BRAC University
MAT-215
Exercise Sheet #1

Part A

1. Perform each of the indicated operations:

(i) $(i-2)\{2(1+i)-3(i-1)\}$ (ii) $\dfrac{(2+i)(3-2i)(1-i)}{(1-i)^2}$ (iii) $(2i-1)^2\left\{\dfrac{4}{1-i}+\dfrac{2-i}{1+i}\right\}$

(iv) $3\left(\dfrac{1+i}{1-i}\right)^2-2\left(\dfrac{1-i}{1+i}\right)^3$ (v) $\dfrac{3i^{10}-i^{19}}{2i-1}$ (vi) $\dfrac{i^4-i^9+i^{16}}{2-i^5+i^{10}-i^{15}}$

2. Show that (i) $(5+3i)+\{(-1+2i)+(7-5i)\}$ and (ii) $\{(5+3i)+(-1+2i)\}+(7-5i)$ illustrate the associative law of addition.

3. If $z_1=1-i,\ z_2=-2+4i,\ z_3=\sqrt{3}-2i,$ evaluate each of the following:

(i) $|2z_2-3z_1|^2$ (ii) $\left|\dfrac{z_1+z_2+1}{z_1-z_2+i}\right|$ (iii) $\overline{(z_2+z_3)(z_1-z_3)}$ (iv) $\operatorname{Re}\{2z_1^3+3z_2^3-5z_3^2\}$

(v) $\operatorname{Im}\left\{\dfrac{z_1z_2}{z_3}\right\}$ (vi) $z_1^2+2z_1-3$ (vii) $|z_1\overline{z_2}+z_2\overline{z_1}|$ (viii) $\dfrac{1}{2}\left(\dfrac{z_3}{\overline{z_3}}+\dfrac{\overline{z_3}}{z_3}\right)$

(ix) $(z_3-\overline{z_3})^5$

4. Express each of the following complex number in polar form and show them graphically.

(i) $2+2\sqrt{3}\,i$ (ii) $2\sqrt{2}+2\sqrt{2}\,i$ (iii) $-2\sqrt{3}-2i$ (iv) $-1+\sqrt{3}\,i$ (v) $-\sqrt{6}-\sqrt{2}i$

5. Prove that: (i) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ (ii) $|z_1z_2|=|z_1||z_2|$ (iii) $|z_1\pm z_2|\le|z_1|+|z_2|$
(iv) $|z_1\pm z_2|\ge|z_1|-|z_2|$.

6. State and prove the **De Moivre's Theorem**

7. Evaluate each of the following by **De Moivre's Theorem:**

(i) $\dfrac{(8cis40^0)^3}{(2cis60^0)^4}$ (ii) $\dfrac{(3e^{\frac{\pi i}{6}})(2e^{\frac{-5\pi i}{4}})(6e^{\frac{5\pi i}{3}})}{(4e^{\frac{2\pi i}{3}})^2}$ (iii) $(5cis20^0)(3cis40^0)$ (iv) $(2cis50^0)^6$

8. Find all the roots of the following equations.

(i) $(-1+i)^{\frac{1}{3}}$ (ii) $z^5=-4+4i$ (iii) $z^4=-16i$ (iv) $z^6=64$ (v) $z^4+z^2+1=0$.

$(vi)\ (-4+4i)^{1/5}$ $(vii)\ (-2\sqrt{3}-2i)^{1/4}$

Part B

1. Perform the indicated operations analytically and graphically.

   (a) $(2+3i)+(4-5i)$ (b) $(7+i)-(4-2i)$

2. Describe geometrically the set of points z satisfying the following conditions:

   (a) $Re(z) > 1$

   (b) $|2z+3| > 4$

   (c) $Re(\frac{1}{z}) > 1$

   (d) $1 < |z-2i| < 2$

   (e) $|z+1-i| \leq |z-1+i|$

   (f) $Re(z) \geq 0$

   (g) $|z-4| \geq |z|$

   (h) $|z-2| \leq |z+2|$

   (i) $Re(1/z) \leq 1/2$

   (j) $\pi/2 < \arg z < 3\pi/2$, $|z| > 2$

3. Using the properties of conjugate and modulus, show that:

   I. $\overline{\bar{z}+3i} = z-3i$

   II. $|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}\ |2z+5|$

   III. $|2z+3\bar{z}| \leq 4|Re(z)|+|z|$

4. Find the modulus and argument of the following complex numbers:

   (i) $\frac{2-i}{2+i}$

   (ii) $\frac{\sqrt{3}+i}{\sqrt{3}-i}$

   (iii) $(\frac{1+\sqrt{3}i}{1-\sqrt{3}i})^2$

5. Prove that $|z-i| = |z+i|$ represents a straight line.
6. Prove that $|z+2i|+|z-2i| = 6$ represents an ellipse.
7. Find an equation of a circle center at (2,3) with radius 3.
8. Sketch the region in xy- plane represented by the following set of points:

   $Re(\bar{z}-1) = 2$

   $Im(z^2) = 4$

   $|\frac{2z-3}{2z+3}| = 1$

   $Re(z)+Im(z) = 0$