Qiskit Global Summer School 2024

Characterizing noise on quantum hardware

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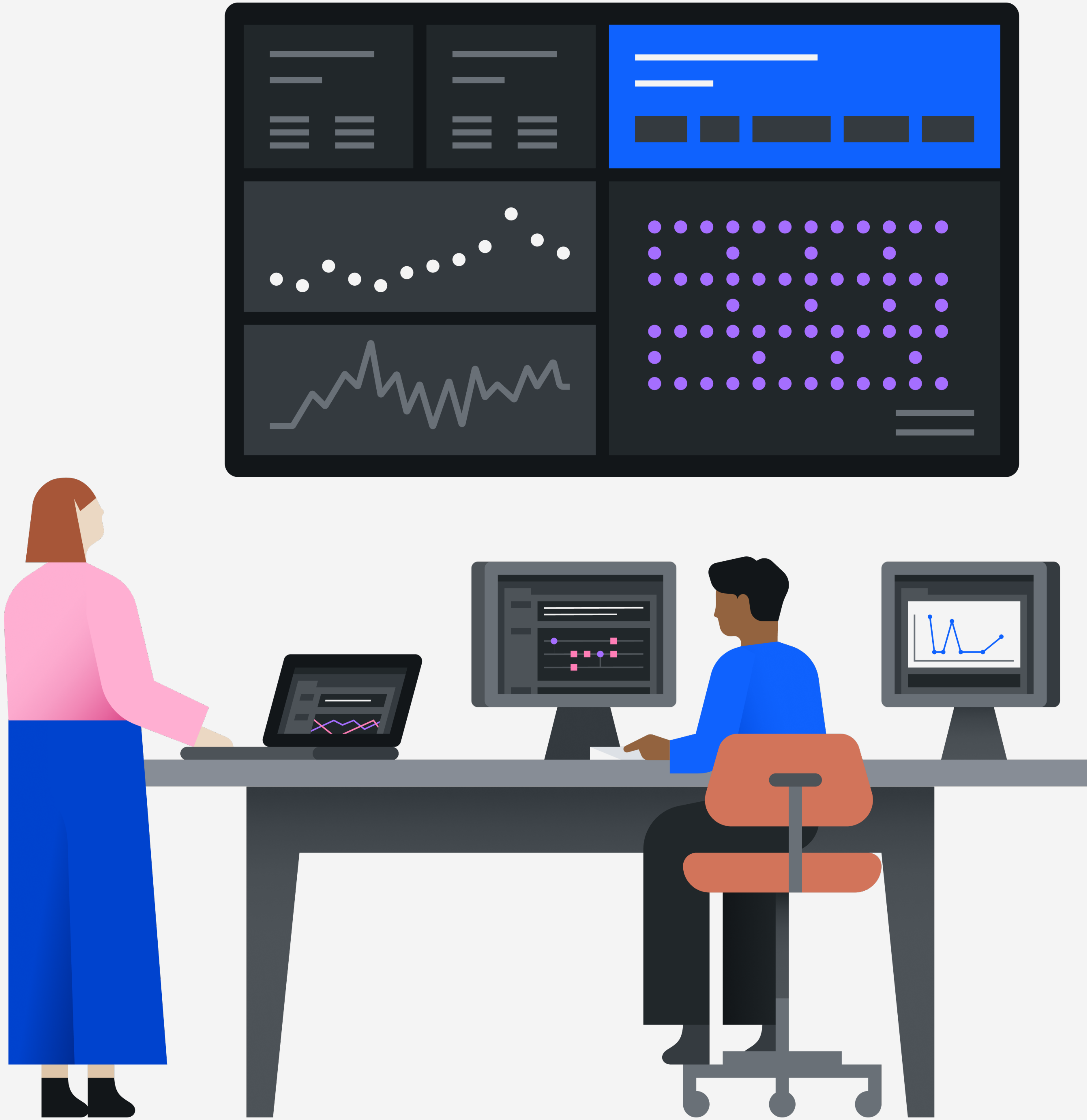


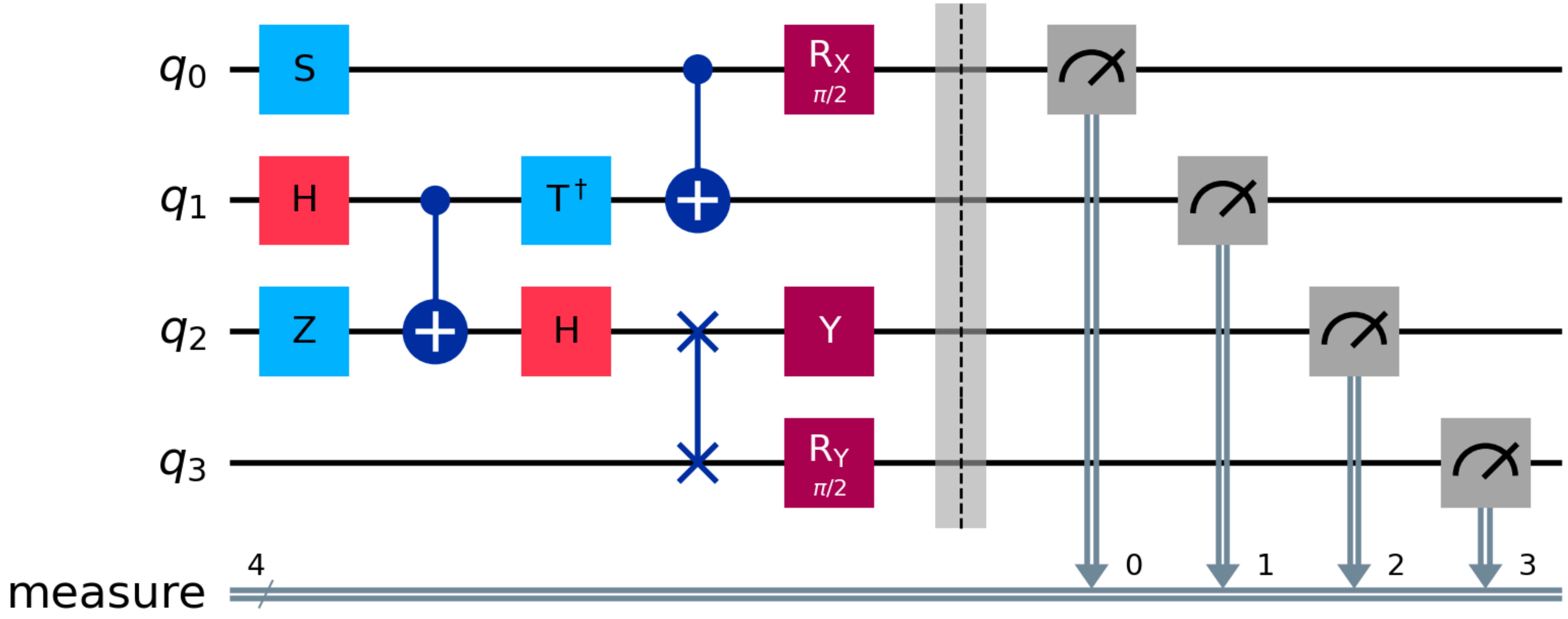
Start with a Quantum circuit

A Quantum circuit consists of unitary gates applied to qubits.

Qubit initialization

Apply quantum gates
Basis gates: ECR, ID, RZ, SX, X,
They form a universal gate set.

Measurement



Noise in today’s quantum hardware

Quantum circuits are compiled and run physically on the quantum hardware;

The performance and power of today quantum processors are still largely limited by noise;

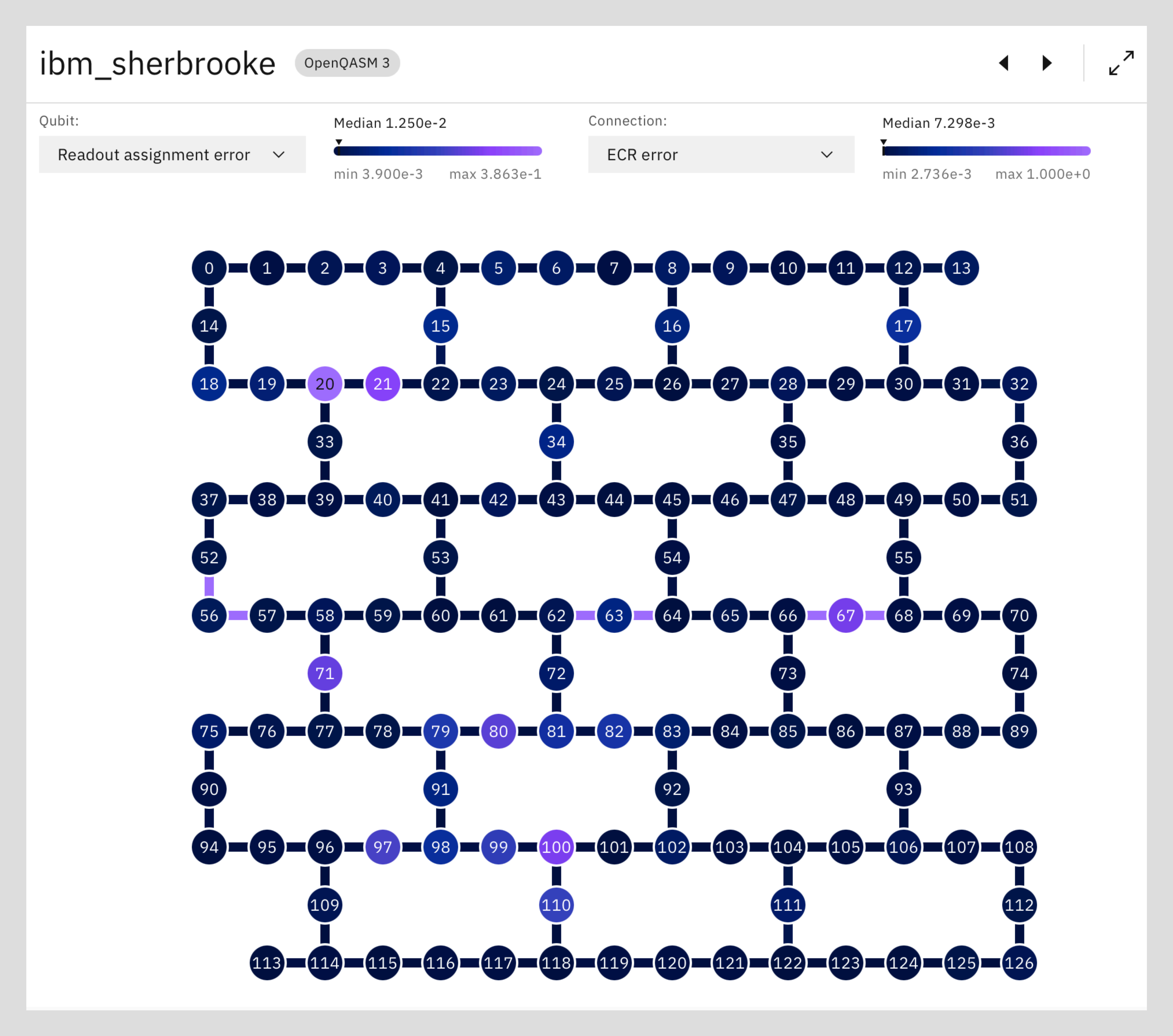
We want to understand the limitation of quantum hardware due to noise so we can make best use of it.

In this lecture:

Different sources of noise;

Where to read the noise-related metrics from the IBM Quantum Platform;

How they are experimentally characterized.



Start with a Quantum circuit

Rotation gates generated by a Hamiltonian

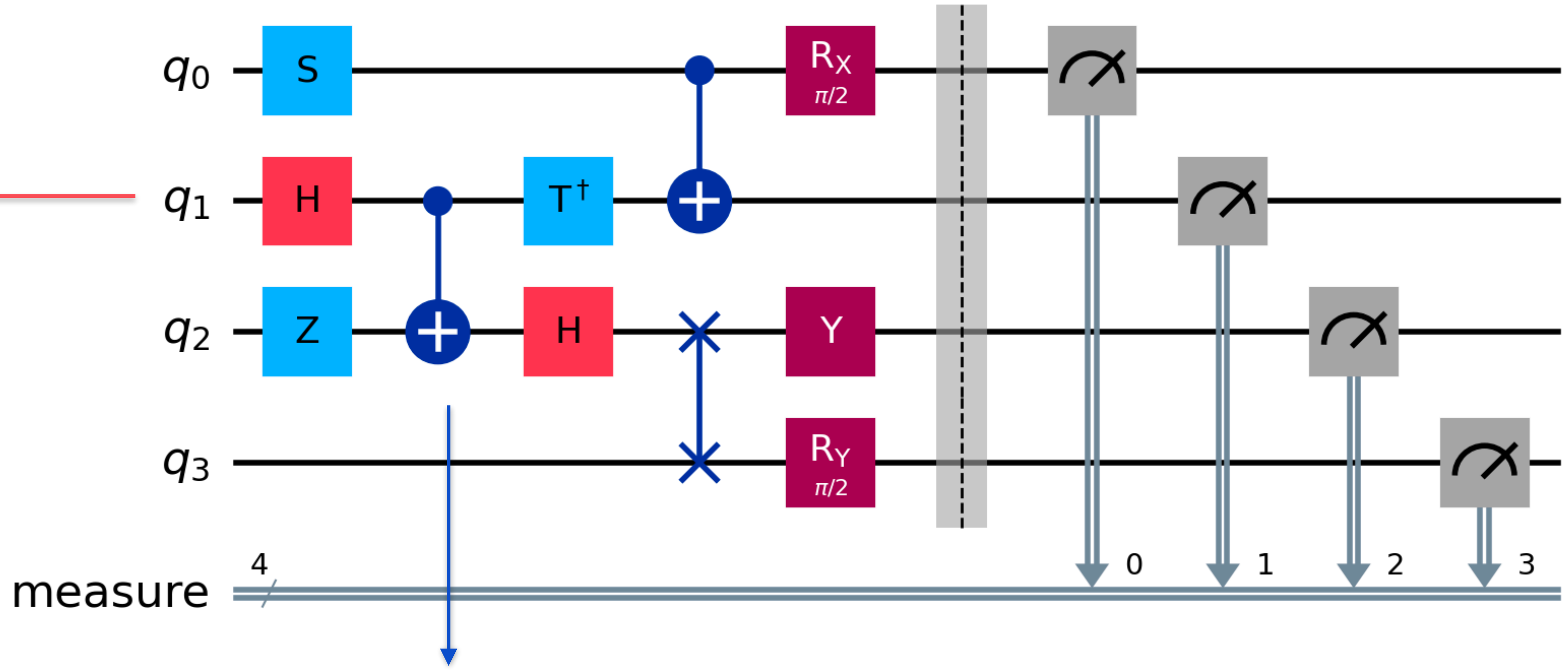
$$R_x(\theta) = \exp(-i\frac{\theta}{2}\sigma_x)$$
$$= \exp(-iH_{\text{drive}}t)$$

where $H_{\text{drive}} = v\sigma_x, \theta = vt$

Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|\psi_{out}\rangle = H|\psi_{in}\rangle$$



Measure in the qubit computational basis.
For a single qubit, measurement operators $P_0 = |0\rangle\langle 0|$, $P_1 = |1\rangle\langle 1|$

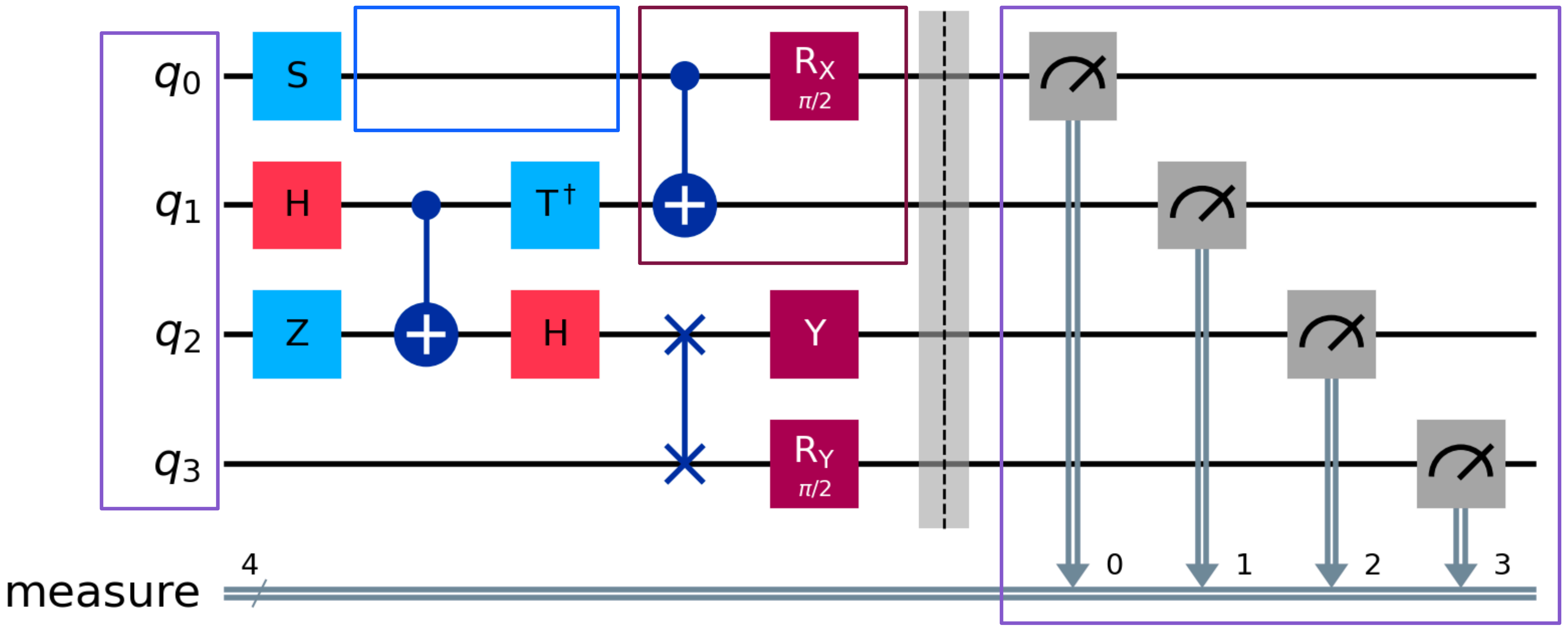
Two-qubit gate generates entanglement.

$$CNOT \cdot (H \otimes I)|0\rangle|0\rangle = (|0\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2}$$

What are the sources of noise?

Incoherent errors:
Loss of quantum information in the form of superposition and entanglement

Coherent errors in gates:
Incorrect Hamiltonian evolution



State preparation and measurement (**SPAM**) errors

Representing a quantum state

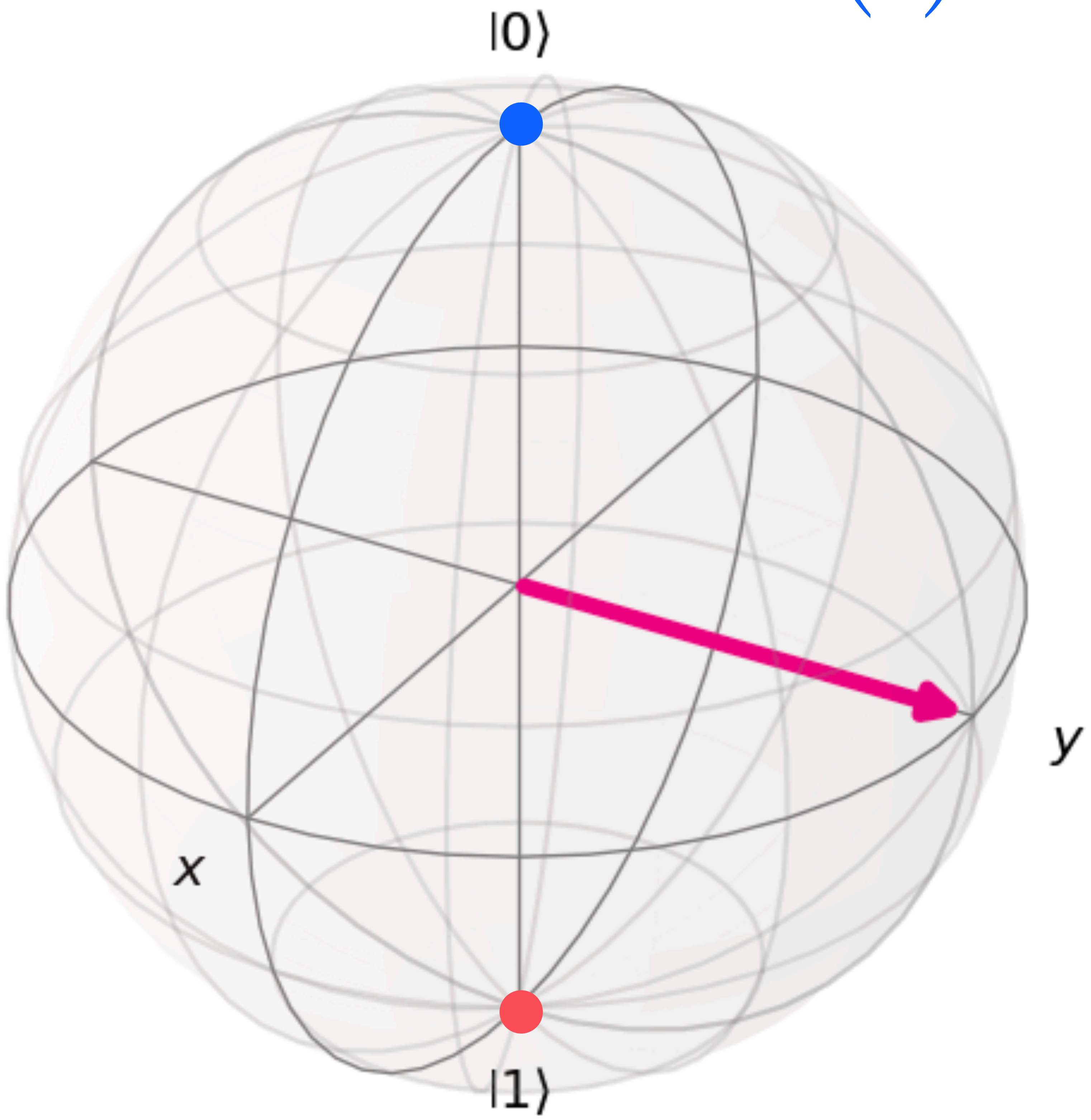
A single-qubit quantum state can be represented by a state vector in the 2-dimensional Hilbert state.

Ground state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Excited state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Representing a quantum state

Ground state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



Excited state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Coherent superposition state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \alpha, \beta \in \mathbb{C}$$

$|\alpha|^2$: probability in ground state

$|\beta|^2$: probability in excited state

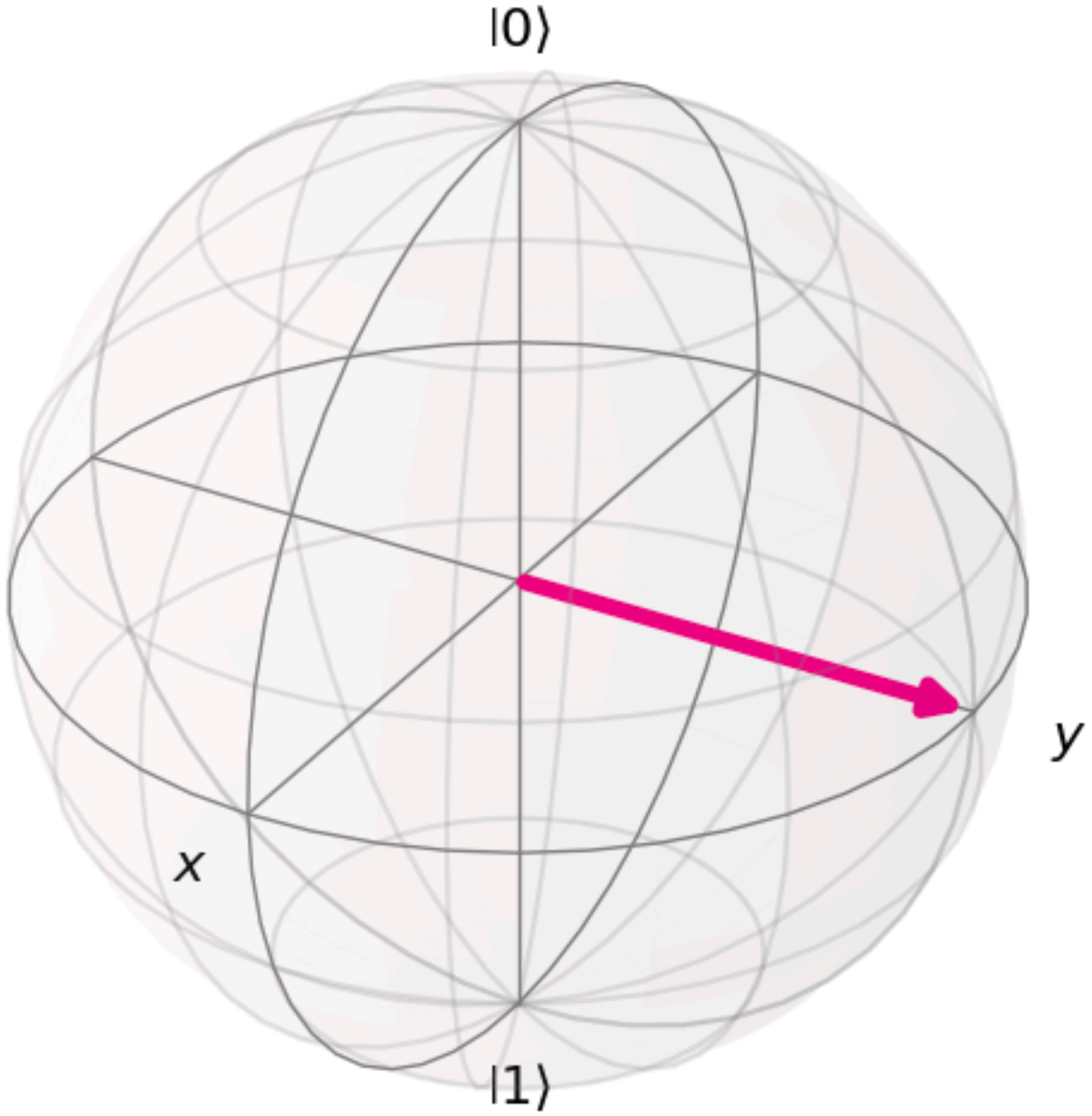
$$|\alpha|^2 + |\beta|^2 = 1$$

Representing a quantum bit

Density matrix representation

Pure state:

$$\rho = |\psi\rangle\langle\psi|$$



Mixed state:

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|,$$

describes an ensemble of pure states

$$\{q_i, |\psi_i\rangle\}.$$

Representing a quantum bit

Bloch vector representation

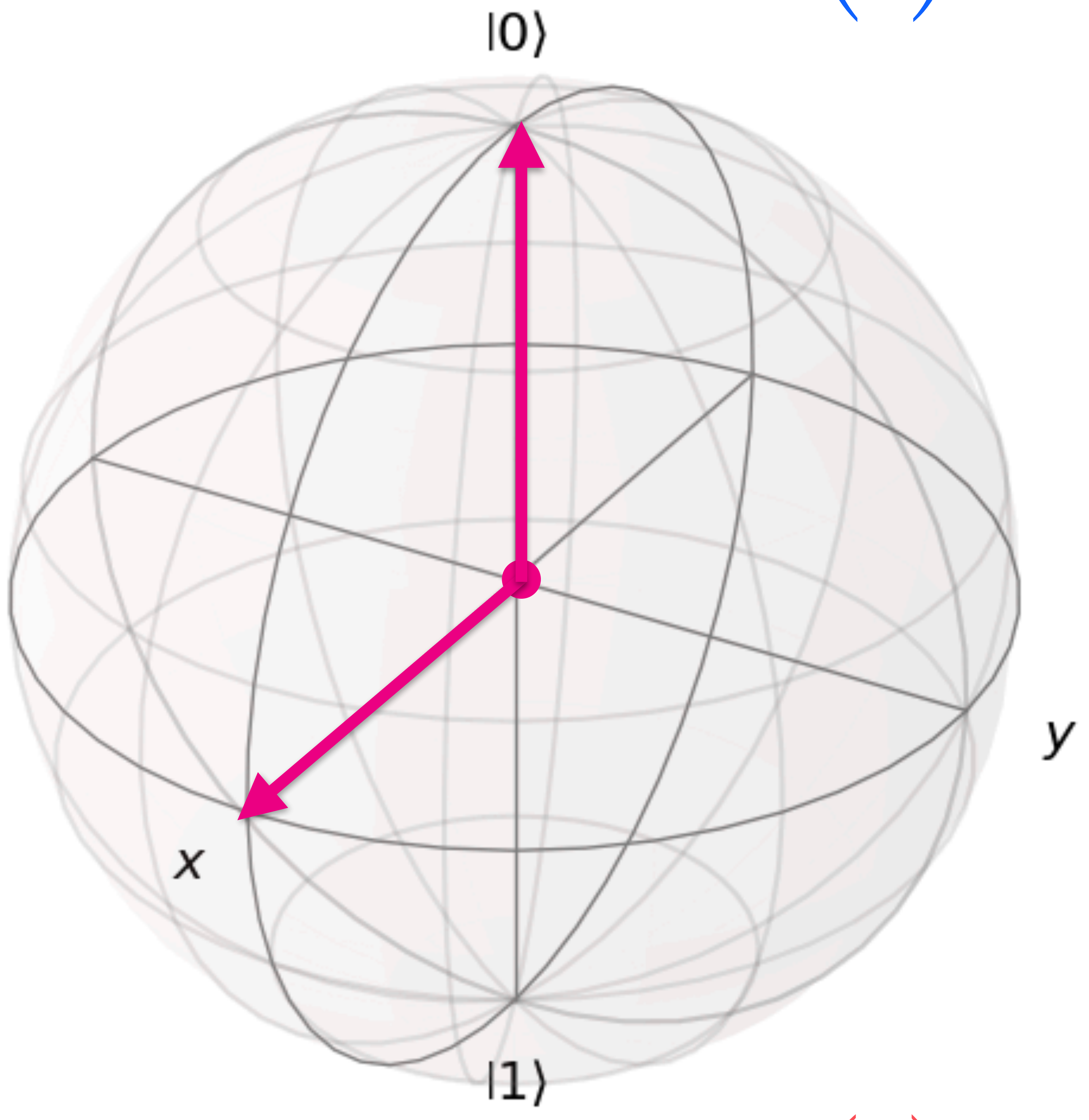
$$\rho = \frac{1}{2}(I + \vec{v} \cdot \vec{\sigma})$$

where

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z),$$

$$\vec{v} = (v_x, v_y, v_z).$$

Ground state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



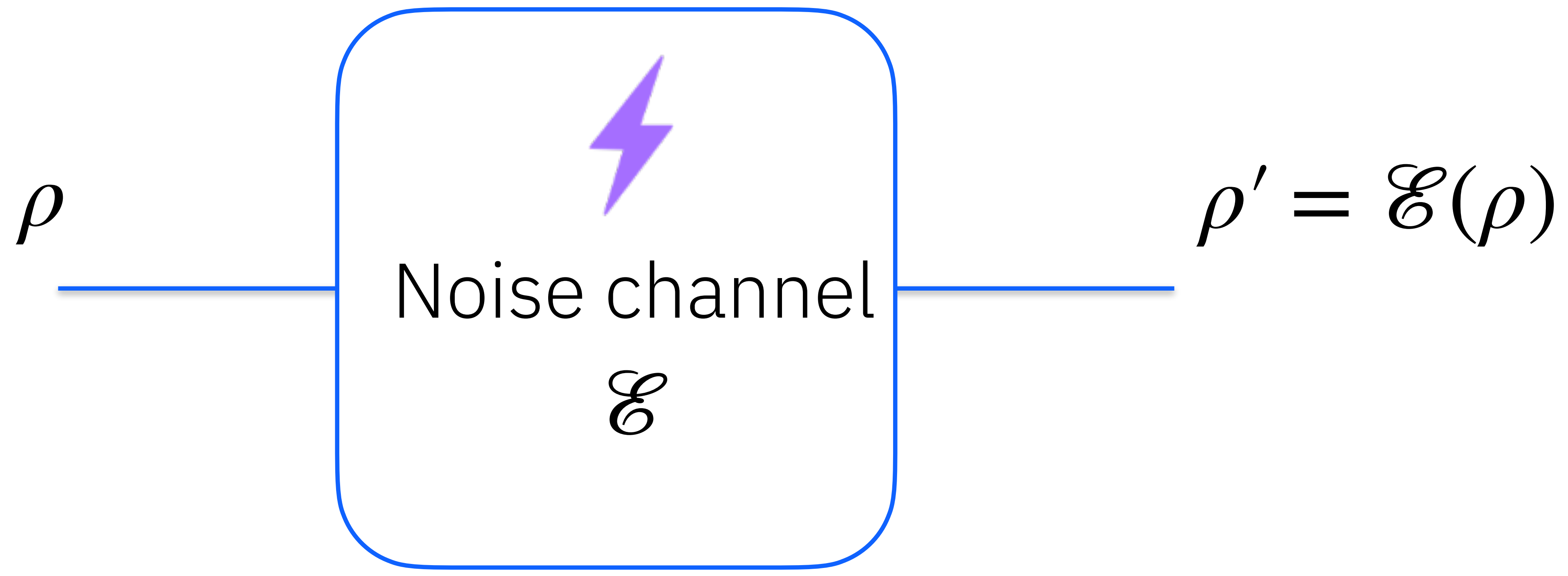
Excited state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

X poles: $|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$

Z poles: ground and excited states

Center: maximally mixed state $\rho = \frac{I}{2}$

Effect of noise



	Ideal	**Noisy**
State	Pure state $\|\vec{v}\| = 1$	Mixed state $\|\vec{v}\| < 1$
Operations	Unitaries $\|\psi\rangle \rightarrow U\|\psi\rangle$	Noise channel $\mathscr{E}$ $\rho \rightarrow \mathscr{E}(\rho)$

SPAM errors

Example of a mix state:

e.g. imperfect state preparation

Measurement (readout) error results in imperfect acquisition of the classical output of the quantum circuit.

In practice, it is difficult to tell them apart.

Mathematically described by:

$$\vec{p}_{\text{noisy}} = A\vec{p}_{\text{ideal}}$$

p_i: probability of measuring bitstring i

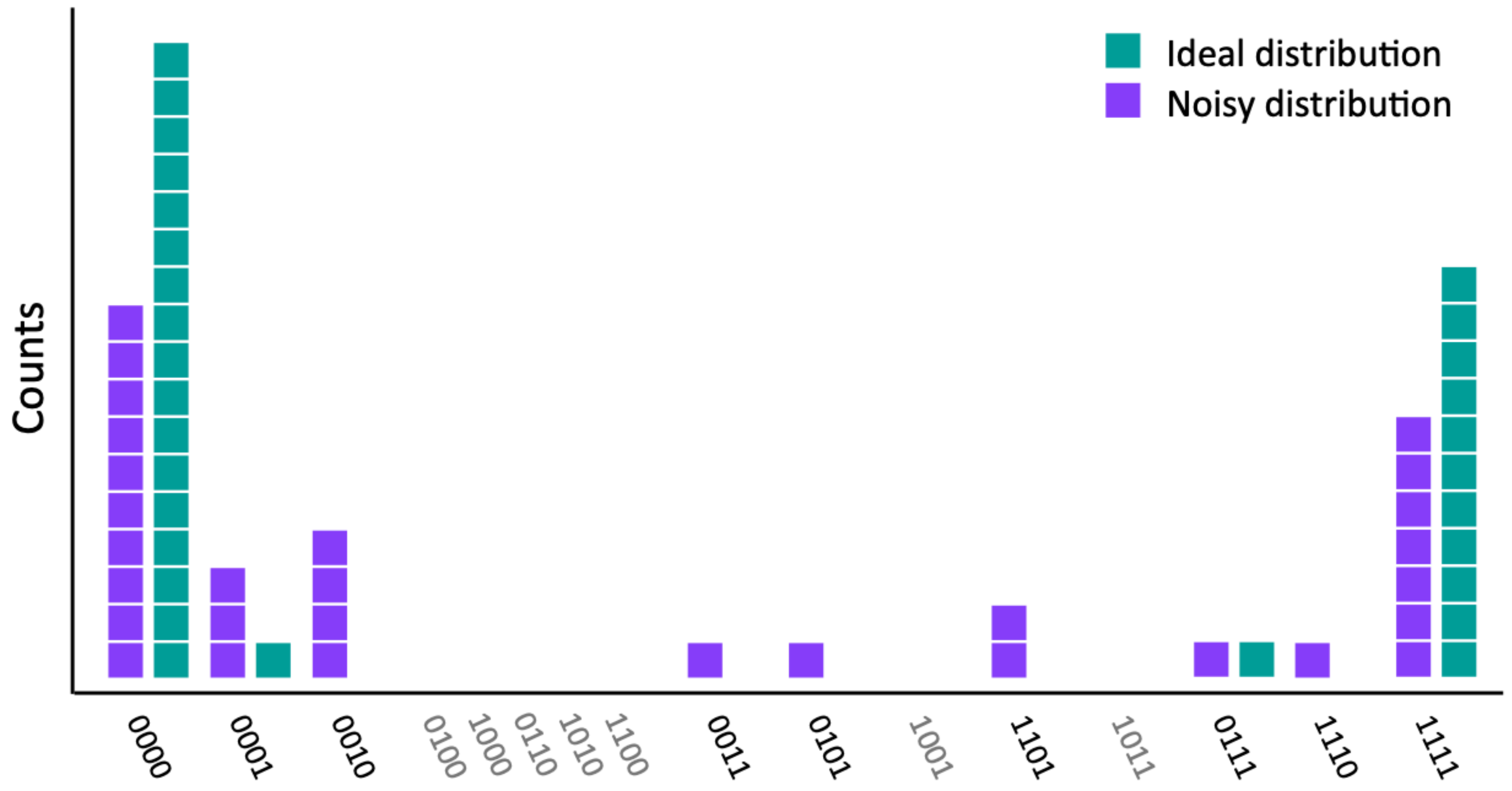
A_{ij}: the probability of observing state j while being prepared in state i.

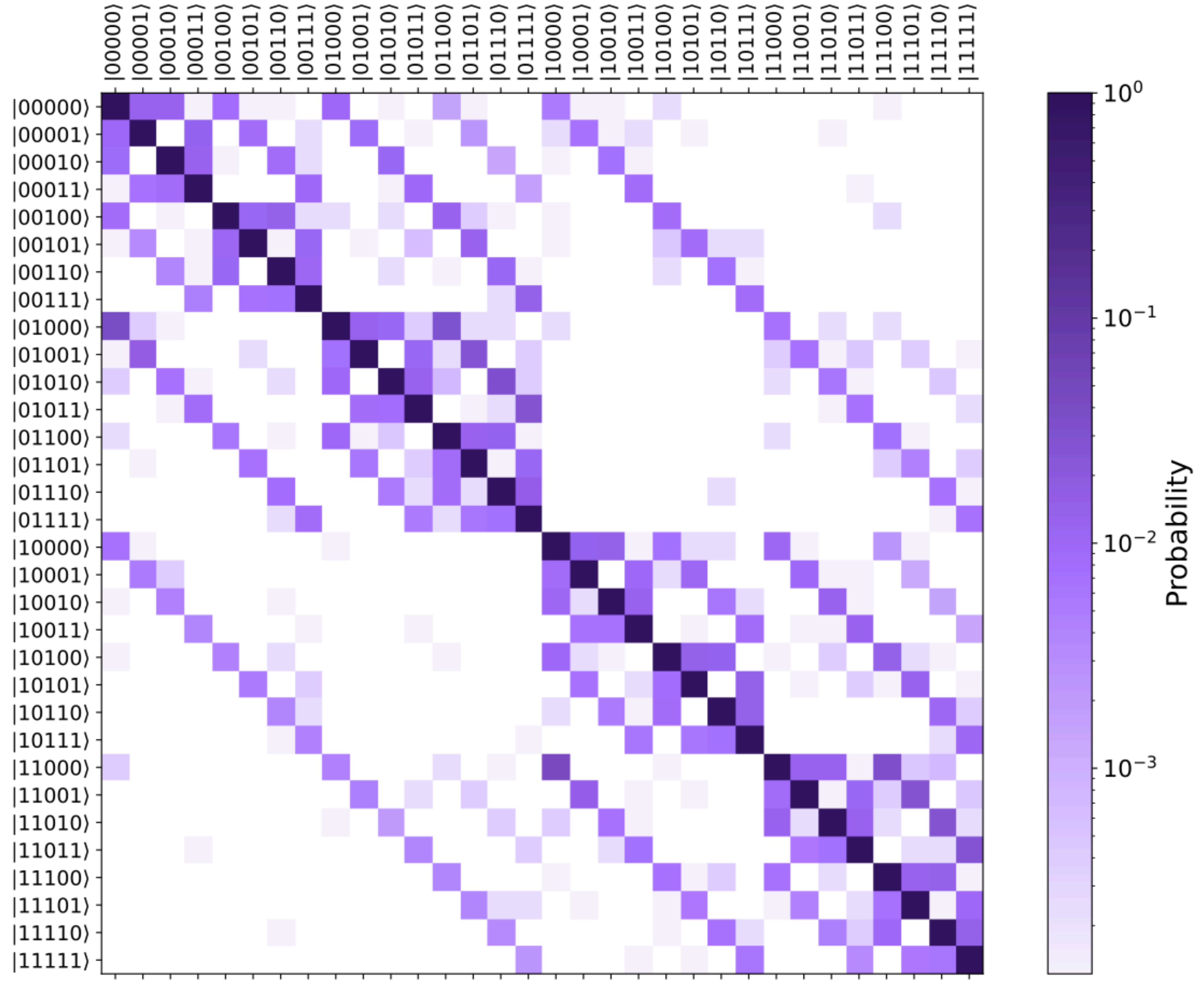
Challenging to characterize at scale: (2^n)^2 elements.

Reported on backend qubit by qubit.

Could be correlated between qubits.

Scalable Mitigation of Measurement Errors on Quantum Computers, PRX Quantum 2, 040326 (2021)





A matrix for five qubits

SPAM errors

Example of a mix state:

e.g. imperfect state preparation

Measurement (readout) error results in imperfect acquisition of the classical output of the quantum circuit.

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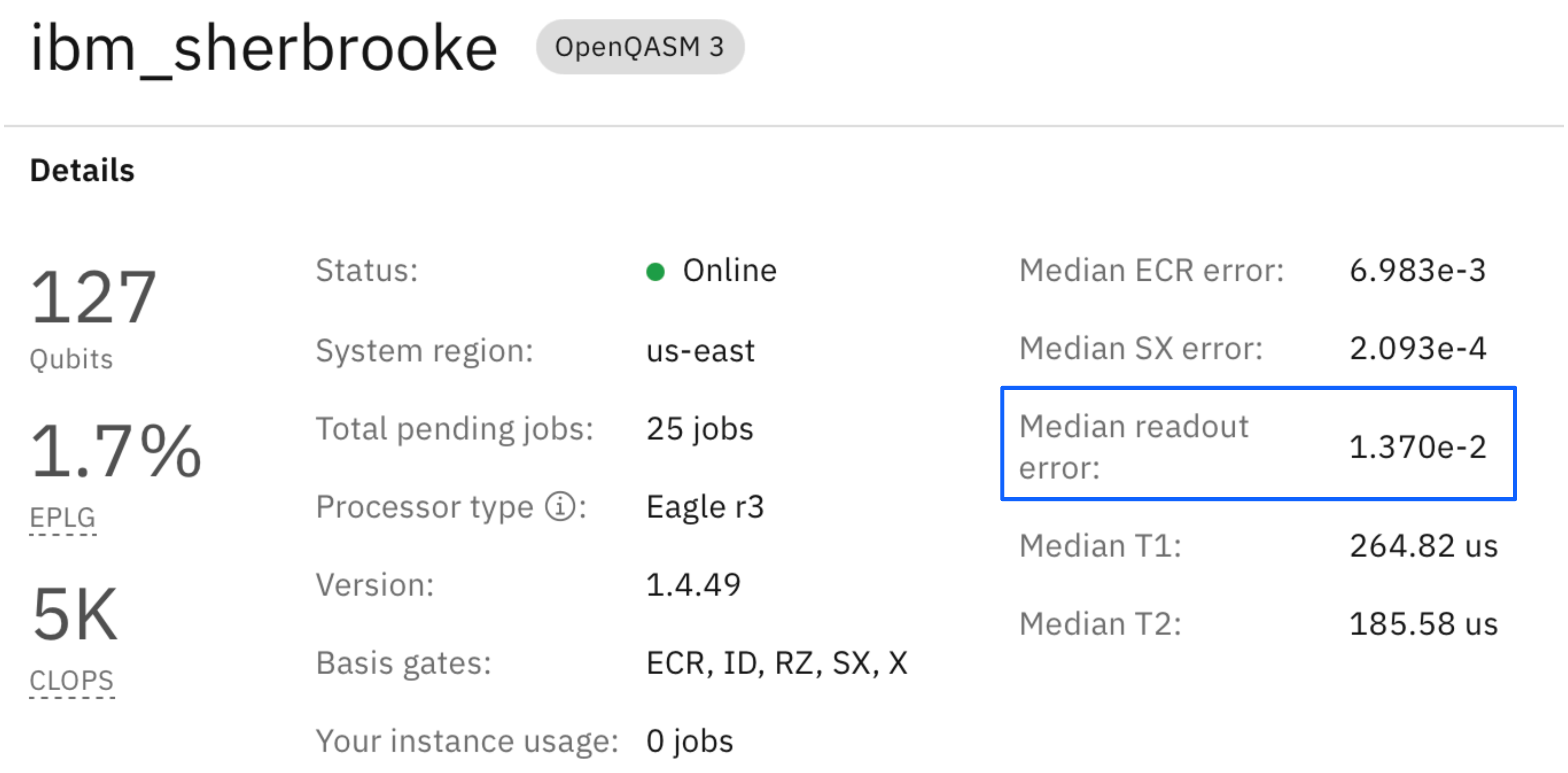
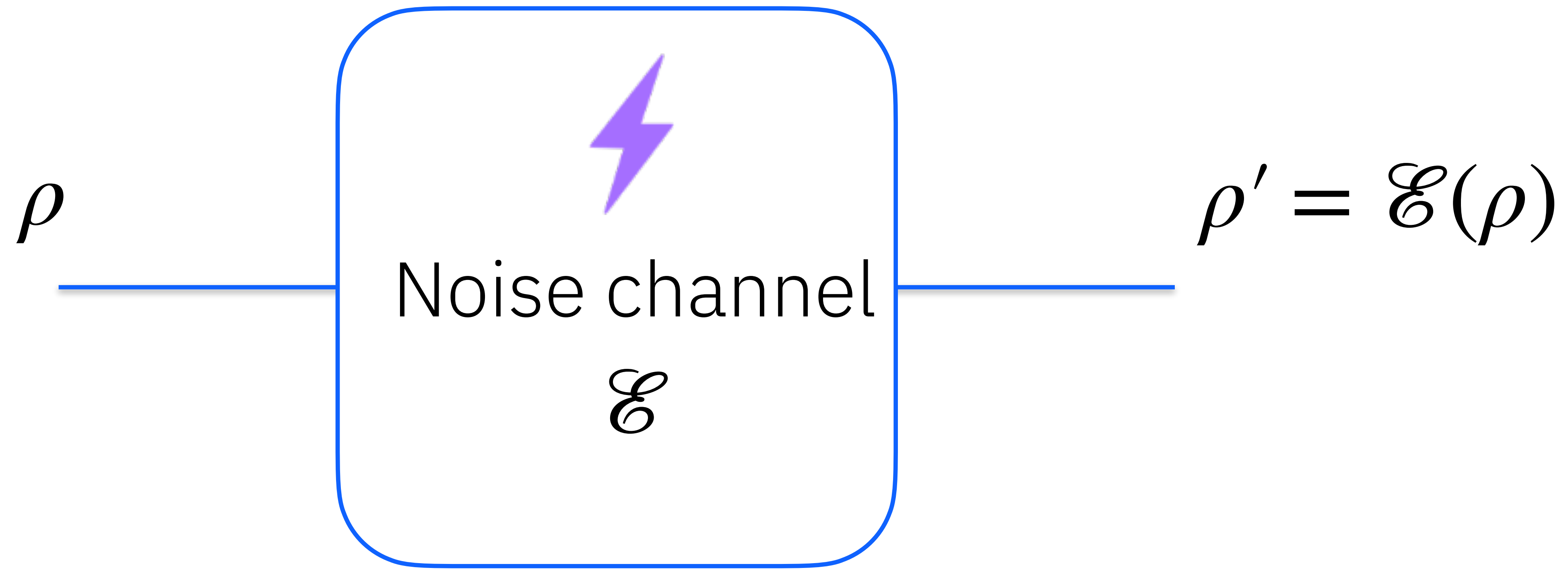


Scalable Mitigation of Measurement Errors on Quantum Computers, PRX Quantum 2, 040326 (2021)

Noise channels



Mathematical representation (Kraus):

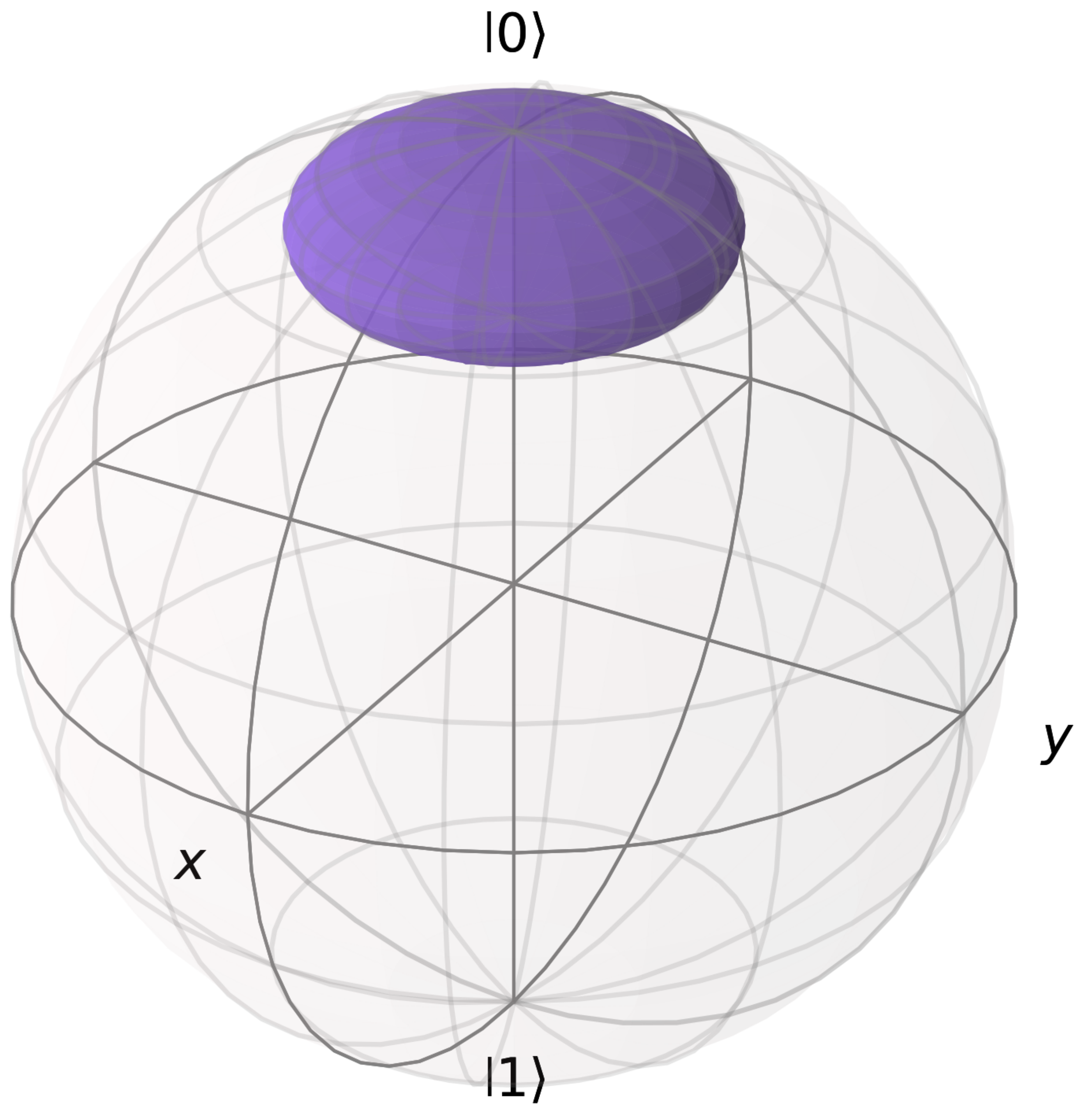
$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger \qquad \sum_i K_j^\dagger K_j = I$$

Linear
Complete positive
Trace preserving

Further reading: *Quantum Channels*, John Watrous, IBM Quantum Learning

Incoherent errors

Action of an amplitude damping channel on the Bloch sphere.



Ground state becomes the steady state of the superconducting qubit, largely unaffected by relaxation errors.

Energy relaxation:

Superconducting qubits operate at low temperatures, typically around 20 mK with qubit frequency between 4-6 GHz.

Hence there is much stronger decay from the excited state $|1\rangle$ to the ground state $|0\rangle$ compared with the excitation from $|0\rangle$ to $|1\rangle$.

Energy relaxation is characterized the timescale T1;
The probability of a relaxation error is given by $1 - e^{-t/T_1}$ for circuit evolution time t.

Amplitude damping channel:

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix},$$

$$K_1 = \sqrt{p}\,|0\rangle\langle 1|.$$

Equivalently:

$|0\rangle \mapsto |0\rangle$ with probability 1

$|1\rangle \mapsto |0\rangle$ with probability p

Incoherent errors

Dephasing:

Superposition states become classical mixtures

Dephasing is characterized by the timescale T2,

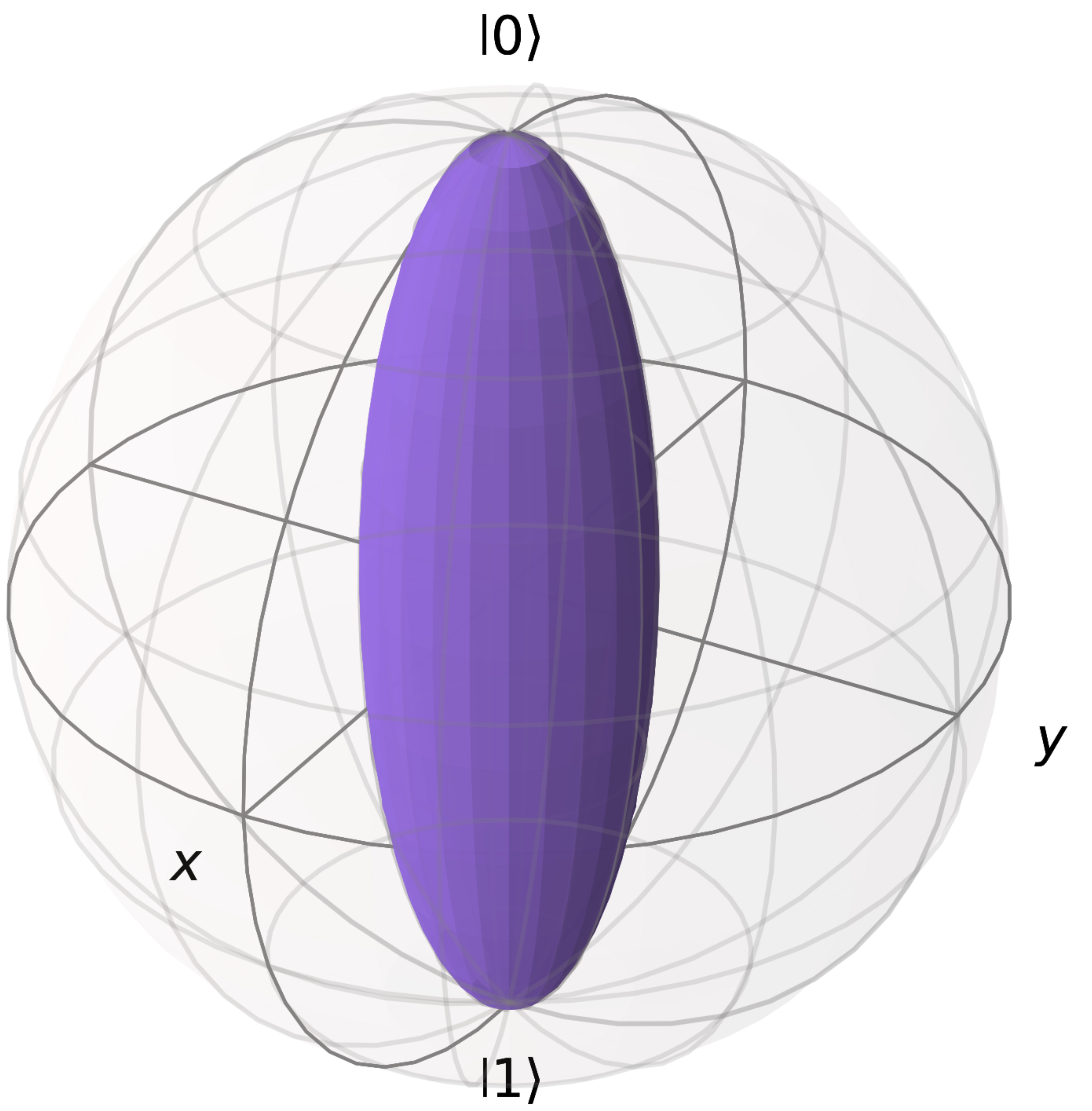
The probability of a dephasing error is given by $1 - e^{-t/T_2}$ for circuit evolution time t.

Both relaxation and dephasing error turn quantum information classical!

Phase flip channel:

$$K_0 = \sqrt{1-p}\, I,$$
$$K_1 = \sqrt{p}\, Z.$$

Action of a phase damping channel on the Bloch sphere.



Incoherent errors

Pauli noise channel

$$\mathcal{E}(\rho) = \sum_j p_j P_j \rho P_j$$
$$= p_I I\rho I + p_X X\rho X + p_Y Y\rho Y + p_Z Z\rho Z$$

P_j: Pauli operators

p_j: the error rate associated with P_j

Sparse Pauli noise channel is used to model noisy quantum processors in practice [1]

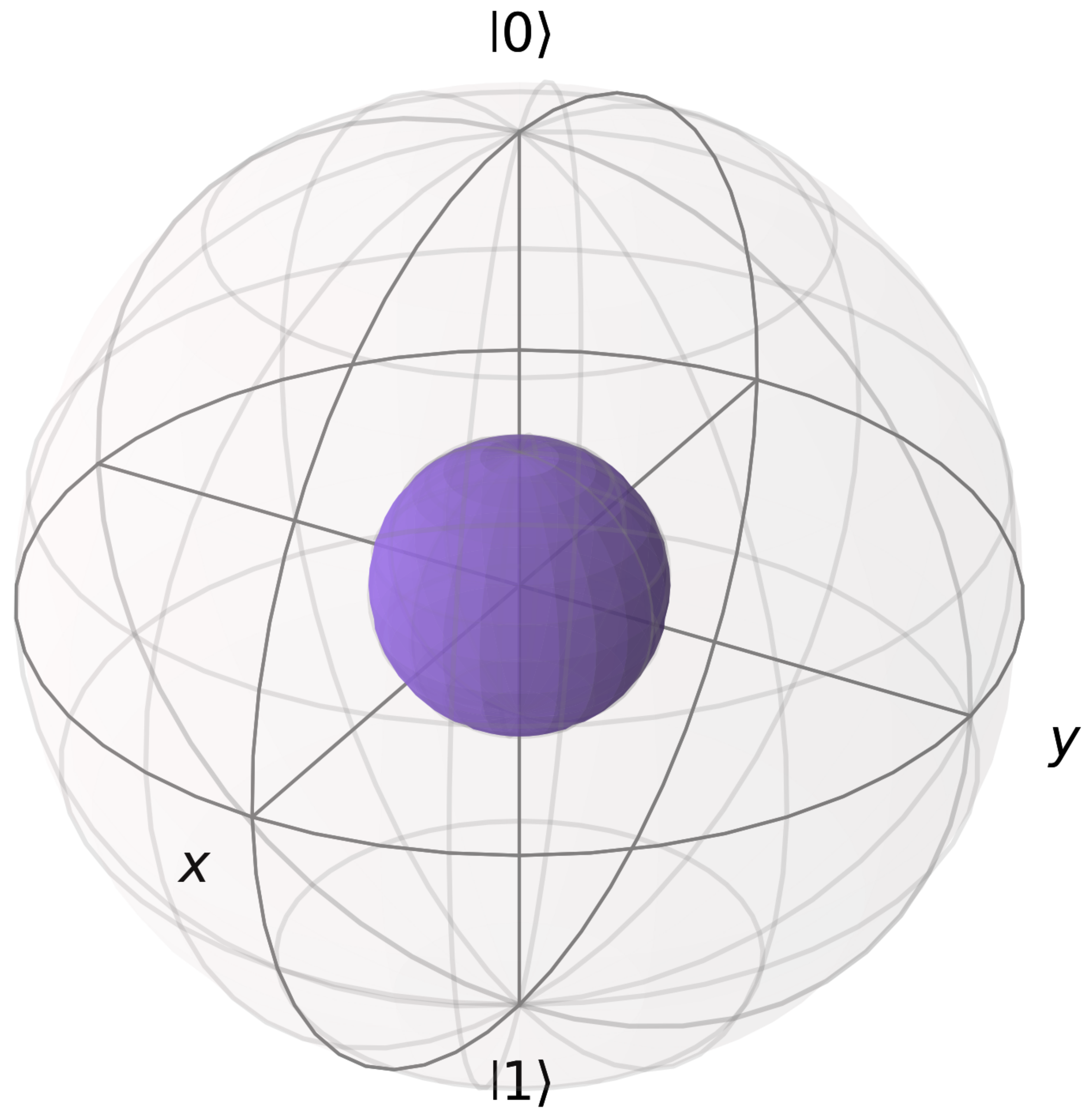
[1] E. Berg et al., Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors. Nature Physics, pages 1–6, 2023.

depolarizing channel

A special case when $p_x = p_y = p_z$

$$\mathcal{E}(\rho) = p\frac{I}{2} + (1-p)\rho$$

$$\mathcal{E}(\rho) = \left(1 - \frac{3p}{4}\right)\rho + \frac{p}{4}\left(X\rho X + Y\rho Y + Z\rho Z\right)$$



Pauli noise channels are *unital*, meaning that it maps the maximally mixed state to itself.

Coherent errors

Gates are generated by turning on certain Hamiltonians that we can control and engineer.

Let’s look at the single-qubit case:

$$RX(\theta) = \exp\left(- i\frac{\theta}{2}\sigma_x\right) = \exp(-iHt)$$
$$H = v\sigma_x, \theta = 2vt$$

What if we engineered the wrong Hamiltonian?

Example: incorrect phase in a microwave generator

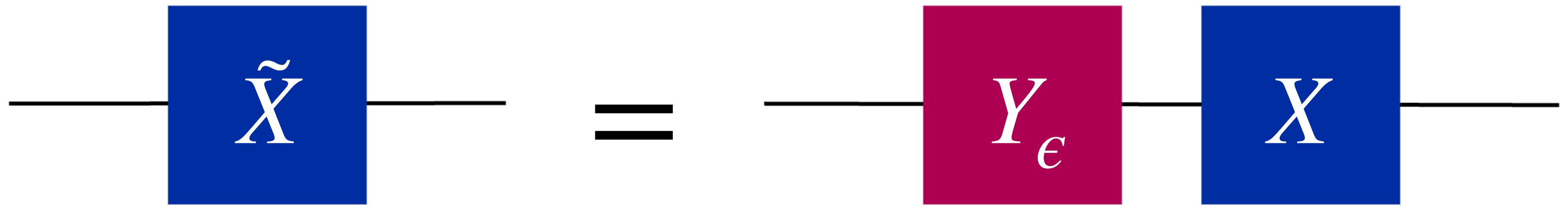
$$H = v\sigma_x + \epsilon\sigma_y.$$

Example: poorly-calibrated pulse durations.

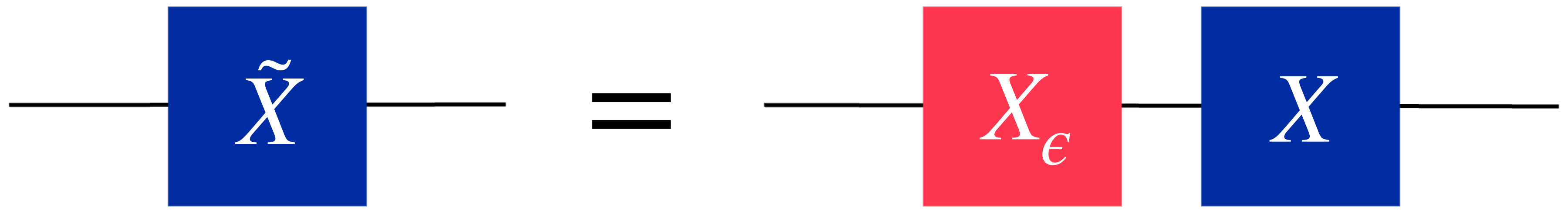
$$\theta = 2v(t+t')$$

Coherent errors can be modeled by unwanted unitary gates in the circuit.

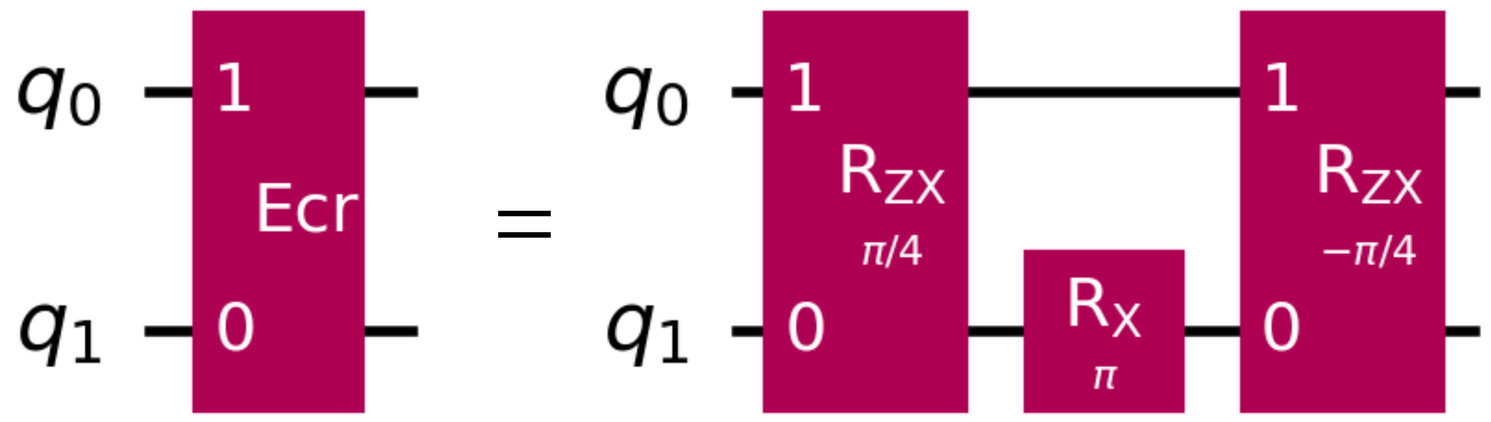
Misaligned rotation



Over/under rotation



Two-qubit errors (crosstalk)



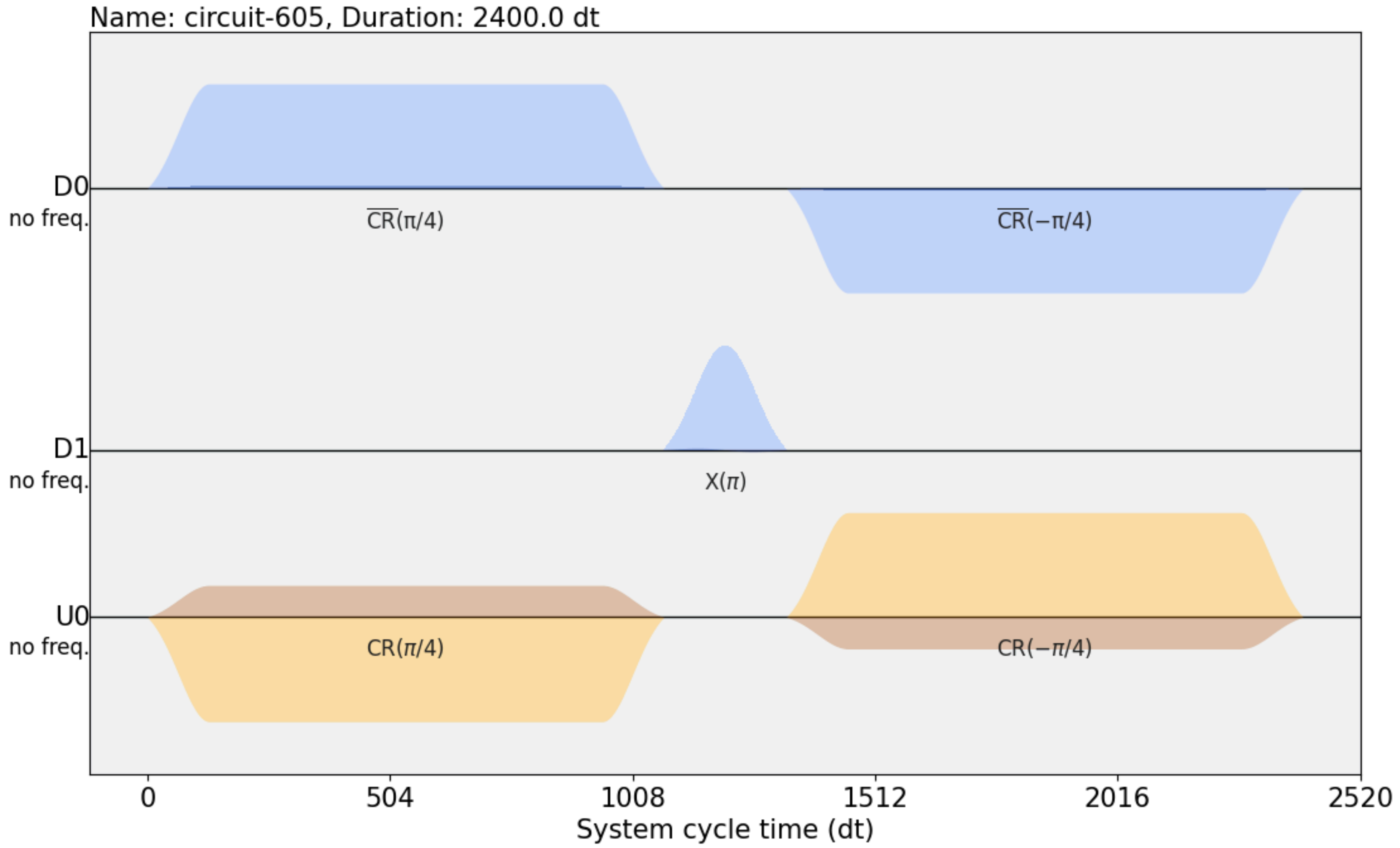
Echoed cross-resonance (ECR) gate is equivalent to a CNOT up to single-qubit rotations.

It implements the following Hamiltonian:

$$H_{\text{eff}} = \boxed{\omega_{ix}\frac{IX}{2} + \omega_{iz}\frac{IZ}{2} + \omega_{zi}\frac{ZI}{2}} + \omega_{zx}\frac{ZX}{2} + \boxed{\omega_{zz}\frac{ZZ}{2}}$$

The ZZ static capacitive coupling results in unwanted crosstalk errors to the connected qubits

An order of magnitude longer gate durations compared with single-qubit gates.



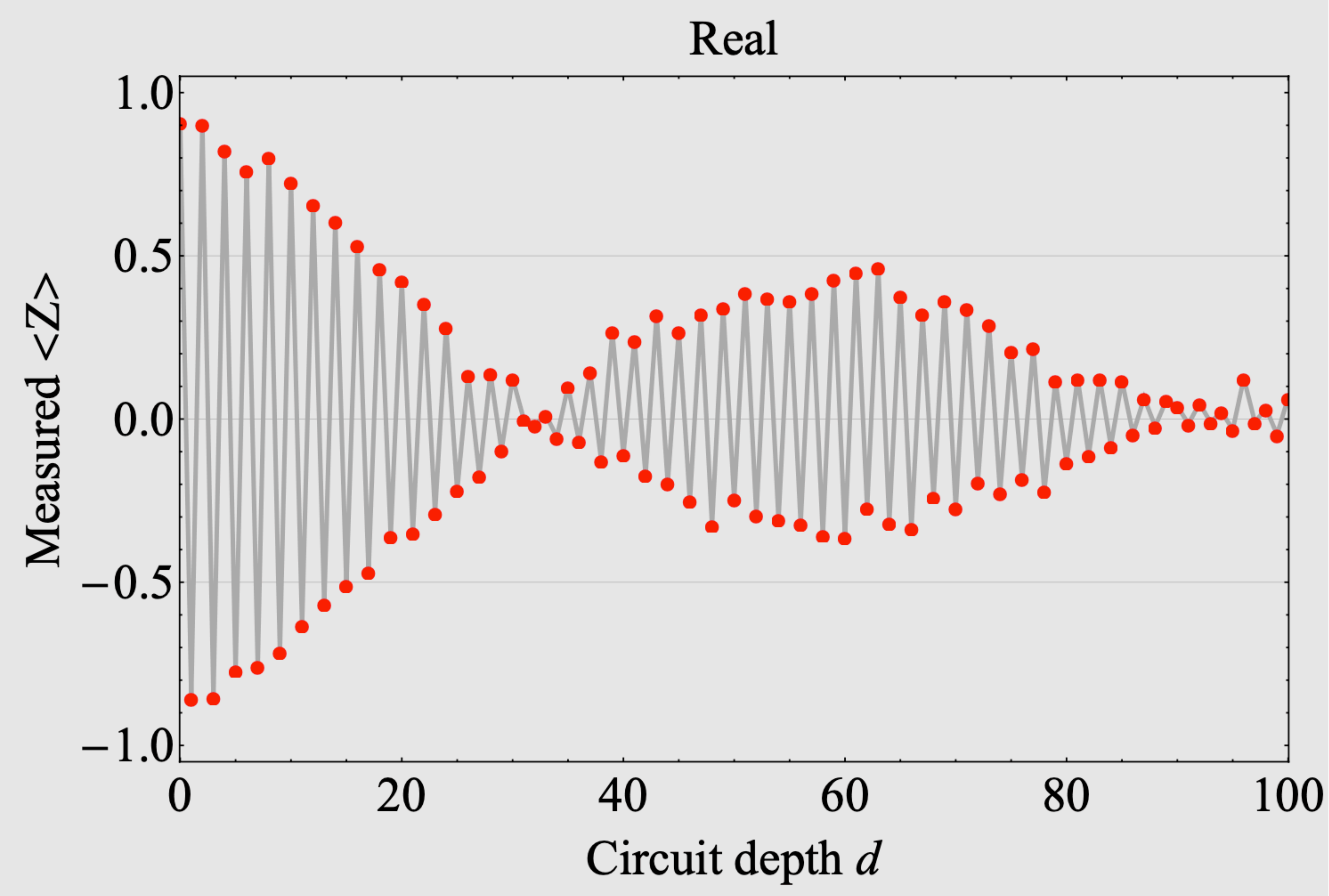
Coherent noise summary

Coherent errors often times result in oscillations in signals.

It can build up much more rapidly than incoherent errors.

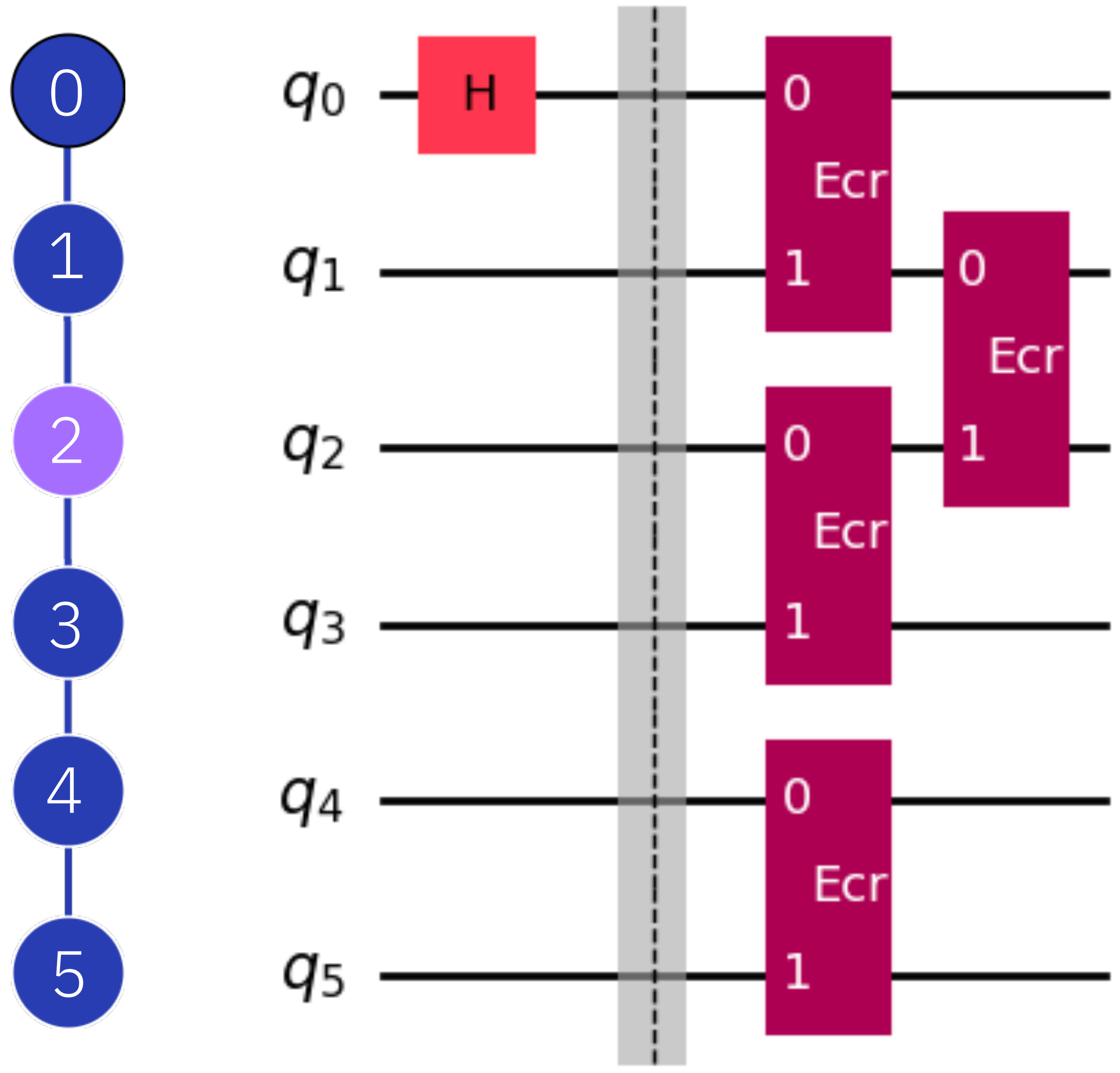
A number of solutions for coherent errors are know: dynamical decoupling, Twirling (random compiling).

Single-qubit over rotation for d cycles

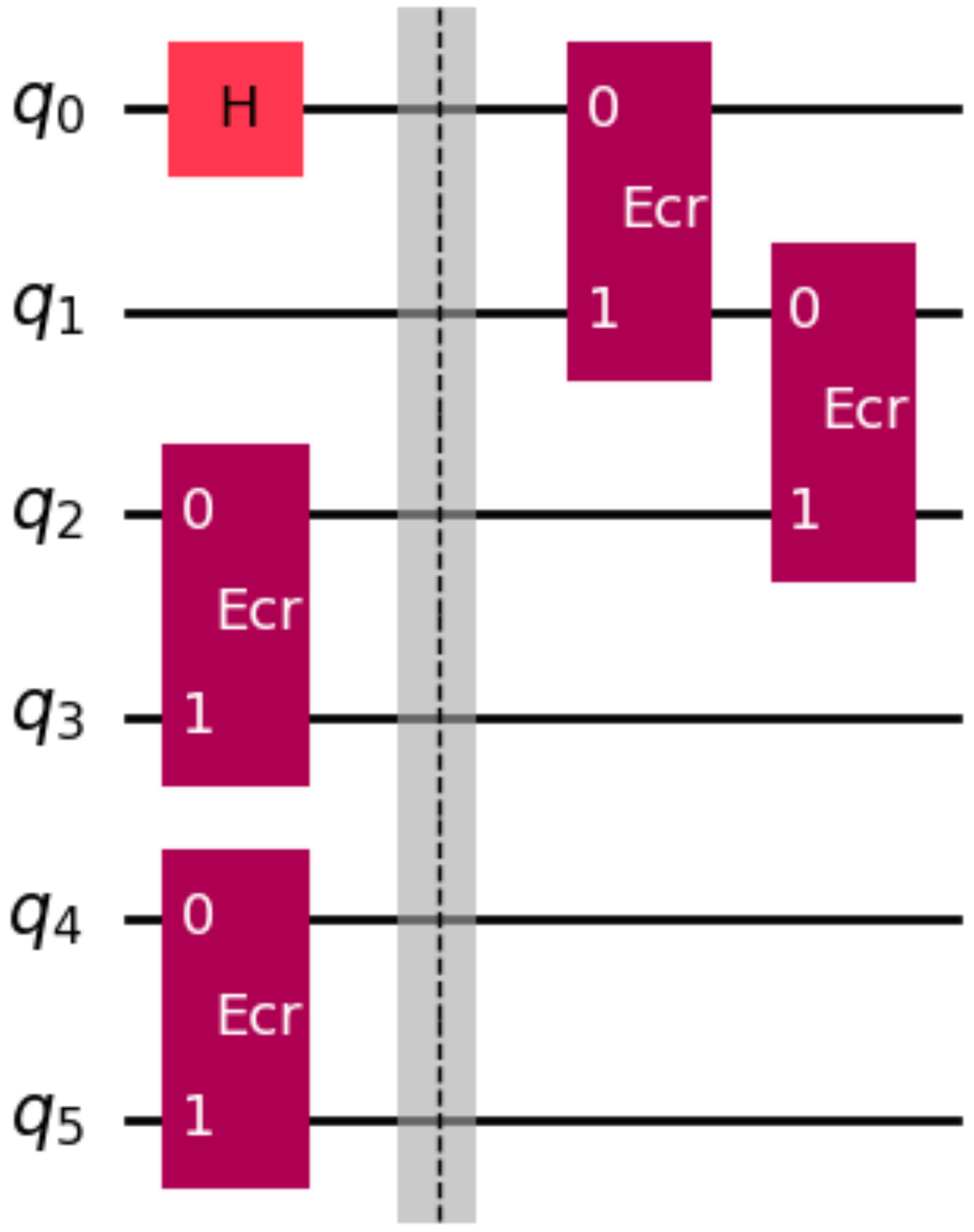


Example in Qiskit Global Summer School 2023, lecture by Dr. Zlatko Minev

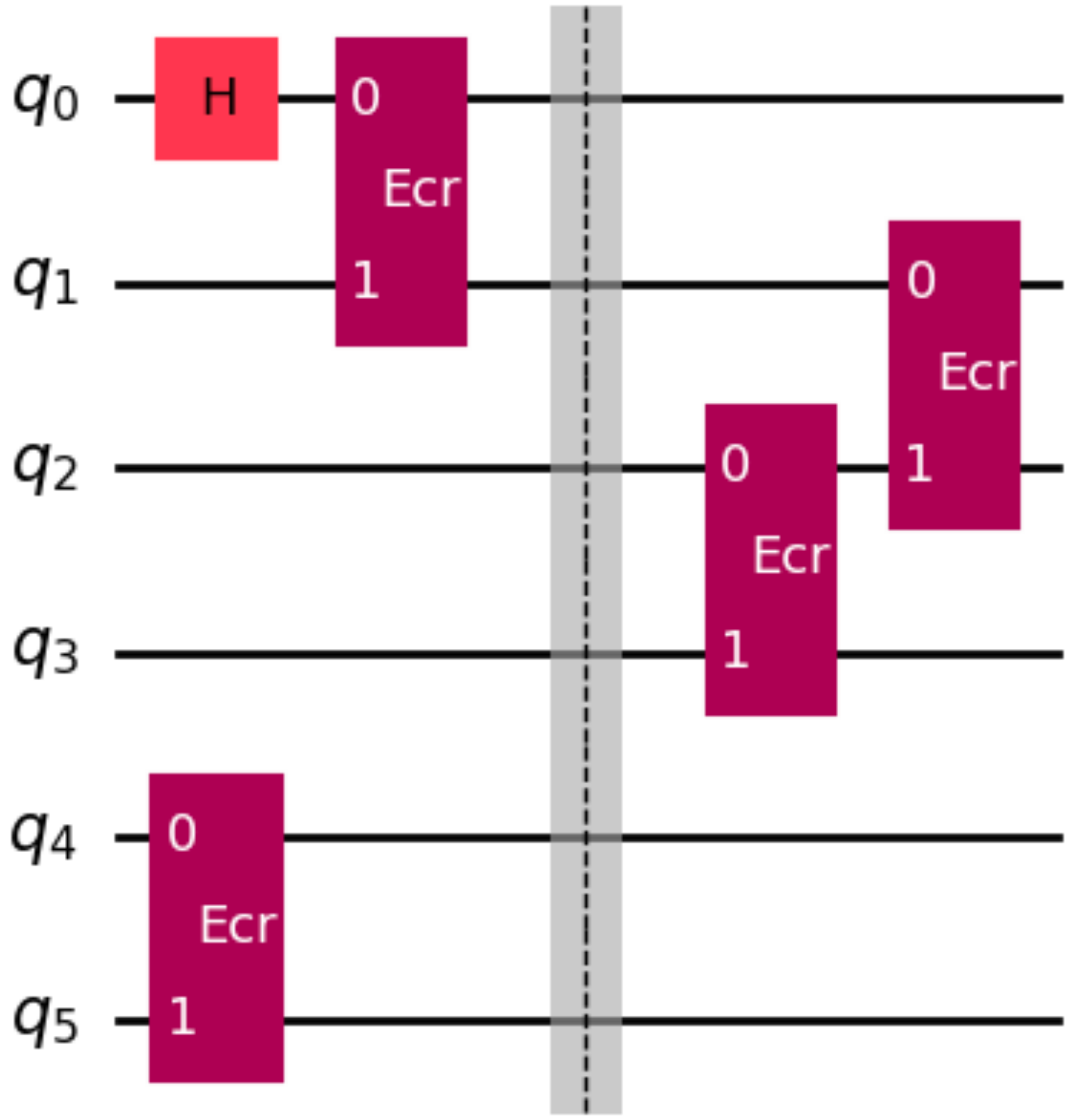
Consider a chain of physically connected qubits:



Crosstalk



Decoherence



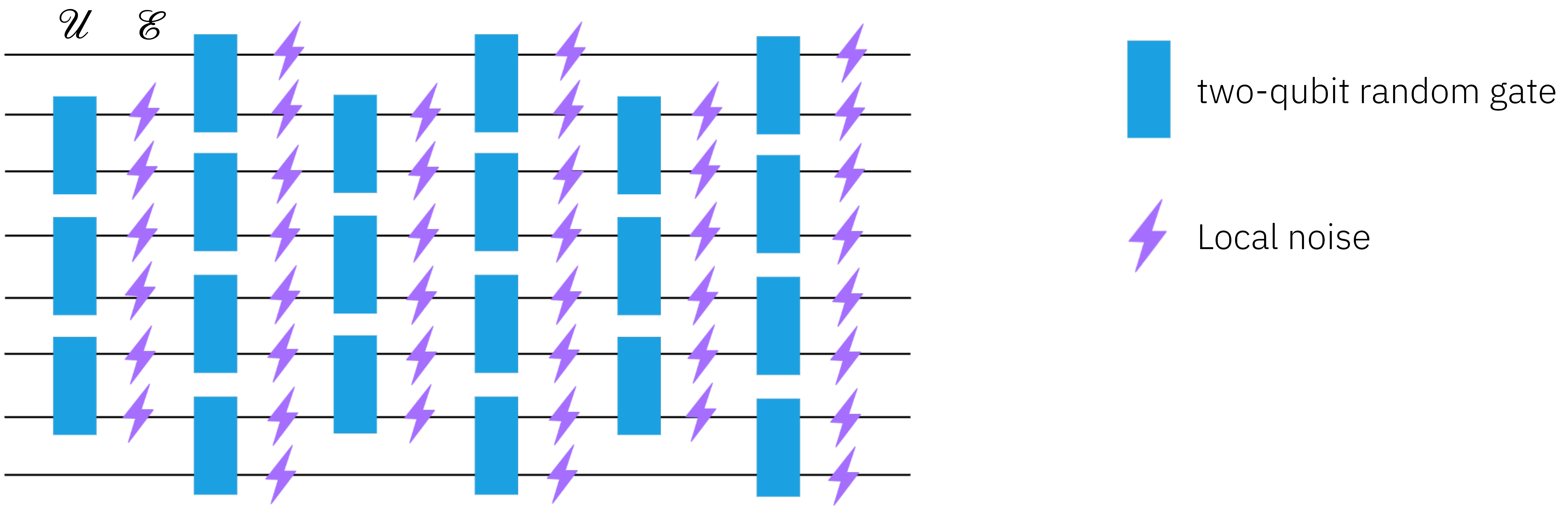
As-late-as-possible (ALAP) scheduling

P. Murali et al., Software Mitigation of Crosstalk on Noisy Intermediate-Scale Quantum Computers, ASPLOS '20

How noise propagates in time and space?

Many applications of near-term quantum computing involve layered circuits

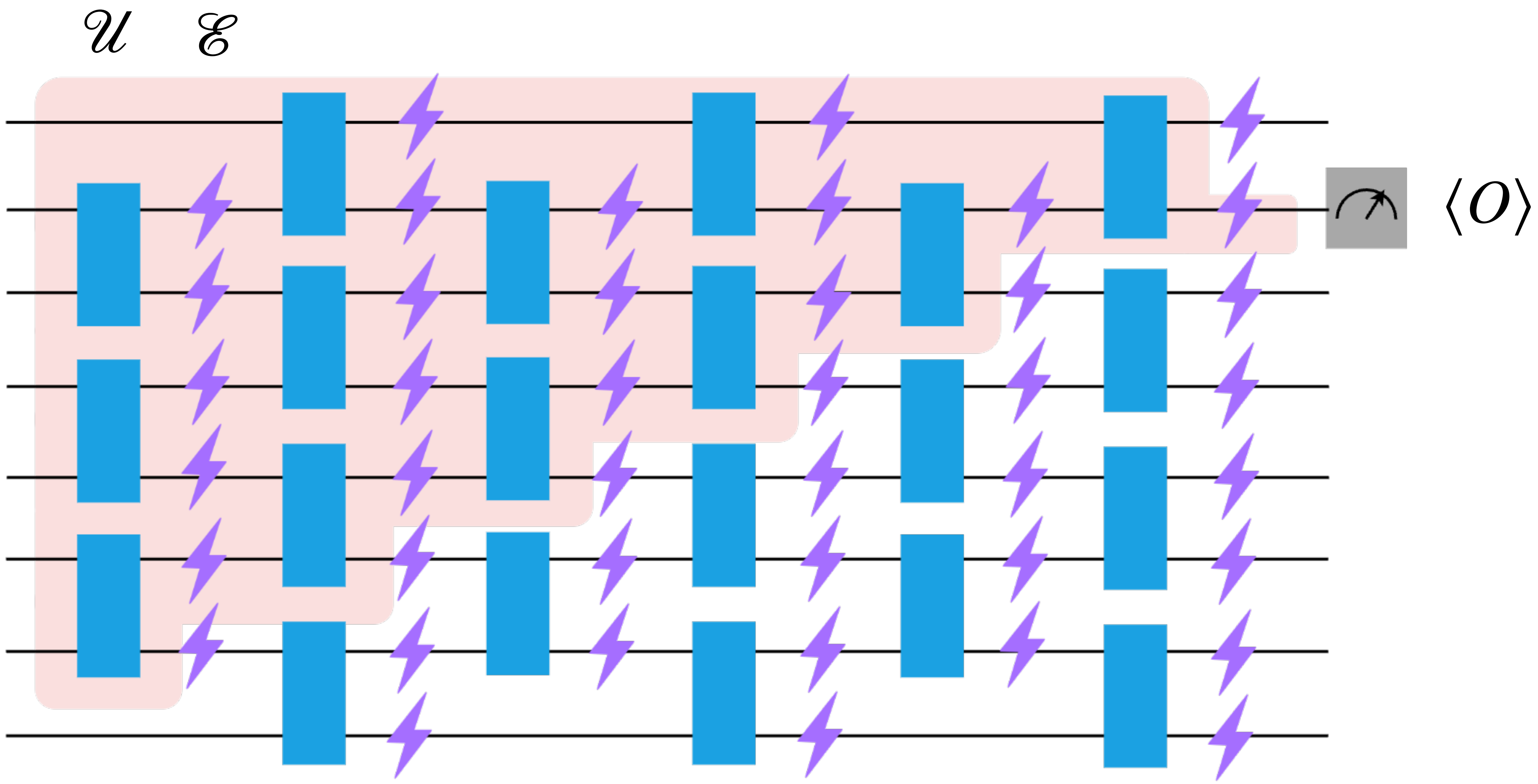
Brickwork random circuit on a 1D chain



1. The output of the circuit converges exponentially fast to the maximally mixed state,
2. Noise in the deep depth limit can be characterized by the global depolarizing noise.

K. Tsubouchi et al., Universal cost bound of quantum error mitigation based on quantum estimation theory, arXiv: 2208.09385 (2023)

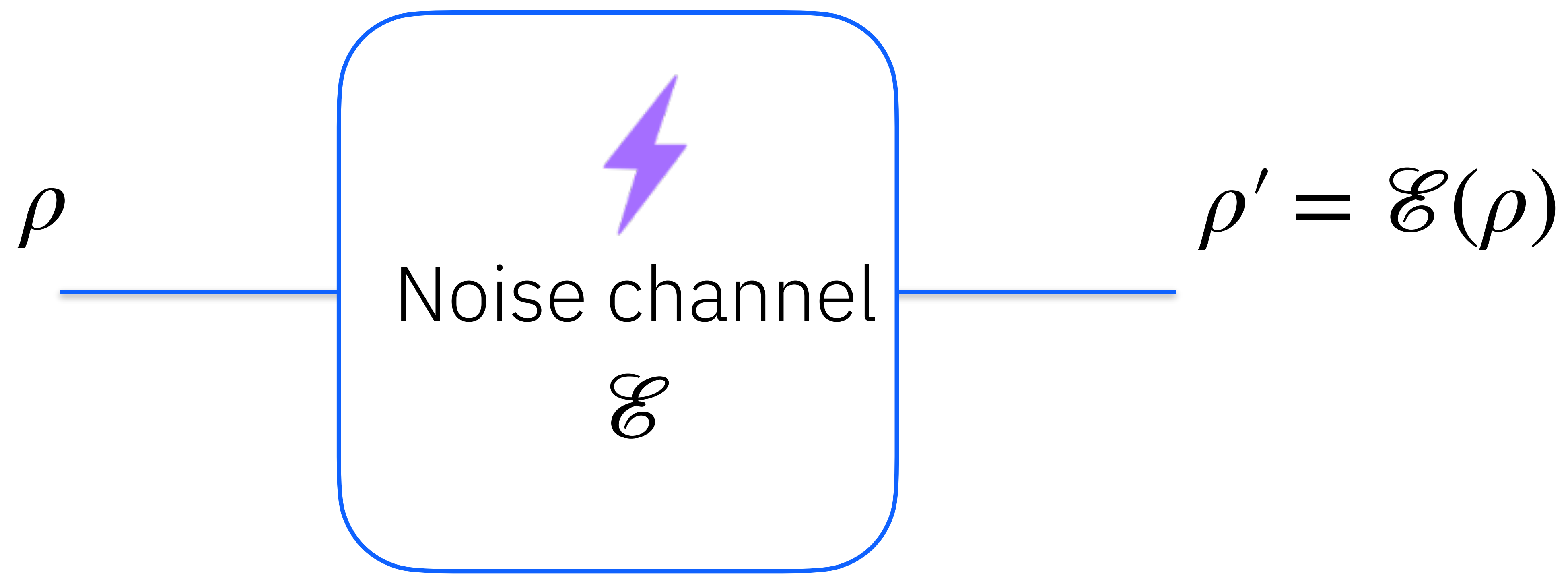
How noise propagates in time and space?



In the case of estimating expectation values, only errors in the backwards lightcone (shaded pink) contribute.

M. Tran, K, Sharma, K, Temme, Locality and Error Mitigation of Quantum Circuits, arXiv:2303.06496, 2023

Noise characterization



Quantum state tomography

Learn an unknown quantum state ρ from experiments

For a single qubit, we can expand ρ as:

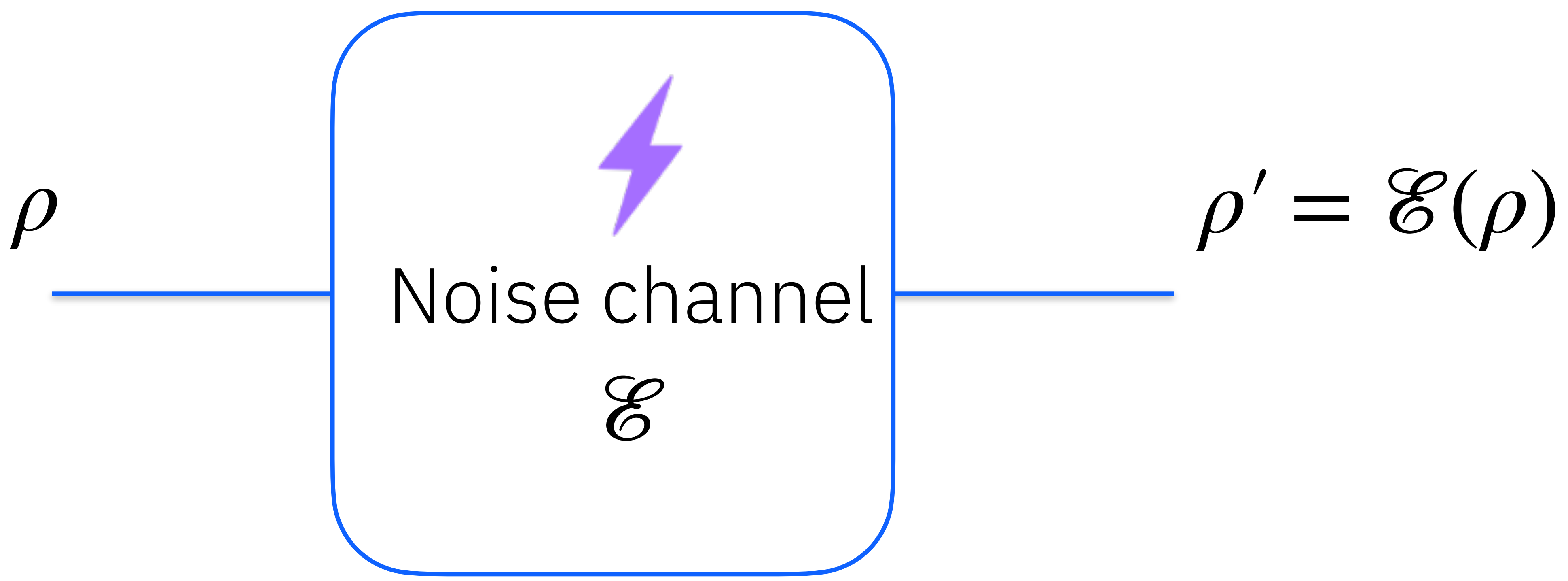
$$\rho = \frac{\mathrm{tr}(\rho)I + \mathrm{tr}(X\rho)X + \mathrm{tr}(Y\rho)Y + \mathrm{tr}(Z\rho)Z}{2}$$

Measure the expectation value of the unknown state with all elements of a complete basis
Needs many copies of the unknown state
Needs to do $4^n - 1$ different measurements for n qubits
Exponentially hard; active area of research to make this easier.

Noise characterization

Quantum process tomography

Learn an unknown quantum process $\mathscr{E}$ from experiments



Prepare 4^n input states: $|n\rangle$

$$|m\rangle$$
$$|+\rangle = (|n\rangle + |m\rangle)/\sqrt{2}$$
$$|-\rangle = (|n\rangle + i|m\rangle)/\sqrt{2}$$

Measure $\mathscr{E}(\rho)$ in 4^n complete basis

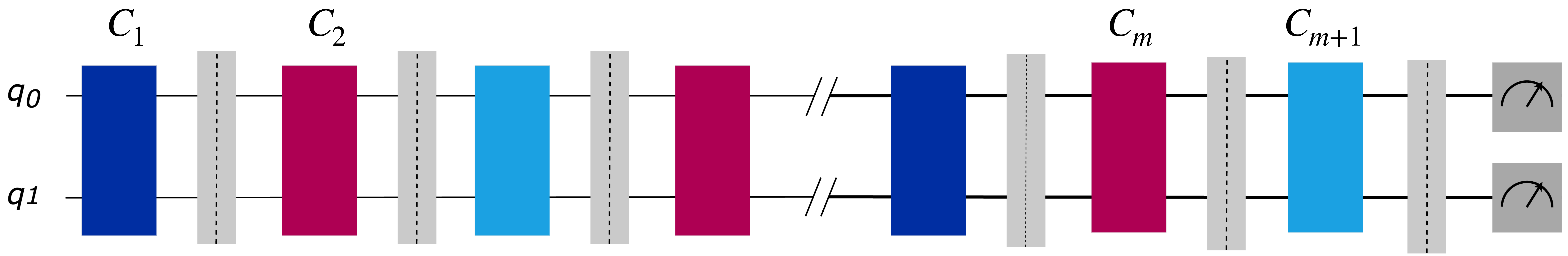
$\mathscr{E}$ has $d^4 - d^2$ independent real parameters

16^n experiments, even harder than state tomography; active area of research to make it scalable.

How noise is characterized: randomized benchmarking (RB)

Can we quantify how much error happens *per gate* in a circuit?

Single/Two-qubit randomized benchmarking



C_i is a single/two-qubit random gate sampled from a finite Clifford gate set;

C_{m+1} is the performed to make the total sequence equal to identity operation

Measure the probabilities to get back to the ground state at the end of the sequence.

Vary sequence length, fit the fidelity decay to an exponential curve to report Error Per Clifford.

Extract error rate per gate (EPG) in RB

Fit the survival probability to an exponential

$$A\alpha^m + B$$

Report average gate error:

$$\epsilon = \frac{d-1}{d}(1-\alpha)$$

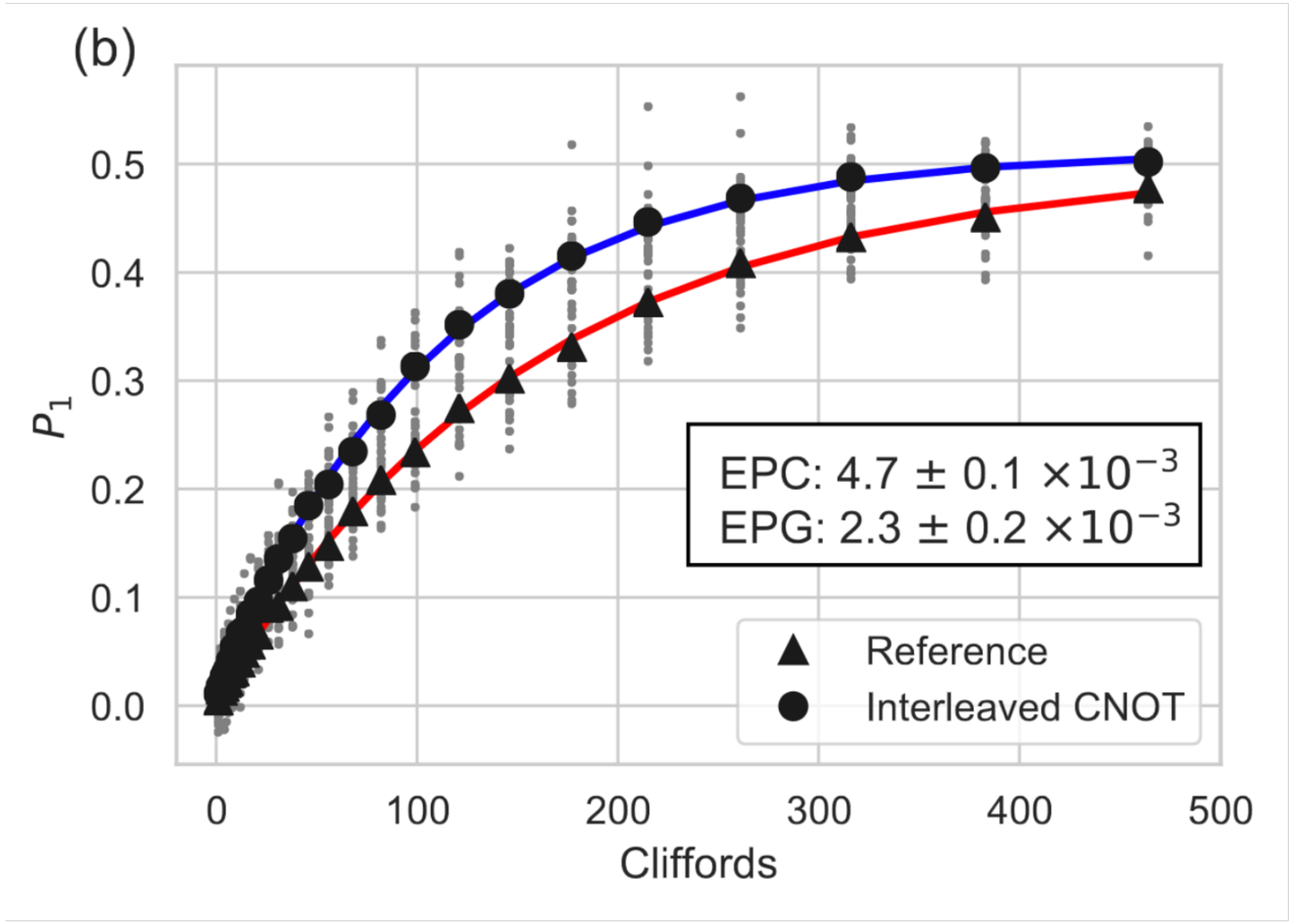
Average gate fidelity

$$F_g = \frac{dF_p + 1}{d+1}$$

In RB, we report the average gate error per Clifford gate

$$\epsilon_g = 1 - F_g$$

Limitation of RB: it does not capture coherent or crosstalk error



Example from Rhys. Rev. Lett. 127.130501 (2021)

Reported gate errors

ibm_sherbrooke

OpenQASM 3

Details

127
Qubits

1.7%
EPLG

5K
CLOPS

Status:	Online
System region:	us-east
Total pending jobs:	25 jobs
Processor type ⓘ:	Eagle r3
Version:	1.4.49
Basis gates:	ECR, ID, RZ, SX, X
Your instance usage:	0 jobs

Median ECR error:	6.983e-3
Median SX error:	2.093e-4
Median readout error:	1.370e-2
Median T1:	264.82 us
Median T2:	185.58 us

Layer fidelity: error per layer gate (EPLG)

Layer fidelity expands on randomized benchmarking

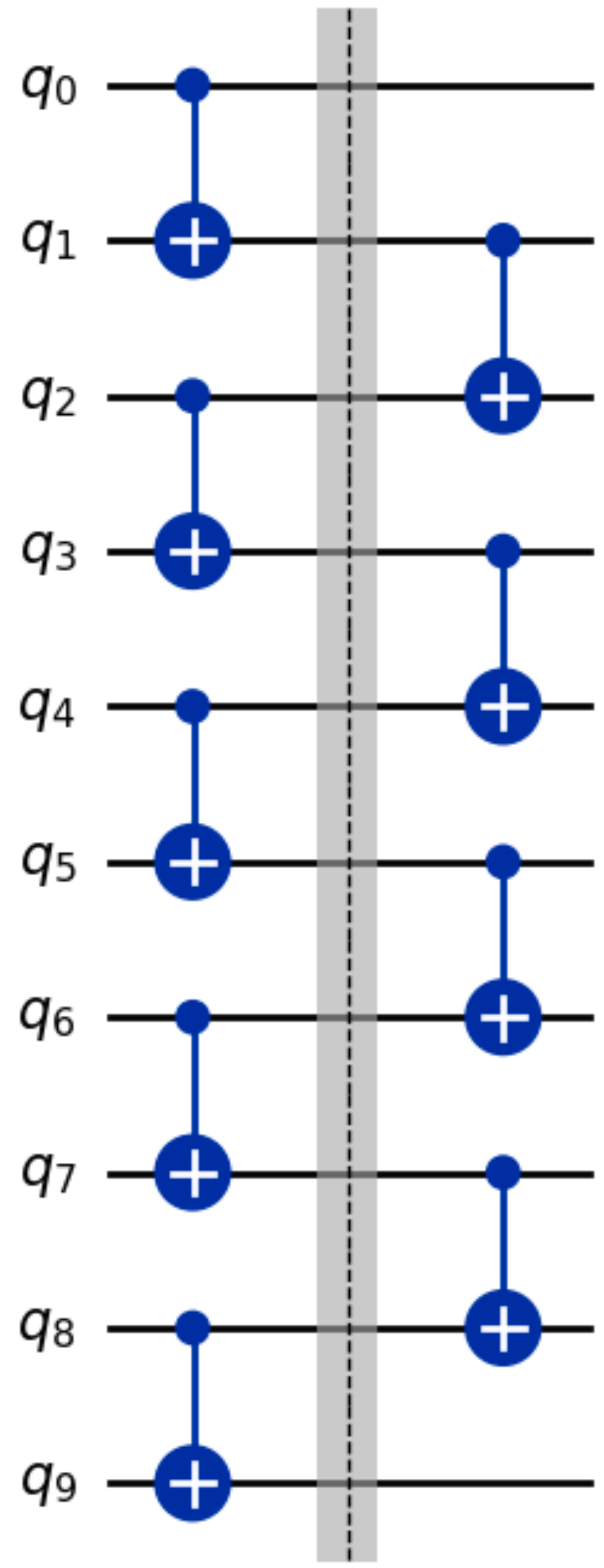
Layer structures are the building blocks for a lot of circuits we care about

Includes crosstalk

Relates to other metrics to infer the error mitigation overhead

N connected qubits
A set of connecting gates on that device from the entangling Clifford gates

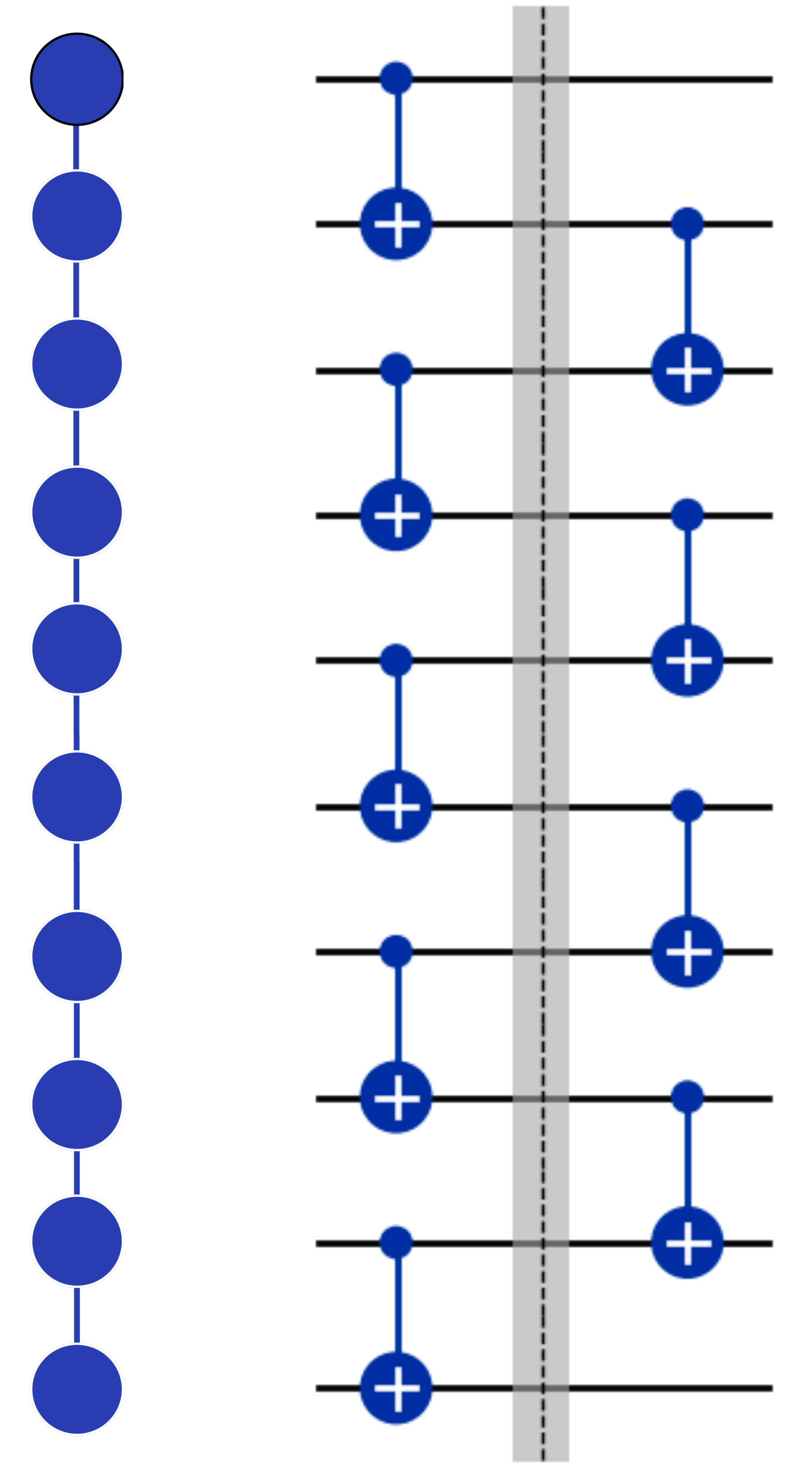
We ask, what is the process fidelity of this layer of gates?



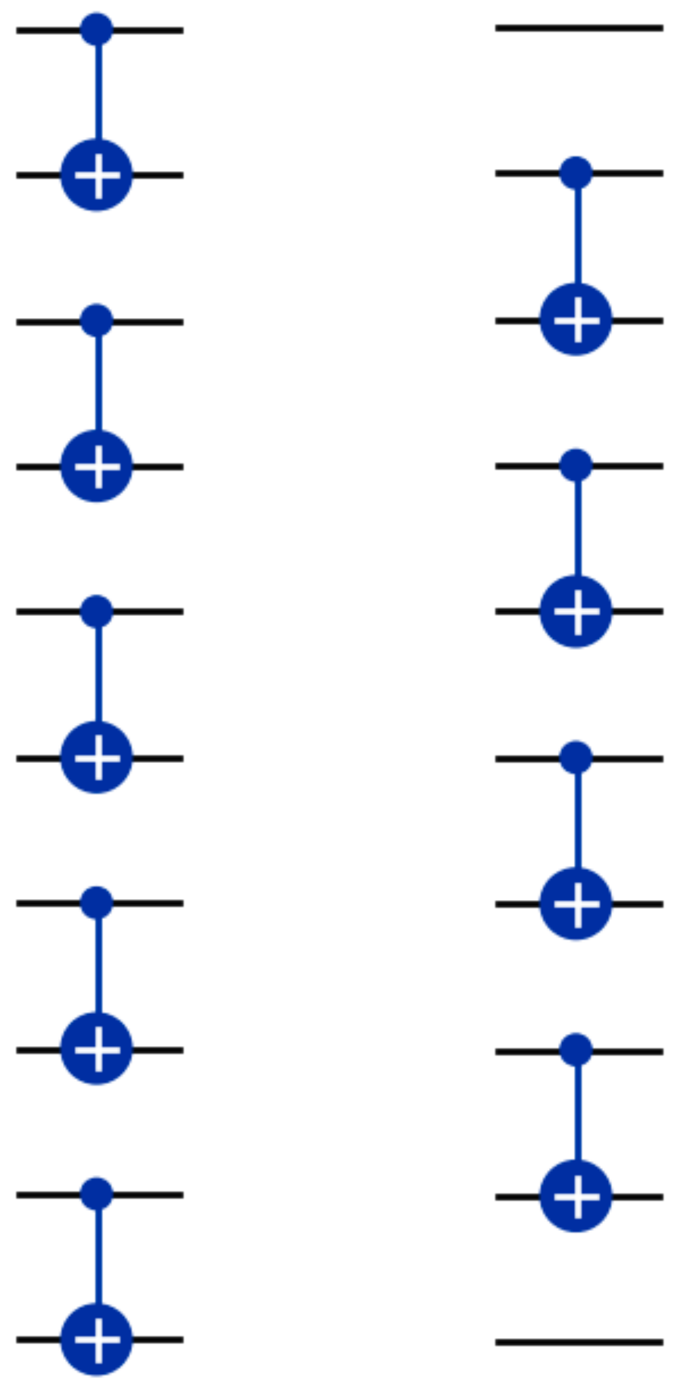
D. McKay et al., Benchmarking Quantum Processor Performance at Scale, arXiv: 2311.05933 (2023)

Layer fidelity circuits

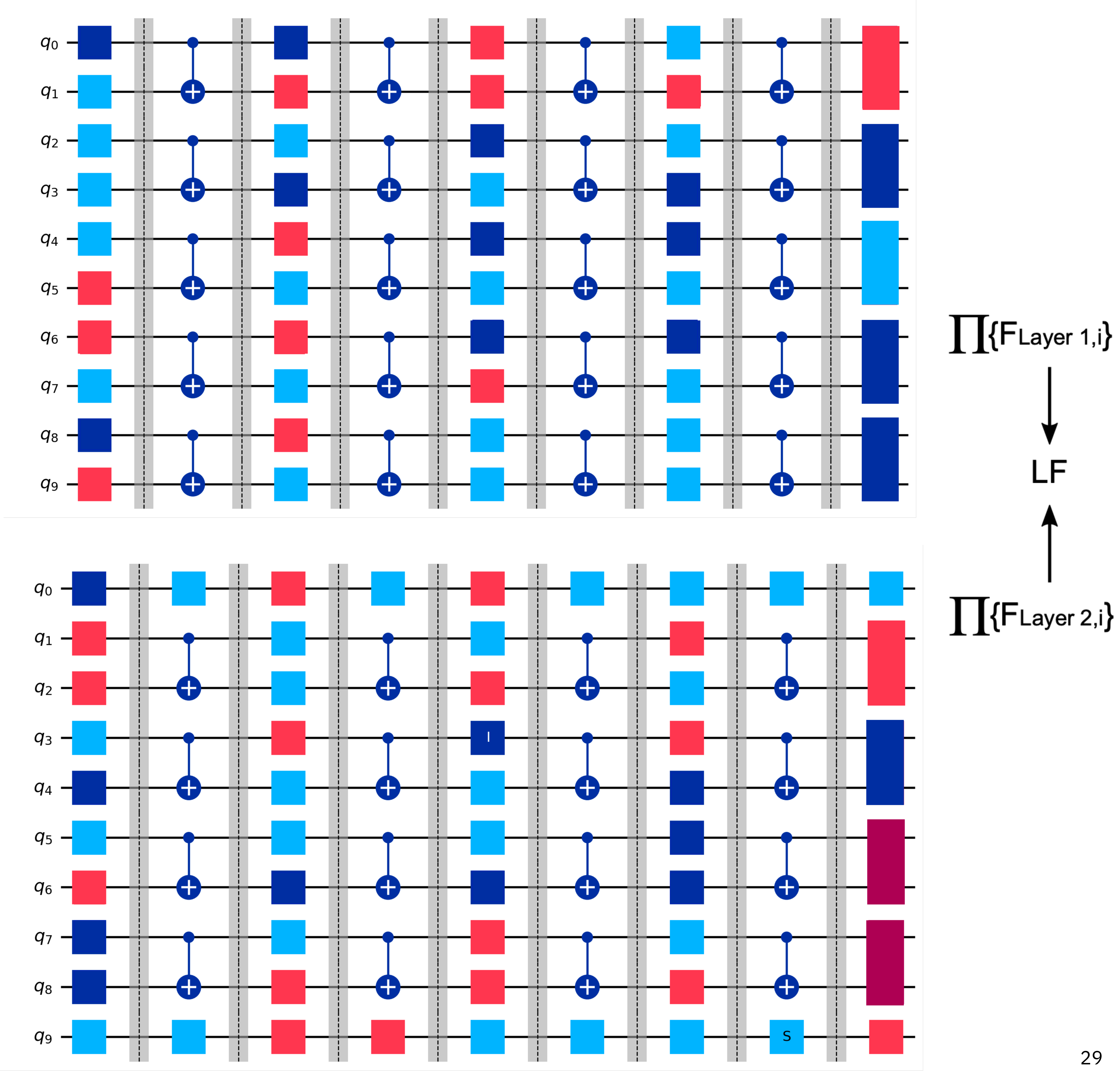
1 Define layer structure over physically connected qubits



2 Separating the layer into disjoint sets and measure with RB

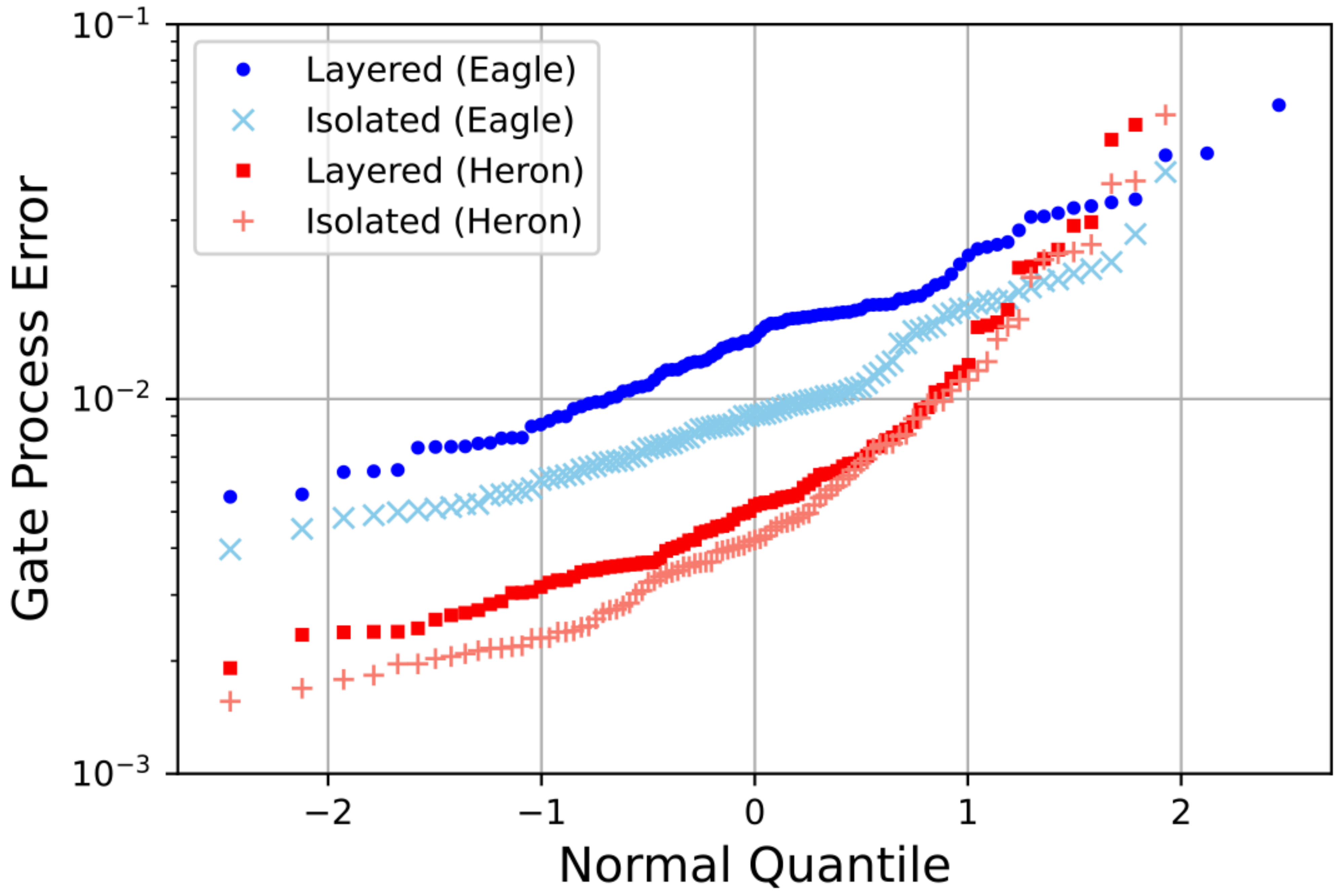


3 1Q/2Q Simultaneous, Direct RB with Barriers

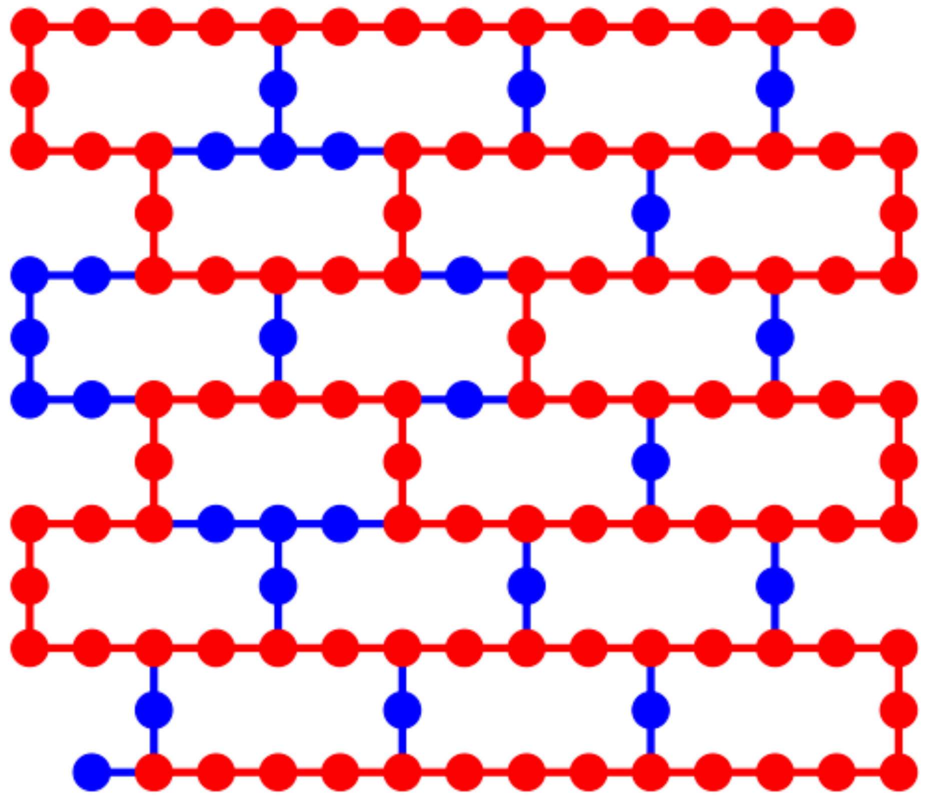


EPLG vs RB (EPG)

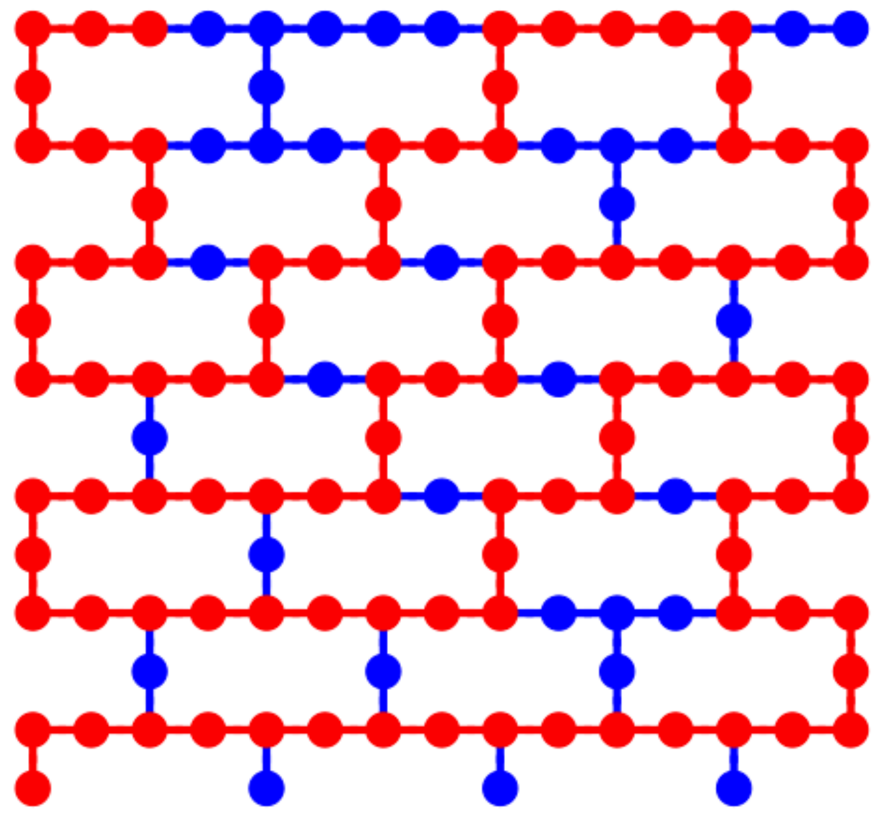
$$EPLG = 1 - LF^{1/n_{2q}}$$



Eagle processor



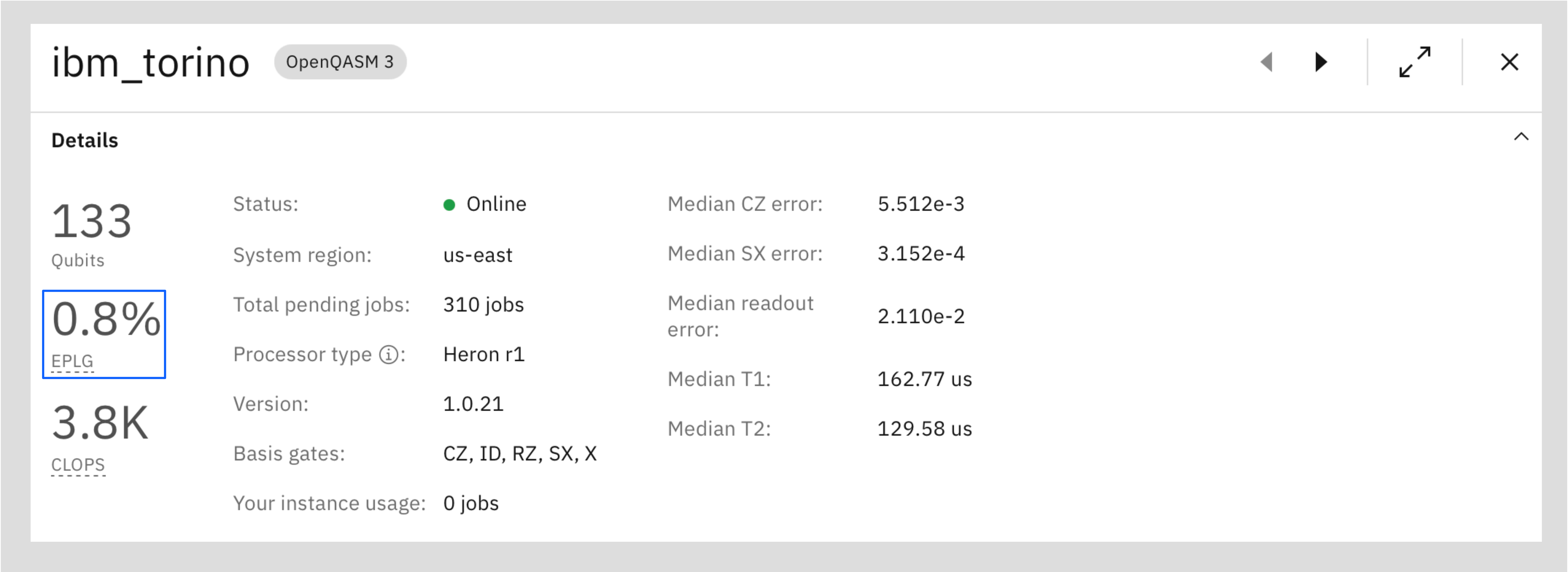
Heron processor



Quantile plot of the individual gate errors measured from the best 100 qubit chain from

simultaneous direct RB ("layered") versus the backend reported gate errors ("isolated")

EPLG metric reported on backends



Lab session: layer fidelity experiment via Qiskit Runtime led by Samantha Barron.

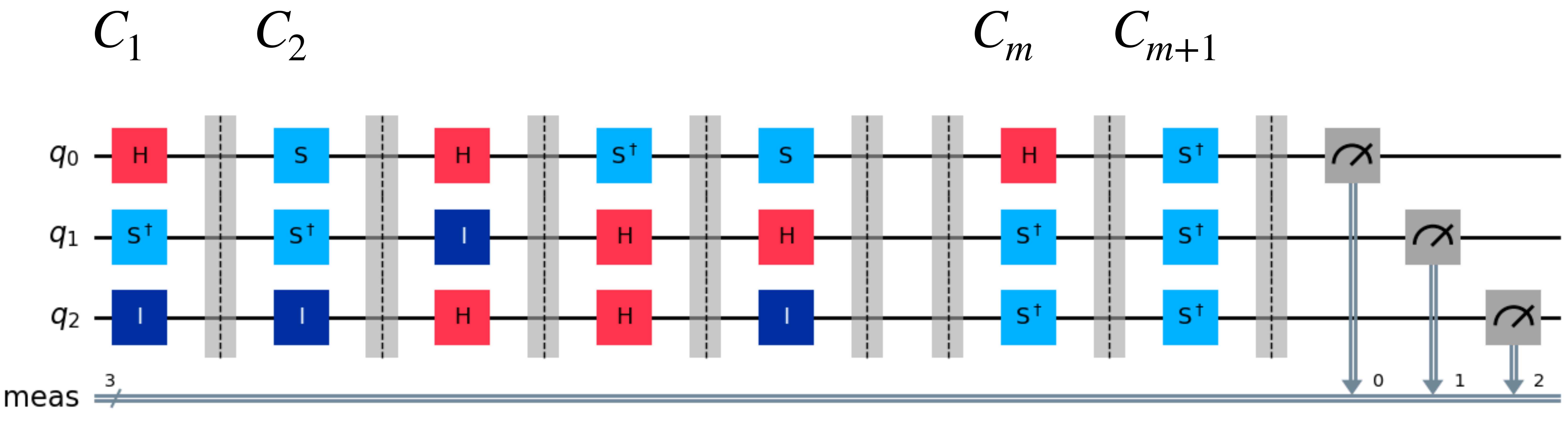
https://docs.quantum.ibm.com/run/system-information#system-configuration-values

IBM

How noise is characterized: randomized benchmarking

Can we quantify how much error happens *per gate* in a circuit?

Single-qubit randomized benchmarking



Use the idea of randomization

For sequence length m

Step 1: fix $m \leq M - 1$ and generate K_m sequences consisting of $m + 1$ quantum operations;

Step 2: For each of the K_m sequences, measure the survival probability

Step 3: Average over the K_m random realizations to find the averaged sequence fidelity

Repeat Steps 1 through 3 for difference values of m, and fit for the model

C_i is a single-qubit random gate sampled from a finite Clifford gate set;
C_{m+1} is the performed to make the total sequence equal to identity operation.

Navigating the IBM Quantum Platform

Noise-related metrics

T1

T2

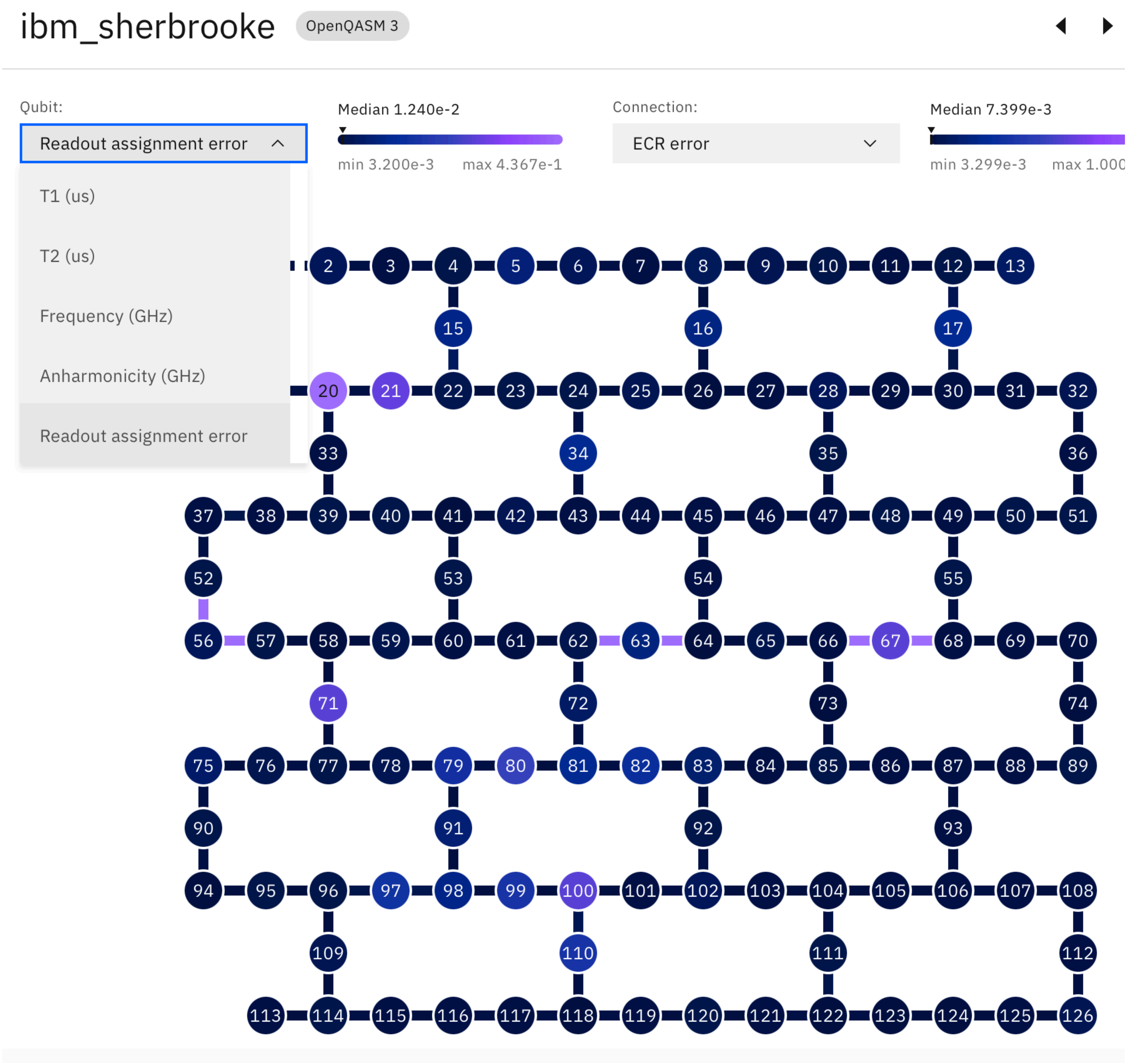
Readout assignment error

ID error

sx error

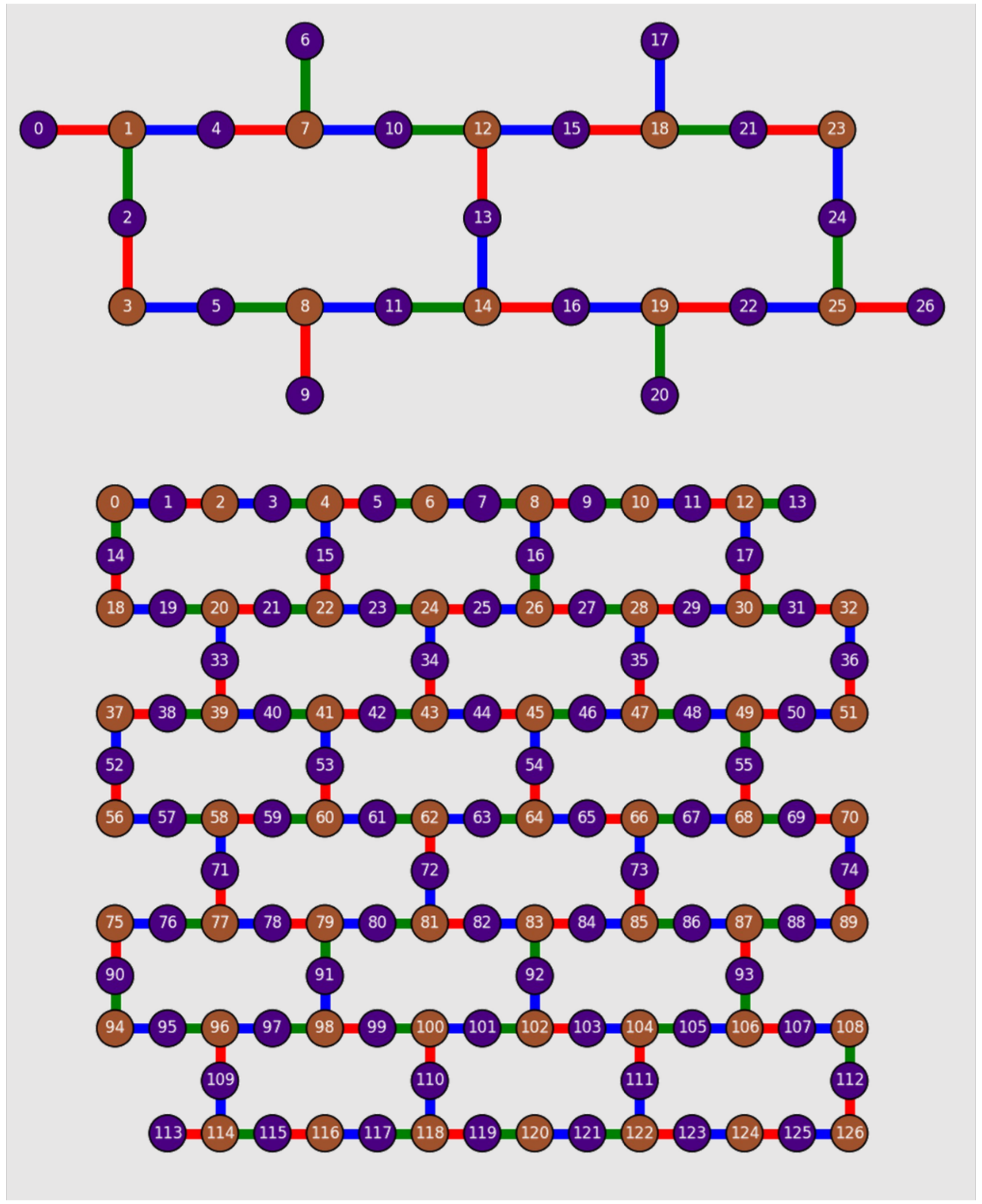
Pauli-x error

ECR error



Characterize noise by layer: layer fidelity

EPLG



Can we quantify how much error happens per layer in a circuit?

Edge-coloring problem of the coupling graph:
Assign colors to edges of the graph so that no two incident edges have the same color.

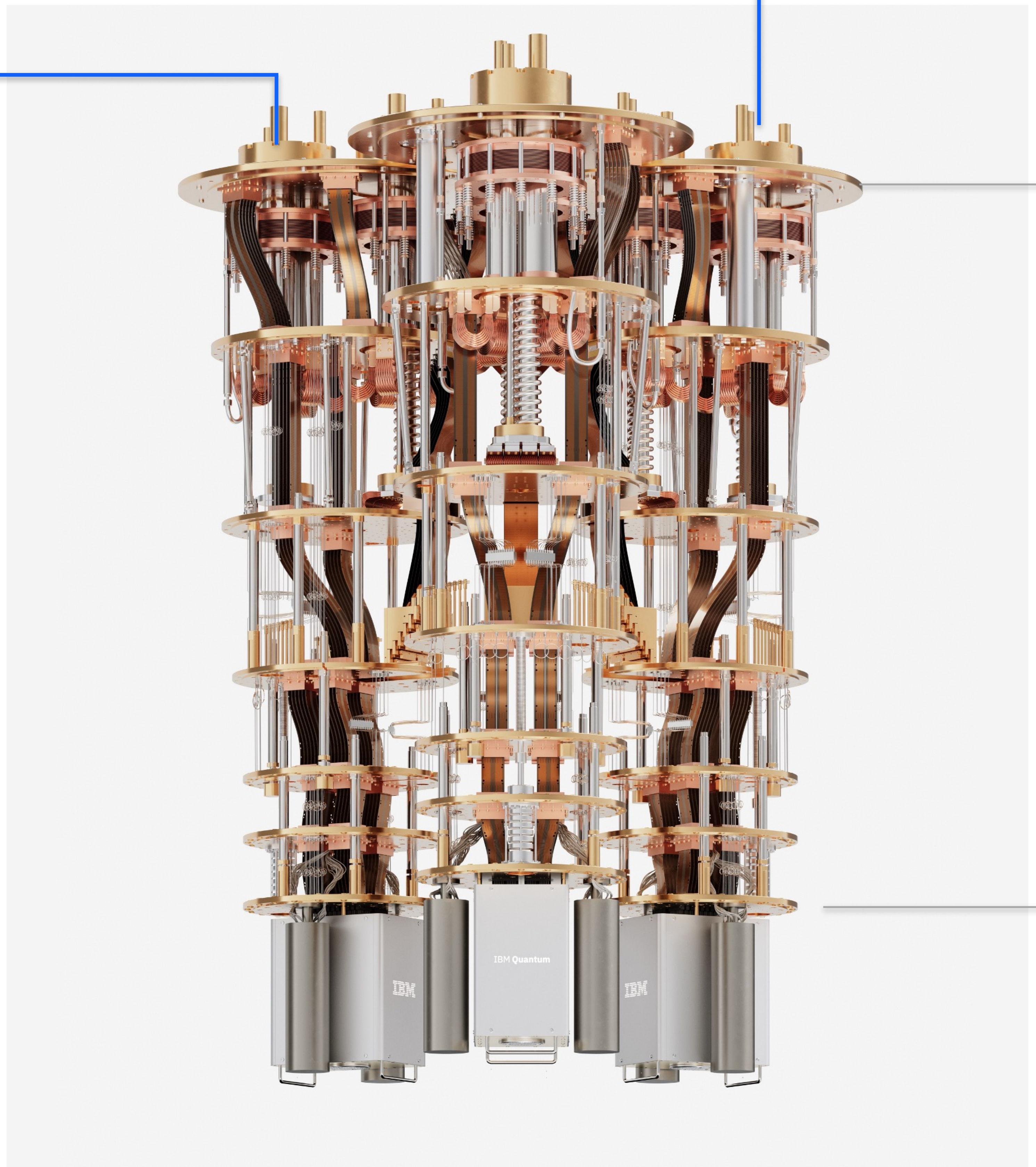
Quantum hardware is

a device that computes an output from input data
using fundamental rules of quantum mechanics.

microwave pulse schedules

readout signal

room temperature



20 mK

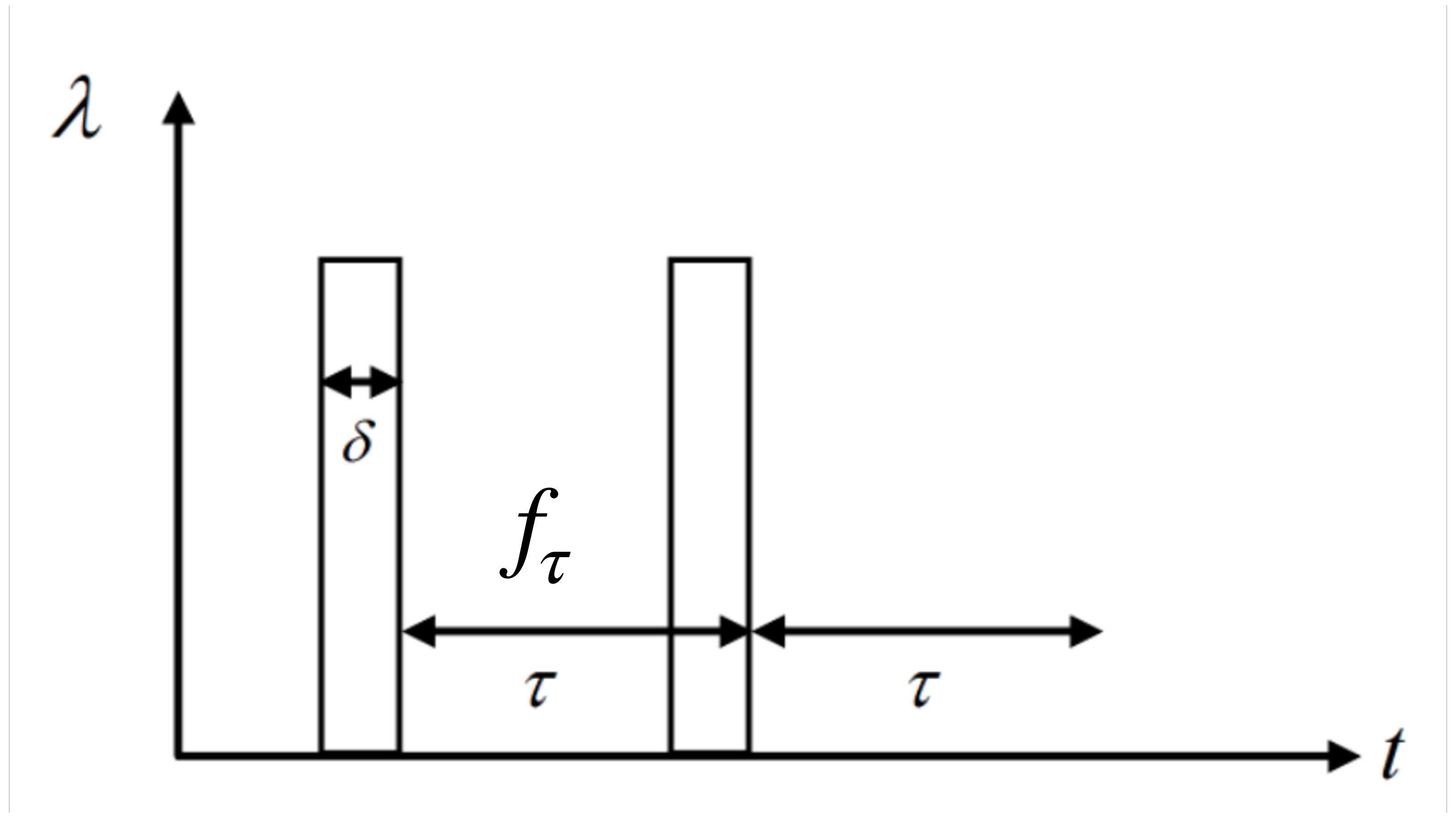
quantum circuits

quantum algorithms

Quantum information is stored in a quantum bit, made of **superconducting circuits**

Noise can be suppressed using single-qubit control gates: dynamical decoupling

A sequence of pulses applied with the goal to remove unwanted system-bath interactions



λ: amplitude

δ: duration

Example: a pure dephasing system-bath coupling

$$H_{SB} = \sigma^z \otimes B^z$$

Apply a time-dependent control on the system:

$$H_S = \lambda(t)\sigma^x$$

Ideal pulse: $\delta \to 0, \lambda \to \infty$

Free evolution: $f_\tau \equiv e^{-i\tau H_{SB}}$

Pulse: $X \equiv e^{-i\delta\lambda\sigma^x} \otimes I_B = e^{-i\frac{\pi}{2}\sigma^x} \otimes I_B = -i\sigma^x \otimes I_B$

Evaluate $Xf_\tau Xf_\tau$ at time $t = 2\tau$:

$$Xf_\tau Xf_\tau = \sigma_x e^{-i\tau H_{SB}} \sigma_x e^{-i\tau H_{SB}} = e^{-i\tau\sigma_x H_{SB}\sigma_x} e^{-i\tau H_{SB}} = e^{+i\tau H_{SB}} e^{-i\tau H_{SB}} = I$$

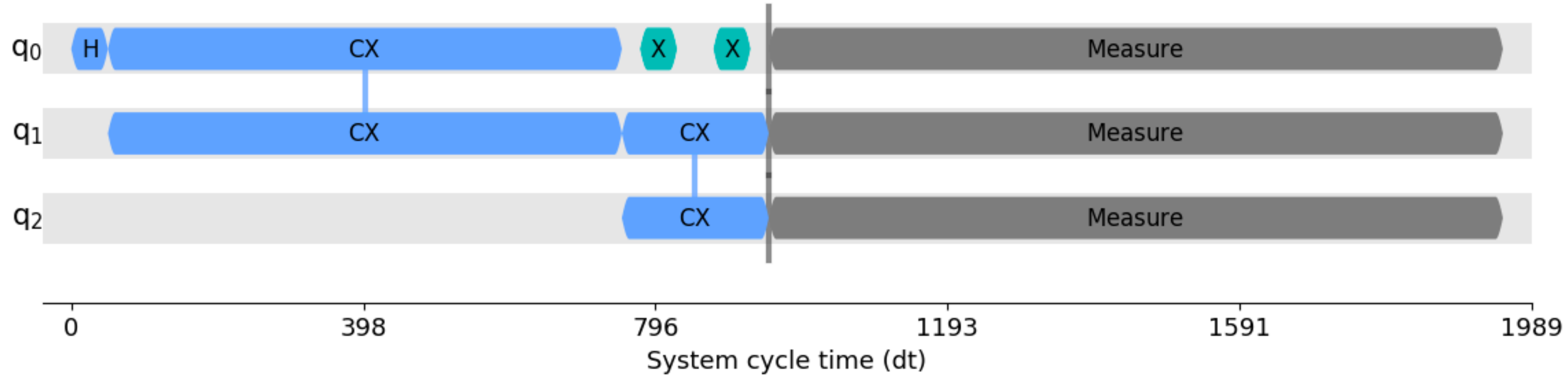
Bath has no effect on the system at the instant $t = 2\tau$!

Noise can be suppressed using single-qubit control gates: dynamical decoupling

In Qiskit, DD is added to circuit evolution by a transpilation pass

Example in Qiskit Docs: https://docs.quantum.ibm.com/api/qiskit/qiskit.transpiler.passes.DynamicalDecoupling

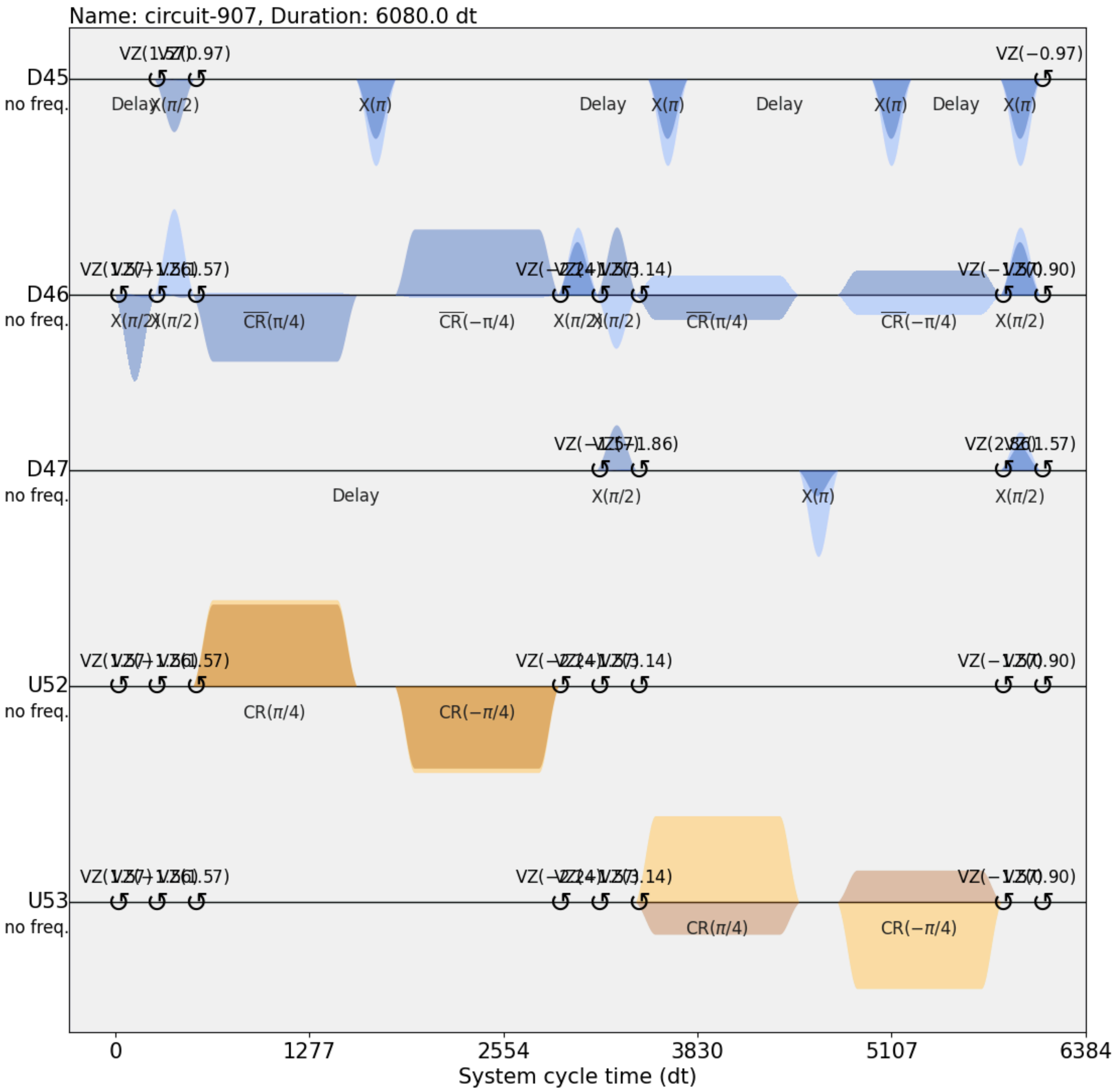
```python
import numpy as np
from qiskit.circuit import QuantumCircuit
from qiskit.circuit.library import XGate
from qiskit.transpiler import PassManager, InstructionDurations
from qiskit.transpiler.passes import ALAPSchedule, DynamicalDecoupling
from qiskit.visualization import timeline_drawer

# Because the legacy passes do not propagate the scheduling information correctly, it is
# necessary to run a no-op "re-schedule" before the output circuits can be drawn.
def draw(circuit):
    from qiskit import transpile

    scheduled = transpile(
        circuit,
        optimization_level=0,
        instruction_durations=InstructionDurations(),
        scheduling_method="alap",
    )
    return timeline_drawer(scheduled)

circ = QuantumCircuit(4)
circ.h(0)
circ.cx(0, 1)
circ.cx(1, 2)
circ.cx(2, 3)
circ.measure_all()
durations = InstructionDurations(
    [("h", 0, 50), ("cx", [0, 1], 700), ("reset", None, 10),
     ("cx", [1, 2], 200), ("cx", [2, 3], 300),
     ("x", None, 50), ("measure", None, 1000)]
)
# balanced X-X sequence on all qubits
dd_sequence = [XGate(), XGate()]
pm = PassManager([ALAPSchedule(durations),
                  DynamicalDecoupling(durations, dd_sequence)])
circ_dd = pm.run(circ)
draw(circ_dd)
```



Noise can be suppressed using single-qubit control gates: dynamical decoupling

In practice, pulse has finite duration

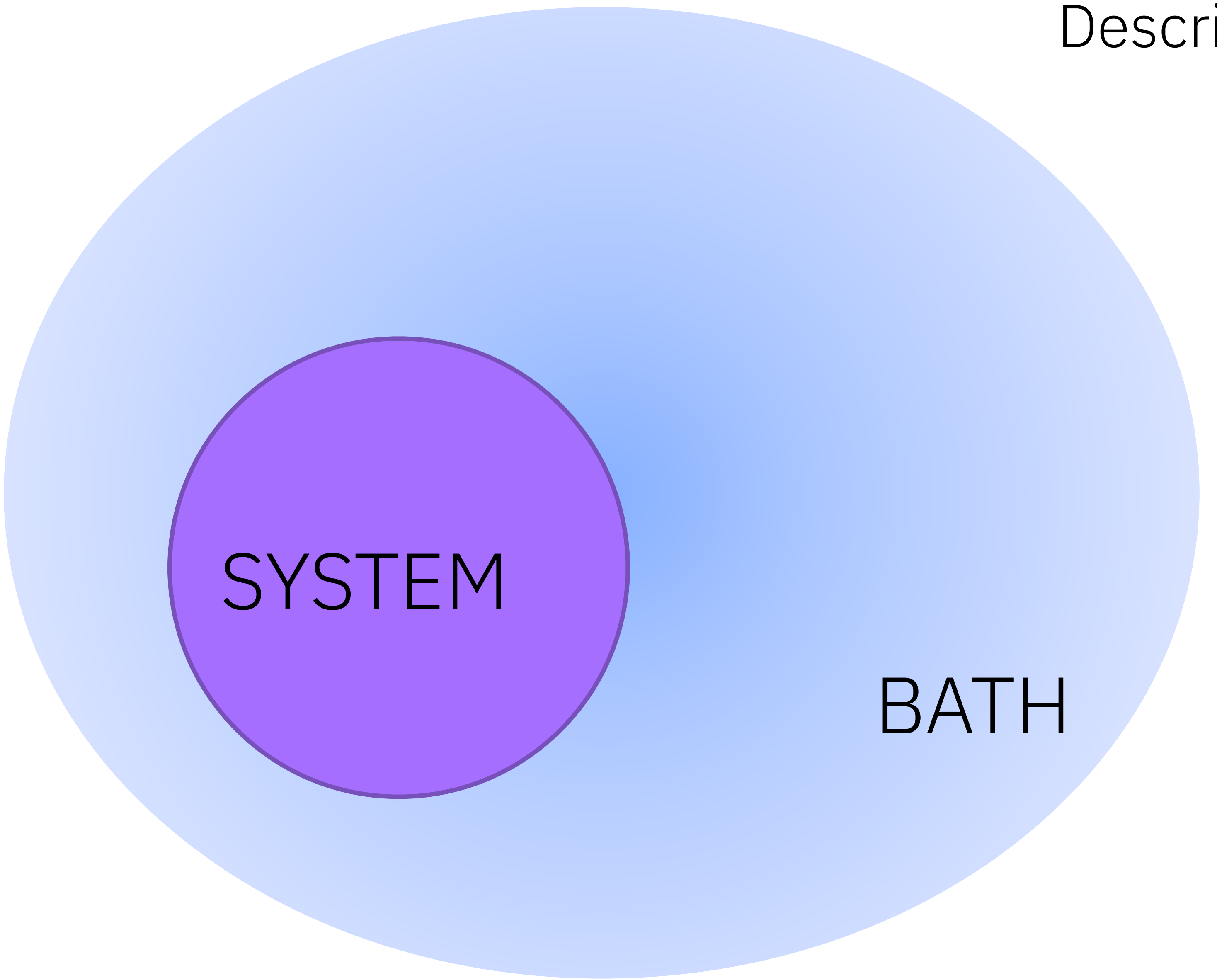
Example in Qiskit Docs: https://docs.quantum.ibm.com/api/qiskit/qiskit.transpiler.passes.DynamicalDecoupling



(Readout pulses omitted)

Summary

$$H = H_S + H_B + H_{SB}$$



Describes thermalization

H_{SB}: system-bath coupling

$$\tau = |\psi\rangle\langle\psi|$$

Pure state

An ensemble of pure quantum states:

$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

This is called the density matrix

Maximally mixed state:

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Mixed state

Source: 1. Lorem source name
Source: 2. Lorem second source name

Visualizing a quantum state

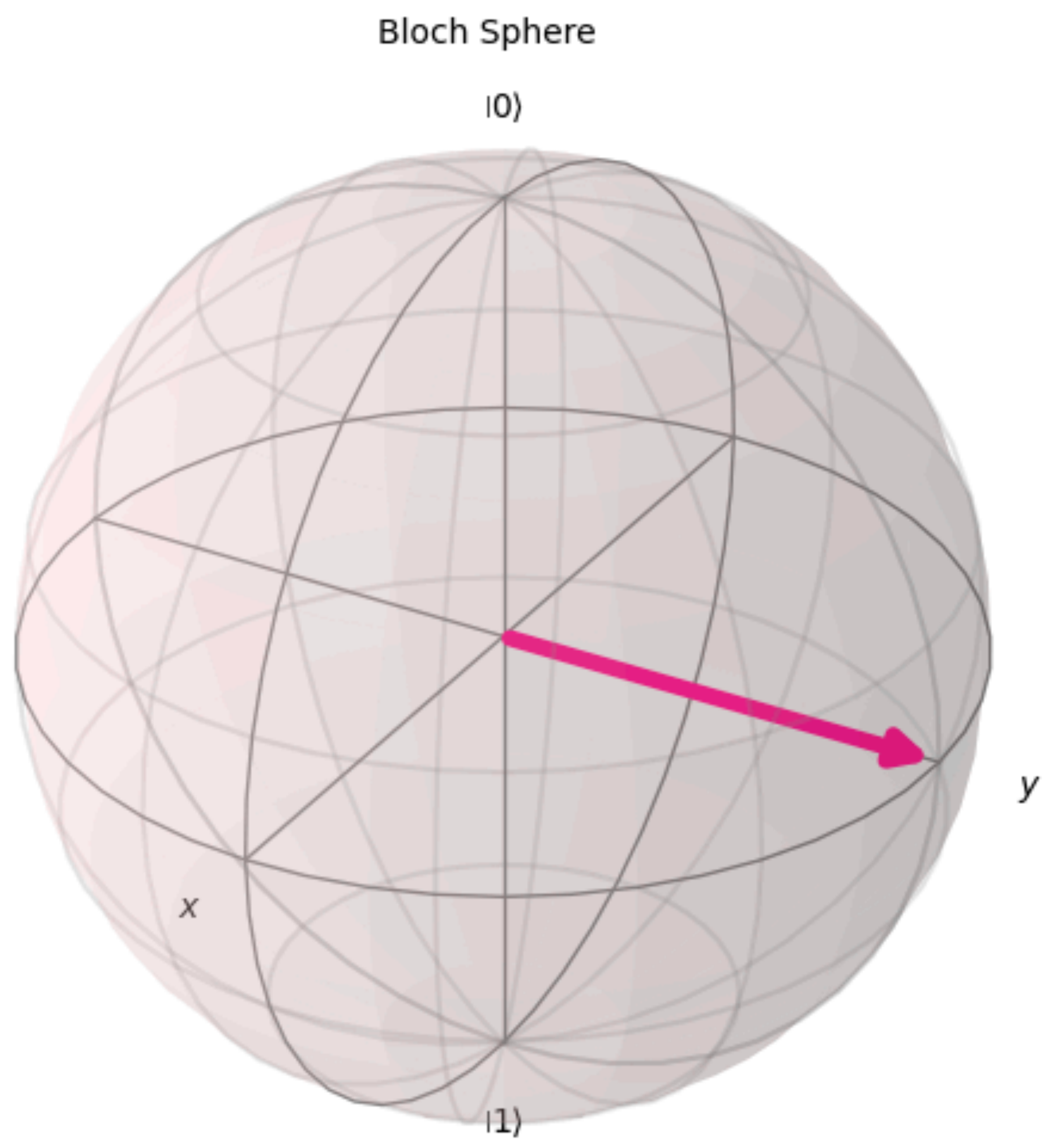
Bloch sphere representation

$$\rho = \frac{1}{2}\left(I + \overrightarrow{v} \cdot \overrightarrow{\sigma}\right)$$

where

$$\overrightarrow{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right),$$

$$\overrightarrow{v} = \left(v_x, v_y, v_z\right).$$

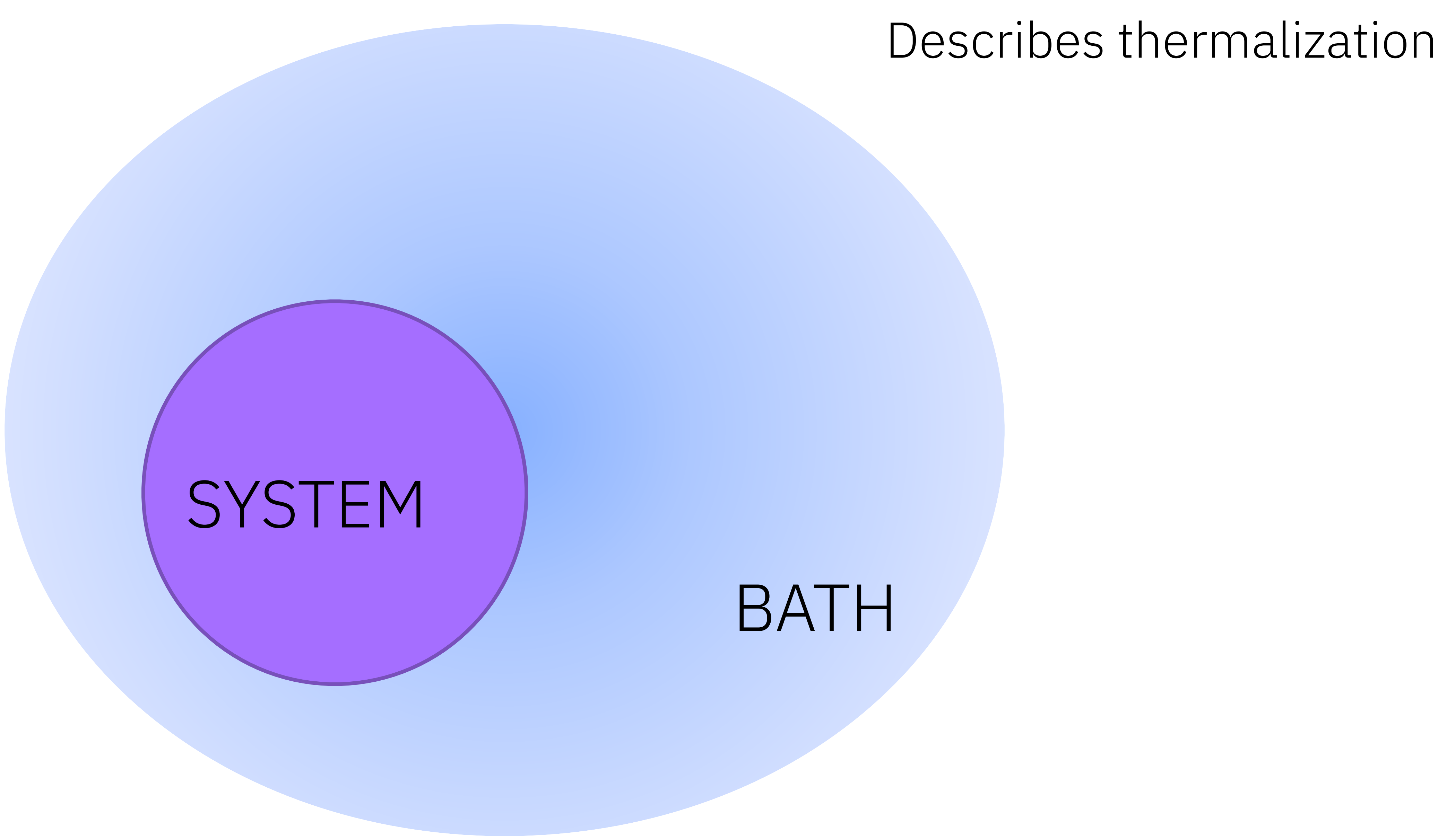


For pure states, $\|\overrightarrow{v}\| = 1$,

For mixed states, $\|\overrightarrow{v}\| < 1$

How to model a quantum computer?

$$H = H_S + H_B + H_{SB}$$



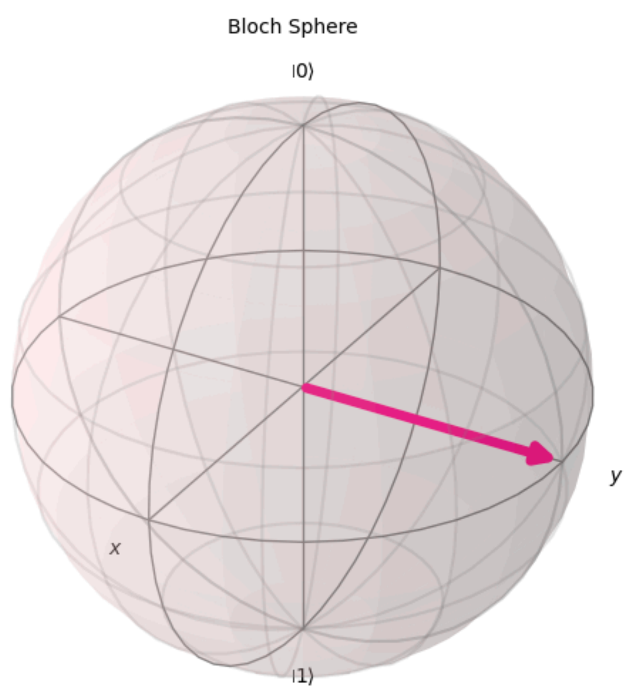
H_{SB}: system-bath coupling

Quantum map

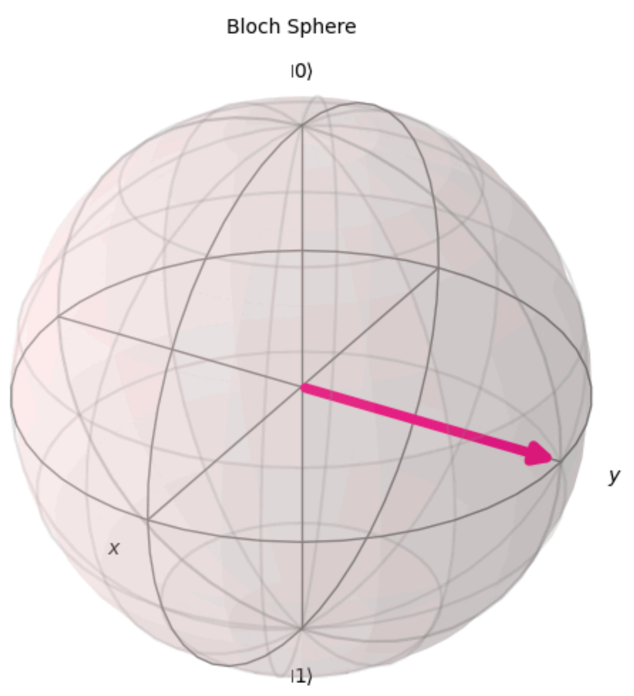
Transform one quantum state to another

Example:

$$\Phi : \rho \mapsto \rho'$$



$\mapsto$



Pauli transfer matrix

Example: the dephasing map

$$\rho \mapsto \rho' = \begin{cases} \frac{1}{2}I & \text{w/ prob. } p \\ \rho & \text{w/ prob. } 1-p \end{cases}$$

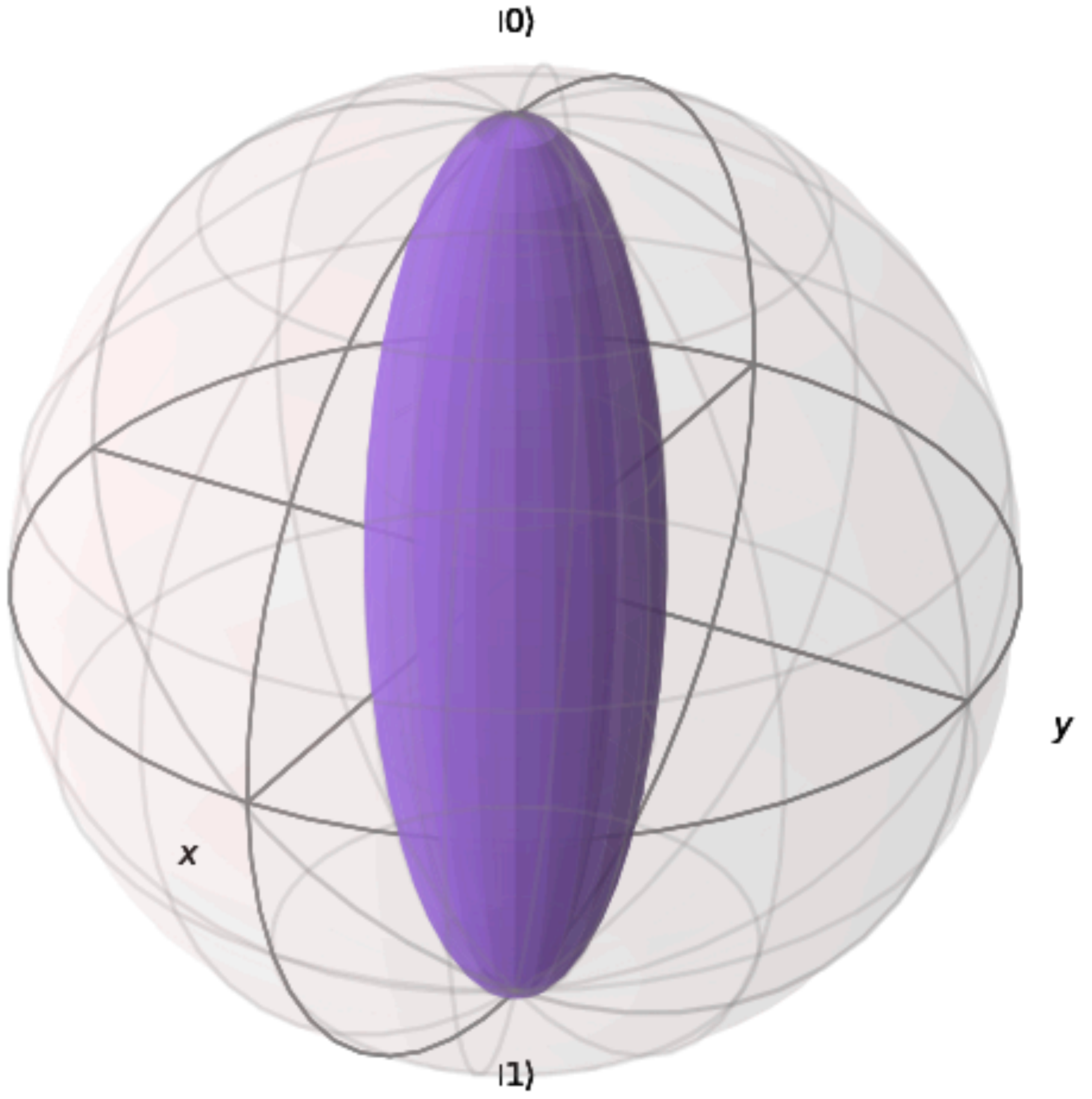
Equivalently,

$$\rho' = p\frac{I}{2} + (1-p)\rho$$

Write in the form of Kraus operators,

$$K_0 = \sqrt{1 - \frac{3}{4}p}I,$$

$$K_i = \sqrt{\frac{p}{4}}\sigma_i, \text{ for } i = 1,2,3$$

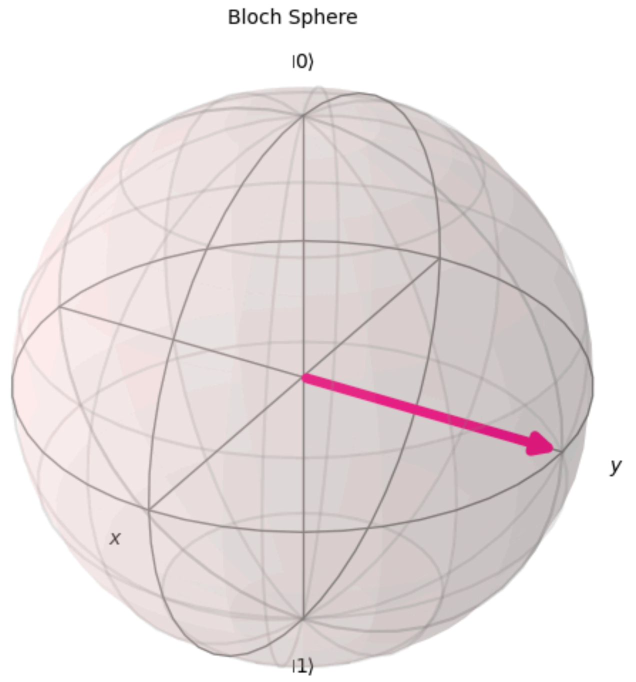


Noise channel $\mathcal{N}$:

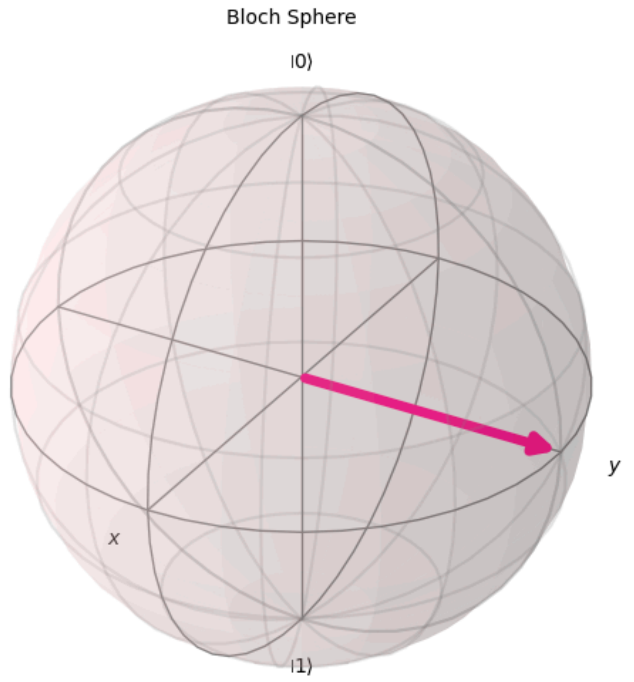
Quantum map that transforms one quantum state to another

Example:

$$\Phi : \rho \mapsto \rho'$$



$$\mapsto$$



Fidelity of a state passing through a noise channel

Example: the depolarizing map

$$\rho \mapsto \rho' = \begin{cases} \frac{1}{2}I & \text{w/ prob. } p \\ \rho & \text{w/ prob. } 1-p \end{cases}$$

Equivalently,

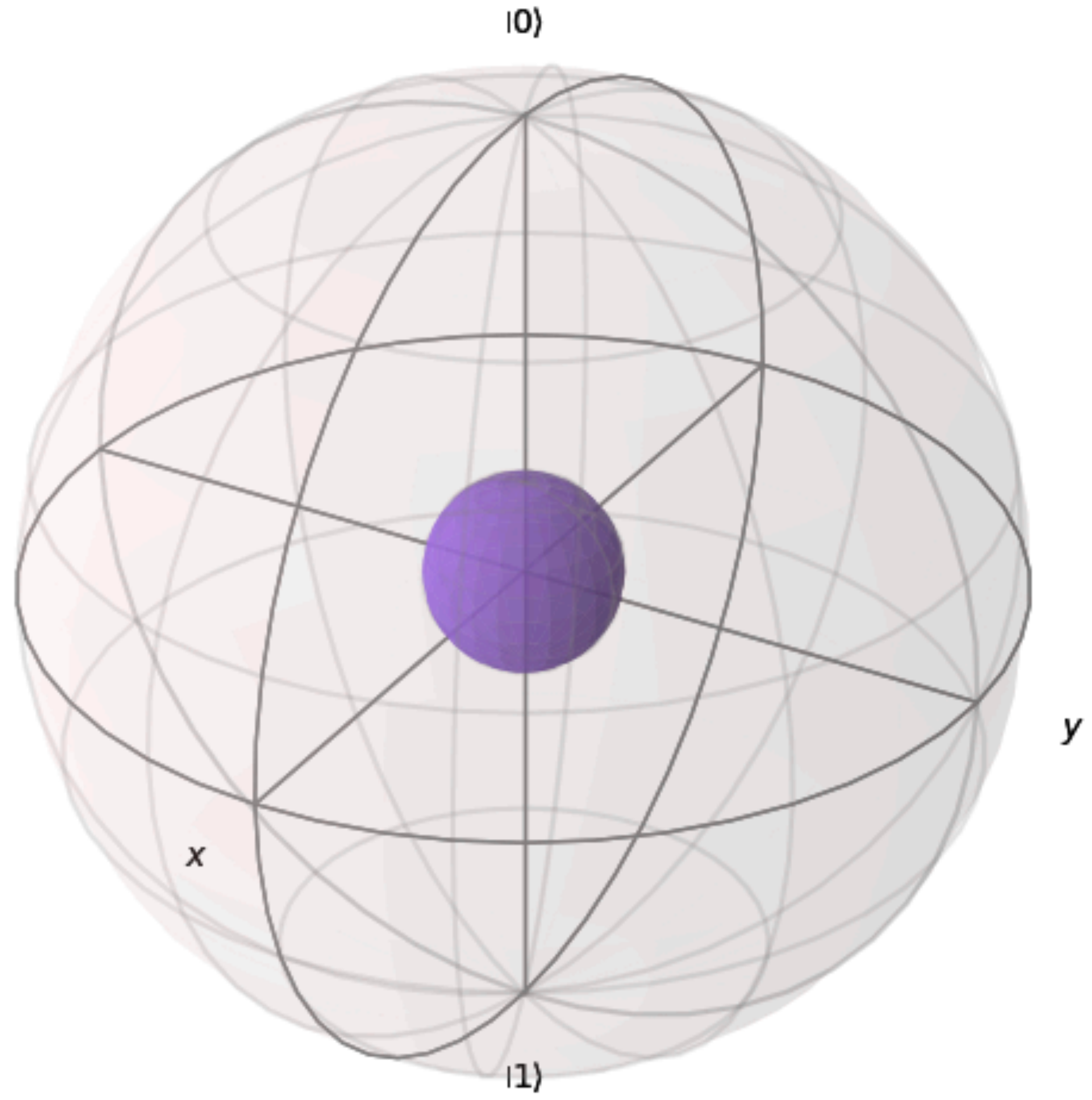
$$\rho' = (1-p)\rho + p\frac{I}{2}$$

Write in the form of Kraus operators,

$$K_0 = \sqrt{1 - \frac{3}{4}pI},$$

$$K_i = \sqrt{\frac{p}{4}}\sigma_i, \text{ for } i = 1,2,3$$

The depolarizing map is a unital map



Example: amplitude damping map

$|0\rangle \mapsto |0\rangle$ with probability 1

$|1\rangle \mapsto |0\rangle$ with probability p

The amplitude damping map is NOT a unital map

Equivalently,

$$\rho' = p\frac{I}{2} + (1-p)\rho$$

Write in the form of Kraus operators,

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix},$$

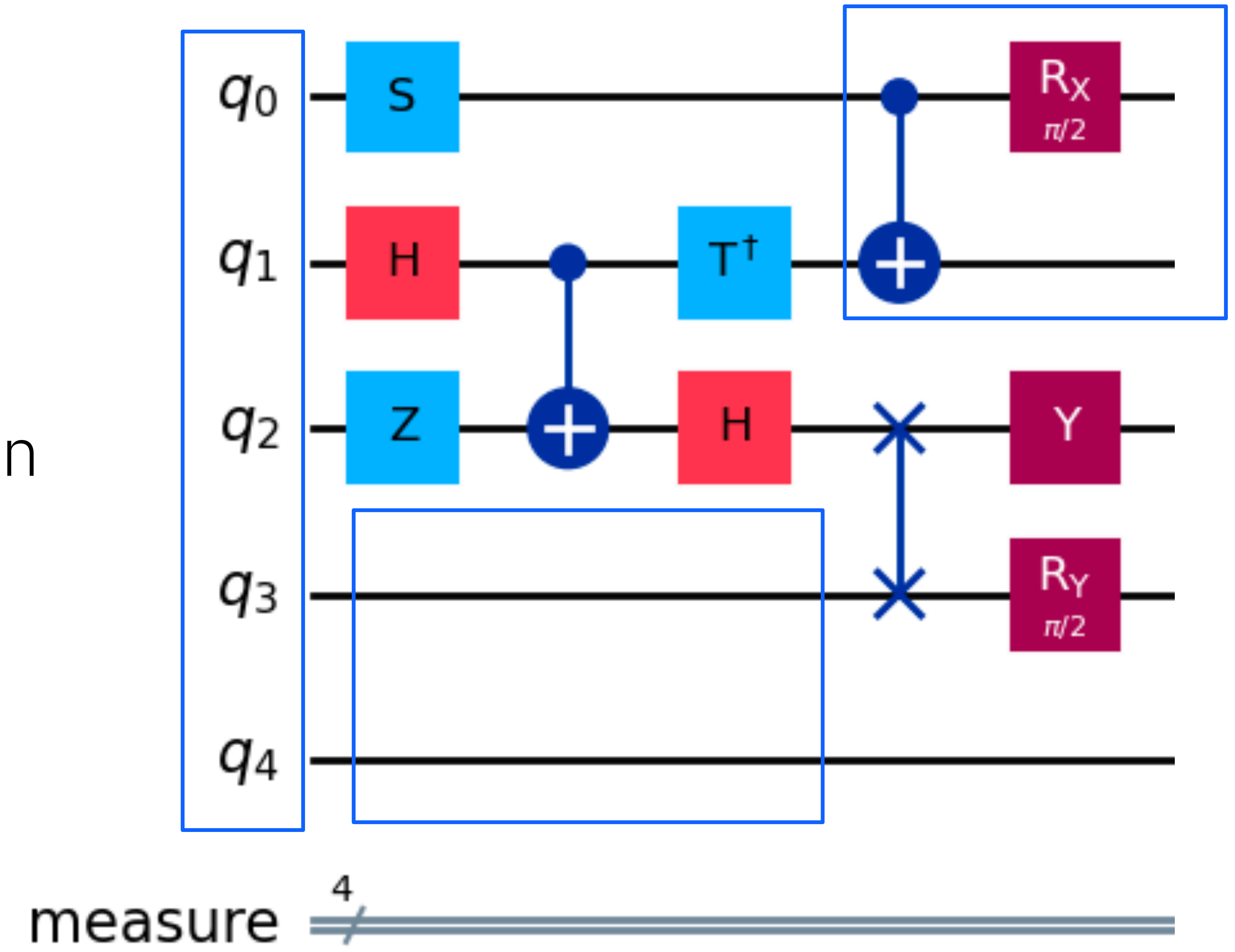
$$K_1 = \sqrt{p}\,|0\rangle\langle 1|.$$

Summary of noise examples

	Kraus operator representation		
Bit flip			
Phase flip			
Amplitude damping			
Depolarizing			
Pauli			

How do quantum computation fail

Incoherent errors:
Loss of quantum information in the form of superposition and entanglement



Coherent errors:
Incorrect Hamiltonian evolution

1

Qubits have finite lifetime

Qubit T1 time
Qubit T2 time

2

Gates and their errors

Single-qubit coherent noise
Two-qubit crosstalk

3

Noise channel examples

4

Noise characterization

Process tomography
Randomized benchmarking

Hardware connectivity: heavy-hex lattice

The connectivity dictations how noise propagates spatially

The coupling map of the device:
Nodes: qubits
Edges: native two-qubit entangle gates are allowed