Summary of Key Concepts

Grover's Algorithm

Week of February 11th, 2024

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Resources

- QXQ YLC Week 16 Lab Notebook [STUDENT].ipynb
- Original Grover's Algorithm Paper by Lov Grover



Key Terms

Key Term	Definition
Unstructured Search	Searching through items such that the items have no meaningful structure or order that can help us search more efficiently.
Oracle	An oracle is a part of an algorithm that can perform a specific computation "for free". The tradeoff is that we can't know how they do it. In this case, the oracle can mark the state we're looking for, often by flipping its phase.
Diffusion Operator	An operator that increases the likelihood of measuring states that have been "marked" and decreases the likelihood of measuring the rest.
Amplitude Amplification	The combination of an oracle and a diffusion operator that Lov Grover first showed could be used to achieve a quantum speedup for unstructured search.



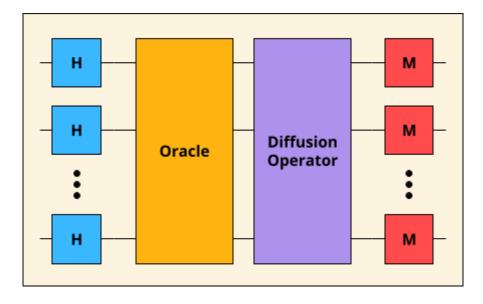
Lecture

Learning Objectives

- 1. Recognize what the unstructured search problem is.
- 2. Recognize how Grover's algorithm can solve this problem.
- 3. Recognize what Amplitude Amplification is.

Key Ideas

- Unstructured data has no underlying structure or pattern that allows us to organize it. When we search through this data, it is called unstructured search.
 This is fundamentally quite difficult, since there are no meaningful structures to help us search more efficiently. In unstructured search:
 - a. We know what we're looking for, we just don't know where it is.
 - b. The items in the list are not in any particular order.
- 2. **Grover's algorithm** is a quantum algorithm and provably the most efficient possible algorithm for unstructured search. It can be broken down into 4 key steps:
 - a. Prepare equal superposition.
 - b. Mark states that match criteria.
 - c. Constructively interfere marked states, destructively interfere the rest.
 - d. Once we have repeated the middle two steps enough times to be confident, we measure possible solutions.





Grover's Algorithm

- 3. **Amplitude amplification** is the fundamental technique behind Grover's algorithm. It is the combination of steps 2 3.
 - a. It accomplishes: increasing the amplitude of marked states, while decreasing the amplitude of the rest.
 - b. Lov Grover realized we can build a working diffusion operator by flipping around the average amplitude.
 - c. Amplitude amplification offers a quadratic speed up over the classical counterpart generically: $O(\sqrt{n})_{\cdot}$

