Quantum Computing for the Quantum Curious

4.0 Creating Superposition: Stern–Gerlach

4.0 Objectives

- Explain why electron spin could serve as an example of a qubit.
- Show how the Stern–Gerlach experiment illustrates spin quantization, superposition, and measurement collapse.
- Define what is meant by a measurement basis and convert a given spin to a

  different basis.
- Compute the probability of an electron passing through one or more Stern-Gerlach apparatuses.

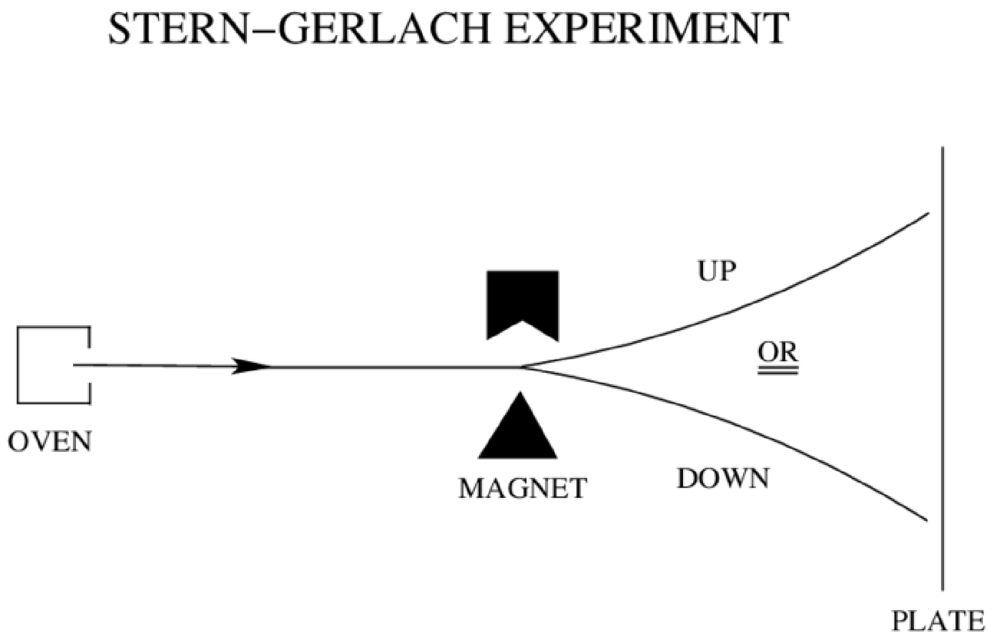
Key Terms: spin, Stern–Gerlach experiment, measurement basis, orthogonal states, no-cloning theorem

4.0 Introduction

- In the previous discussion, a photon in an interferometer was identified as a prototype for a qubit.
- Electron as a Qubit: An electron, another particle, can also serve as a qubit due to its two-state property called spin.
- Electron's Measurable Properties: An electron has several measurable properties like energy, mass, and momentum, but for qubit purposes, spin is the most relevant.
- Understanding how an electron's spin can be used as a qubit helps explore key quantum phenomena such as superposition and measurement.

4.1 Stern–Gerlach Apparatus

The SGA demonstrates that electron spin is quantized to only two values, which can be used to measure electron spin.



Watch This video Simulator

4.1 Stern–Gerlach Apparatus

The SGA, when oriented vertically (z-direction), measures the spin as either up or down, corresponding to qubit states $|0\rangle$ (spin up) and $|1\rangle$ (spin down).

Orientation of SGA: The orientation of the SGA determines the measurement outcome. A horizontally oriented SGA measures spin as either left or right, while one rotated by 45° measures spin as either diagonally up or down.

Understanding how an electron's spin can be used as a qubit helps explore key quantum phenomena such as superposition and measurement.

4.1 Stern–Gerlach Apparatus

Question 1 Open up the [Stern–Gerlach simulator] and try sending electrons of various initial spins into the Stern–Gerlach apparatus (SGA).

Are the results what you would expect? The "up" and "down" directions are defined by the orientation of the apparatus, as in Fig. 4.2. There is nothing inherently special about the z-direction compared to the x- or y-direction. An SGA rotated horizontally would measure either spin left or spin right. An SGA rotated by 45◦ would measure the spin to be either diagonally up or diagonally down. What is particularly interesting is if we send a single spin up electron into a horizontally oriented SGA

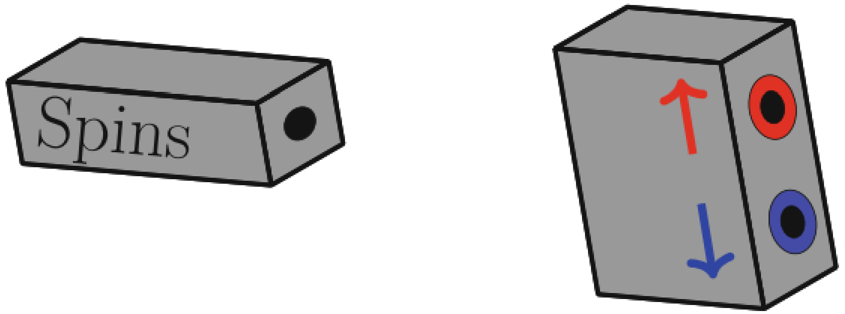


Fig. 4.2 A cartoon picture of the Stern–Gerlach Apparatus. Electron spin produces a magnetic field either in the up or down direction.

4.1 Stern–Gerlach Apparatus

Question 2 Where would you expect a spin up electron to land in Fig. 4.3 after passing through a horizontal SGA?

Sending a spin up electron through a horizontal SGA puts the electron in a superposition state of left and right.

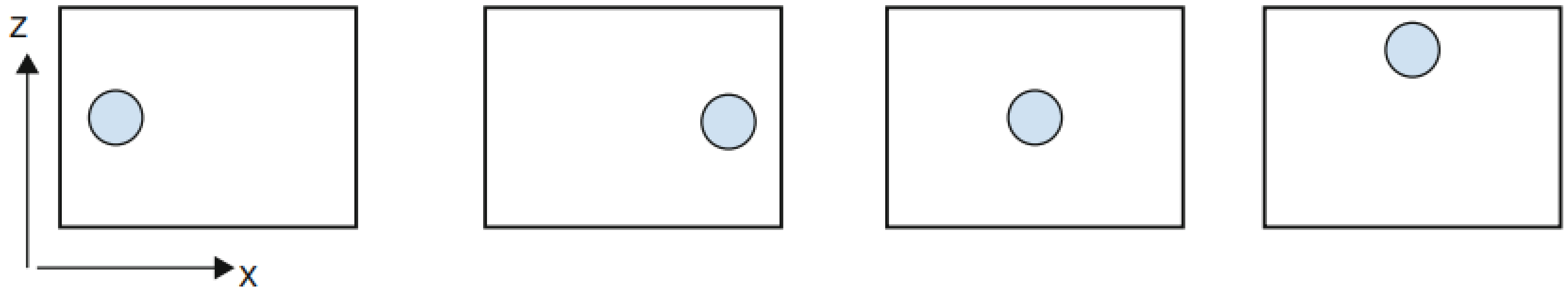


Fig. 4.3 Choices for Question 2.

4.2 Measurement Basis

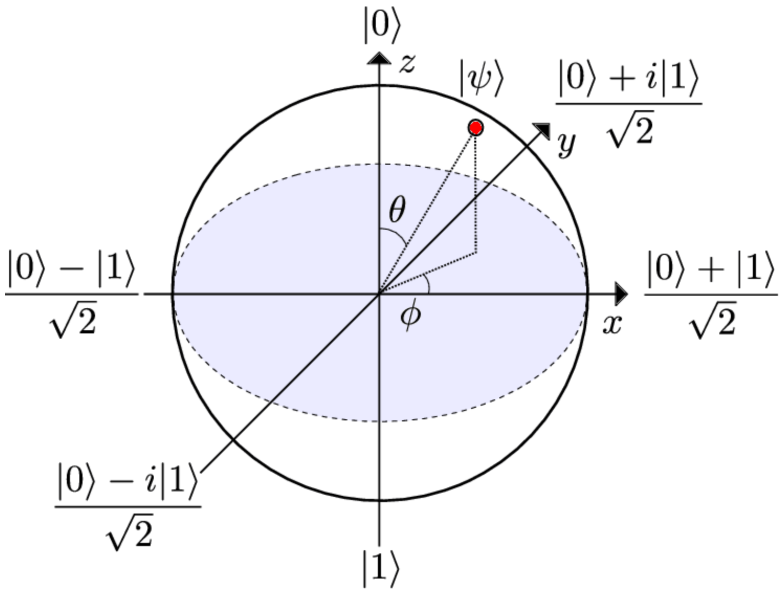
What is a Basis?

A basis in quantum mechanics is a set of states that are used to describe any possible state of a quantum system.

These states are analogous to a coordinate system in classical mechanics.

Most Common Basis states:

1. **Z-Basis (Computational Basis)States:** $|0\rangle$ and $|1\rangle$
2. **X-Basis States:** $|+\rangle$ and $|-\rangle$
3. **Y-Basis States:** $|i+\rangle$ and $|i-\rangle$



Kockum et al. (2018)

4.2 Measurement Basis

Mathematical Properties of Bases

1. Orthogonality:
In each basis, the states are orthogonal to each other, meaning that the inner product of any two different basis states is zero:

$$\langle 0|1\rangle = 0, \quad \langle +|-\rangle = 0, \quad \langle i+|i-\rangle = 0$$

Orthogonality ensures that the states are distinct and can be used to fully describe any quantum state within that basis.

2. Normalization:
Each basis state is normalized, meaning that the inner product of a state with itself is one:

$$\langle 0|0\rangle = 1, \quad \langle 1|1\rangle = 1,$$

Normalization ensures that the probabilities derived from these states add up to 1.

3. Completeness:
Any quantum state $|\psi\rangle$ can be expressed as a linear combination of the basis states in any of these bases.

4.2 Measurement Basis

- Spin in the vertical direction can be represented as a superposition of spins in the horizontal direction. As shown in the simulation, an electron with vertical spin has a 50% chance of being measured as right or left:

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle,$$
$$|\downarrow\rangle = \frac{1}{\sqrt{2}}|\leftarrow\rangle - \frac{1}{\sqrt{2}}|\rightarrow\rangle.$$

In more traditional qubit notation $\longrightarrow$

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle,$$
$$|1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle.$$

Non-Classical Behavior: quantum states exhibit non-classical behavior. For example, you cannot simply add or subtract horizontal magnetic field vectors to obtain a vertical magnetic field vector. This non-classical behavior is illustrated with an analogy of a coin: if you observe a coin from the side, it appears to be in a superposition of heads and tails.

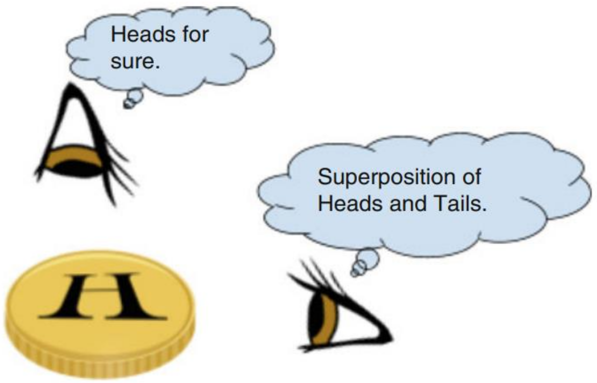


Fig. 4.4 Analogy for how a definite vertical spin is seen as a superposition in the horizontal direction.

4.2 Measurement Basis

- spins in one direction can be written as a superposition of spins in another direction.

Example Write the $|+\rangle$ state in terms of $|0\rangle$ and $|1\rangle$.

Solution Adding Eqs. (4.3) and (4.4) we find

$$|0\rangle + |1\rangle = \frac{2}{\sqrt{2}}|+\rangle. \tag{4.5}$$

Rearranging, we get

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle. \tag{4.6}$$

Similarly, by subtracting Eqs. (4.3) and (4.4), we find

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle. \tag{4.7}$$

4.4 Effect of Measurement

- You learned that measuring a qubit collapses its superposition state into one of two possibilities.
- To appreciate the truly strange nature of quantum measurement, let’s see what happens when electrons are sent through multiple Stern–Gerlach devices in a row

Question 4 Open the PhET Stern–Gerlach simulator6 and send electrons with randomly oriented spins through a vertical SGA as in Fig. 4.7. What is the spin of the electrons that pass through the hole?

(a) +z

(b) −z

(c) Superposition of +z and −z

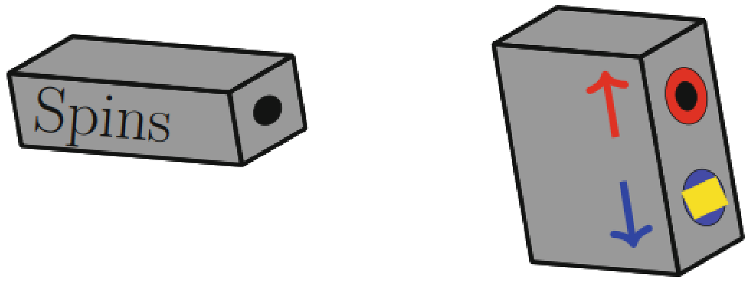


Fig. 4.7 The z-axis SGA lets through spin up electrons but blocks spin down electrons.

4.4 Effect of Measurement

Question 5 Add a second SGA, oriented horizontally as in Fig. 4.8. What is the spin of the electrons before entering the second SGA?

(a) +x

(b) −x

(c) Superposition of +x and −x

Question 6 What is the spin of the electrons after passing through the second SGA?

(a) +x

(b) −x

(c) Superposition of +x and −x

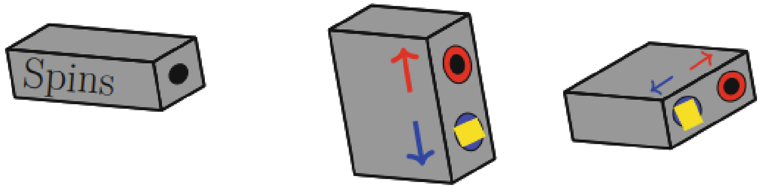


Fig. 4.8 The z and x-axis SGA.

4.4 Effect of Measurement

Question 7 What is the z-spin of the electron coming out of the second SGA? Design an experiment to confirm this in the simulation.

(a) +z

(b) −z

(c) Superposition of +z and −z

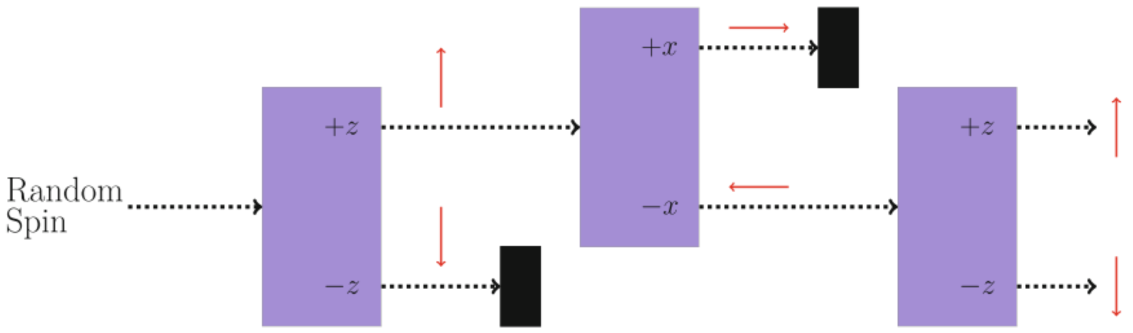


Fig. 4.9 The first SGA selects for $+z$ spin and the second SGA selects for $-x$. The third SGA shows that by measuring the $-x$ in the z-basis then the electron is in a superposition of $+z$ and $-z$.

[Simulator]

4.5 Big Ideas

1. An electron has an intrinsic property called spin, which is quantized into two values called spin-up and spin-down.

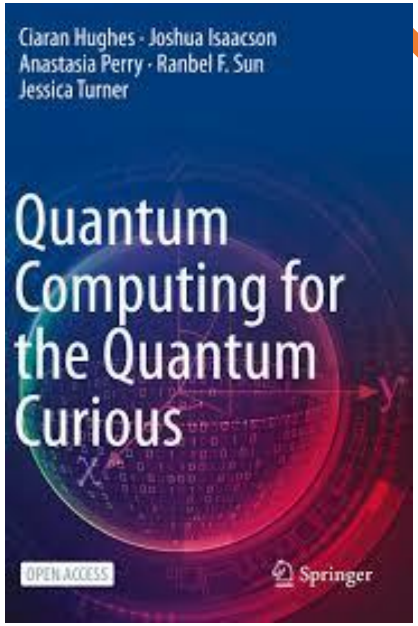
2. The measurement basis is important when interpreting results from experiments on quantum states. Two common basis are the z-basis ($|0\rangle$ and $|1\rangle$) and the x-basis ($|+\rangle$ and $|-\rangle$).

3. The Stern–Gerlach apparatus (SGA) can be used to put the electron into a superposition state. The electron can be used as a qubit, and the SGA as a way to operate on this qubit. Together, they are a simple model of a quantum computer.

Reference Book

Quantum Computing for the Quantum Curious

This open access book makes quantum computing more accessible than ever before. A fast-growing field at the intersection of physics and computer science, quantum computing promises to have revolutionary capabilities far surpassing “classical” computation. Getting a grip on the science behind the hype can be tough: at its heart lies quantum mechanics, whose enigmatic concepts can be imposing for the novice. This classroom-tested textbook uses simple language, minimal math, and plenty of examples to explain the three key principles behind quantum computers: superposition, quantum measurement, and entanglement.



Hughes, C., Isaacson, J., Perry, A., Sun, R. F., & Turner, J. (2021). Quantum computing for the quantum curious. In Springer eBooks. https://doi.org/10.1007/978-3-030-61601-4

Slides prepared by Roua Alimam