+\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty}\left(\frac{t}{n+1}\right)^2 e^{-t}\frac{dt}{n+1}$$

$$=\Gamma(3)+\sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)^3}\int_0^{\infty}(t)^2 e^{-t}dt$$

$$=\Gamma(3)+\sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)^3}\Gamma(3)=2\left(1+\sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)^3}\right)$$

$$=-2\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(n+1)^3}=-2\eta(3)$$

$$=-\left(1-2^{1-3}\right)\zeta(3)=-\frac{3}{4}\zeta(3)$$

$$SO, \boxed{\int_1^{\infty}\left(\frac{\log^2(x)}{x(x+1)}\right)dx=-\frac{3}{4}\zeta(3)}$$

247 Calculate integral $J=\int_0^{\frac{\pi}{2}}\sin^2(x)\sin^2(2x)dx$

Answer

They give $$J=\int_0^{\frac{\pi}{2}}\sin^2(x)\sin^2(2x)dx \quad (*)$$

$$=\int_0^{\frac{\pi}{2}}\sin^2\left(\frac{\pi}{2}-x\right)\sin^2\left(\pi-2x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\cos^2(x)\sin^2(2x)dx \quad (**)$$

$$Take:(*)+(**) That: 2J=\int_0^{\frac{\pi}{2}}\sin^2(x)\sin^2(2x)dx+\int_0^{\frac{\pi}{2}}\cos^2(x)\sin^2(2x)dx$$

$$=\int_0^{\frac{\pi}{2}}\left[\sin^2(x)+\cos^2(x)\right]\sin^2(2x)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin^2(2x)dx$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(1-\cos(4x)\right)dx=\frac{\pi}{4}$$

$$SO, \boxed{\int_0^{\frac{\pi}{2}}\sin^2(x)\sin^2(2x)dx=\frac{\pi}{4}}$$

248 Calculate integral $K=\int_0^{\frac{\pi}{4}}\frac{\sin(x)+\cos(x)}{9+16\sin(2x)}dx$

Answer

They give
$$K=\int_0^{\frac{\pi}{4}}\frac{\sin(x)+\cos(x)}{9+16\sin(2x)}dx$$
$$=\int_0^{\frac{\pi}{4}}\frac{\sin(x)+\cos(x)}{9+16\left[1-\left(\sin(x)-\cos(x)\right)^2\right]}dx$$

$Let: y=\sin(x)-\cos(x)\Rightarrow dy=\left(\sin(x)+\cos(x)\right)dx\ ,\ If: x\in(0,\frac{\pi}{4})\Rightarrow y\in(-1,0)$

$$\Rightarrow K=\int_{-1}^{0}\frac{1}{9+16\left(1-y^2\right)}dx=\int_{-1}^{0}\frac{1}{25-16y^2}dx$$
$$=\frac{1}{16}\int_{-1}^{0}\frac{1}{(5/4)^2-y^2}dx=\frac{1}{40}\log\left(\frac{5+4y}{5-4y}\right)\Bigg|_{-1}^{0}=\frac{3}{40}\log(3)$$

SO, $\boxed{\int_0^{\frac{\pi}{4}}\frac{\sin(x)+\cos(x)}{9+16\sin(2x)}dx=\frac{3}{40}\log(3)}$

249 Calculate integral $I=\int_0^{\infty}\frac{e^{-\pi x}-e^{-ex}}{x}dx$

Answer

They give
$$I=\int_0^{\infty}\frac{e^{-\pi x}-e^{-ex}}{x}dx$$
$$=\int_0^{\infty}\frac{1-e^{-(e-\pi)x}}{x}e^{-\pi x}dx$$
$$\Rightarrow I(a)=\int_0^{\infty}\frac{1-e^{-(e-\pi)x}}{x}e^{-ax}dx$$
$$\Rightarrow I'(a)=\int_0^{\infty}\frac{\left(1-e^{-(e-\pi)x}\right)(-x)e^{-ax}}{x}dx$$
$$=\int_0^{\infty}\left(1-e^{-(e-\pi)x}\right)e^{-ax}dx=\int_0^{\infty}e^{-(e-\pi+a)x}dx-\int_0^{\infty}e^{-ax}dx$$
$$=-\frac{1}{e-\pi+a}e^{-(e-\pi+a)x}\Bigg|_0^{\infty}+\frac{1}{a}e^{-ax}\Bigg|_0^{\infty}$$
$$=\frac{1}{e-\pi+a}-\frac{1}{a}$$

$$\Rightarrow I(a)=\int\left(\frac{1}{e-\pi+a}-\frac{1}{a}\right)da=\log\left(\frac{a+e-\pi}{a}\right)+C$$

$$If:a=\pi\Rightarrow I(\pi)=I=\log\left(\frac{\pi+e-\pi}{\pi}\right)+C=\log\left(\frac{e}{\pi}\right)+C$$

$$If:a=\infty\Rightarrow I(\infty)=0=\lim_{a\to\infty}\left[\log\left(\frac{\pi+e-\pi}{\pi}\right)+C\right]=0+C\Rightarrow C=0$$

$$They\ Have:I=\log\left(\frac{e}{\pi}\right)$$

$$SO,\quad \boxed{\int_0^\infty\frac{e^{-\pi x}-e^{-ex}}{x}dx=\log\left(\frac{e}{\pi}\right)}$$

$$\oplus OR:I=\int_0^\infty\frac{e^{-(\pi-e)x}-1}{x}e^{-ex}dx$$

$$\Rightarrow I(a=e)=\int_0^\infty\frac{e^{-(\pi-e)x}-1}{x}e^{-ax}dx=\log\left(\frac{e}{\pi}\right)$$

250 Calculate integral $J=\int_{-1}^{1}x\tan(x)\tan\left(\frac{1}{x}\right)dx$

Answer

$$They\ give\qquad J=\int_{-1}^{1}x\tan(x)\tan\left(\frac{1}{x}\right)dx$$

$$=\int_{-1}^{1}x\tan^{-1}(x)\cot^{-1}(x)dx$$

$$=\int_{-1}^{1}(-x)\tan^{-1}(-x)\cot^{-1}(-x)dx$$

$$Take:\tan^{-1}(-x)=-\tan^{-1}(x),\cot^{-1}(-x)=\pi-\cos^{-1}(x),\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}(x)$$

$$\Rightarrow J=\int_{-1}^{1}x\tan^{-1}(x)\left[\pi-\cot^{-1}(x)\right]dx$$

$$=\pi\int_{-1}^{1}x\tan^{-1}(x)dx-\int_{-1}^{1}x\tan^{-1}(x)\cot^{-1}(x)dx$$

$$\Rightarrow J=\frac{\pi}{2}\underbrace{\int_{-1}^{1}x\tan^{-1}(x)dx}_{is\,an\,even\ function}=\pi\int_0^1x\tan^{-1}(x)dx$$

$$Let:u=\tan^{-1}(x)\Rightarrow du=\frac{1}{1+x^2}dx,v=\int xdx=\frac{1}{2}x^2$$

$$\Rightarrow J=\pi\left[\frac{1}{2}x^2\tan^{-1}(x)\Big|_0^1-\frac{1}{2}\int_0^1\frac{x^2}{1+x^2}dx\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \int_0^1 \frac{1+x^2-1}{1+x^2}dx\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx\right] = \frac{\pi(\pi-2)}{4}$$

SO, $\boxed{\int_{-1}^{1} x\tan(x)\tan\left(\frac{1}{x}\right)dx = \frac{\pi(\pi-2)}{4}}$

251 Calculate integral $K = \int_0^{\infty} \frac{\log(2e^x-1)}{e^x-1}dx$

Answer

They give $K = \int_0^{\infty} \frac{\log(2e^x-1)}{e^x-1}dx$

$Let: t = e^x \Rightarrow dx = \frac{1}{t}dt, If: x \in (0,\infty) \Rightarrow t \in (1,\infty)$

$$\Rightarrow K = \int_1^{\infty} \frac{\log(2t-1)}{t(t-1)}dt$$

$Let: t = \frac{1}{y} \Rightarrow dt = -\frac{1}{y^2}dy, If: t \in (1,\infty) \Rightarrow y \in (1,0)$

$$\Rightarrow K = -\int_1^0 \frac{\log\left(\frac{2}{y}-1\right)}{\frac{1}{y}\left(\frac{1}{y}-1\right)} \times \frac{1}{y^2}dy = \int_0^1 \frac{\log(2-y)-\log(y)}{(1-y)}dy$$

$Let: z = 1-y \Leftrightarrow y = 1-z \Rightarrow dz = -dy, If: y \in (0,1) \Rightarrow z \in (1,0)$

$$\Rightarrow K = -\int_1^0 \frac{\log(1+z)-\log(1-z)}{z}dz$$

$$= \int_0^1 \frac{\left(z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots\right) - \left(-z - \frac{z^2}{2} - \frac{z^3}{3} - \frac{z^4}{4} - \dots\right)}{z}dz$$

$$= \int_0^1 \frac{\left(2z + \frac{2z^3}{3} + \frac{2z^5}{5} + \frac{2z^7}{7} + \dots\right)}{z}dz$$

$$= 2\int_0^1 \left(1 + \frac{z^2}{3} + \frac{z^4}{5} + \frac{z^6}{7} + \dots\right)dz$$

$$= 2\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots\right)$$

$$= 2\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{4} \quad , Because: \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

$SO,$ $$\boxed{\int_0^{\infty} \frac{\log(2e^x - 1)}{e^x - 1} dx = \frac{\pi^2}{4}}$$

252 Calculate integral $I = \int_0^{\pi} \frac{x\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx$

Answer

They give $$I = \int_0^{\pi} \frac{x\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx$$

$$= \int_0^{\pi} \frac{(\pi - x)\sin^{2026}(\pi - x)}{\cos^{2026}(\pi - x) + \sin^{2026}(\pi - x)} dx$$

$$= \int_0^{\pi} \frac{\pi\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx - \underbrace{\int_0^{\pi} \frac{x\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx}_{I}$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx \quad (1)$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}\left(\frac{\pi}{2} - x\right)}{\cos^{2026}\left(\frac{\pi}{2} - x\right) + \sin^{2026}\left(\frac{\pi}{2} - x\right)} dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\cos^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx \quad (2)$$

Take $(1) + (2)$ That: $$2I = \pi \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx + \pi \int_0^{\frac{\pi}{2}} \frac{\cos^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\cos^{2026}(x) + \sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx = \frac{\pi^2}{4}$$

$SO,$ $$\boxed{\int_0^{\pi} \frac{x\sin^{2026}(x)}{\cos^{2026}(x) + \sin^{2026}(x)} dx = \frac{\pi^2}{4}}$$

253 Calculate integral $J=\int_1^e\left(\frac{\tan^{-1}(x)}{x}+\frac{\log(x)}{x^2+1}\right)dx$

Answer

They give $J=\int_1^e\left(\frac{\tan^{-1}(x)}{x}+\frac{\log(x)}{x^2+1}\right)dx$

$$=\int_1^e\left[\left(\log(x)\right)'\tan^{-1}(x)+\left(\tan^{-1}(x)\right)'\log(x)\right]dx$$

$$=\int_1^e\left[\left(\log(x)\tan^{-1}(x)\right)'\right]dx \qquad ,\left(Use:(uv)'=u'v+v'u\right)$$

$$=\log(x)\tan^{-1}(x)\Big|_1^e=\tan^{-1}(e) \qquad ,\left(Use:\int_a^b f'(x)dx=f(a)-f(b)\right)$$

$$=\tan^{-1}(e)$$

SO, $\boxed{\int_1^e\left(\frac{\tan^{-1}(x)}{x}+\frac{\log(x)}{x^2+1}\right)dx=\tan^{-1}(e)}$

254 Calculate integral $I=\int_0^2\sqrt{x+\sqrt{x+\sqrt{x+...}}}\,dx$

Answer

They give $I=\int_0^2\sqrt{x+\sqrt{x+\sqrt{x+...}}}\,dx$

$Let: y_{(y>0\forall x\in(0,2))}=\sqrt{x+\sqrt{x+\sqrt{x+...}}}$

$$\Leftrightarrow y=\sqrt{x+y}$$

$$\Leftrightarrow y^2-y-x=0\Rightarrow y=\frac{1\pm\sqrt{1+4x}}{2}=\begin{cases}y=\dfrac{1+\sqrt{1+4x}}{2}\\ y=\dfrac{1-\sqrt{1+4x}}{2} & \text{(Do not take)}\end{cases}$$

$$\Rightarrow I=\int_0^2\frac{1+\sqrt{1+4x}}{2}$$

$$=\int_0^2\frac{1}{2}dx+\frac{1}{2}\int_0^2\sqrt{1+4x}\,dx=\frac{19}{6}$$

SO, $\boxed{\int_0^2\sqrt{x+\sqrt{x+\sqrt{x+...}}}\,dx=\frac{19}{6}}$

255 Calculate integral $K=\int_0^{\infty}\frac{1}{\sqrt{e^{2x}+e^x+1}}dx$

Answer

They give $$K=\int_0^{\infty}\frac{1}{\sqrt{e^{2x}+e^x+1}}dx$$

$$Let: y=e^x \Rightarrow dx=\frac{1}{y}dy, If: x\in(0,\infty)\Rightarrow y\in(1,\infty)$$

$$\Rightarrow K=\int_1^{\infty}\frac{1}{y\sqrt{y^2+y+1}}dy$$

$$Let: t=\frac{1}{y}\Rightarrow dy=-\frac{1}{t^2}dt, If: y\in(1,\infty)\Rightarrow t\in(1,0)$$

$$\Rightarrow K=-\int_1^0\frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}}\frac{1}{t^2}dt=\int_0^1\frac{1}{\sqrt{t^2+t+1}}dt=\int_0^1\frac{1}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}}dt$$

$$Let: t+\frac{1}{2}=\sqrt{\frac{3}{4}}\tan(u)\Rightarrow dt=\sqrt{\frac{3}{4}}\sec^2(u)du, If: t\in(0,1)\Rightarrow u\in(\frac{\pi}{6},\frac{\pi}{3})$$

$$\Rightarrow K=\sqrt{\frac{3}{4}}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\sqrt{\frac{3}{4}\left(\tan^2(u)+1\right)}}\sec^2(u)du$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\sqrt{\sec^2(u)}}\sec^2(u)du$$

$$=\log\left(\sec(u)+\tan(u)\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=\log\left(1+\frac{2\sqrt{3}}{3}\right)$$

$$\oplus OR: \int_0^1\frac{1}{\sqrt{t^2+t+1}}dt=\int_0^1\frac{1}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}}dt=\log\left(t+\frac{1}{2}+\sqrt{1+t+t^2}\right)\Big|_0^1=\log\left(1+\frac{2\sqrt{3}}{3}\right)$$

SO, $$\boxed{\int_0^{\infty}\frac{1}{\sqrt{e^{2x}+e^x+1}}dx=\log\left(1+\frac{2\sqrt{3}}{3}\right)}$$

256 Calculate integral $I=\int_0^1\left\lfloor\frac{1}{\sqrt{x}}\right\rfloor dx$

Answer

They give $$I=\int_0^1\left\lfloor\frac{1}{\sqrt{x}}\right\rfloor dx$$

$$Let: y=\frac{1}{\sqrt{x}}\Rightarrow dx=-\frac{2}{y^3}dy, If: x\in(0,1)\Rightarrow y\in(\infty,1)$$

$$\Rightarrow I=-2\int_\infty^1\frac{\lfloor y\rfloor}{y^3}dy=2\int_1^\infty\frac{\lfloor y\rfloor}{y^3}dy$$

$$=2\sum_{n=1}^{\infty}\int_n^{n+1}\frac{n}{y^3}dy=-\sum_{n=1}^{\infty}n\left(\frac{1}{y^2}\right)\Bigg|_n^{n+1}$$

$$=\sum_{n=1}^{\infty}n\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{(2n+1)}{n(n+1)^2}\right)$$

$$=2\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n(n+1)^2}\right)$$

$$=2\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}-\frac{1}{(n+1)^2}\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)+\lim_{m\to\infty}\sum_{n=1}^{m}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)+\lim_{m\to\infty}\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\dots\frac{1}{m}-\frac{1}{m+1}\right)$$

$$=\zeta(2)-1+\lim_{m\to\infty}\left(\frac{1}{1}-\frac{1}{m+1}\right)$$

$$=\frac{\pi^2}{6}-1+1-0=\frac{\pi^2}{6}$$

SO, $$\boxed{\int_0^1\left\lfloor\frac{1}{\sqrt{x}}\right\rfloor dx=\frac{\pi^2}{6}}$$

257 Calculate integral $J=\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx$

Answer

They give $$J=\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx$$

$$=\int_0^{\infty}\log\left(\frac{1+e^{-x}}{1-e^{-x}}\right)dx$$

$$Let: u=\log\left(\frac{1+e^{-x}}{1-e^{-x}}\right)\Rightarrow du=-\frac{2e^{-x}}{1-e^{-2x}}dx, dv=dx\Rightarrow v=x$$

$$\Rightarrow J=\underbrace{x\log\left(\frac{1+e^{-x}}{1-e^{-x}}\right)\Bigg|_0^{\infty}}_{0}+\int_0^{\infty}\frac{2xe^{-x}}{1-e^{-2x}}dx$$

$$=\int_0^{\infty}\left(2xe^{-x}\sum_{n=0}^{\infty}e^{-2nx}dx\right)$$

$$=2\sum_{n=0}^{\infty}\left(\int_0^{\infty}xe^{-(2n+1)x}dx\right)$$

$$Let: z=(2n+1)x\Leftrightarrow x=\frac{z}{(2n+1)}\Rightarrow dx=\frac{1}{(2n+1)}dz, If: x\in(0,\infty)\Rightarrow z\in(0,\infty)$$

$$\Rightarrow J=2\sum_{n=0}^{\infty}\left[\int_0^{\infty}\left(\frac{z}{(2n+1)}\right)e^{-z}\times\frac{1}{(2n+1)}dz\right]$$

$$=2\sum_{n=0}^{\infty}\left(\frac{1}{(2n+1)^2}\int_0^{\infty}ze^{-z}dz\right)$$

$$=2\sum_{n=0}^{\infty}\left(\frac{1}{(2n+1)^2}\times\Gamma(2)\right)$$

$$=2\times\frac{\pi^2}{8}\times 1! \quad ,\left(By:\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}\right)$$

$$=\frac{\pi^2}{4}$$

SO, $\boxed{\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx=\frac{\pi^2}{4}}$

258 Calculate integral $K=\int_0^{\infty} e^{-\lfloor x\rfloor(1+\{x\})}dx$

Answer

They give $$K=\int_0^{\infty} e^{-\lfloor x\rfloor(1+\{x\})}dx$$
$$=\int_0^{1} e^{-\lfloor x\rfloor(1+\{x\})}dx+\int_1^{\infty} e^{-\lfloor x\rfloor(1+\{x\})}dx$$

$By:\{x\}=x-\lfloor x\rfloor,(\{x\}: is\, the\ fraction\ part\ function)$

$That:-\lfloor x\rfloor(1+\{x\})=-\lfloor x\rfloor(1+x-\lfloor x\rfloor)$

$if\ :0<x<1\Rightarrow-\lfloor x\rfloor(1+x-\lfloor x\rfloor)=0$

$if\ :(\forall n>1)That:n\le x<n+1,\lfloor x\rfloor=n$

$They\, have:-\lfloor x\rfloor(1+x-\lfloor x\rfloor)=-n(1+x-n)=(n^2-nx-n)$

$$\Rightarrow K=\int_0^1 e^0dx+\sum_{n=1}^{\infty}\int_n^{n+1}e^{(n^2-n-nx)}dx$$
$$=1+\sum_{n=1}^{\infty}e^{(n^2-n)}\int_n^{n+1}e^{-nx}dx$$
$$=1-\sum_{n=1}^{\infty}e^{(n^2-n-)}\frac{e^{-nx}\Big|_n^{n+1}}{n}$$
$$=1-\sum_{n=1}^{\infty}e^{(n^2-n)}\frac{\left(e^{-(n^2+n)}-e^{-n^2}\right)}{n}$$
$$=1+\sum_{n=1}^{\infty}\frac{\left(e^{-n}-e^{-2n}\right)}{n}$$
$$=1+\sum_{n=1}^{\infty}\frac{e^{-n}}{n}-\sum_{n=1}^{\infty}\frac{e^{-2n}}{n}\quad,\left(By:\sum_{n=1}^{\infty}\left(\frac{x^n}{n}\right)=-\log(1-x)\ ,1\le x<1\right)$$
$$\Rightarrow K=1-\log(1-e^{-1})+\log(1-e^{-2})$$
$$=1-\log\left(\frac{e-1}{e}\right)+\log\left(\frac{e^2-1}{e^2}\right)$$
$$=1-\log(e-1)-\log(e)+\log(e^2-1)+\log(e^2)$$
$$=1-\log(e-1)-1+\log(e-1)+\log(e+1)+2$$
$$=\log(e+1)$$

SO, $\boxed{\int_0^{\infty} e^{-\lfloor x\rfloor(1+\{x\})}dx=\log(e+1)}$

259 Calculate integral $I=\int_0^{\infty} i^{ix^2}dx\ \ ,i=\sqrt{-1}$

Answer

They give $\qquad I=\int_0^{\infty} i^{ix^2}dx\ \ ,i=\sqrt{-1}$

$$\Rightarrow I=\int_0^{\infty}\left(e^{i\frac{\pi}{2}}\right)^{ix^2}dx=\int_0^{\infty}e^{-\frac{\pi}{2}x^2}dx\ ,\left(By: i=\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)=e^{i\frac{\pi}{2}}\right)$$

$$Let: y=\frac{\pi}{2}x^2\Leftrightarrow x=\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{y}\Rightarrow dx=\frac{\sqrt{2}}{2\sqrt{\pi}}\times\frac{1}{\sqrt{y}}dy, If: x\in(0,\infty)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow I=\frac{\sqrt{2}}{2\sqrt{\pi}}\int_0^{\infty}e^{-y}\times\frac{1}{\sqrt{y}}dy=\frac{\sqrt{2}}{2\sqrt{\pi}}\int_0^{\infty}y^{\frac{1}{2}-1}e^{-y}dy=\frac{\sqrt{2}}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{2}}{2}$$

SO, $\boxed{\int_0^{\infty} i^{ix^2}dx=\frac{\sqrt{2}}{2}}$

260 Calculate integral $\qquad J=\int_0^1\frac{1}{x^2+1}dx$

Answer

They give $\qquad J=\int_0^1\frac{1}{x^2+1}dx$

$$Take: J'=\int\frac{1}{x^2+1}dx=\int\frac{1}{(x-i)(x+i)}dx$$

$$=\frac{1}{2i}\int\left(\frac{1}{x-i}-\frac{1}{x+i}\right)dx=\frac{1}{2i}\log\left(\frac{x-i}{x+1}\right)+C$$

$$By: Z=a+bi, That: Z=|r|e^{i\tan^{-1}\left(\frac{b}{a}\right)}\Rightarrow\begin{cases}x-i=\sqrt{x^2+1}e^{i\tan^{-1}\left(-\frac{1}{x}\right)}\\ x+i=\sqrt{x^2+1}e^{i\tan^{-1}\left(\frac{1}{x}\right)}\end{cases}$$

$$That: J'=\frac{1}{2i}\log\left(\frac{\sqrt{x^2+1}e^{i\tan^{-1}\left(-\frac{1}{x}\right)}}{\sqrt{x^2+1}e^{i\tan^{-1}\left(\frac{1}{x}\right)}}\right)+C=\frac{1}{2i}\log\left(e^{-i\tan^{-1}\left(\frac{1}{x}\right)}\times e^{-i\tan^{-1}\left(\frac{1}{x}\right)}\right)+C$$

$$=\frac{1}{2i}\left(-2i\tan^{-1}\left(\frac{1}{x}\right)\right)+C=-\tan^{-1}\left(\frac{1}{x}\right)+C$$

$$=-\cot^{-1}(x)+C=-\left(\pm\frac{\pi}{2}-\tan^{-1}(x)\right)+C$$

$$=\tan^{-1}(x)\underbrace{\mp\frac{\pi}{2}+C}_{C}=\tan^{-1}(x)+C$$

$$They\ Have: J=\int_0^1\frac{1}{x^2+1}dx=\tan^{-1}(x)\Big|_0^1=\frac{\pi}{4}$$

$$SO,\ \boxed{\int_0^1\frac{1}{x^2+1}dx=\frac{\pi}{4}}$$

261 Calculate integral $K=\int_0^1\frac{\log^3(1-x^2)}{x}dx$

Answer

$$They\ give\qquad K=\int_0^1\frac{\log^3(1-x^2)}{x}dx$$

$$=\frac{1}{2}\int_0^1\frac{\log^3(1-x^2)}{x^2}d(x^2)=\frac{1}{2}\int_0^1\frac{\log^3\left(1-(1-x^2)\right)}{1-x^2}d(x^2)$$

$$=\frac{1}{2}\int_0^1\frac{\log^3(x^2)}{1-x^2}d(x^2)$$

$$Let: x^2=e^u\Rightarrow d(x^2)=e^u du, If: x^2\in(0,1)\Rightarrow u\in(-\infty,0)$$

$$\Rightarrow J=\frac{1}{2}\int_{-\infty}^0\frac{u^3e^u}{1-e^u}du$$

$$Let: u=-z\Rightarrow du=-dz, If: u\in(-\infty,0)\Rightarrow z\in(\infty,0)$$

$$\Rightarrow J=-\frac{1}{2}\int_\infty^0\frac{(-z)^3e^{-z}}{1-e^{-z}}dz=-\frac{1}{2}\int_0^\infty\frac{z^3}{e^z-1}dz$$

$$=-\frac{1}{2}\int_0^\infty\frac{z^{4-1}}{e^z-1}dz=-\frac{1}{2}\Gamma(4)\zeta(4)=-\frac{\pi^4}{30}$$

$$SO,\ \boxed{\int_0^1\frac{\log^3(1-x^2)}{x}dx=-\frac{\pi^4}{30}}$$

262 Calculate integral $I=\int_0^1\cos\left(\log(x)\right)dx$

Answer

$$They\ give\qquad I=\int_0^1\cos\left(\log(x)\right)dx$$

$$Take\qquad J=\int_0^1\sin\left(\log(x)\right)dx$$

$$That:\qquad I+iJ=\int_0^1\left[\cos\left(\log(x)\right)+i\sin\left(\log(x)\right)\right]dx$$

$$=\int_0^1 e^{i\log(x)}dx=\int_0^1 x^i dx=\frac{1}{1+i}=\frac{1}{2}-\frac{1}{2}i$$

$$\Leftrightarrow I+iJ=\frac{1}{2}-\frac{1}{2}i\quad ,That: I=\frac{1}{2}, J=-\frac{1}{2}$$

$$SO,\ \boxed{\int_0^1\cos\left(\log(x)\right)dx=\frac{1}{2}}$$

263 Calculate integral $J=\int_0^{\pi} e^x\sin(x)dx$

Answer

They give $J=\int_0^{\pi} e^x\sin(x)dx$

Take $K=\int_0^{\pi} e^x\cos(x)dx$

That $K+iJ=\int_0^{\pi}\left(e^x\cos(x)+ie^x\sin(x)\right)dx=\int_0^{\pi} e^x\left(\cos(x)+i\sin(x)\right)dx$

$$=\int_0^{\pi} e^x\times e^{ix}dx=\int_0^{\pi} e^{(i+1)x}dx$$

$$=\frac{1}{1+i}e^{(i+1)x}\Big|_0^{\pi}=\frac{1}{2}(1-i)\left(e^{i\pi+\pi}-1\right)$$

$$=\frac{1}{2}(1-i)\left(e^{i\pi}e^{\pi}-1\right)=\frac{1}{2}(1-i)\left(-e^{\pi}-1\right)$$

$$=\frac{1}{2}(i-1)\left(e^{\pi}+1\right)=\frac{1}{2}\left(e^{\pi}+1\right)i-\frac{1}{2}\left(e^{\pi}+1\right)$$

$$\Leftrightarrow K+iJ=\frac{1}{2}\left(e^{\pi}+1\right)i-\frac{1}{2}\left(e^{\pi}+1\right)$$

That Have : $K=-\frac{1}{2}\left(e^{\pi}+1\right)$ *And* $J=\frac{1}{2}\left(e^{\pi}+1\right)$

SO, $\boxed{\int_0^{\pi} e^x\sin(x)dx=\frac{1}{2}\left(e^{\pi}+1\right)}$

264 Calculate integral $K=\int_0^1\frac{\eta(x)}{\zeta(x)}dx$

Answer

They give $K=\int_0^1\frac{\eta(x)}{\zeta(x)}dx$

By : $\eta(x)=(1-2^{1-x})\zeta(x)\Rightarrow\frac{\eta(x)}{\zeta(x)}=(1-2^{1-x})$

$$\Rightarrow K=\int_0^1(1-2^{1-x})dx$$

$$=\int_0^1(1-2e^{-x\log 2})dx=1-2\int_0^1 e^{-x\log 2}dx$$

$$=1+\frac{2}{\log(2)}\int_0^1(-x\log 2)'e^{-x\log 2}dx=1-\log_2(e)$$

SO, $\boxed{\int_0^1\frac{\eta(x)}{\zeta(x)}dx=1-\log_2(e)}$

265 Calculate integral $I=\int_{-1}^{\infty}\frac{9x+4}{4x^5+3x^2+x}dx$

Answer

They give $$I=\int_{-1}^{\infty}\frac{9x+4}{4x^5+3x^2+x}dx$$

$$=\int_{-1}^{\infty}\frac{9x+4}{4x^5+3x^2+x}\times\frac{x^{-5}}{x^{-5}}dx$$

$$=\int_{-1}^{\infty}\frac{9x^{-4}+4x^{-5}}{4+3x^{-3}+x^{-4}}dx$$

$$Let: u=4+3x^{-3}+x^{-4}\Rightarrow -du=(9x^{-4}+4x^{-5})dx, If: x\in(-1,\infty)\Rightarrow u\in(2,1)$$

$$\Rightarrow I=-\int_{2}^{1}\frac{1}{u}du=\int_{1}^{2}\frac{1}{u}du=\log(2)$$

$$SO, \boxed{\int_{-1}^{\infty}\frac{9x+4}{4x^5+3x^2+x}dx=\log(2)}$$

266 Calculate integral $J=\int_0^1\log(x)\log(1-x)dx$

Answer

They give $$J=\int_0^1\log(x)\log(1-x)dx$$

$$=-\int_0^1\log(x)\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}dx$$

$$=-\sum_{n=0}^{\infty}\frac{1}{n+1}\underbrace{\int_0^1 x^{n+1}\log(x)dx}_{(Use\ partial\ integral)}$$

$$=-\sum_{n=0}^{\infty}\frac{1}{n+1}\left(\underbrace{\frac{x^{n+2}\log(x)}{n+2}\Big|_0^1}_{0}-\int_0^1\frac{x^{n+2}}{n+2}\times\frac{1}{x}dx\right)$$

$$=\sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)^2}\left(\underbrace{x^{n+2}\Big|_0^1}_{1}\right)$$

$$=\sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)^2}=\sum_{n=1}^{\infty}\frac{1}{(n)(n+1)^2}$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}-\frac{1}{(n+1)^2}\right)=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^2}\right)$$

$$=\lim_{m\to\infty}\sum_{n=1}^{m}\left(\frac{1}{n}-\frac{1}{n+1}\right)-\left(\sum_{n=0}^{\infty}\left(\frac{1}{(n+1)^2}\right)-1\right)$$

$$=\lim_{m\to\infty}\left(1-\frac{1}{m+1}\right)-(\zeta(2)-1)=1-(\zeta(2)-1)=2-\frac{\pi^2}{6}$$

$$SO,\ \boxed{\int_0^1 \log(x)\log(1-x)dx=2-\frac{\pi^2}{6}}$$

267 Calculate integral $J=\int_0^1\frac{\tan^{-1}(x)}{x+1}dx$

Answer

$$\textit{They give}\qquad J=\int_0^1\frac{\tan^{-1}(x)}{x+1}dx$$

$$Let: u=\tan^{-1}(x)\Rightarrow du=\frac{1}{x^2+1}dx, v=\int\frac{1}{x+1}dx=\log(x+1)$$

$$\Rightarrow J=\tan^{-1}(x)\log(x+1)\Big|_0^1-\int_0^1\frac{\log(x+1)}{x^2+1}dx$$

$$=\frac{\pi}{4}\log(2)-\int_0^1\frac{\log(x+1)}{x^2+1}dx\quad(*)$$

$$Take: J'=\int_0^1\frac{\log(x+1)}{x^2+1}dx\quad,\begin{cases}Let: x=\tan(u)\Rightarrow dx=\sec^2(u)du\\ If\ : x\in(0,1)\Rightarrow u\in\left(0,\frac{\pi}{4}\right)\end{cases}$$

$$That:\ J'=\int_0^{\frac{\pi}{4}}\frac{\log\left(\tan(u)+1\right)}{\tan^2(u)+1}\sec^2(u)du=\int_0^{\frac{\pi}{4}}\log\left(\tan(u)+1\right)du$$

$$=\int_0^{\frac{\pi}{4}}\log\left(\tan\left(\frac{\pi}{4}-u\right)+1\right)du=\int_0^{\frac{\pi}{4}}\log\left(\frac{1-\tan(u)}{1+\tan(u)}+1\right)du$$

$$=\int_0^{\frac{\pi}{4}}\log\left(2\right)du-\int_0^{\frac{\pi}{4}}\log\left(\tan(u)+1\right)du$$

$$\Leftrightarrow\ J'=\frac{\pi}{4}\log(2)-J'\Rightarrow\ J'=\frac{\pi}{8}\log(2)$$

$$Take: (*) They\ Have: J=\frac{\pi}{4}\log(2)-\frac{\pi}{8}\log(2)=\frac{\pi}{8}\log(2)$$

$$SO,\ \boxed{\int_0^1\frac{\tan^{-1}(x)}{x+1}dx=\frac{\pi}{8}\log(2)}$$

268 Calculate integral $K=\int_0^{\pi}\frac{x+\sin(x)-\cos(x)-1}{e^x+\sin(x)+x}dx$

Answer

They give $$K=\int_0^{\pi}\frac{x+\sin(x)-\cos(x)-1}{e^x+\sin(x)+x}dx$$

$Let: f(x)=e^x+\sin(x)+x \Rightarrow f'(x)=e^x+\cos(x)+1$

$If: x\to 0\Rightarrow f(0)=1 , If: x\to\pi\Rightarrow f(\pi)=e^{\pi}+\pi$

$That: f(x)-f'(x)=x+\sin(x)-\cos(x)-1$

$$\Rightarrow K=\int_0^{\pi}\frac{f(x)-f'(x)}{f(x)}dx=\left(x-\log|f(x)|\right)\Big|_0^{\pi}$$

$$=\pi-\log\left|\frac{e^{\pi}+\pi}{1}\right|=\pi-\log\left(e^{\pi}+\pi\right)$$

$SO,$ $$\boxed{\int_0^{\pi}\frac{x+\sin(x)-\cos(x)-1}{e^x+\sin(x)+x}dx=\pi-\log\left(e^{\pi}+\pi\right)}$$

269 Calculate integral $I=\int_0^{\pi}\frac{\sin(x)}{\sin(x)+\cos(x)}dx$

Answer

They give $$I=\int_0^{\pi}\frac{\sin(x)}{\sin(x)+\cos(x)}dx$$

$Let: f(x)=\sin(x)+\cos(x)\Rightarrow f'(x)=\cos(x)-\sin(x)$

$If: x\to 0\Rightarrow f(0)=1 , If: x\to\pi\Rightarrow f(\pi)=-1$

$That: \frac{1}{2}\left(f(x)-f'(x)\right)=\sin(x)$

$$\Rightarrow I=\frac{1}{2}\int_0^{\pi}\frac{f(x)-f'(x)}{f(x)}dx$$

$$=\frac{1}{2}\left(x-\log|f(x)|\right)\Big|_0^{\pi}$$

$$=\frac{1}{2}\left(\pi-\log\left|\frac{-1}{1}\right|\right)=\frac{\pi}{2}$$

$SO,$ $$\boxed{\int_0^{\pi}\frac{\sin(x)}{\sin(x)+\cos(x)}dx=\frac{\pi}{2}}$$

270 Calculate integral $J=\int_0^{\log(2)}\sqrt{\frac{e^x-1}{e^x+1}}\,dx$

Answer

They give $$J=\int_0^{\log(2)}\sqrt{\frac{e^x-1}{e^x+1}}\,dx$$

Let $: e^x=\sec(2u)\Leftrightarrow x=\log(\sec(2u))\Rightarrow dx=-2\tan(2u)du$, *If* $: x\in\left(0,\log(2)\right)\Rightarrow u\in\left(0,\frac{\pi}{6}\right)$

$$\Rightarrow J=-2\int_0^{\frac{\pi}{6}}\sqrt{\frac{\sec(2u)-1}{\sec(2u)+1}}\tan(2u)du$$

$$=-2\int_0^{\frac{\pi}{6}}\sqrt{\frac{1-\cos(2u)}{1+\cos(2u)}}\tan(2u)du=-2\int_0^{\frac{\pi}{6}}\sqrt{\frac{2\sin^2(u)}{2\cos^2(u)}}\tan(2u)du$$

$$=-2\int_0^{\frac{\pi}{6}}\sqrt{\tan^2(u)}\tan(2u)du=-2\int_0^{\frac{\pi}{3}}\left|\tan(u)\right|\tan(2u)du$$

$$=-2\int_0^{\frac{\pi}{3}}\tan(u)\tan(2u)du=-4\int_0^{\frac{\pi}{3}}\frac{\sin^2(u)}{\cos^2(u)-\sin^2(u)}du$$

$$=-4\int_0^{\frac{\pi}{3}}\frac{\tan^2(u)}{\left(1-\tan^2(u)\right)\left(1+\tan^2(u)\right)}\left(1+\tan^2(u)\right)du$$

$$=4\int_0^{\frac{\pi}{3}}\frac{\tan^2(u)}{\left(\tan^2(u)-1\right)\left(1+\tan^2(u)\right)}d\left(\tan(u)\right)$$

$$=2\int_0^{\frac{\pi}{3}}\left(\frac{1}{\left(\tan^2(u)-1\right)}+\frac{1}{\left(1+\tan^2(u)\right)}\right)d\left(\tan(u)\right)$$

$$=2\int_0^{\frac{\pi}{3}}\left(\frac{1}{\left(\tan^2(u)-1\right)}\right)d\left(\tan(u)\right)+2\int_0^{\frac{\pi}{3}}\frac{1}{\left(1+\tan^2(u)\right)}d\left(\tan(u)\right)$$

$$=\log\left|\frac{\tan(u)-1}{\tan(u)+1}\right|\Bigg|_0^{\frac{\pi}{3}}+2\tan^{-1}\left(\tan(u)\right)\Big|_0^{\frac{\pi}{3}}$$

$$=\log\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right|-\log\left|\frac{0-1}{0+1}\right|+2\left(\frac{\pi}{3}-0\right)=\log\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{2\pi}{3}$$

SO, $$\boxed{\int_0^{\log(2)}\sqrt{\frac{e^x-1}{e^x+1}}\,dx=\frac{2\pi}{3}+\log\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$$

271 Calculate integral $K=\int_0^{\infty}\frac{x^n}{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots}dx$

Answer

They give $K=\int_0^{\infty}\frac{x^n}{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots}dx$

$By: e^x = 1+x++\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

$$\Rightarrow K=\int_0^{\infty}\frac{x^n}{e^x}dx$$

$$=\int_0^{\infty}x^n e^{-x}dx=\Gamma(n+1)=n!$$

SO, $\boxed{\int_0^{\infty}\frac{x^n}{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}}dx=n!}$

272 Calculate integral $I=\int_0^{\infty}\frac{x^n}{x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots}dx$

Answer

They give $I=\int_0^{\infty}\frac{x^n}{x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots}dx$

$By: e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots \Leftrightarrow e^x-1=x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

$$\Rightarrow I=\int_0^{\infty}\frac{x^n}{e^x-1}dx$$

$$=\int_0^{\infty}\frac{x^{(n+1)-1}}{e^x-1}dx$$

$$=\Gamma(n+1)\zeta(n+1)$$

SO, $\boxed{\int_0^{\infty}\frac{x^n}{x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots}dx=\Gamma(n+1)\zeta(n+1)}$

273 Calculate integral $J=\int_0^{\infty}\frac{\log(x)}{x^2+y^2}dx$

Answer

They give $$J=\int_0^{\infty}\frac{\log(x)}{x^2+y^2}dx$$

$Let: x=yz\Rightarrow dx=ydz, If: x\in(0,\infty)\Rightarrow z\in(0,\infty)$

$$\begin{aligned}
\Rightarrow J&=\int_0^{\infty}\frac{\log(yz)}{y^2z^2+y^2}ydz\\
&=\frac{1}{y}\int_0^{\infty}\frac{\log(y)+\log(z)}{z^2+1}dz\\
&=\frac{1}{y}\int_0^{\infty}\frac{\log(y)}{z^2+1}dz+\frac{1}{y}\underbrace{\int_0^{\infty}\frac{\log(z)}{z^2+1}dz}_{J'}\\
&=\frac{\log(y)}{y}\tan^{-1}(x)\Big|_0^{\infty}+\frac{1}{y}J'\\
&=\frac{\pi}{2y}\log(y)+\frac{1}{y}J'
\end{aligned}$$

$$\begin{aligned}
For:\ J'&=\int_0^{\infty}\frac{\log(z)}{z^2+1}dz\\
&=\int_0^{1}\frac{\log(z)}{z^2+1}dz+\underbrace{\int_1^{\infty}\frac{\log(z)}{z^2+1}dz}_{J''}
\end{aligned}$$

$$For:\ J''=\int_1^{\infty}\frac{\log(z)}{z^2+1}dz\quad,\begin{cases}Let: z=\frac{1}{t}\Rightarrow dz=-\frac{1}{t^2}dt\\ If: z\in(1,\infty)\Rightarrow z\in(1,\infty)\end{cases}$$

$$That: J''=-\int_1^{\infty}\frac{\log(t^{-1})}{t^{-2}+1}\times\frac{1}{t^2}dt=-\int_1^{\infty}\frac{\log(t)}{t^2+1}dt$$

$$They\ Have: J'=\int_0^{1}\frac{\log(z)}{z^2+1}dz-\int_1^{\infty}\frac{\log(t)}{t^2+1}dt=0$$

$$That: J=\frac{\pi}{2y}\log(y)+\frac{1}{y}\times 0=\frac{\pi}{2y}\log(y)$$

$$SO,\quad\boxed{\int_0^{\infty}\frac{\log(x)}{x^2+y^2}dx=\frac{\pi}{2y}\log(y)}$$

274 Calculate integral $K=\int_0^1 \frac{\log(x)}{\sqrt{1-x^2}}dx$

Answer

They give $$K=\int_0^1 \frac{\log(x)}{\sqrt{1-x^2}}dx$$

$$Let: x=\sin(u) \Rightarrow dx=\cos(u)du \, , If\ : x\in(0,1) \Rightarrow z\in(0,\frac{\pi}{2})$$

$$\Rightarrow K=\int_0^{\frac{\pi}{2}} \frac{\log(\sin(u))}{\sqrt{1-\sin^2(u)}}\times\cos(u)du$$

$$=\int_0^{\frac{\pi}{2}} \frac{\log(\sin(u))}{|\cos(u)|}\times\cos(u)du$$

$$=\int_0^{\frac{\pi}{2}} \frac{\log(\sin(u))}{\cos(u)}\times\cos(u)du=-\frac{\pi}{2}\log(2)$$

SO, $\boxed{\int_0^1 \frac{\log(x)}{\sqrt{1-x^2}}dx=-\frac{\pi}{2}\log(2)}$

275 Calculate integral $I=\int_0^1 \frac{\log(x)}{1+x}dx$

Answer

They give $$I=\int_0^1 \frac{\log(x)}{1+x}dx$$

$$=\int_0^1 \log(x)\sum_{n=0}^{\infty}(-1)^n x^n dx=\sum_{n=0}^{\infty}(-1)^n \underbrace{\int_0^1 x^n\log(x)dx}_{(Use\ partial\ integral)}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left(\underbrace{x^{n+1}\log(x)\Big|_0^1}_{0}-\int_0^1 x^{n+1}\times\frac{1}{x}dx\right)$$

$$=-\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left(\int_0^1 x^n dx\right)=-\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^2}$$

$$=-\eta(2)=-\frac{\pi^2}{12}$$

SO, $\boxed{\int_0^1 \frac{\log(x)}{1+x}dx=-\frac{\pi^2}{12}}$

276 Calculate integral $J=\int_0^1 \frac{\log(x)}{1-x}dx$

Answer

They give
$$J=\int_0^1 \frac{\log(x)}{1-x}dx$$
$$=\int_0^1 \log(x)\sum_{n=0}^{\infty} x^n dx$$
$$=\sum_{n=0}^{\infty} \underbrace{\int_0^1 x^n \log(x)dx}_{(\textit{Use partial integral})}$$
$$=\sum_{n=0}^{\infty}\frac{1}{n+1}\left(\underbrace{x^{n+1}\log(x)\Big|_0^1}_{0}-\int_0^1 x^{n+1}\times\frac{1}{x}dx\right)$$
$$=-\sum_{n=0}^{\infty}\frac{1}{n+1}\left(\int_0^1 x^n dx\right)=-\sum_{n=0}^{\infty}\frac{1}{(n+1)^2}=-\zeta(2)=-\frac{\pi^2}{6}$$

SO, $\boxed{\int_0^1 \frac{\log(x)}{1+x}dx=-\frac{\pi^2}{12}}$

277 Calculate integral $K=\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx$

Answer

They give
$$K=\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx$$
$$=\int_0^{\infty}\log\left(\frac{1+e^{-x}}{1-e^{-x}}\right)dx=\int_0^{\infty}\left[\log\left(1+e^{-x}\right)-\log\left(1-e^{-x}\right)\right]dx$$
$$=\int_0^{\infty}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}e^{-nx}}{n}\right)dx-\int_0^{\infty}\left(-\sum_{n=1}^{\infty}\frac{e^{-nx}}{n}\right)dx$$
$$=\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n}\right)\int_0^{\infty}e^{-nx}dx+\left(\sum_{n=1}^{\infty}\frac{1}{n}\right)\int_0^{\infty}e^{-nx}dx$$
$$=\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n^2}\right)+\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)=\eta(2)+\zeta(2)=\frac{\pi^2}{4}$$

SO, $\boxed{\int_0^{\infty}\log\left(\frac{e^x+1}{e^x-1}\right)dx=\frac{\pi^2}{4}}$

278 Calculate integral $I=\int_0^{\infty} x^2e^{-x}\cos(x)dx$

Answer

They give $I=\int_0^{\infty} x^2e^{-x}\cos(x)dx$

Take $J=\int_0^{\infty} x^2e^{-x}\sin(-x)dx$

That $I+iJ=\int_0^{\infty} x^2e^{-x}e^{-xi}dx=\int_0^{\infty} x^2e^{-(i+1)x}dx$

$$Let: t=(i+1)x \Rightarrow dx=\frac{1}{i+1}dt, If: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow I+iJ=\int_0^{\infty}\left(\frac{t}{i+1}\right)^2 e^{-t}\frac{1}{i+1}dt=\frac{1}{(i+1)^3}\int_0^{\infty} t^2e^{-t}dx=\frac{\Gamma(3)}{(2i)(i+1)}=-\frac{1}{2}-\frac{1}{2}i$$

$$\Leftrightarrow I+iJ=-\frac{1}{2}-\frac{1}{2}i \quad ,That: I=-\frac{1}{2}$$

$SO,$ $\boxed{\int_0^{\infty} x^2e^{-x}\cos(x)dx=-\frac{1}{2}}$

279 Calculate integral $J=\int_0^{\infty}(-1)^{ix^2}dx\ ,i=\sqrt{-1}$

Answer

They give $J=\int_0^{\infty}(-1)^{ix^2}dx\ ,i=\sqrt{-1}$

$By: -1=\cos(\pi)+i\sin(\pi)=e^{\pi i}$

That: $J=\int_0^{\infty}(e^{\pi i})^{ix^2}dx=\int_0^{\infty} e^{-\pi x^2}dx$

$$Let: t=\pi x^2 \Leftrightarrow x=\frac{1}{\sqrt{\pi}}\sqrt{t}\Rightarrow dx=\frac{t^{-\frac{1}{2}}}{2\sqrt{\pi}}dt, If: x\in(0,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow J=\int_0^{\infty} e^{-t}\times\frac{t^{-\frac{1}{2}}}{2\sqrt{\pi}}dt=\frac{1}{2\sqrt{\pi}}\int_0^{\infty} t^{-\frac{1}{2}}e^{-t}dt$$

$$=\frac{1}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)=\frac{1}{2} \quad ,Note:\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$SO,$ $\boxed{\int_0^{\infty}(-1)^{ix^2}dx=\frac{1}{2}}$

280 Calculate integral $K=\int_0^{\frac{\pi}{2}}\frac{x\sin(x)}{\cos(2x)+1}dx$

Answer

They give $$K=\int_0^{\pi}\frac{x\sin(x)}{\cos(2x)+1}dx$$

$$=\frac{1}{2}\int_0^{\pi}\frac{x\sin(x)}{\cos^2(x)}dx$$

$$Let: u=x\Rightarrow du=dx\ And: v=\int\frac{\sin(x)}{\cos^2(x)}dx=\frac{1}{\cos(x)}=\sec(x)$$

$$\Rightarrow K=\frac{1}{2}\times\frac{x}{\cos(x)}\Bigg|_0^{\pi}-\frac{1}{2}\int_0^{\pi}\sec(x)dx$$

$$=-\frac{\pi}{2}-\underbrace{\log\left|\sec(x)+\tan(x)\right|\Big|_0^{\pi}}_{0}=-\frac{\pi}{2}$$

SO, $\boxed{\int_0^{\pi}\frac{x\sin(x)}{\cos(2x)+1}dx=-\frac{\pi}{2}}$

281 Calculate integral $K'=\int_{-1}^{1}\frac{1}{8^x+1}dx$

Answer

They give $$K'=\int_{-1}^{1}\frac{1}{8^x+1}dx$$

$$=\int_{-1}^{1}\frac{8^x+1-8^x}{8^x+1}dx=\int_{-1}^{1}\left(1-\frac{8^x}{8^x+1}\right)dx$$

$$=2-\frac{1}{\log(8)}\int_{-1}^{1}\left(\frac{\left(8^x+1\right)'}{8^x+1}\right)dx$$

$$=2-\frac{1}{\log(8)}\log\left(8^x+1\right)\Big|_{-1}^{1}$$

$$=2-\frac{1}{\log(8)}\log\left(\frac{8+1}{8^{-1}+1}\right)$$

$$=2-\frac{1}{\log(8)}\log\left(8\right)=1$$

SO, $\boxed{\int_{-1}^{1}\frac{1}{8^x+1}dx=1}$

282 Calculate integral $I=\int_1^{\infty}\frac{\log^3(x)}{x^2(x-1)}dx$

Answer

They give $$I=\int_1^{\infty}\frac{\log^3(x)}{x^2(x-1)}dx$$

$$Let: t=\log(x)\Leftrightarrow x=e^t\Rightarrow dx=e^t dt\,, If: x\in(1,\infty)\Rightarrow t\in(0,\infty)$$

$$\Rightarrow I=\int_0^{\infty}\frac{t^3}{e^{2t}(e^t-1)}\times e^t dt=\int_0^{\infty}\frac{t^3e^{-2t}}{(1-e^{-t})}dt$$

$$=\int_0^{\infty}\left(t^3e^{-2t}\sum_{n=0}^{\infty}e^{-nt}\right)dt=\sum_{n=0}^{\infty}\left(\int_0^{\infty}t^3e^{-(n+2)t}dt\right)$$

$$Let: y=(n+2)t\Leftrightarrow t=\frac{y}{(n+2)}\Rightarrow dt=\frac{1}{(n+2)}dy\,, If: t\in(0,\infty)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow I=\sum_{n=0}^{\infty}\left(\int_0^{\infty}\left(\frac{y}{n+2}\right)^3e^{-y}\times\frac{1}{(n+2)}dy\right)=\sum_{n=0}^{\infty}\left(\frac{1}{(n+2)^4}\int_0^{\infty}(y)^3e^{-y}dy\right)$$

$$=\sum_{n=0}^{\infty}\left(\frac{\Gamma(4)}{(n+2)^4}\right)=6\left[\sum_{n=0}^{\infty}\left(\frac{1}{(n+1)^4}\right)-1\right]=6\left(\frac{\pi^4}{40}-1\right)$$

SO, $$\boxed{\int_1^{\infty}\frac{\log^3(x)}{x^2(x-1)}dx=6\left(\frac{\pi^4}{40}-1\right)}$$

283 Calculate integral $J=\int_{-\pi}^{\pi}\frac{x(\sin(x)+1)}{\cos^2(x)+1}dx$

Answer

They give $$J=\int_{-\pi}^{\pi}\frac{x\left(\sin(x)+1\right)}{\cos^2(x)+1}dx$$

$$=\underbrace{\int_{-\pi}^{\pi}\frac{x\sin(x)}{\cos^2(x)+1}dx}_{is\,an\,even\,\,function}+\underbrace{\int_{-\pi}^{\pi}\frac{x}{\cos^2(x)+1}dx}_{is\,an\,odd\,\,function}$$

$$=2\int_0^{\pi}\frac{x\sin(x)}{\cos^2(x)+1}dx=2\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{\cos^2(\pi-x)+1}dx$$

$$=2\int_0^{\pi}\frac{\pi\sin(x)}{\cos^2(x)+1}dx-2\int_0^{\pi}\frac{x\sin(x)}{\cos^2(x)+1}dx$$

$$\Leftrightarrow J=-\pi\int_0^{\pi}\frac{1}{\cos^2(x)+1}d\left(\cos(x)\right)=-\pi\left(-\frac{\pi}{2}\right)=\frac{\pi^2}{2}$$

SO, $$\boxed{\int_{-\pi}^{\pi}\frac{2x\left(\sin(x)+1\right)}{\cos^2(x)+1}dx=\frac{\pi^2}{2}}$$

284 Calculate integral $K=\int_0^{\pi} x\sin^6(x)dx$

Answer

They give $$K=\int_0^{\pi} x\sin^6(x)dx$$

$$=\int_0^{\pi}(\pi-x)\sin^6(\pi-x)dx=\int_0^{\pi}\pi\sin^6(x)dx-\int_0^{\pi}x\sin^6(x)dx$$

$$\Rightarrow 2K=\pi\int_0^{\pi}\sin^6(x)dx$$

$$\Rightarrow K=\frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}}\sin^6(x)dx \quad ,\left(Take:\begin{cases}\int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx\\ f(2a-x)=f(x)\end{cases}\right)$$

$$=\pi\left(\frac{\pi}{2}\times\frac{5\times3\times1}{6\times4\times2}\right)=\frac{5\pi^2}{32}$$

SO, $\boxed{\int_0^{\pi} x\sin^6(x)dx=\frac{5\pi^2}{32}}$

285 Calculate integral $I=\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx$

Answer

They give $$I=\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx$$

$$=\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx+\underbrace{\int_1^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx}_{I'}$$

$$For: I'=\int_1^{\infty}\left(\frac{\log(x)}{1+x}\right)^2 dx \quad ,\begin{cases}Let: t=\frac{1}{x}\Rightarrow dx=-\frac{1}{t^2}dt\\ If: x\in(1,\infty)\Rightarrow t\in(1,0)\end{cases}$$

$$That: I'=-\int_1^{0}\left(\frac{\log(t^{-1})}{1+t^{-1}}\right)^2\times\frac{1}{t^2}dx=\int_0^{1}\left(\frac{\log(t)}{1+t}\right)^2 dx=\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx$$

$$\Rightarrow I=\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx+\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx$$

$$=2\int_0^{1}\left(\frac{\log(x)}{1+x}\right)^2 dx$$

$$Let: t=-\log(x)\Leftrightarrow x=e^{-t}\Rightarrow dx=-e^{-t}dt, If: x\in(0,1)\Rightarrow t\in(\infty,0)$$

$$\Rightarrow I=-2\int_{\infty}^{0}\left(\frac{-t}{1+e^{-t}}\right)^2 e^{-t}dt=2\int_0^{\infty}t^2\frac{e^{-t}}{(1+e^{-t})^2}dt$$

$$=2\int_0^{\infty}t^2\sum_{n=1}^{\infty}(-1)^{n-1}ne^{-nt}dt=2\sum_{n=1}^{\infty}\left((-1)^{n-1}n\int_0^{\infty}t^2e^{-nt}dt\right)$$

$$Let: y=nt\Leftrightarrow t=\frac{y}{n}\Rightarrow dt=\frac{dy}{n}, If: t\in(0,\infty)\Rightarrow y\in(0,\infty)$$

$$\Rightarrow I=2\sum_{n=1}^{\infty}\left((-1)^{n-1}n\int_0^{\infty}\left(\frac{y}{n}\right)^2e^{-y}\frac{dy}{n}\right)=2\sum_{n=1}^{\infty}\left(\frac{(-1)^{n-1}}{n^2}\int_0^{\infty}(y)^2e^{-y}dy\right)$$

$$=2\sum_{n=1}^{\infty}\left(\frac{(-1)^{n-1}}{n^2}\times 2!\right)=4\eta(2)=\frac{\pi^2}{3}$$

$$SO,\quad \boxed{\int_0^{\infty}\left(\frac{\log(x)}{1+x}\right)^2dx=\frac{\pi^2}{3}}$$

286 Calculate integral $J=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[(x)!+(x-1)!\right](-x)!dx$

Answer

$$They\ give\quad J=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[(x)!+(x-1)!\right](-x)!dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\Gamma(x+1)+\Gamma(x)\right]\Gamma(1-x)dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\Gamma(x+1)\Gamma(1-x)+\Gamma(x)\Gamma(1-x)\right)dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x\Gamma(x)\Gamma(1-x)+\Gamma(x)\Gamma(1-x)\right)dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x+1)\Gamma(x)\Gamma(1-x)dx$$

$$=\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{(x+1)}{\sin(x)}dx=\underbrace{\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x}{\sin(x)}dx}_{is\ an\ even\ function}+\underbrace{\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{\sin(x)}dx}_{is\ an\ odd\ function}$$

$$=2\pi\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)}dx+0=2\pi\int_0^{\frac{\pi}{2}}\frac{x}{\sin(x)}dx$$

$$Let: u=x\Rightarrow du=dx, v=\int\frac{1}{\sin(x)}dx=\log\left(\frac{1-\cos(x)}{\sin(x)}\right)=\log\left(\tan\left(\frac{x}{2}\right)\right)$$

$$\Rightarrow J = \underbrace{2\pi x \log\left(\frac{1-\cos(x)}{\sin(x)}\right)\Bigg|_0^{\frac{\pi}{2}}}_{0} - 2\pi\int_0^{\frac{\pi}{2}} \log\left(\frac{1-\cos(x)}{\sin(x)}\right)dx$$

$$= -2\pi\int_0^{\frac{\pi}{2}} \log\left(\tan\left(\frac{x}{2}\right)\right)dx$$

$$Let: u = \frac{x}{2} \Rightarrow dx = 2du, If: x \in \left(0, \frac{\pi}{2}\right) \Rightarrow u \in \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow J = -4\pi\int_0^{\frac{\pi}{4}} \log(\tan(u))dx = 4\pi\left(-\int_0^{\frac{\pi}{4}} \log(\tan(u))dx\right)$$

$$= 4\pi G \quad ,(Note: Where\ "G"\ is\ Catalan's\ constant)$$

$$SO, \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [(x)! + (x-1)!](-x)!dx = 4\pi G}$$

287 Calculate integral $K = \int_0^{\infty} \frac{\log(x)}{x^2+2x+4}dx$

Answer

They give $\qquad K = \int_0^{\infty} \frac{\log(x)}{x^2+2x+4}dx$

$$Let: x = \frac{4}{u} \Rightarrow dx = -\frac{4}{u^2}du, If: x \in (0,\infty) \Rightarrow u \in (\infty, 0)$$

$$\Rightarrow K = -\int_{\infty}^{0} \frac{\log(4/u)}{\left(\frac{4}{u}\right)^2 + 2\left(\frac{4}{u}\right) + 4} \times \frac{4}{u^2}du$$

$$= \int_0^{\infty} \frac{\log(4) - \log(u)}{u^2+2u+4}du$$

$$= \int_0^{\infty} \frac{\log(4)}{u^2+2u+4}du - \int_0^{\infty} \frac{\log(u)}{u^2+2u+4}du$$

$$\Rightarrow K = \frac{\log(4)}{2}\int_0^{\infty} \frac{1}{u^2+2u+4}du$$

$$= \log(2)\int_0^{\infty} \frac{1}{(u+1)^2+3}du = \frac{\pi\log(2)}{3\sqrt{3}}$$

$$SO, \boxed{\int_0^{\infty} \frac{\log(x)}{x^2+2x+4}dx = \frac{\pi\log(2)}{3\sqrt{3}}}$$

288 Calculate integral $I=\int_0^{\infty} e^{-x^2}dx$

Answer

They give $I=\int_0^{\infty} e^{-x^2}dx$

$$=\int_0^{\infty} x^{-1}e^{-x^2}xdx$$

$$=\frac{1}{2}\int_0^{\infty}(x^2)^{-\frac{1}{2}}e^{-x^2}2xdx$$

$Let: u=x^2 \Rightarrow du=d(x^2), If: x\in(0,\infty)\Rightarrow u\in(0,\infty)$

$$=\frac{1}{2}\int_0^{\infty}(u)^{-\frac{1}{2}}e^{-u}du=\frac{1}{2}\int_0^{\infty}(u)^{\frac{1}{2}-1}e^{-u}du$$

$$=\frac{1}{2}\Gamma\left(\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2}$$

SO, $\boxed{\int_0^{\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}}$

289 Calculate integral $J=\int_0^{\infty}\frac{\log(x+x^{-1})}{x^2+1}dx$

Answer

They give $J=\int_0^{\infty}\frac{\log(x+x^{-1})}{x^2+1}dx$

$$=\int_0^{\infty}\frac{\log(x^2+1)-\log(x)}{x^2+1}dx=\int_0^{\infty}\frac{\log(x^2+1)}{x^2+1}dx-\int_0^{\infty}\frac{\log(x)}{x^2+1}dx$$

$Let: x=\tan(u)\Rightarrow dx=\sec^2(u)du, If: x\in(0,\infty)\Rightarrow u\in(0,\frac{\pi}{2})$

$$\Rightarrow J=\int_0^{\frac{\pi}{2}}\frac{\log(\tan^2(u)+1)}{\tan^2(u)+1}\times\sec^2(u)du-\int_0^{\frac{\pi}{2}}\frac{\log(\tan(u))}{\tan^2(u)+1}\times\sec^2(u)du$$

$$=\int_0^{\frac{\pi}{2}}\log(\tan^2(u)+1)du-\int_0^{\frac{\pi}{2}}\log(\tan(u))du$$

$$=\int_0^{\frac{\pi}{2}}\log(\sec^2(u))du-\int_0^{\frac{\pi}{2}}\log\left(\frac{\sin(u)}{\cos(u)}\right)du$$

$$=2\int_0^{\frac{\pi}{2}}\log(\cos^{-1}(u))du-\int_0^{\frac{\pi}{2}}\log\left(\frac{\sin(u)}{\cos(u)}\right)du$$

$$= -2\int_0^{\frac{\pi}{2}} \log(\cos(u))du - \int_0^{\frac{\pi}{2}} \log(\sin(u))du + \int_0^{\frac{\pi}{2}} \log(\cos(u))du$$

$$= -\int_0^{\frac{\pi}{2}} \log(\cos(u))du - \int_0^{\frac{\pi}{2}} \log\left(\sin(\frac{\pi}{2} - u)\right)du$$

$$= -\int_0^{\frac{\pi}{2}} \log(\cos(u))du - \int_0^{\frac{\pi}{2}} \log(\cos(u))du$$

$$= -2\int_0^{\frac{\pi}{2}} \log(\cos(u))du = \pi\log(2)$$

$$SO, \boxed{\int_0^{\infty} \frac{\log(x+x^{-1})}{x^2+1}dx = \pi\log(2)}$$

290 Calculate integral $K = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan^4(x)+1}}dx$

Answer

They give $\quad K = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan^4(x)+1}}dx \quad (*)$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan^4\left(\frac{\pi}{2} - x\right)+1}}dx = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\frac{1}{\tan^4(x)}+1}}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\tan^2(x)}{\sqrt{\tan^4(x)+1}}dx \quad (**)$$

$$Take: (*)+(**) They\ have: 2K = \int_0^{\frac{\pi}{2}} \frac{\tan^2(x)+1}{\sqrt{\tan^4(x)+1}}dx$$

$$\Rightarrow K = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan^4(x)+1}}d\left(\tan(x)\right)$$

$$Let: y = \tan^4(x)+1 \Rightarrow d\left(\tan(x)\right) = \frac{1}{4}(y-1)^{\frac{1}{4}-1}dy, If: x \in (0,\frac{\pi}{2}) \Rightarrow y \in (1,\infty)$$

$$\Rightarrow K = \frac{1}{2\times 4}\int_1^{\infty} \frac{(y-1)^{\frac{1}{4}-1}}{\sqrt{y}}dy = \frac{1}{2\times 4}\int_1^{\infty} y^{-\frac{1}{2}}(y-1)^{\frac{1}{4}-1}dy$$

$$Let: y = \frac{1}{u} \Rightarrow dy = -\frac{1}{u^2}du, If: y \in (1,\infty) \Rightarrow u \in (1,0)$$

$$\Rightarrow K = -\frac{1}{2\times 4}\int_1^0 \left(u^{-1}\right)^{-\frac{1}{2}}(u^{-1}-1)^{\frac{1}{4}-1}dy$$

$$= \frac{1}{8}\int_0^1 u^{\frac{1}{4}-1}(1-u)^{\frac{1}{4}-1}du$$

$$= \frac{1}{8}B\left(\frac{1}{4},\frac{1}{4}\right)$$

$$= \frac{1}{8}\times\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{2}\right)} = \frac{\Gamma^2\left(\frac{1}{4}\right)}{8\sqrt{\pi}}$$

SO, $$\boxed{\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{\tan^4(x)+1}}dx = \frac{\Gamma^2\left(\frac{1}{4}\right)}{8\sqrt{\pi}}}$$

291 Calculate integral $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\sin(x)\sin(2x)\right)^2 dx$

Answer

They give $$I = \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\sin(x)\sin(2x)\right)^2 dx}_{\text{is an even function}}$$

$$= 2\int_0^{\frac{\pi}{2}} 4\cos^2(x)\sin^4(x)dx$$

$$= 8\int_0^{\frac{\pi}{2}}\left(1-\sin^2(x)\right)\sin^4(x)dx$$

$$= 8\int_0^{\frac{\pi}{2}}\left(\sin^4(x)-\sin^6(x)\right)dx$$

$$= 8\left[\int_0^{\frac{\pi}{2}}\sin^4(x)dx - \int_0^{\frac{\pi}{2}}\sin^6(x)dx\right]$$

$$= 8\times\frac{\pi}{2}\left(\frac{3\times 1}{4\times 2}-\frac{5\times 3\times 1}{6\times 4\times 2}\right) = \frac{\pi}{4}$$

SO, $$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\sin(x)\sin(2x)\right)^2 dx = \frac{\pi}{4}}$$

292 Calculate integral $J=\int_0^{\infty}\frac{1}{(x^9+1)(x^2+1)}dx$

Answer

They give $$J=\int_0^{\infty}\frac{1}{(x^9+1)(x^2+1)}dx \quad (*)$$

$$Let: x=\frac{1}{u}=u^{-1}\Rightarrow dx=-\frac{1}{u^2}du, If: x\in(0,\infty)\Rightarrow u\in(\infty,0)$$

$$\Rightarrow J=-\int_{\infty}^{0}\frac{1}{(u^{-9}+1)(u^{-2}+1)}\times\frac{1}{u^2}du$$

$$=\int_0^{\infty}\frac{u^9}{(u^9+1)(u^2+1)}du \quad (**)$$

$$Take: (*)+(**) They\ have: 2J=\int_0^{\infty}\frac{u^9+1}{(u^9+1)(u^2+1)}du=\tan^{-1}(u)\Big|_0^{\infty}=\frac{\pi}{2}$$

$$\Rightarrow J=\frac{\pi}{4}$$

SO, $\boxed{\int_0^{\infty}\frac{1}{(x^9+1)(x^2+1)}dx=\frac{\pi}{4}}$

293 Calculate integral $K=\int_0^{\infty}2^{-3x^2}dx$

Answer

They give $$K=\int_0^{\infty}2^{-3x^2}dx$$

$$=\int_0^{\infty}e^{-3x^2\log(2)}dx$$

$$Let: u=3x^2\log(2)\Rightarrow dx=\frac{1}{2\sqrt{3\log(2)}}u^{\frac{1}{2}-1}du, If: x\in(0,\infty)\Rightarrow u\in(0,\infty)$$

$$\Rightarrow K=\frac{1}{2\sqrt{3\log(2)}}\int_0^{\infty}u^{\frac{1}{2}-1}e^{-u}du$$

$$=\frac{\Gamma(1/2)}{2\sqrt{3\log(2)}}=\frac{\sqrt{\pi}}{2\sqrt{3\log(2)}}$$

SO, $\boxed{\int_0^{\infty}2^{-3x^2}dx=\frac{\sqrt{\pi}}{2\sqrt{3\log(2)}}}$

294 Calculate integral $I=\int_0^{2\pi}\frac{1}{2+\cos(x)}dx$

Answer

They give

$$I=\int_0^{2\pi}\frac{1}{2+\cos(x)}dx$$

$$=\int_0^{2\pi}\frac{1}{1+2\cos^2(x/2)}dx=2\int_0^{2\pi}\frac{1}{1+2\cos^2(x/2)}d(x/2)$$

$$=2\int_0^{\pi}\frac{1}{1+2\cos^2(x)}dx=4\int_0^{\frac{\pi}{2}}\frac{1}{\sin^2(x)+3\cos^2(x)}d(x)$$

$$=4\int_0^{\frac{\pi}{2}}\frac{1}{\tan^2(x)+3}\times\frac{1}{\cos^2(x)}d(x)=4\int_0^{\frac{\pi}{2}}\frac{1}{\tan^2(x)+3}d(\tan(x))$$

$$=\frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)\Bigg|_0^{\frac{\pi}{2}}=\frac{4}{\sqrt{3}}\times\frac{\pi}{2}=\frac{2\pi}{\sqrt{3}}$$

SO, $\boxed{\int_0^{2\pi}\frac{1}{2+\cos(x)}dx=\frac{2\pi}{\sqrt{3}}}$

295 Calculate integral $J=\int_0^1\log\left(1-x^4\right)dx$

Answer

They give

$$J=\int_0^1\log\left(1-x^4\right)dx$$

$$=\int_0^1\log\left((1+x^2)(1-x^2)\right)dx$$

$$=\int_0^1\log\left((1+x^2)(1-x)(1+x)\right)dx$$

$$=\int_0^1\log\left(1+x\right)dx+\int_0^1\log\left(1-x\right)dx+\int_0^1\log\left(1+x^2\right)dx$$

$$=J_1+J_2+J_3\quad(*)$$

$$For:\ J_1=\int_0^1\log\left(1+x\right)dx\ ,\begin{cases}Let: x+1=t\Rightarrow dx=dt\\ If\ : x\in(0,1)\Rightarrow t\in(1,2)\end{cases}$$

$$That: J_1=\int_1^2\log(t)dt=t\left(\log(t)-1\right)\Big|_1^2=\log(4)-1$$

$$For:\ J_2=\int_0^1\log\left(1-x\right)dx\ ,\begin{cases}Let: 1-x=t\Rightarrow dx=-dt\\ If\ : x\in(0,1)\Rightarrow t\in(1,0)\end{cases}$$

$$That: J_2=\int_0^1\log(t)dt=t\left(\log(t)-1\right)\Big|_0^1=-1$$

$$For:\ J_3=\int_0^1 \log\left(1+x^2\right)dx\ ,\begin{cases} Let: u=\log\left(1+x^2\right)\Rightarrow du=\frac{2x}{1+x^2}dx \\ dv=dx\Rightarrow v=x \end{cases}$$

$$That: J_3 = x\log\left(1+x^2\right)\Big|_0^1 - 2\int_0^1 \frac{x^2}{1+x^2}dx$$

$$=\log(2)-2\int_0^1\left(\frac{1}{1+x^2}-1\right)dx$$

$$=\log(2)+2\left(\tan^{-1}(x)-x\right)\Big|_0^1=\log(2)+2\left(\frac{\pi}{4}-1\right)$$

$$Take:(*)That: J=\left(\log(4)-1\right)-1+\left(\log(2)+\frac{\pi}{2}-2\right)=\frac{\pi}{2}+3\log(2)-4$$

$$SO,\ \boxed{\int_0^1\log\left(1-x^4\right)dx=\frac{\pi}{2}+3\log(2)-4}$$

296 Calculate integral $K=\int_0^{\pi}\sec(x)\log\left(1+\frac{1}{2}\cos(x)\right)dx$

Answer

$$They\ give\qquad K=\int_0^{\pi}\sec(x)\log\left(1+\frac{1}{2}\cos(x)\right)dx$$

$$\Rightarrow K(a)=\int_0^{\pi}\sec(x)\log\left(1+a\cos(x)\right)dx$$

$$\Rightarrow K'(a)=\int_0^{\pi}\frac{\sec(x)\cos(x)}{1+a\cos(x)}dx$$

$$=\int_0^{\pi}\frac{1}{1+a\cos(x)}dx$$

$$Let: t=\tan\left(\frac{x}{2}\right)\Rightarrow dx=\frac{2}{1+t^2}dt, If\ : x\in(0,\pi)\Rightarrow t\in(0,\infty)\ ,\cos(x)=\frac{1-t^2}{t^2+1}$$

$$=\int_0^{\infty}\frac{1}{1+a\frac{1-t^2}{t^2+1}}\times\frac{2}{1+t^2}dt=2\int_0^{\infty}\frac{1}{(1+t^2)+a(1-t^2)}dt$$

$$=2\int_0^{\infty}\frac{1}{(1+a)+(1-a)t^2}dt=\frac{2}{1-a}\int_0^{\infty}\frac{1}{\sqrt{\left(\frac{1+a}{1-a}\right)^2+t^2}}dt$$

$$=\frac{2}{1-a}\times\frac{\sqrt{1-a}}{\sqrt{1+a}}\tan^{-1}\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}t\right)\Bigg|_0^{\infty}=\frac{\pi}{\sqrt{1-a^2}}$$

$$\Rightarrow K(a)=\int\frac{\pi}{\sqrt{1-a^2}}da=\pi\sin^{-1}(a)+C$$

$$If\ : a=0\Rightarrow K(0)=0=\pi\sin^{-1}(0)+C\Rightarrow C=0$$

$$If\ : a=\frac{1}{2}\Rightarrow K\left(\frac{1}{2}\right)=I=\pi\sin^{-1}\left(\frac{1}{2}\right)+0=\pi\times\frac{\pi}{6}=\frac{\pi^2}{6}$$

$$SO,\ \boxed{\int_0^{\pi}\sec(x)\log\left(1+\frac{1}{2}\cos(x)\right)dx=\frac{\pi^2}{6}}$$

297 Calculate integral $I=\int_0^3\sqrt{\frac{x}{3-x}}dx$

Answer

They give $\quad I=\int_0^3\sqrt{\frac{x}{3-x}}dx$

Mathod : 1 $\quad I=\int_0^3\sqrt{\frac{x}{3-x}}dx=\int_0^3 x^{\frac{1}{2}}(3-x)^{-\frac{1}{2}}dx$

$$Let : x=3u\Rightarrow dx=3du\,,If\ : x\in(0,3)\Rightarrow u\in(0,1)$$

$$That : I=\int_0^1(3u)^{\frac{1}{2}}(3-3u)^{-\frac{1}{2}}3du=3\int_0^1(u)^{\left(1+\frac{1}{2}\right)-1}(1-u)^{\frac{1}{2}-1}du$$

$$=3B\left(1+\frac{1}{2},\frac{1}{2}\right)=3\times\frac{\Gamma\left(1+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(1+\frac{1}{2}+\frac{1}{2}\right)}=\frac{3\pi}{2}$$

Mathod : 2 $\quad I=\int_0^3\sqrt{\frac{x}{3-x}}dx$

$$Let : x=3\sin^2(u)\Rightarrow dx=6\sin(u)\cos(u)du\,,If\ : x\in(0,3)\Rightarrow u\in(0,\frac{\pi}{2})$$

$$That : I=6\int_0^{\frac{\pi}{2}}\sqrt{\frac{3\sin^2(u)}{3-3\sin^2(u)}}\times\sin(u)\cos(u)du=6\int_0^{\frac{\pi}{2}}|\tan(u)|\times\sin(u)\cos(u)du$$

$$=6\int_0^{\frac{\pi}{2}}\frac{\sin(u)}{\cos(u)}\times\sin(u)\cos(u)du=6\int_0^{\frac{\pi}{2}}\sin^2(u)du=\frac{3\pi}{2}$$

$$SO,\ \boxed{\int_0^3\sqrt{\frac{x}{3-x}}dx=\frac{3\pi}{2}}$$

298 Calculate integral $J=\int_0^3\{x\}^{\lfloor x\rfloor}dx$

Answer

They give $J=\int_0^3\{x\}^{\lfloor x\rfloor}dx$

$$=\int_0^3\left(x-\lfloor x\rfloor\right)^{\lfloor x\rfloor}dx\qquad\left(\{x\}:\text{is the fraction part function}\right)$$

$$=\int_0^1\left(x-\lfloor x\rfloor\right)^{\lfloor x\rfloor}dx+\int_1^2\left(x-\lfloor x\rfloor\right)^{\lfloor x\rfloor}dx+\int_2^3\left(x-\lfloor x\rfloor\right)^{\lfloor x\rfloor}dx$$

$$By:\left(x-\lfloor x\rfloor\right)^{\lfloor x\rfloor}=\begin{cases}1 & 0\le x\le 1\\(x-1) & 1\le x\le 2\\(x-2)^2 & 2\le x\le 3\end{cases}$$

$$\Rightarrow J=\int_0^1 1dx+\int_1^2(x-1)dx+\int_2^3(x-2)^2dx=\frac{11}{6}$$

SO, $\boxed{\int_0^3\{x\}^{\lfloor x\rfloor}dx=\frac{11}{6}}$

299 Calculate integral $K=\int_0^1\frac{x^4-1}{x(x^4-5)(x^5-5x+1)}dx$

Answer

They give $K=\int_{-1}^1\frac{x^4-1}{x(x^4-5)(x^5-5x+1)}dx$

$$=\frac{1}{5}\int_{-1}^1\frac{5(x^4-1)}{(x^5-5x)(x^5-5x+1)}dx$$

$$Let:x^5-5x=t\Rightarrow 5(x^4-1)dx=dt,\ If\ :x\in(-1,1)\Rightarrow t\in(4,-4)$$

$$\Rightarrow K=\frac{1}{5}\int_4^{-4}\frac{1}{t(t+1)}dx$$

$$=\frac{1}{5}\log\left|\frac{t}{t+1}\right|\Bigg|_4^{-4}$$

$$=\frac{1}{5}\log\left(\frac{5}{3}\right)$$

SO, $\boxed{\int_{-1}^1\frac{x^4-1}{x(x^4-5)(x^5-5x+1)}dx=\frac{1}{5}\log\left(\frac{5}{3}\right)}$

300 Calculate integral $I=\int_0^1 \{x\}^x\,dx$

Answer

They give $$I=\int_0^1 \{x\}^x\,dx$$

$$=\int_0^1 (x-\lfloor x\rfloor)^x\,dx=\int_0^1 (x-0)^x\,dx \quad ,\forall x\in(0,1)\Rightarrow \lfloor x\rfloor=0$$

$$=\int_0^1 x^x dx$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)^{n+1}}=1-\frac{1}{2^2}+\frac{1}{3^3}-\frac{1}{4^4}+\dots$$

SO, $$\boxed{\int_0^1 \{x\}^x\,dx=1-\frac{1}{2^2}+\frac{1}{3^3}-\frac{1}{4^4}+\dots}$$

301 Calculate integral $J=\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{\cos(2x)}}{\sin(x)}\right)dx$

Answer

They give $$J=\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{\cos(2x)}}{\sin(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{\cos^2(x)-\sin^2(x)}}{\sin(x)}\right)dx$$

$$=\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{1-\tan^2(x)}}{\tan(x)}\right)dx$$

$$Let:\cos(u)=\tan(x)\Rightarrow \sec^2(x)dx=-\sin(u)du\,, If\ :x\in\left(0,\frac{\pi}{4}\right)\Rightarrow u\in\left(\frac{\pi}{2},0\right)$$

$$\Rightarrow J=-\int_{\frac{\pi}{2}}^{0}\tan^{-1}\left(\frac{\sqrt{1-\cos^2(u)}}{\cos(u)}\right)\sin(u)du=\int_0^{\frac{\pi}{2}}\tan^{-1}\left(\frac{|\sin(u)|}{\cos(u)}\right)\sin(u)du$$

$$=\int_0^{\frac{\pi}{2}}\tan^{-1}(\tan(u))\sin(u)du=\int_0^{\frac{\pi}{2}}u\sin(u)du=1$$

SO, $$\boxed{\int_0^{\frac{\pi}{4}}\sec^2(x)\tan^{-1}\left(\frac{\sqrt{\cos(2x)}}{\sin(x)}\right)dx=1}$$

302 Calculate integral $K=\int_1^{\infty}\left(\left(\frac{x}{x+1}\right)^2\left(\frac{x-1}{x+1}\right)\left(\frac{1}{x+1}\right)\right)^2 dx$

Answer

They give $$K=\int_1^{\infty}\left(\left(\frac{x}{x+1}\right)^2\left(\frac{x-1}{x+1}\right)\left(\frac{1}{x+1}\right)\right)^2 dx$$

$$=\int_1^{\infty}\left(\frac{x}{x+1}\right)^4\left(\frac{x-1}{x+1}\right)^2\left(\frac{1}{x+1}\right)^2 dx=\frac{1}{16}\int_1^{\infty}\left(\frac{2x}{x+1}\right)^4\left(\frac{x-1}{x+1}\right)^2\left(\frac{1}{x+1}\right)^2 dx$$

$$=\frac{1}{16}\int_1^{\infty}\left(1-\frac{x-1}{x+1}\right)^4\left(\frac{x-1}{x+1}\right)^2\left(\frac{1}{x+1}\right)^2 dx$$

$$Let: y=\frac{x-1}{x+1}\Rightarrow dy=\frac{1}{(x+1)^2}dx, If\ : x\in(1,\infty)\Rightarrow y\in(0,1)$$

$$\Rightarrow K=\frac{1}{16}\int_0^1(1-y)^4(y)^2dy=\frac{1}{16}B(5,3)=\frac{1}{560}$$

SO, $$\boxed{\int_1^{\infty}\left(\left(\frac{x}{x+1}\right)^2\left(\frac{x-1}{x+1}\right)\left(\frac{1}{x+1}\right)\right)^2 dx=\frac{1}{560}}$$

303 Calculate integral $I=\int_{\frac{1}{2}}^{2}\sqrt{\log^2(x)}dx$

Answer

They give $$I=\int_{\frac{1}{2}}^{2}\sqrt{\log^2(x)}dx$$

$$=\int_{\frac{1}{2}}^{2}\left|\log(x)\right|dx=\int_{\frac{1}{2}}^{1}\left|\log(x)\right|dx+\int_1^2\left|\log(x)\right|dx$$

$$=-\int_{\frac{1}{2}}^{1}\log(x)dx+\int_1^2\log(x)dx$$

$$=-x\left(\log(x)-1\right)\Big|_{\frac{1}{2}}^{1}+x\left(\log(x)-1\right)\Big|_1^2$$

$$=\frac{1}{2}\left(\log(2)-1\right)$$

SO, $$\boxed{\int_{\frac{1}{2}}^{2}\sqrt{\log^2(x)}dx=\frac{1}{2}\left(\log(2)-1\right)}$$

304 Calculate integral $J=\int_0^{\infty}\frac{\sin^{2n+1}}{x}dx \quad ,\forall n\in\mathbb{N}$

Answer

They give $$J=\int_0^{\infty}\frac{\sin^{2n+1}(x)}{x}dx \quad ,\forall n\in\mathbb{N}$$

$$=\int_0^{\infty}\frac{\sin(x)}{x}\times\sin^{2n}(x)dx$$

$By: f(x)=\sin^{2n}(x)\Leftrightarrow f(x)=f(\pi-x)=f(\pi+x)$

$$\Rightarrow J=\int_0^{\infty}\frac{\sin(x)}{x}\times\sin^{2n}(x)dx=\int_0^{\frac{\pi}{2}}\sin^{2n}(x)dx$$

$$=\frac{\pi}{2}\times\frac{(2n-1)(2n-3)(2n-5)...(3)(1)}{(2n)(2n-2)...(4)(2)}$$

SO, $$\boxed{\int_0^{\infty}\frac{\sin^{2n+1}(x)}{x}dx=\frac{\pi}{2}\times\frac{(2n-1)(2n-3)(2n-5)...(3)(1)}{(2n)(2n-2)...(4)(2)}}$$

305 Calculate integral $K=\int_0^{\pi}\frac{\sin(x)+x/2}{e^x+\sin(x)+\cos(x)+x+1}dx$

Answer

They give $$K=\int_0^{\pi}\frac{\sin(x)+x/2}{e^x+\sin(x)+\cos(x)+x+1}dx$$

$Let: f(x)=e^x+\sin(x)+\cos(x)+x+1\Rightarrow f'(x)=e^x+\cos(x)-\sin(x)+1$

$That: f(x)-f'(x)=2\sin(x)+x\Leftrightarrow \frac{1}{2}\left(f(x)-f'(x)\right)=\sin(x)+\frac{x}{2}$

$If: x\to 0\Rightarrow f(0)=3\,, If: x\to\pi\Rightarrow f(\pi)=e^{\pi}+\pi$

$$\Rightarrow K=\frac{1}{2}\int_0^{\pi}\frac{f(x)-f'(x)}{f(x)}dx$$

$$=\left(x-\log\left|f(x)\right|\right)\Big|_0^{\pi}$$

$$=\frac{1}{2}\left(\pi-\log\left|\frac{e^{\pi}+\pi}{3}\right|\right)=\frac{\pi}{2}-\log\left(\sqrt{\frac{e^{\pi}+\pi}{3}}\right)$$

SO, $$\boxed{\int_0^{\pi}\frac{\sin(x)+x/2}{e^x+\sin(x)+\cos(x)+x+1}dx=\frac{\pi}{2}-\log\left(\sqrt{\frac{e^{\pi}+\pi}{3}}\right)}$$

306 Calculate integral $I=\int_0^2 Max\left\{x,x^2\right\}dx$

Answer

They give $I=\int_0^2 Max\left\{x,x^2\right\}dx$

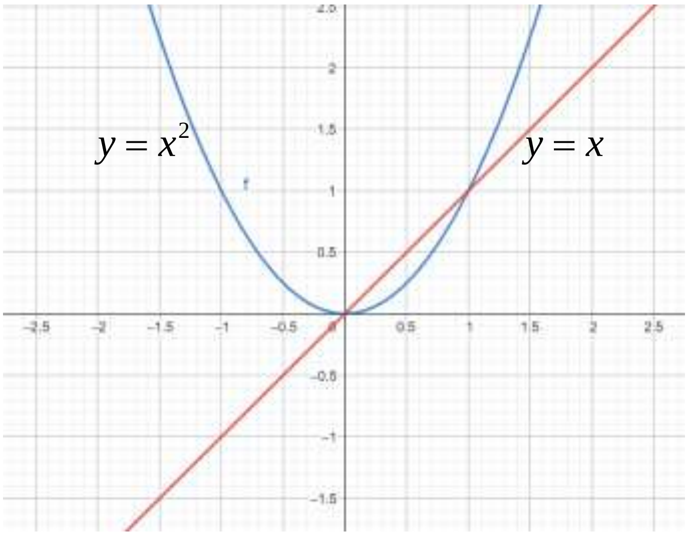
$\forall x\in(0,2)$, $x\cap x^2$ Timing $x=0, x=1$

$If: x\in(0,1)\ That\ x^2\le x\ \ They\ Have\ Max\left\{x,x^2\right\}=x\quad ,\forall x\in(0,1)$

$If: x\in(1,2)\ That\ x\le x^2\ \ They\ Have\ Max\left\{x,x^2\right\}=x^2\quad ,\forall x\in(1,2)$

$$\Rightarrow I=\int_0^1 xdx+\int_1^2 x^2dx$$

$$=\frac{1}{2}+\frac{1}{3}x^3\Big|_1^2=\frac{17}{6}$$



SO, $\boxed{\int_0^2 Max\left\{x,x^2\right\}dx=\frac{17}{6}}$

$If: I=\int_0^2 Min\left\{x,x^2\right\}dx$

$\forall x\in(0,2)$, $x\cap x^2$ Timing $x=0, x=1$

$If: x\in(0,1)\ That\ x^2\le x\ They\ Have\ Min\left\{x,x^2\right\}=x^2\quad ,\forall x\in(0,1)$

$If: x\in(1,2)\ That\ x\le x^2\ \ They\ Have\ Min\left\{x,x^2\right\}=x\quad ,\forall x\in(1,2)$

$$\Rightarrow I=\int_0^1 x^2dx+\int_1^2 xdx$$

$$=\frac{1}{3}+\frac{1}{2}x^2\Big|_1^2=\frac{11}{6}$$

SO, $\boxed{\int_0^2 Min\left\{x,x^2\right\}dx=\frac{11}{6}}$

$\otimes$*Note*

$\oplus Min\{x,y\}=\begin{cases} x & When\ \ x\le y \\ y & When\ \ y\le x\end{cases}$ On the space (a,b)

$\oplus Max\{x,y\}=\begin{cases} x & When\ \ x\ge y \\ y & When\ \ y\ge x\end{cases}$ On the space (a ,b)

307 Calculate integral $J=\int_{-2}^{2} Max\left\{2x^2, x^2+1\right\} dx$

Answer

They give $J=\int_{-2}^{2} Max\left\{2x^2, x^2+1\right\} dx$

$\forall x \in (-2,2)$, $2x^2 \cap x^2+1$ Timing $x=-1,\ x=1$

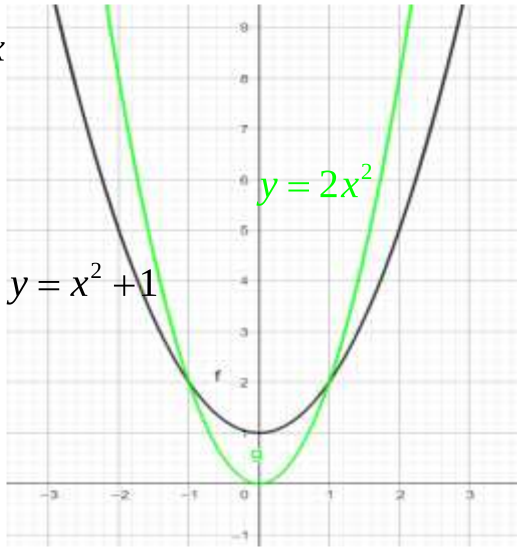
If $: x \in (-2,-1)$ *That* $2x^2 \geq x^2+1$ *They Have* $Max\left\{2x^2, x^2+1\right\}=2x^2 \quad ,\forall x \in (-2,-1)$

If $: x \in (-1,1)$ *That* $2x^2 \leq x^2+1$ *They Have* $Max\left\{2x^2, x^2+1\right\}=x^2+1 \quad ,\forall x \in (-1,1)$

If $: x \in (1,2)$ *That* $2x^2 \geq x^2+1$ *They Have* $Max\left\{2x^2, x^2+1\right\}=2x^2 \quad ,\forall x \in (1,2)$

$$\Rightarrow J=\underbrace{\int_{-2}^{-1} 2x^2 dx}_{Let: x=-x}+\underbrace{\int_{-1}^{1}(x^2+1) dx}_{is\ an\ even\ function}+\int_{1}^{2} 2x^2 dx$$

$$=2\int_{1}^{2} 2x^2 dx+2\int_{0}^{1}(x^2+1) dx=12$$



SO, $\boxed{\int_{-2}^{2} Max\left\{2x^2, x^2+1\right\} dx=12}$

If $: J=\int_{-2}^{2} Min\left\{2x^2, x^2+1\right\} dx$

$\forall x \in (-2,2)$, $2x^2 \cap x^2+1$ Timing $x=-1,\ x=1$

If $: x \in (-2,-1)$ *That* $2x^2 \geq x^2+1$ *They Have* $Min\left\{2x^2, x^2+1\right\}=x^2+1 \quad ,\forall x \in (-2,-1)$

If $: x \in (-1,1)$ *That* $2x^2 \leq x^2+1$ *They Have* $Min\left\{2x^2, x^2+1\right\}=2x^2 \quad ,\forall x \in (-1,1)$

If $: x \in (1,2)$ *That* $2x^2 \geq x^2+1$ *They Have* $Min\left\{2x^2, x^2+1\right\}=x^2+1 \quad ,\forall x \in (1,2)$

$$\Rightarrow J=\underbrace{\int_{-2}^{-1}(x^2+1) dx}_{Let: x=-x}+\underbrace{\int_{-1}^{1} 2x^2 dx}_{is\ an\ even\ function}+\int_{1}^{2}(x^2+1) dx$$

$$=2\int_{1}^{2}(x^2+1) dx+2\int_{0}^{1} 2x^2 dx=\frac{20}{3}$$

$$=2\left(\frac{1}{3}x^3+x\right)\Big|_{1}^{2}+\frac{4}{3}x^3\Big|_{1}^{2}$$

$$=\frac{20}{3}$$

SO, $\boxed{\int_{-2}^{2} Min\left\{2x^2, x^2+1\right\} dx=\frac{20}{3}}$

ឯកសារយោងនិងឯកសារស្នើរអានបន្ថែម

1. ដកស្រង់ចេញពីវិការប្រលង Massachusetts Institute of Technology (MIT)
2. ដកស្រង់ចេញពីអ៊ីនធើណេត (youtube Google facebook) និងការនិពន្ធពីខ្ញុំផ្ទាល់។
3. ដកស្រង់ចេញពីសៀវភៅអាំងតេក្រាលបរទេសដូចជា វៀតណាម បារាំង ... ជាដើម។
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8. Improper Riemann Integrals (Ioannis M... (Z-Library).pdf
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11. A_S_Demidov_Equations_of_Mathematical_Physics_Generalized_Functions.pdf
12. Almost Impossible Integrals, Sums and Series.pdf
13. G_Plonka,_D_Potts,_G_Steidl,_M_Tasche.pdf
14. Integrals and Series [Vol 2 - Spl Functions] - PDF Room.pdf
15. Series Involving Euler's Eta Function.pdf

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