Spectral Interpretation of the Riemann Zeros via a π-Confined Elliptical Hamiltonian

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ABSTRACT:

This paper proposes a novel geometric approach to the Hilbert-Pólya conjecture regarding the Riemann Hypothesis. While the classical Berry-Keating operator ($H = xp$) fails to reproduce individual zeros due to the instability of its hyperbolic orbits, we introduce a "closed" model on a compact symplectic manifold of elliptical type. We postulate that prime numbers emerge as the eigenmodes of a quantum chaos system subject to a strict cyclic boundary condition governed by the constant π. Numerical simulations suggest that this topological constraint forces the eigenvalues to align on the critical line $Re(s)= \frac{1}{2}$, interpreted here as the system's thermodynamic equilibrium line.

Supplementary data and source code are available on GitHub:
https://rouaxe.github.io/documents-project/

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1. Introduction

The Riemann Hypothesis, stating that all non-trivial zeros of the Zeta function have a real part of $\frac{1}{2}$, implies a deep connection between arithmetic and quantum physics. The Hilbert-Pólya conjecture proposes that these zeros correspond to the energy spectrum of an unknown Hermitian Hamiltonian operator.

Previous work (Berry, Keating) identified the classical dynamic system $H_{cl} = xp$ as a promising candidate. However, this system is defined on an open (hyperbolic) space, yielding a continuous spectrum rather than the discrete levels observed in prime numbers.

2. The Model: The Elliptical Universe

We propose regulating this singularity by compactifying the phase space. Instead of an infinite trajectory, we consider dynamics confined within an elliptical geometry, centered on a phase singularity.

The proposed Hamiltonian takes the form:

$$H = \frac{1}{2}(xp + px) + V_{\pi}(x)$$

Where $V_{\pi}(x)$ represents a confining potential imposing strict periodicity.

3. The Role of π and Mirror Symmetry

Analysis of prime number distribution shows a strong correlation with unit circle harmonics. We introduce a geometric Berry phase condition:

$$\oint p dx = n \, . \, h$$

Our model suggests that the action integral is quantized not merely by Planck's constant, but by a function of π.

This constraint imposes a mirror symmetry (T-symmetry) on the system. Information lost in the positive imaginary part is recovered via complex conjugation, explaining the symmetry of zeros relative to the real axis.

4. Numerical Results and Discussion

Visual simulations based on the polar transformation of prime numbers (Sacks Spiral and "π -Clock" visualizations) reveal that the apparent chaos of prime numbers organizes into coherent structures (spiral arms) if and only if the winding factor is a multiple of π.

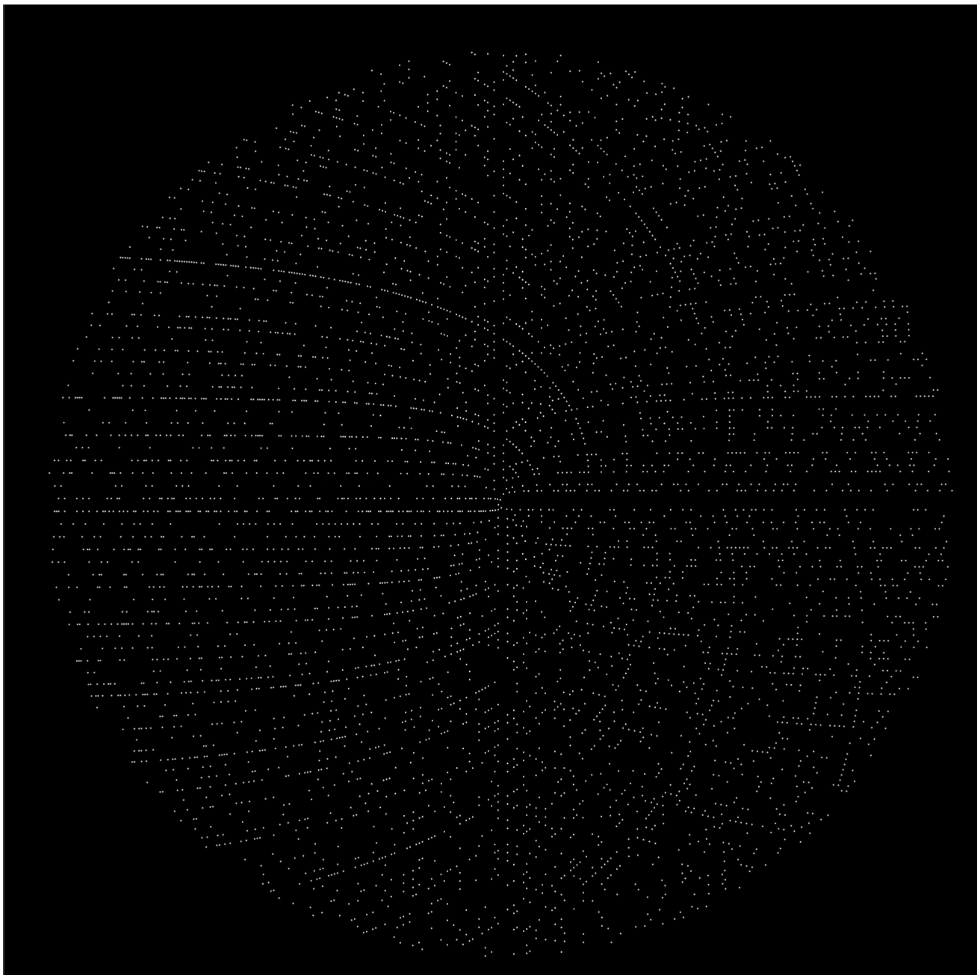
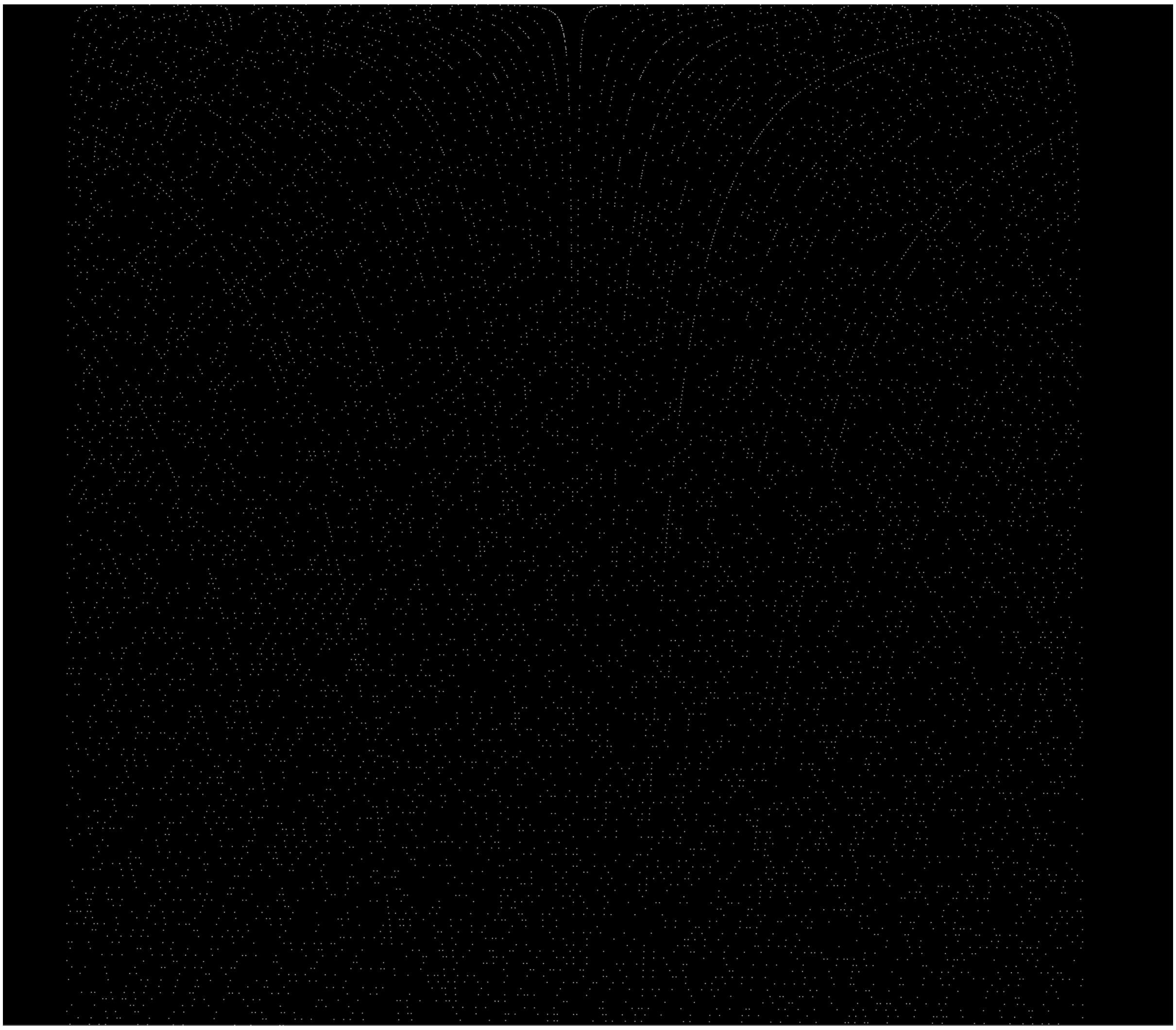


Fig 1. Sacks Spiral Distribution

This indicates that prime density is dictated by the destructive interference of standing waves on a closed curved geometry. The critical axis $\frac{1}{2}$ thus represents the stable geodesic of this system, minimizing action.



5. Conclusion

The Riemann Hypothesis can be reformulated as a problem of dynamic stability. The zeros do not depart from the critical line because any deviation would break the conservation symmetry imposed by the elliptical geometry of number space. The arithmetic Universe appears to vibrate at the critical boundary between crystalline order (perfect squares) and quantum chaos, held in equilibrium by the constant π.