



# Small cracks—nucleation, growth and implication to fatigue life

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## Abstract

The nucleation and early propagation of fatigue cracks are smoothly consequential processes, which can occupy a major part of the fatigue life of structural materials. Nevertheless, their quantitative description is at present not yet sufficient to perform reliable life calculations. The paper summarizes the present state of the art in the recent advancement of nucleation and early propagation of cracks in metals both as for the mechanisms and as for their quantitative description. The following items are treated: Sites of crack nucleation, prerequisites and mechanisms of the nucleation, end of nucleation stage and transition to propagation stage, plasticity of small cracks and growth of microstructurally and mechanically small cracks.

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## 1. Introduction

Nucleation of fatigue cracks has been systematically studied for at least last fifty years. Application of high-resolution experimental techniques, especially electron microscopy, considerably enlarged the knowledge on details of the crack nucleation sites and on the morphology of the small cracks. In the 1960s the linear elastic fracture mechanics (LEFM) was for the first time used for the description of long cracks kinetics [1]; in the 1970s the LEFM description became generally accepted both for propagation rate of the long cracks and for their thresholds. Soon afterwards it was shown experimentally that the growth rate of the small cracks cannot be described in the same way as the growth rate of the long cracks. The growth rate of microstructurally small cracks (crack size smaller than or comparable to grain size or other characteristic dimension) oscillates and the growth rate of mechanically small cracks (cracks several times longer than grain size or other characteristic microstructural dimension) is significantly faster than that of the long cracks under the same nominal stress intensity factor range  $\Delta K$  [2].

The nucleation and early propagation of fatigue cracks are smoothly consequential processes without any clear border between them. That is why the end of the nucleation is most often identified simply with the occurrence of the first detectable cracks. In this conception the end of the nucleation depends on the experimental technique used. Nevertheless, even when using such an inexact definition, it is possible to assess the relative importance of the nucleation and propagation stages for the total fatigue life. While the crack nucleation can represent only a small fraction of the total fatigue life in the low cycle fatigue region, it can represent a substantial part of the life in the high cycle fatigue (HCF) region and the major part in the gigacycle fatigue region. For example, Wang et al. [3] came to the conclusion that the portion of life attributed to subsurface crack initiation in high strength steels cycled in the gigacycle region may exceed 99%.

The aim of the paper is to offer an overview of the present state of the art in the recent advancement of nucleation of fatigue cracks and of their growth before they reach the size when the classical LEFM is applicable.

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## 2. Nucleation of cracks

### 2.1. Where do the cracks nucleate

Macroscopically, the fatigue cracks in flaw-free materials and structures nucleate most often at free surface. In materials containing defects, the cracks originate preferably at these defects. Stress concentrations leading to higher local cyclic plastic strains are the reason for this behaviour. In engineering components, higher surface stresses result from either surface notches or bending and twisting, both of which lead to stress gradients with the highest stress on the surface. Even in nominal uniformly stressed parts, a small degree of eccentricity in the axial load is practically unavoidable, again leading to small bending or twisting moments, and consequently to stress concentration in the surface layer. Even in homogeneous materials loaded perfectly uniaxially the cracks would nucleate at the surface. This is due to the fact that the crack nucleation is controlled by the cyclic plastic deformation. The cyclic plastic deformation at or near the surface is higher than the average one due to the reduced constraint of the surface layer. This can be best seen in a polycrystalline metal. In the bulk material, each interior grain is under constraints imposed by neighbouring grains. Surface grains have a lower number of neighbouring grains than the interior grains and, consequently, the constraint is relaxed and the cyclic plastic deformation is easier. In materials with more or less homogeneously distributed pores, cavities or second phase particles, the cracks nucleate at these defects lying at or near the surface (in uniformly stressed parts), because the stress concentrating effect of a near-surface defect is higher than that of a defect in the interior.

The fact that the fatigue cracks nucleate at free surface has been repeatedly demonstrated by the observation that when a specimen is fatigued for a substantial fraction of its fatigue life and its surface is then removed by electro-polishing, the specimen in a subsequent test exhibits a fatigue life as long as that of a virgin specimen (e.g. Mughrabi [4]). Moreover, it is well established that the fatigue process is very sensitive to surface state and is influenced by the surface finish and surface treatment.

There are several noticeable exceptions to the rule that the fatigue cracks originate at the surface. The most important of them seems to be subsurface crack nucleation in the gigacycle fatigue region. In a number of engineering materials containing inclusions such a low amplitude cycling (number of stress cycles in the range  $10^7$ – $10^{10}$  cycles) leads to crack nucleation at the inclusions below the surface. The same materials cycled by stress amplitudes in the high cycle fatigue region, i.e. by amplitudes leading to failure after less than about  $10^7$  cycles, show surface nucleation (e.g. Wang et al. [3]). A further exception is the behaviour of some titanium alloys. Under suitable loading conditions in the standard

HCF region the cracks in these alloys nucleate under the surface without any evident connection with defects (e.g. Tokaji and Kariya [5]) both at cryogenic and room temperatures.

Direct microscopic observations of surfaces have shown that there are three basic types of nucleation sites: (i) fatigue slip bands, (ii) grain and twin boundaries and (iii) inclusions and other inhomogeneities. Very often a combination or simultaneous occurrence of these types has been observed. For example, in polycrystalline materials the nucleation at slip bands and grain boundaries cannot be treated separately, as the intergranular cracks nucleate at places where the slip bands (or homogeneous cyclic slip) impinge the grain boundaries (e.g. Wang et al. [6]). The choice of prevailing site of crack nucleation in polycrystalline materials (slip bands vs. boundaries) is strongly influenced by microstructure, loading conditions and environment. The nucleation at inclusions (and fibres or particles in composites) can be understood as cyclic slip localization due to the stress-concentrating effect of the inclusion, leading either to decohesion of the inclusion-matrix interface or to cracking of the inclusion.

An example of two possible crack nucleation sites at a pore is shown in Fig. 1. This micrograph shows the persistent slip bands (PSBs) and activated  $\gamma$  channels observed by using scanning electron microscopy (SEM) of the surface of superalloy single crystal CMSX-4 cycled in LCF region [7]. In one of the PSBs a crack is formed at the edge of the pore (marked by circle). The cyclic plastic deformation in this two-phase  $\gamma/\gamma'$  crystal is taking place mainly by the slip activity of more slip systems of the type  $\{111\} <011>$  in the  $\gamma$  channels perpendicular to the stress axis. This activity results in “squeezing out” or “sucking in” of the  $\gamma$  matrix in between the harder  $\gamma'$  particles. In Fig. 1 approximately

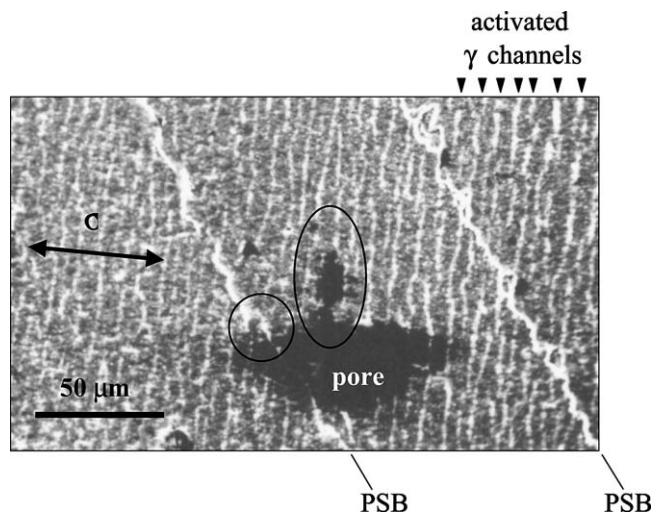


Fig. 1. Crack nucleation at a pore in CMSX-4 superalloy single crystal cycled in LCF region at 700 °C.

every 20<sup>th</sup> horizontal  $\gamma$  channel produces a visible surface relief. Interaction of one of the activated  $\gamma$  channels with the pore also leads also to formation of a crack (oval).

## 2.2. Why do the cracks nucleate

Fatigue crack nucleation is preceded by cyclic slip localization. The forms of the cyclic slip localization are different for different types of metals, for different loading conditions and environment and for different nucleation sites. As already mentioned, the cracks often start from the surface intrusions, i.e. from surface micronotches formed by cyclic plastic deformation. There is an abundance of papers showing gradual formation of surface relief. There are also many models of surface relief formation. In some metals it is formation of coarse slip surface bands lying along the slip planes and having a marked valley and hill topography, in other metals it is rather homogeneous fine slip with a less expressive topography. Further, the coarse slip surface bands are in some cases related to zones of specific dislocation structures, in other cases there is no difference between the dislocation structure underneath the surface coarse slip bands and in the remaining volume of the metal. In the former case the bands are persistent. PSBs in the sense of zones with specific dislocation structure ending on the surface with extrusions and intrusions have been studied very thoroughly for about 35 years. Their importance in fatigue has been overestimated. There are many metals in which the PSBs have never been seen. Examples are martensitic and bainitic steels and metals with difficult cross-slip. Even in the case of the simplest metals like copper, the zones of specific dislocation structure (PSBs) can be observed only under some conditions. Table 1 delimitates the regions of strain or stress amplitude, stress cycle asymmetry, orientation in the case of single crystals, grain size in the case of polycrystals and temperature in which the PSBs are formed during cycling of copper.

Several models have been proposed for the gradual formation of surface relief by the activity of the PSBs. These models are based on the slip processes, on the formation and migration of point defects, or on a combination of both. Perhaps most widely accepted is the

model by Essmann et al. [13], which is based on the role of vacancies and their agglomerates produced by non-conservative jog dragging and opposite-edge dislocation annihilation.

As already mentioned, the coarse-slip markings on the surface are not always related to the zones of specific dislocation structure that differ from the structure of the surrounding matrix. For example, asymmetrical cycling never produces PSBs even when the symmetrical cycling of the same metal does so [10]. In wavy slip metals like copper, the typical resulting dislocation structure is a cell structure in the whole loaded volume. Tensile mean stress causes cyclic creep, i.e. permanent elongation of the cycled specimen. The creep strain need not be high, but it cannot be accommodated by just one slip system. The slip activity of more slip systems inevitably leads to the formation of dislocation cells. The plastic deformation takes place by the motion of the whole slabs of the cells along the slip planes. Gradual formation of coarse slip markings on the surface is due to this repeated and only partly reversible sliding of the whole slabs of cells along the slip planes. The microcracks nucleate again in the surface slip intrusions. In the gigacycle regime it is the cyclic slip irreversibility, which very slowly builds up the surface relief [9]. In difficult-cross-slip metals, the irreversible part of the motion of dislocations in planar arrays leads to the planar hill and valley surface topography. The common denominator of all these cases is the nucleation of microcracks in surface intrusions.

In cyclically loaded material exposed to chemically aggressive media preferential attack of the environment at the selected locations may enhance crack nucleation. Corrosion pits are often observed at slip steps or intrusions at the surface. Another effect related to the presence of environment is its influence on the slip reversibility. Reduction in slip displacement during unloading due to oxidation is observed. Generally, environment affects the crack nucleation due to the influence on processes related to slip irreversibility and material dissolution. The details and intensity of these processes depend on the particular material and environment. Moreover, they need not to be fixed on the free surface only. In super long life fatigue regime environ-

Table 1  
Occurrence of PSBs in cycled copper

	Yes	No
Strain or stress amplitude [8,9]	Low and medium	Very low (gigacycle region) and high (LCF region)
Stress cycle asymmetry [10]	Symmetrical cycling	Asymmetrical cycling
Orientation of single crystals [6]	Single-slip	Some of multiple-slip orientations
Grain size of polycrystals [11]	Above several microns	Below several microns
Temperature [12]	Below $\sim 0.5 T/T_m$	Above $\sim 0.5 T/T_m$

mental effect associated with the hydrogen trapped at non-metallic inclusions has been shown to influence the crack nucleation [14].

The crack nucleation is an irreversible process, which has to be preceded by irreversible dislocation processes in the critical volumes. This leads to the conclusion that the necessary prerequisites for the microcrack nucleation are (i) expressive notch-peak topography, (ii) locally higher cyclic plastic strain at the intrusion root, and (iii) presence of suitable dislocation configuration around the surface intrusions. The notch-peak topography causes geometrical stress concentration. This in itself would not suffice; the intensity of the irreversible cyclic plastic deformation, i.e. the irreversible part of the dislocation glide, has to be higher at the intrusion root than in other places. This requires such a dislocation structure that prevents local stress relaxation by unimpeded glide of dislocations out of the critical volume and such dislocation movement which contributes to the intrusion root sharpening and not to its blunting. For these purposes the presence of suitable dislocation configuration is needed.

### 2.3. Models of crack nucleation

Several tens of crack-nucleation models have been proposed over the last five decades. Many of them have been already disproved by confrontation with the experimental observations. At present, it seems to be generally accepted that the fatigue crack nucleation is a process of cyclic plastic deformation, i.e. a process given by the generation and motion of dislocations and other lattice defects. There seems to be no qualitative border between the intrusion and the crack. For example, Ahmed et al. [15] state “the only distinction that can be made between an intrusion and a short stage I crack is the angle between the two free surfaces”. The continuous growth of intrusion into the depth of the crystal may happen by repeated slip on one or more slip systems. The basic idea for the one-slip-system models is the relative motion of parallel cards. Computer simulations of random irreversible slip represent the modern version of such models [16]. Neumann [17] proposed a model for the formation of cracks by coarse slip on alternating parallel slip planes. In all the proposed models the continuously growing intrusions eventually are deep enough to be termed cracks. Besides this terminological difference there is no distinction between the intrusion and the crack. Slightly different class of models is based on the accumulation of lattice defects leading to formation of microvoids [18]. Nucleation of cracks in grain boundaries is modelled as the consequence of the interaction of slip within the grain with the grain boundary [19].

### 2.4. End of nucleation

The simplest way how to define the end of the nucleation stage and thus the beginning of the propa-

gation stage is to use a characteristic crack size or other geometrical parameter. For example, Rosenbloom and Laird [16] define the “initiation” as an intrusion of  $4\text{ }\mu\text{m}$  depth. Repetto and Ortiz [20] identify the nucleation event in the continuous growth of a surface intrusion with the moment when the radius of the intrusion tip becomes zero. Because usually a whole system of surface intrusions and microcracks develops during cycling and some of them can stop, it does not seem plausible to relate the end of nucleation stage with the appearance of the first detectable microcracks. This transition is rather the transition from the system of microcracks governed by the cyclic plastic strain to fatal cracks propagation governed by the fracture mechanics. The first steps in this direction were already done by the idea of critical degree of strain relaxation [21]. Based on the fact that the mean spacing between cracks initiated in Cu single crystal decreases with increasing number of cycles and increasing plastic strain amplitude, strain relaxation has to be expected when critical mean spacing is reached. Shallower and more relaxed cracks stop growing and the strain redistribution accelerates the growth of the crack that becomes fatal.

Initiated cracks do interact when the critical spacing is reached. The crack distribution varies according to the material, slip systems etc. The mechanics of interaction of cracks has been theoretically analyzed (e.g. by Kachanov [22]) with the result that interaction effects may fluctuate even qualitatively from shielding to amplification; the exact position of microcracks in a short range zone is decisive. At present, it seems to be no quantitative description of these processes with general validity.

## 3. Behaviour of small cracks

### 3.1. Small crack plasticity

In the case of long cracks, the plastic zone can be detected by many experimental techniques, as the plastic zone exhibits a microstructure different from that in the surrounding material. Thus, for example, the observation of dislocation cell structure around the tip of a long crack clearly indicates high degree of cyclic plasticity [23]. In the case of small cracks it is usually assumed that the plastic zone is large in comparison to the crack size; direct experimental evidence is missing. Fig. 2 shows a TEM micrograph of extremely small cracks nucleated at the surface intrusions of a copper single crystal. The crystal was cycled under stress control with a long stress ramp at the beginning. The saturated shear plastic strain amplitude was  $3 \times 10^{-3}$  and the number of cycles to failure was  $1 \times 10^5$ . At this plastic strain amplitude the dislocation cell structure was formed right at the beginning of the cycling and the microcracks were nucleated later. There is no visible crack tip plastic zone in Fig.

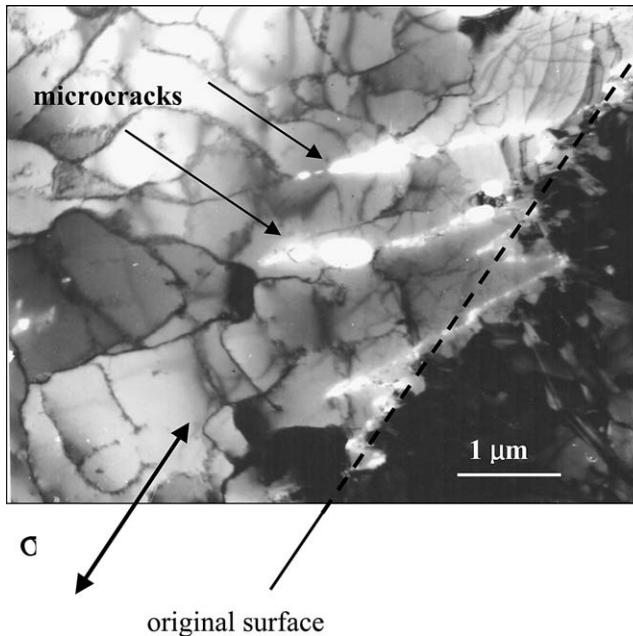


Fig. 2. Natural microcracks starting at surface intrusions in cycled copper single crystal.

2. It can be speculated that the crack had been developing and propagating by a single slip on slip planes emanating from the crack tip. Because the TEM observation on foils reveals only the “dead” structures, this type of slip activity might not be visualised at all. Nevertheless, the microcracks grew in the material subjected to cyclic plastic deformation of  $3 \times 10^{-3}$ . Thus the material surrounding the whole cracks including their tips did not behave elastically and the application of the LEFM to these cracks would not be justified. Another case is shown in Fig. 3. This is the case of high temperature cycling of superalloy single crystal CMSX-4 cycled with a low stress amplitude at a high mean stress [24]; under these conditions the material creeps. The TEM micrograph presented in Fig. 3 shows a crack running in “zig-zag” manner along the two sets of the persistent slip bands. Similarly as in the preceding case, there is no visible crack tip plastic zone in the fracture mechanical sense. Contrary to Fig. 2, the slip bands exhibit the specific dislocation structure. Detailed observation reveals a dislocation network in these bands, which witnesses to the slip on two coplanar slip systems. The creep plastic strain exceeds 5% and thus the plastic deformation, this time monotonic plastic deformation, excludes application of the LEFM. These two cases represent extremes. For cracks nucleated at stresses just above the fatigue limit the extent of plasticity is lower. Plastic strain amplitude corresponding to the fatigue limit is typically of the order of  $10^{-5}$  [25]. Stress concentrating effect of the intrusion (from which the microcrack starts) and later the stress concentrating effect of the microcrack certainly increases the local plastic strain

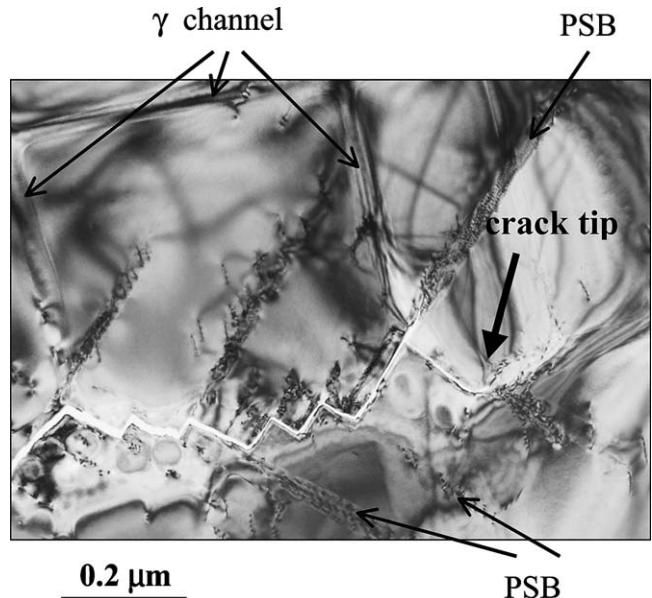


Fig. 3. Crack, which follows in “zig-zag” manner persistent slip bands running along different {111} planes in CMSX-4 superalloy single crystal cycled asymmetrically at 850 °C.

amplitude. Generally it is not possible to estimate how much. For the nucleation in the PSBs in ductile materials like copper a rough evaluation is possible. The plastic strain amplitude within the PSBs is about 100 times higher than that in the matrix. The microcrack nucleated and propagated within a PSB is thus connected with the cyclic plasticity of the order of at least  $10^{-3}$ ; the length of the PSB in which the microcrack grows can be many times larger than the crack size and thus the application of LEFM is not possible.

This paragraph can be summarized as follows. The application of LEFM to the description of the microstructurally small cracks is not possible, as these cracks grow under general yield conditions.

### 3.2. Growth of small cracks

The number of microcracks depends not only on the material and loading parameters, but also on the number of loading cycles. This is shown schematically in Fig. 4. First, the number of microcracks increases; some of them grow, some of them stop. Then either their number remains constant (no coalescence) or they coalesce and their number decreases. Measurement of the crack propagation rate showed that the grain boundaries retard or even arrest the growth of small cracks—the  $da/dN$  vs.  $a$  curve exhibits maxima and minima and the growth is fluctuating or intermittent. For small cracks several times longer than the grain size the growth curve can be considered smooth in the first approximation. One example is shown in Fig. 5. The cylindrical specimen of low carbon steel having a diameter  $D$  of 6 mm and a small flat and well polished area in the middle of the gauge length

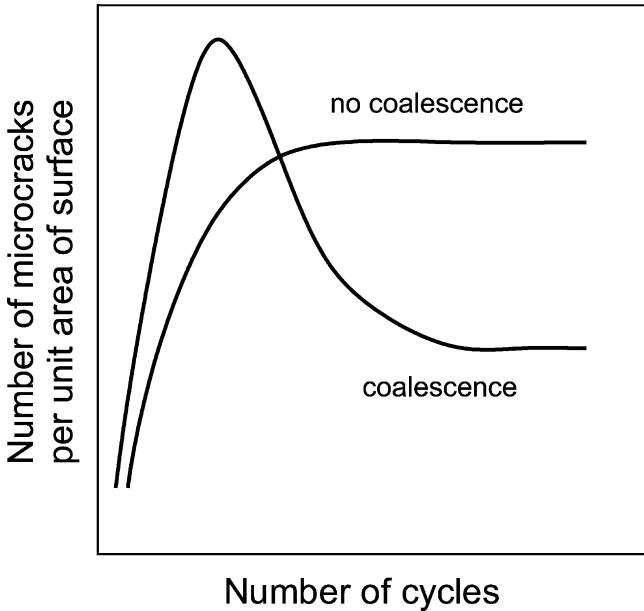


Fig. 4. Density of microcracks in dependence on number of loading cycles.

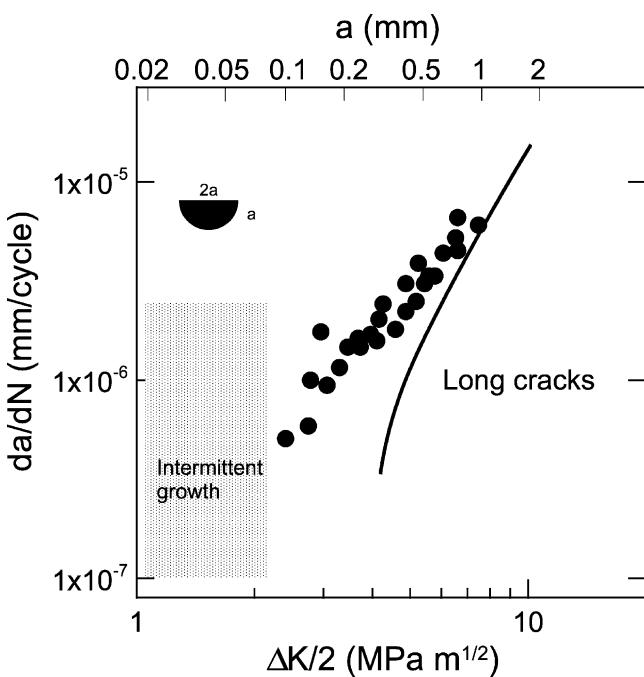


Fig. 5. Crack propagation rate of natural small crack in low carbon steel cycled at stress amplitude slightly exceeding fatigue limit.

was symmetrically cycled at the stress amplitude  $\sigma_a = 208$  MPa (corresponding to fatigue life  $1.8 \times 10^6$  cycles) slightly exceeding plain fatigue limit ( $\sigma_c = 200$  MPa). Mean grain size was  $40 \mu\text{m}$ . It can be seen that the intermittent growth applies roughly to two grains. For the crack lengths higher than  $0.1$  mm the amplitude of the stress intensity factor,  $\Delta K/2$ , was calculated from the expression for a surface semicircular crack with the

shape factor  $Y = 0.65$ . The crack length ratio  $a/D$  for the crack length limit  $0.1$  mm was  $0.016$ . This enables comparison of the small crack data (points) with the long crack data (full line). The propagation rate of the small cracks of the length  $0.1$ – $1$  mm is faster than that of the long cracks under the same  $\Delta K$ . At a length of about  $1$  mm the small crack and the long crack data merge.

There have been many attempts to describe the behaviour of small cracks, especially their propagation rate. A summary is presented in Table 2. Here the grain size or other relevant structural dimension is denoted by  $d$ . There is no generally accepted and generally applicable description of the propagation rate of the microstructurally small cracks. The equation proposed by Miller [26] describes the deceleration of cracks approaching grain boundaries. The problem is the determination of the  $\Delta\gamma$ , which is the local plastic resolved shear strain range. Measurement of the growth rate of large number of microstructurally small cracks on the surface of one specimen [27,28] showed that the  $da/dN$  values of individual propagating cracks at the given crack length can differ by a factor of 1000. Nevertheless it is possible to define an average value, which obeys the second equation in Table 2. The modifications of fracture mechanics presented in Table 2 have been reasonably successful for small cracks about 2–3 times longer than the grain size.

The Unified Approach developed by Vasudevan, Sadananda and co-workers and published in a series of papers (e.g. [32,33]) is promising and deserves a more detailed description. Its starting point is presentation of the long crack threshold data in the form of a dependence of the threshold stress intensity factor range,  $\Delta K_{\text{th}}$ , on the threshold maximum stress intensity factor,  $K_{\max}$ . An example of such dependence is shown in Fig. 6 for carbon steel. A similar curve can be drawn for any constant crack propagation rate. Vasudevan, Sadananda and co-workers see the prerequisites for the development of reliable life prediction model in the recognition that (1) the true material behavior is represented by the long crack growth properties, (2) fatigue damage must be described by two driving force parameters  $\Delta K$  and  $K_{\max}$  instead of one, and (3) the deviations from the long crack growth behavior arise from the internal stresses present ahead of the crack tip [33]. Thus the replacement of the  $\Delta K$  and  $K_{\max}$  applied values by their total values ( $\Delta K_{\text{total}} = \Delta K_{\text{appl}} + \Delta K_{\text{int}}$  and  $K_{\max,\text{total}} = K_{\max,\text{appl}} + K_{\max,\text{int}}$ ) in the diagram of the type shown in Fig. 6 leads to a generally valid diagram also covering small cracks. The problem is then reduced to the determination and/or calculation of the internal components  $\Delta K_{\text{int}}$  and  $K_{\max,\text{int}}$ . The mentioned series of papers contains a number of cases in which the internal components were evaluated. The two-component description of the driving force used in the Unified Approach, i.e. one cyclic and one monotonic, is plausible. So is the assumption of the linear summation of the applied and internal components of the

Table 2  
Quantitative description of small crack behaviour

$a < d$ microstructurally small cracks	$a > d$ mechanically small cracks
Crack propagation rate can be expressed as a function of either local ( $\Delta\gamma$ ) or applied ( $\Delta\varepsilon$ ) cyclic plastic strain range	Modifications and adaptations of linear elastic and elastic-plastic fracture mechanics
<ul style="list-style-type: none"> <li>• <math>da/dN = \text{const. } \Delta\gamma^i (d-a)</math> for decelerating cracks approaching grain boundaries [26]</li> <li>• <math>da/dN = \text{const. } \Delta\varepsilon^m</math> applied either to “equivalent” or to “most damaging” crack [27]</li> </ul>	<ul style="list-style-type: none"> <li>• crack closure approach [29]</li> <li>• crack deflection approach [30]</li> <li>• J-integral [31]</li> </ul>
	Unified Approach [32,33]

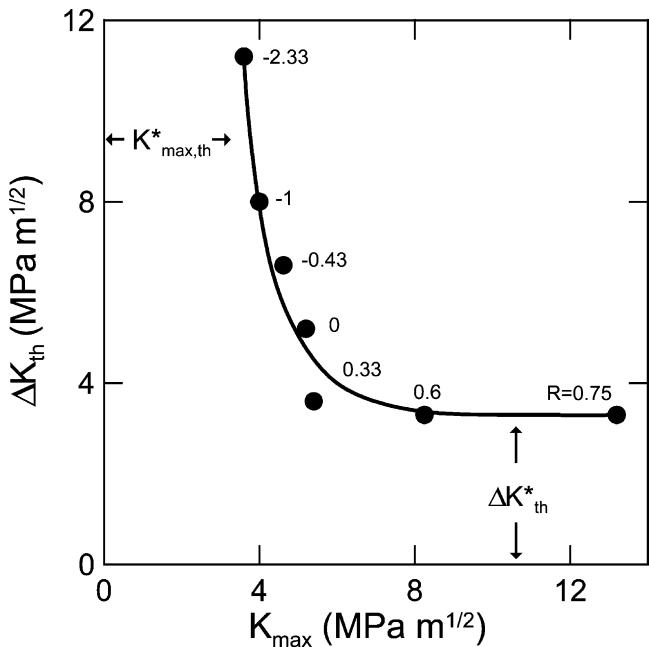


Fig. 6.  $\Delta K$  vs.  $K_{\max}$  curve for long cracks thresholds ( $da/dN < 5 \times 10^{-8}$  mm/cycle) in 0.6 C carbon steel.

driving forces. Debatable is the possibility to express the driving forces via  $K$ -factor. It is highly improbable for microstructurally small cracks as they grow under general yield conditions (see section 3.1), but it is plausible for longer mechanically small cracks. Nevertheless, two minor questions remain: (1) The applied values of the  $K$ -factor are based on remote stress, the internal values on local stress. The compatibility of these  $K$ -factors is not automatic. (2) As the driving force in LEFM is proportional to the square of the  $K$ -factor, rather the squares of the applied and internal  $K$ -factors should be added to get the total driving force. In sum, a rigorous delimitation of the validity of the promising Unified Approach would be needed.

#### 4. Conclusions

The present state of the art in advancement of nucleation and growth of small cracks can be summarized in the following way:

##### 1. Nucleation

(i) Sites of crack nucleation are known. (ii) Prerequisites for the nucleation are also largely known. (iii) Details of the mechanisms of the nucleation are still a matter of speculation. (iv) There is no generally accepted quantitative description of the nucleation process and there is no unequivocal definition of the end of the nucleation process.

##### 2. Growth of microstructurally small cracks

There is no generally accepted quantitative description of the microcrack behaviour, but there are promising attempts to characterize system of microcracks by “average” microcrack.

##### 3. Growth of mechanically small cracks

Modifications and adaptations of linear elastic and elastic-plastic fracture mechanics (crack closure, crack deflection,  $J$ -integral) have been reasonably successful. The United Approach appears promising; a rigorous delimitation of its validity would be needed.

Future development can be expected in the modeling on the basis of mechanisms and quantitative description.

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