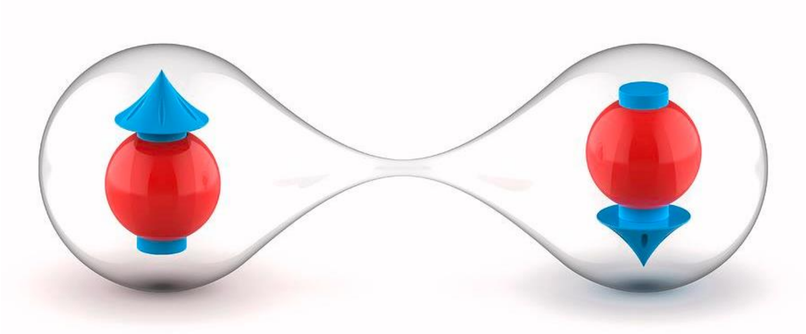
Quantum Computing for the Quantum Curious

5.0 Entanglement

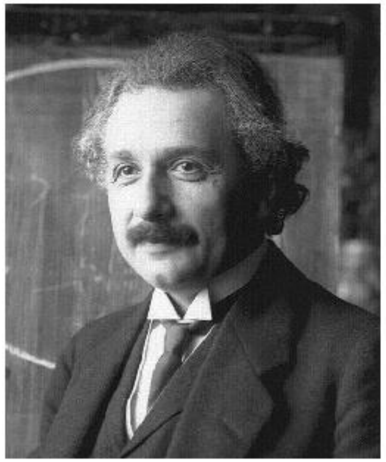
5.1 Quantum Entanglement

What is Quantum Entanglement?

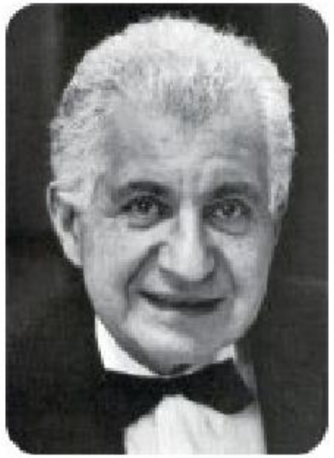
- A mysterious connection between particles.
- Measuring one particle instantly affects the other, no matter the distance
- Fundamental in quantum mechanics



5.1 The hidden variable



Albert Einstein



Boris Podolsky

Nathan Rosen

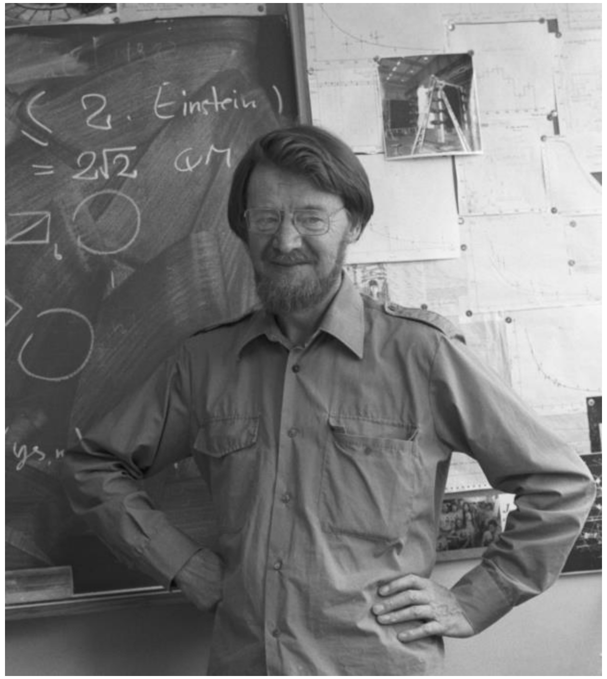
Vs

Bohr

- Proposed the EPR Paradox in 1935
- Suggested quantum mechanics is incomplete and it's violating locality and realism
- Proposed hidden variables to explain the phenomenon

5.2 Bell’s Theorem

- John Bell’s Solution (1964):Bell proposed an inequality to test hidden variables
- Bell’s Inequality: If hidden variables exist, certain limits should be observed
- If quantum mechanics is correct, these limits are violated, S exceeds 2.



$$S = |E(a,b) + E(a',b) + E(a,b') - E(a',b')| \leq 2$$

[Watch this video]

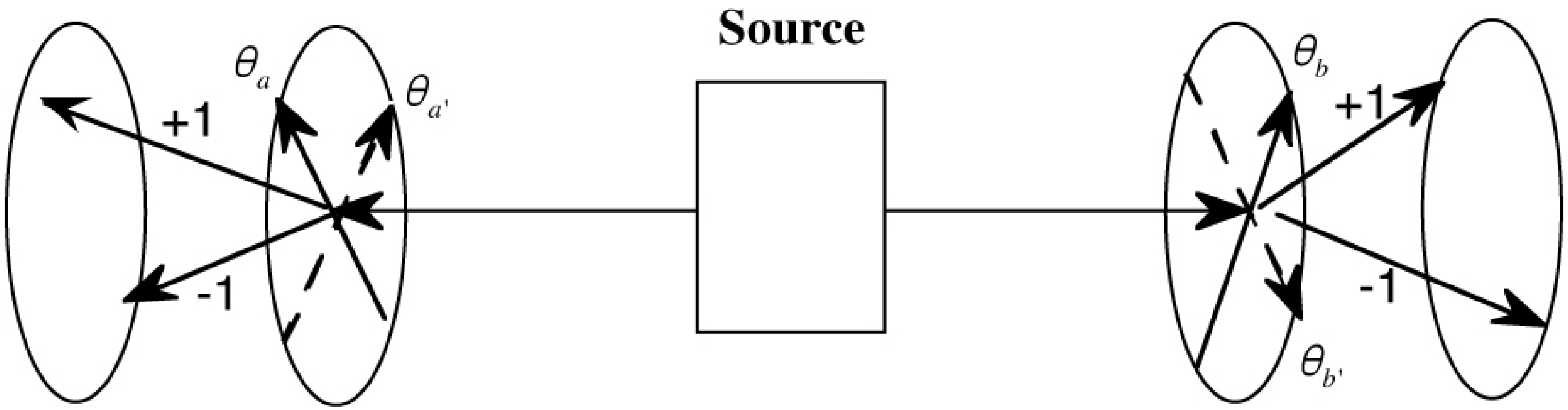
5.2 Experiment set up

Photon pairs are generated and sent to Alice and Bob.

Alice and Bob each randomly select one of three polarizer angles (Z, X, Q).

They measure the polarization of their photons and record the results.

Over many rounds, their **results** show correlations that violate Bell's inequality, indicating the presence of quantum entanglement and ruling out local hidden-variable theories.



5.3 Tensor Product

The **tensor product** is a mathematical operation used to combine two quantum states into a single state, typically used when working with multi-qubit systems. For example, if you have two qubits in states $|\psi\rangle$ and $|\varphi\rangle$, their combined state is written as $|\psi\rangle \otimes |\varphi\rangle$, This operation creates a joint quantum state for the two qubits.

- Example: If $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\phi\rangle = \gamma|0\rangle + \delta|1\rangle$.

$$|\psi\rangle \otimes |\phi\rangle = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

5.3 Separable vs Entangled States

Separable State:

- Product of individual states.
- - No connection between qubits.
- Example: $|0\rangle \otimes |1\rangle = |01\rangle$

Entangled State:

- Cannot be written as a product of individual states.
- Connected: Measuring one qubit affects the other.
- Example: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Check if the state can be factored into individual qubit states.

5.4 Bell States

Bell states are specific quantum states that represent the simplest forms of entangled two-qubit systems.

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

5.5 CHSH Game

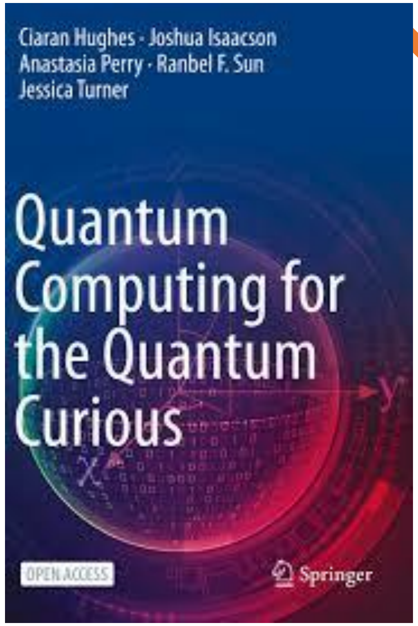
Class Activity

Go through the CHSH game and check whether the bells inequality was violated or not

Reference Book

Quantum Computing for the Quantum Curious

This open access book makes quantum computing more accessible than ever before. A fast-growing field at the intersection of physics and computer science, quantum computing promises to have revolutionary capabilities far surpassing “classical” computation. Getting a grip on the science behind the hype can be tough: at its heart lies quantum mechanics, whose enigmatic concepts can be imposing for the novice. This classroom-tested textbook uses simple language, minimal math, and plenty of examples to explain the three key principles behind quantum computers: superposition, quantum measurement, and entanglement.



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