BRAC University
MAT-215
Exercise Sheet Laplace Transformation

1. Find the Laplace transformation of each of the following function:

$(i)\ 3e^{-2t}$ $(ii)\ 4t^3 - e^{-t}$ $(iii)\ 7\sin 2t - 3\cos 2t$ $(iv)\ (t^2+1)^2$ $(v)\ (4e^{2t}-2)^3$.

2. Evaluate each of the following:

$(i)\ \Lambda\{t^3 e^{-3t}\}$ $(ii)\ \Lambda\{5e^{3t}\sin 4t\}$ $(iii)\ \Lambda\{(t+2)^2 e^t\}$ $(iv)\ \Lambda\{e^{-t}(3\sinh 2t - 5\cosh 2t)\}$

$(v)\ \Lambda\{e^{-4t}\cosh 2t\}$ $(iv)\ \Lambda\{e^{2t}(3\sin 4t - 4\cos 4t)\}$.

3. Determine each of the following:

$(i)\ \Lambda^{-1}\left\{\dfrac{12}{4-3s}\right\}$ $(ii)\ \Lambda^{-1}\left\{\dfrac{2s-5}{s^2-9}\right\}$ $(iii)\ \Lambda^{-1}\left\{\dfrac{23s-15}{s^2+8}\right\}$ $(iv)\ \Lambda^{-1}\left\{\dfrac{1}{s^{3/2}}\right\}$

$(v)\ \Lambda^{-1}\left\{\dfrac{s+1}{s^{4/3}}\right\}$ $(vi)\ \Lambda^{-1}\left\{\dfrac{1}{s^4}\right\}$ $(vii)\ \Lambda^{-1}\left\{\dfrac{1}{\sqrt{2s+3}}\right\}$.

4. Evaluate each of the following using partial fraction:

$(i)\ \Lambda^{-1}\left\{\dfrac{6s-4}{s^2-4s+20}\right\}$ $(ii)\ \Lambda^{-1}\left\{\dfrac{4s+12}{s^2+8s+16}\right\}$ $(iii)\ \Lambda^{-1}\left\{\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

$(iv)\ \Lambda^{-1}\left\{\dfrac{5s^2-15s-11}{(s+1)(s-2)^3}\right\}$ $(v)\ \Lambda^{-1}\left\{\dfrac{3s+1}{(s^2+1)(s-1)}\right\}$ $(vi)\ \Lambda^{-1}\left\{\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

$(vii)\ \Lambda^{-1}\left\{\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}$.

5. Solve the given differential equation: (Initial & Boundary Value Problems)

$(i)\ Y'' - 3Y' + 2Y = 4e^{2t},\quad Y(0) = -3,\ Y'(0) = 5$

$(ii)\ Y'' + 9Y = \cos 2t,\quad Y(0) = 1,\ Y(\pi/2) = -1$

$(iii)\ Y'' + 2Y' + 5Y = e^{-t}\sin t,\quad Y(0) = 0,\ Y'(0) = 1$

$(iv)\ Y''' - 3Y'' + 3Y' - Y = e^t t^2,\quad Y(0) = 0,\ Y'(0) = 0,\quad Y''(0) = -2.$

$(v)\ tY'' + (1-2t)Y' - 2Y = 0,\ Y(0) = 1,\ Y'(0) = 2$

$(vi)\ Y'' - tY' + Y = 1,\ Y(0) = 1,\ Y'(0) = 2$

$(vii)\ tY" + Y' + 4tY = 0,\ Y(0) = 3,\ Y'(0) = 0$

$(viii)$ *Solve the following system*: $\begin{cases} \frac{dx}{dt} + y = sint \\ \frac{dy}{dt} + x = cost \end{cases}$, $X(0) = 2,\ Y(0) = 0.$

6. Find the Laplace Transformation of the following functions:

$(i)\ F(t) = \begin{cases} 0, & 0 < t < 1 \\ t - 1, & 1 < t < 2 \\ t + 1, & t > 2 \end{cases}$

$(ii)\ F(t) = \begin{cases} 2, & 0 < t < 3 \\ t^2, & 3 < t < 5 \\ t + 1, & t > 5 \end{cases}$

$(iii)\ F(t) = te^{2t}(\sinh(t) + \cosh(t))$

7. Evaluate $F(t) = \begin{cases} 0, & 0 \le t < 3 \\ 2, & t \ge 3 \end{cases}$ using the definition $\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st}\, F(t)dt$

8. Use 2nd translation theorem to evaluate:

$(i)\ F(t) = 2 - 3u(t-2) + u(t-3)$

$(ii)\ \mathcal{L}^{-1}\left\{\frac{1}{s-4}e^{-2s}\right\}$

$(iii)\ \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}e^{-\frac{\pi s}{2}}\right\}$

9. Solve the given differential equation:

$Y' + Y = Y(t),\ Y(0) = 5,\ where\ Y(t) = \begin{cases} 0, & 0 \le t < \pi \\ 3cost, & t \ge \pi \end{cases}$