Summary of Key Concepts

Quantum Mechanics II: The Wavefunction

Week of October 22, 2023

Resources

- Virtual Lab by Quantum Flytrap

Key Terms

Key Term	Definition
Wavefunction	How physicists describe what state a quantum object is in or, in other words, everything we can possibly know about it.
Wave	In physics, we describe regular changes or disturbances with waves.
Amplitude	The height or size of a wave is called the amplitude. The amplitude of a quantum wavefunction determines how likely it is to be measured.
Phase	The phase of a wave is how shifted it is relative to another wave.The phase of a quantum wavefunction plays an important role in interference.
Ket Notation	Ket Notation was developed by Quantum Physicists to make quantum calculations easier. Ket Notation is a way to represent different wavefunctions.
Wavefunction Collapse	After quantum measurement, only the part of the wave we directly measure remains. For this reason, we say that quantum measurement collapses wavefunctions.
Born's Rule	Born's Rule allows us to predict the probability of measuring each state of a quantum object.

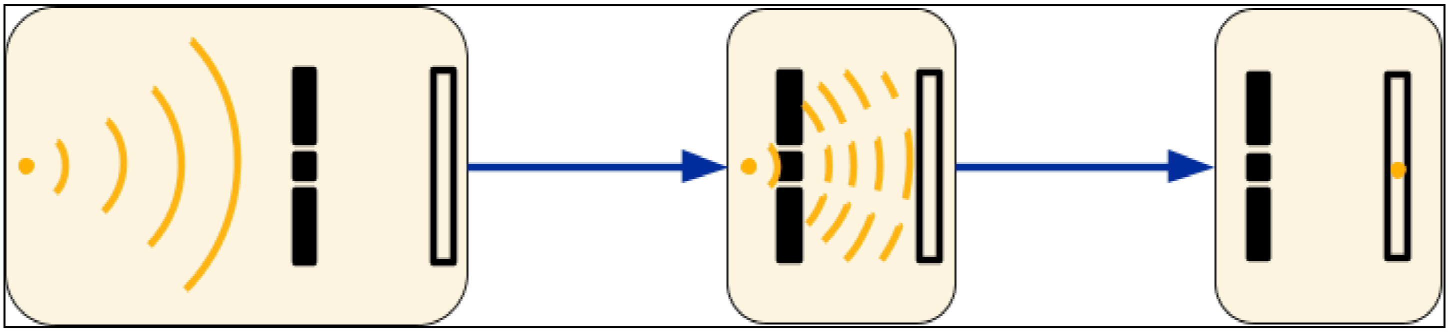
Lecture

Learning Objectives

1. *Recognize* what a **wavefunction** is.
2. *Recognize* what the **phase** of a wavefunction is and **how it affects interference**.
3. *Recognize* what the **amplitude** of a wavefunction is and **how it affects measurement outcomes**.
4. *Recognize* how **kets can represent wavefunctions.**

Key Ideas

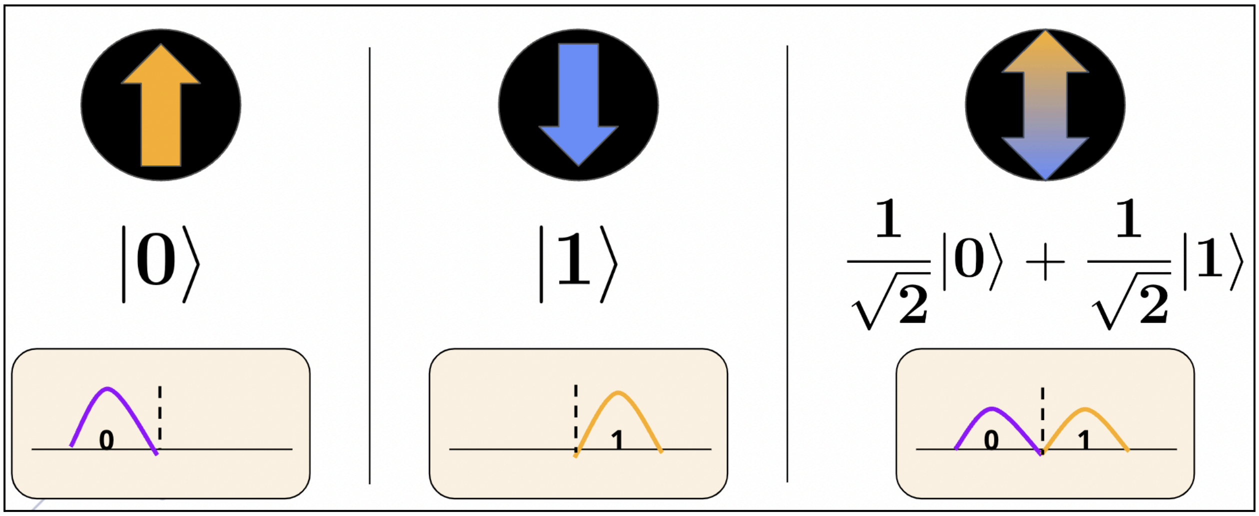
1. The **wavefunction** is how physicists describe what state a quantum object is in, or in other words, everything we can possibly know about it. Here is a visual of the electron's wavefunction at different points in the double slit experiment:



2. In physics, **we describe regular changes or disturbances with waves**. These can occur over:
   a. Time, such as the height of water changing up and down over time.
   b. Space, such as the interference pattern of the double slit experiment.
   c. And more!

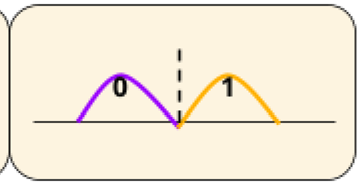
3. Two important characteristics of a wave in quantum mechanics are:
   a. **Amplitude**: The height or size of a wave. The amplitude of a quantum wavefunction **determines how likely it is to be measured**.
   b. **Phase**: How shifted the wave is relative to another wave. The phase of a quantum wavefunction plays an important role in interference.

4. **Qubit wavefunctions** could be visualized and written in ket notation as shown below.
    a. **NOTE**: These wavefunctions are purely for learning purposes and do not represent the real wavefunction of a qubit. To know this, we would need to understand the physical system that we are encoding our qubit into, something we'll learn more about in the second semester.

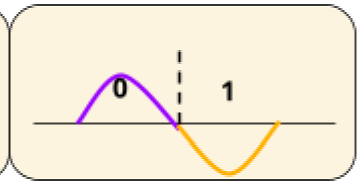


5. We saw two *different* **equal superposition states** for qubits, whose **only difference is phase**:

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$$



$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$$

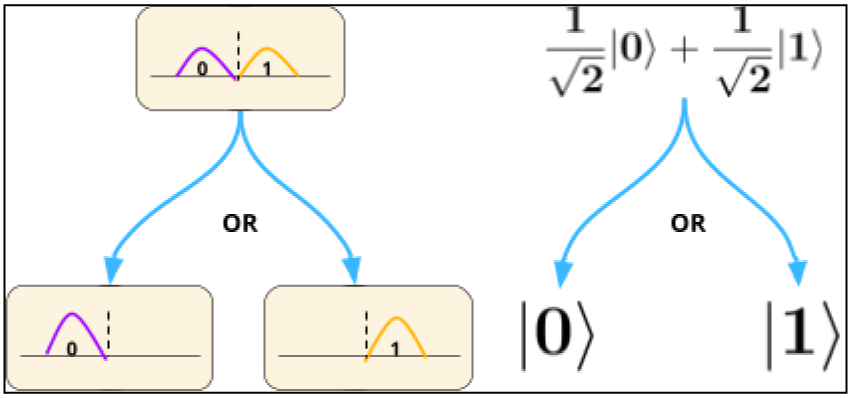


6. We can interpret the **amplitude of a qubit state (0 or 1) as how likely it is to be measured in that state**. In ket form, the amplitudes are given by the numbers in front of each state. For example,

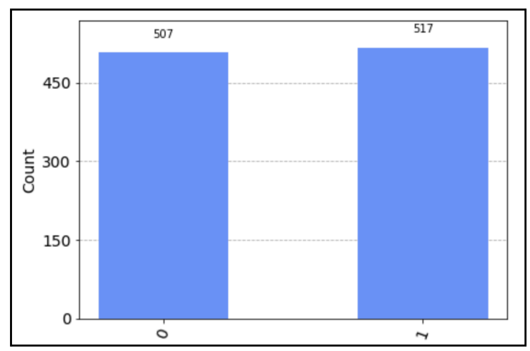
$$\boxed{\frac{\sqrt{2}}{\sqrt{3}}}|0\rangle - \boxed{\frac{1}{\sqrt{3}}}|1\rangle$$

amplitudes

7. **Measuring a quantum object** forces it to choose one outcome. Afterwards, the object is in whatever state was measured. It has forgotten all past information.
    a. **Only the part of the wave we directly measure remains. For this reason, we say that quantum measurement collapses wavefunctions.**



    b. We need to prepare and measure a quantum state many times to understand it. We often **visualize the results using a histogram**.



8. Born's Rule allows us to predict the probability of measuring each state of a quantum object. Born's Rule for a qubit says:

$$\begin{aligned}
|\mathbf{state}\rangle &= \mathbf{a}|\mathbf{0}\rangle + \mathbf{b}|\mathbf{1}\rangle \\
\mathrm{prob}(\mathbf{0}) &= \mathbf{a}^2 \\
\mathrm{prob}(\mathbf{1}) &= \mathbf{b}^2
\end{aligned}$$

Lab

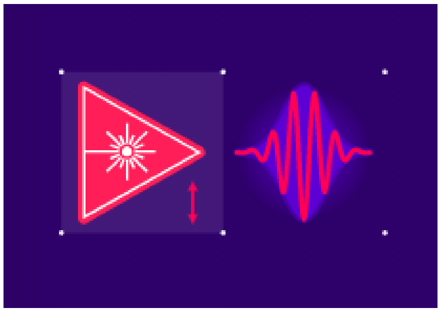
Learning Objectives

1. *Recognize* the wave particle duality with photons.
2. *Recognize* how beamsplitters create superpositions.
3. *Recognize* the nature of quantum measurement.

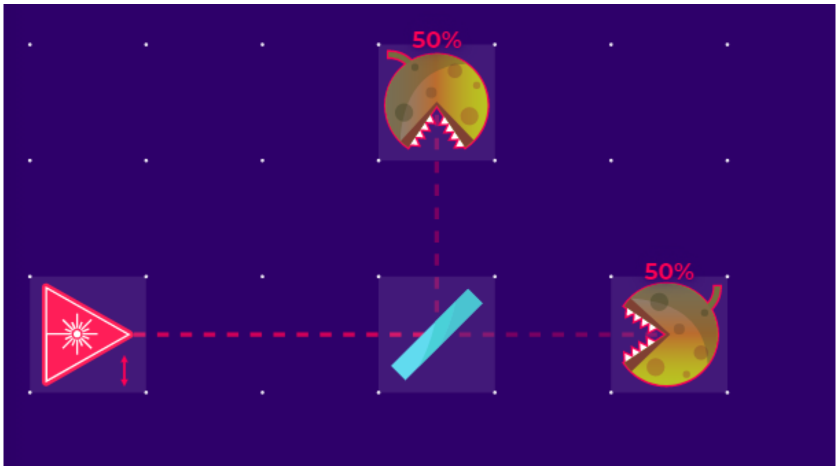
Key Ideas

In this week's lab, we looked at Quantum Flytrap, an online virtual lab to simulate quantum experiments with photons. In Flytrap, we learnt how to use lasers, mirrors, detectors, and beamsplitters. We used these tools to create **superposition** and **interference**, and **observe wave-particle duality** and the **randomness of quantum measurements**.

1. **Wave-particle duality:** We can visualize light as a wavepacket, as shown in the Flytrap screenshot below. **In this representation, the photon is shown as a wave, but the wave is localized like a particle.**

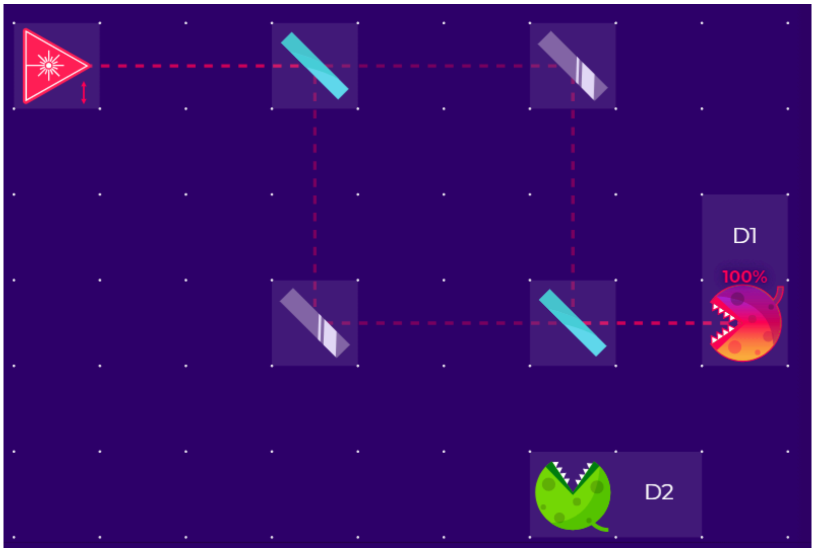


2. **Beamsplitter: A beamsplitter is used to split a beam of light.** It lets some light go through it, and reflects the rest of the light.
3. **Creating superposition**: We can create superposition by passing photons through a beamsplitter. There are two possible paths the photon can take out of the beamsplitter. **The photon is in a superposition of being reflected or transmitted.**



4. **Randomness of quantum measurement:** When we detect a photon coming out of a beamsplitter, the photon gets detected **randomly** along one of the two possible paths. **It is impossible to predict which detector the photon will hit.**

5. **Creating interference:** We can create interference in a Mach-Zehnder interferometer, as shown in the screenshot below:



In this setup, every photon from the laser gets detected on the detector D1. The second detector (D2) does not receive any photons. **The beamsplitter creates constructive interference along the path leading to D1 and destructive interference along the path leading to D2.**

Such interferometers are commonly used as high-precision detectors, because the perfectly constructive and destructive interference gets broken if there is any difference between the two paths that the photon can take in the interferometer.