Rounak Das

SE20UCSE149

CSE-3

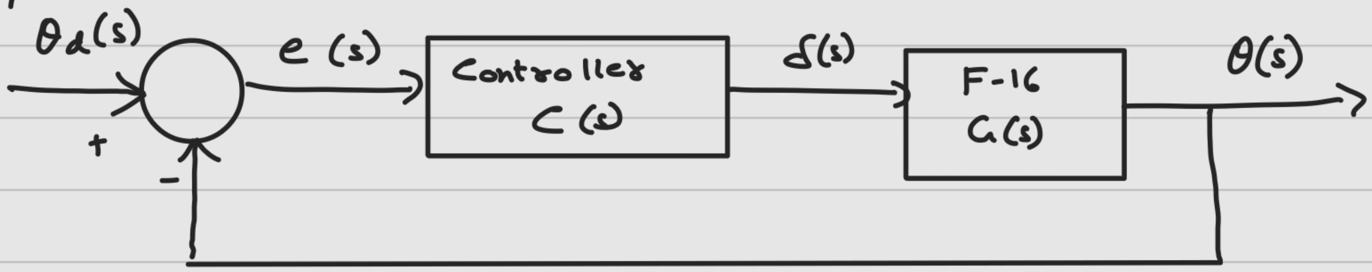
Control Theory Project.

[Screenshots of code and their respective outputs are at last few pages]

1) Unstable System.
(Look at question1.py)

2) (Look at question2.py)

3)



[Steady State error is zero]

$$Y_{ss} = \lim_{s\to 0} s\left[\theta_d(s) - \theta(s)\right].$$

$$= \lim_{s\to 0} s\left[\theta_d(s) - \frac{C(s)G(s)\,\theta_d(s)}{1+C(s)G(s)}\right]$$

$$= \lim_{s\to 0} s\ \theta_d(s)\left(\frac{1+\cancel{C(s)G(s)}-\cancel{C(s)G(s)}}{1+C(s)G(s)}\right).$$

$$= \lim_{s\to 0} s\frac{\theta_d(s)}{1+C(s)G(s)}$$

Now,

$$\lim_{s\to 0} s\ \frac{\theta_d(s)}{1+C(s)G(s)} = 0$$

$$\Rightarrow \lim_{s\to 0} \frac{\cancel{s} \times 1/\cancel{s}}{1+C(s)G(s)} = 0$$

$$\Rightarrow \lim_{s\to 0} \frac{1}{1+C(s)G(s)} = 0$$

$$\lim_{s\to 0} C(s)G(s) \approx \infty$$

$$= \lim_{s\to 0} \frac{\tilde{C}(s)}{s^N} \times \left(\frac{s+23}{s^2+s-1.19}\right)$$

$$\therefore \lim_{s\to 0} \frac{\tilde{C}(s)}{s^N} \text{ must be } \infty$$

$$s^N = 0$$

$\therefore N = 1, 2, 3 \ldots$

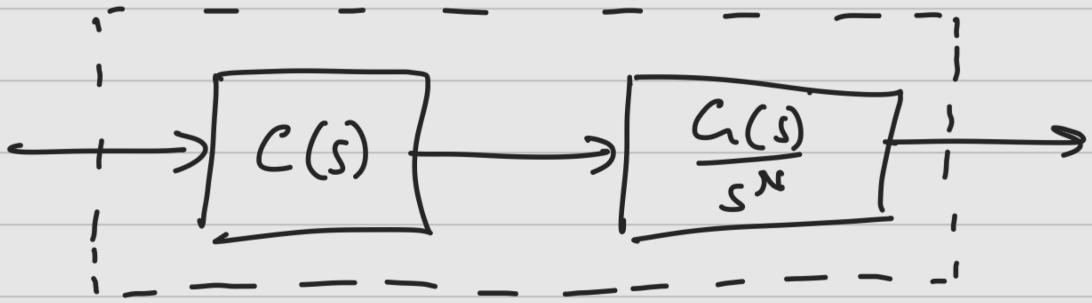
and min of $N = 1$.

$\therefore$ Minimum $N = 1$ ✓

4) $C(s) = \dfrac{\tilde{C}(s)}{s^N}$

Let's plug s^N to $G(s)$

$\therefore$ Our block diagram becomes



i) From the question 3, $N=1$.

Assuming $C(s)$ is K as in P-controller,

Characteristic eqn.:

$$1 + C(s)\frac{G(s)}{s^N} = 0$$

$$\Rightarrow 1 + \frac{C(s)G(s)}{s} = 0.$$

$$\Rightarrow 1 + C(s) \times \frac{1}{s}\left(\frac{s+23}{s^2+s-1.19}\right) = 0.$$

$$\Rightarrow s^3 + s^2 - 1.19s + C(s)s + 23 = 0. \quad \text{—— (1)}$$

Now,

for Max overshoot :

$$\xi = \frac{1}{\sqrt{1 + \left(\frac{\pi}{\log_e (0.2)}\right)^2}} \approx 0.455.$$

for settling time,

$$\frac{4}{\xi \omega_n} = 0.05.$$

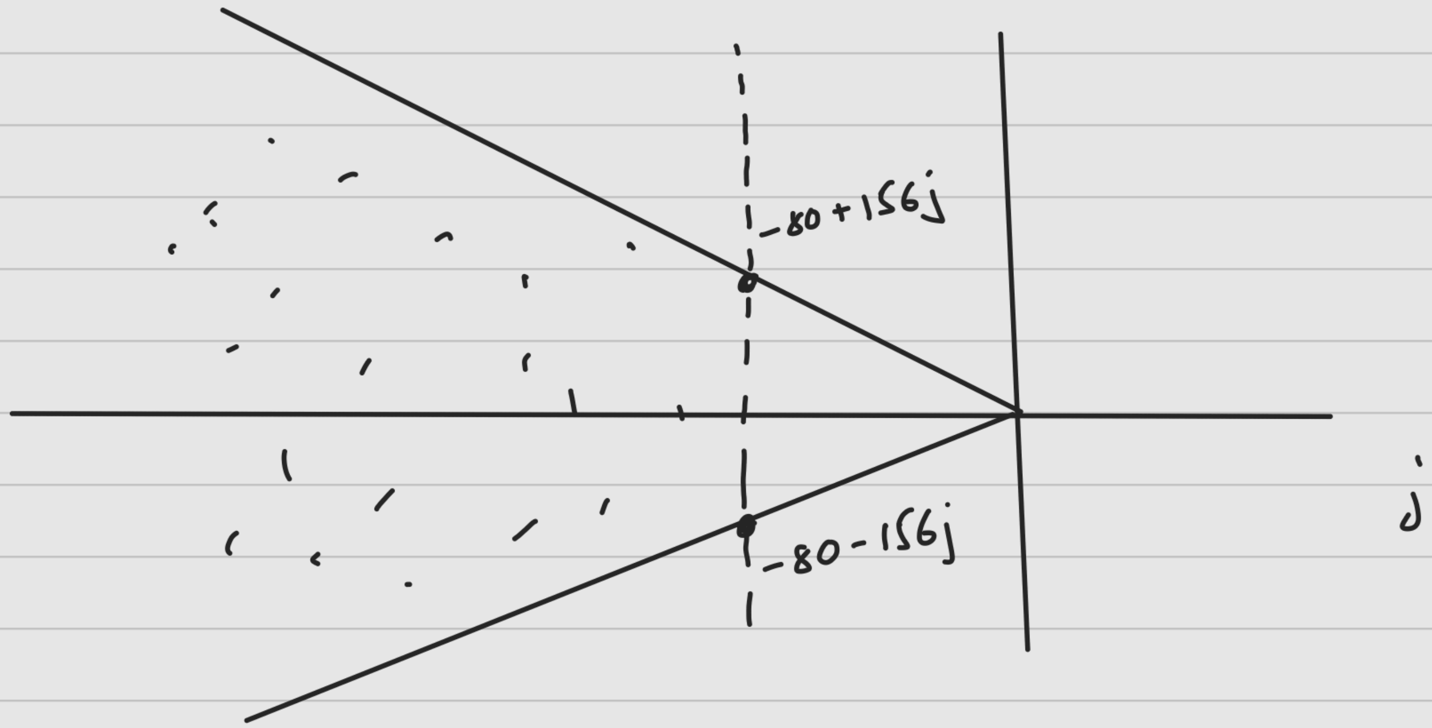
$$\Rightarrow \xi \omega_n = 80.$$

$$\therefore \omega_n = 175.82$$

$$\therefore s = -\xi \omega_n \pm \sqrt{1 - (\xi)^2}\, \omega_n j.$$

$$= -80 \pm \sqrt{0.792975} \times 175.82 j$$

$$= -80 \pm 156.56 j$$



$$s_0 = -80 + 156.56j.$$

$$\Rightarrow \angle G(s_0) = \angle \frac{(s_0+23)}{(s_0^2+s_0-1.19)} \times \left(\frac{1}{s_0}\right)$$

$$= \angle(s_0+23) - \angle(s_0^2+s_0-1.19) - \angle s_0$$

$$= \angle(-80+156.56j+23) - \left((-80+156.56)^2 + (-80+156.56j) - 1.19\right) - \angle s_0$$

$$= 110 + 126. - 117.$$
$$\approx 119^\circ$$

$\therefore$ We need to change our K to

$K G_c(s)$.

$$\boxed{\therefore C(s) = K G_c(s)}$$

Now, our characteristic eqn. becomes.

$$1 + KG(s) = 0$$

where $G(s) = G_c(s) \times \frac{(s+23)}{(s^2+s-1.19)} \times \frac{1}{s}$

Accordingly

$$\angle G(s_0) = \angle G_c(s_0) + 119^\circ.$$

$$\therefore \angle G(s_0) \approx \boxed{61^\circ}$$

Let's consider, $G_c(s) = \frac{s+z}{s+p}$

$p, z \geq 0$. Accordingly,

$$\angle G_c(s_0) = \angle \frac{s_0+z}{s_0+p} = \angle s_0+z - \angle s_0+p.$$

$$\boxed{\therefore \angle s_0+z - \angle s_0+p = -299^\circ.}$$

Assuming $z = 80$, $\angle s_0+z = 90^\circ$

$$90^\circ - \angle s_0+p = 61.$$

$$\Rightarrow \quad 29 = \angle s_0 + p$$

$$\angle(-80 + 156.56j + p) = 29$$

$$\Rightarrow \tan^{-1}\left(\frac{156.56}{-80+p}\right) = 29.$$

$$\Rightarrow p = 362.441 \approx 362$$

$$\therefore G_c(s) = \frac{s+80}{s+362}$$

$$\text{Now, } K = C(s) = \frac{1}{|G(s_0)|}$$

$$= \frac{1}{\left|\left(\frac{s_0+80}{s_0+362}\right) \times \left(\frac{s_0+23}{s_0^2+s_0-1.19}\right)\frac{1}{s}\right|}$$

$$= 67028.47477$$

$$\therefore C(s) = 67028.47 \times \left(\frac{s+80}{s+362}\right)$$

q5) $T(s) = \left[67028.47 \times \frac{(s+80)}{(s+362)} \frac{(s+23)}{(s^2+s-1.19)} \frac{1}{s} \right]$

Open Loop Tf ↗

Closed Loop tf

$$T(s) = \frac{C(s)G(s)}{1+C(s)G(s)}.$$

$$= \frac{x \left(\frac{s+80}{s+362}\right)\left(\frac{s+23}{s^2+s-1.19}\right) 1/s}{1 + x\left(\frac{s+80}{s+362}\right)\left(\frac{s+23}{s^2+s-1.19}\right) 1/s}.$$

$$= \frac{x(s+80)(s+23)}{(s+362)(s^2+s-1.19)s + x(s+80)(s+23)}$$

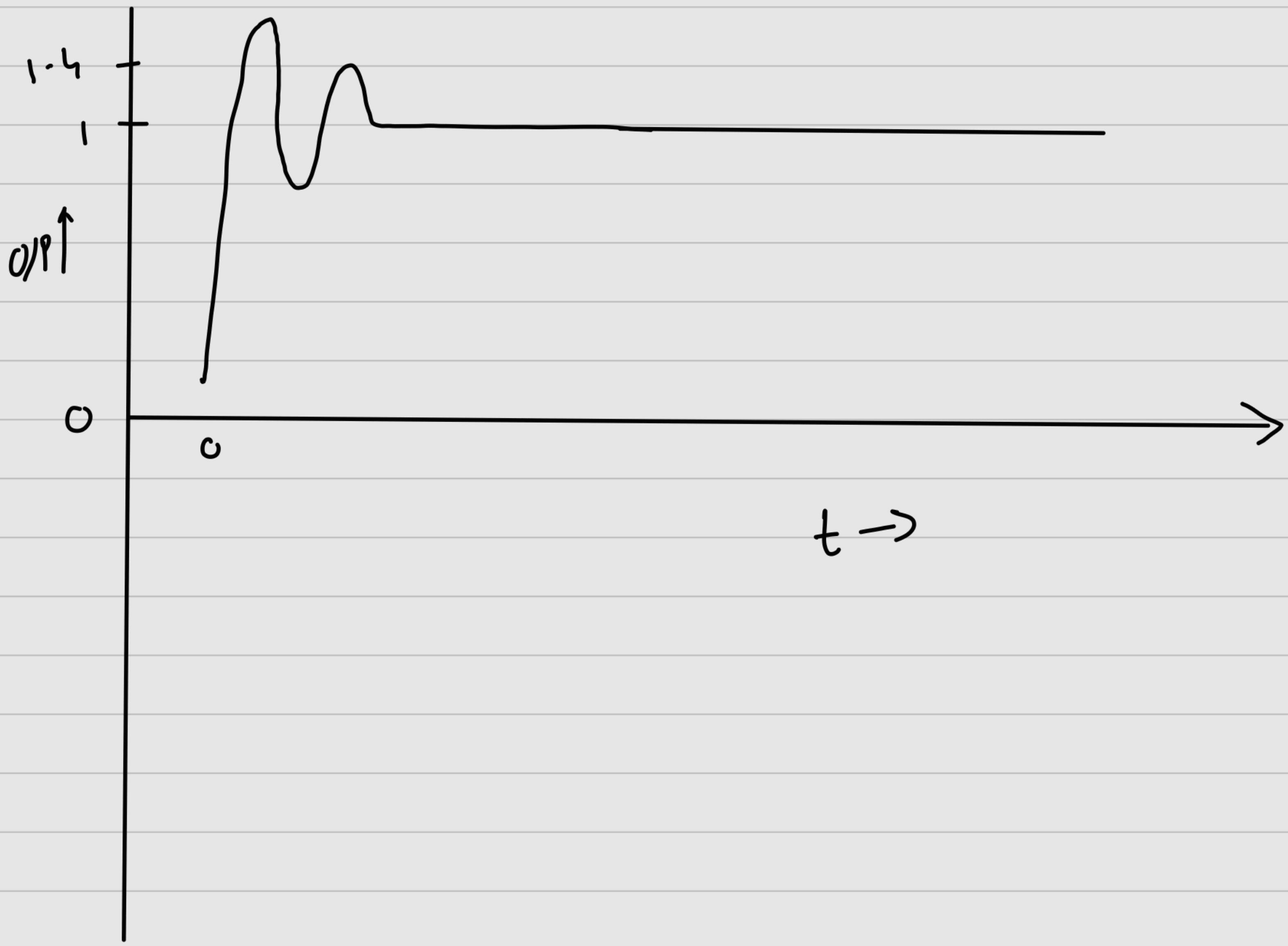
$$= \frac{[s^2 + 103s + 1840]x}{(s+362)(s^3+s^2-1.19s) + x(s^2+103s+1840)}$$

$$= \frac{(s^2+103s+1840)x}{s^4+s^3-1.19s^2+362s^3+362s^2-430.78s - x(s^2+103s+1840)}$$

Q6) Max overshoot is 20%. Therefore, O/P shouldn't exceed $1 + \frac{20}{100} = \frac{6}{5} = 1.2$.

But here it exceeds

So, it does not satisfy controller design.



For pole-zero plot, run

<u>question6.py</u>.

Q7) If we change design requirement for overshoot to 60%, then our closed loop system should satisfy all of the requirements.

$$\xi = \frac{1}{\sqrt{1+\left(\frac{\pi}{\ln(0.6)}\right)^2}}$$

$$\approx 0.16$$

$$\omega_n = \frac{80}{0.16} \approx 500$$

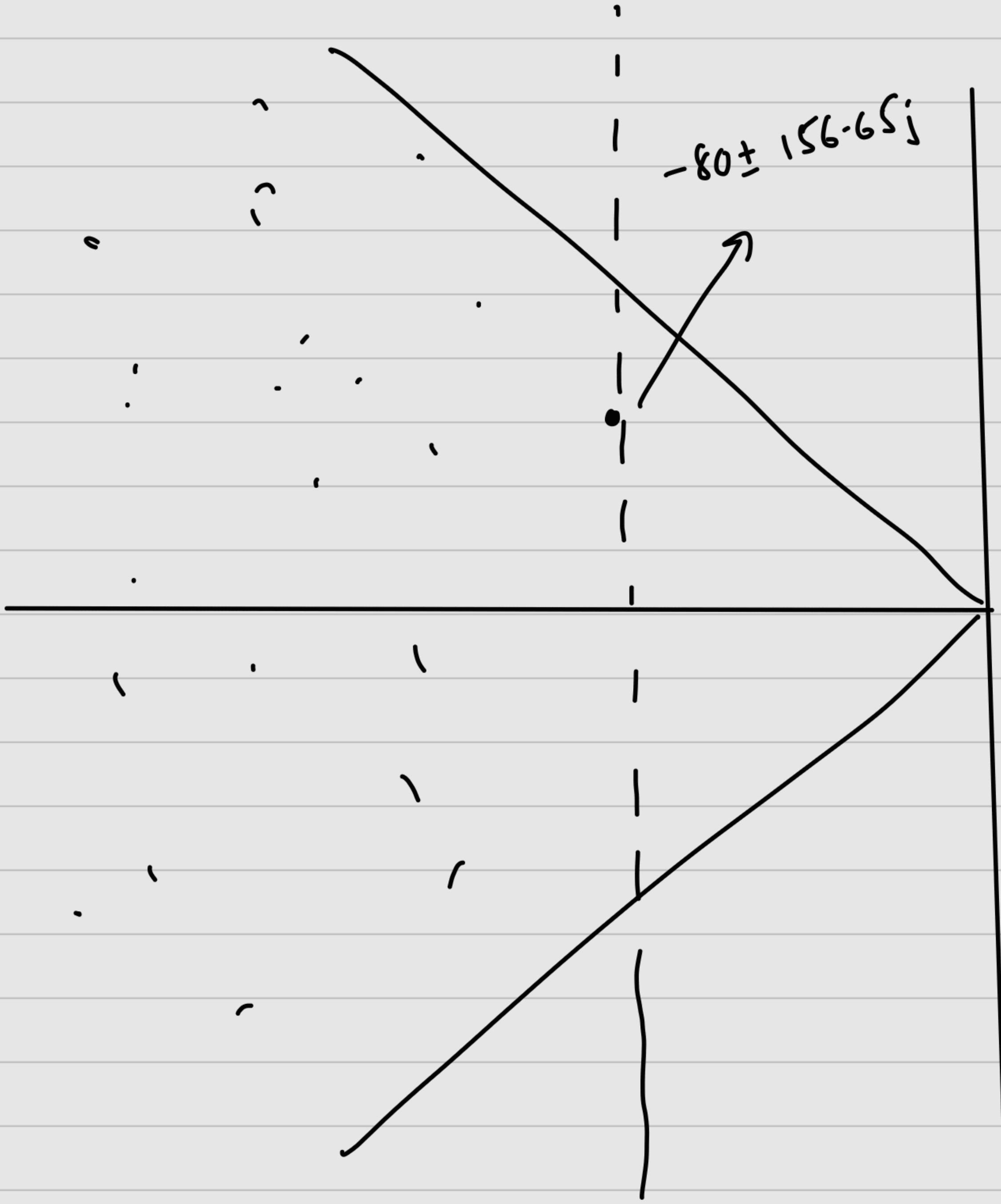
$$S = -\xi\omega_n \pm \sqrt{1-\xi^2}\,\omega_n j$$

$$= -80 \pm 493.55j$$

$\therefore$ $s_0 = -80 \pm 156.65j$ is under the $-80 \pm 493.55j$

Introducing a pole at s_0

satisfies all design requirements.

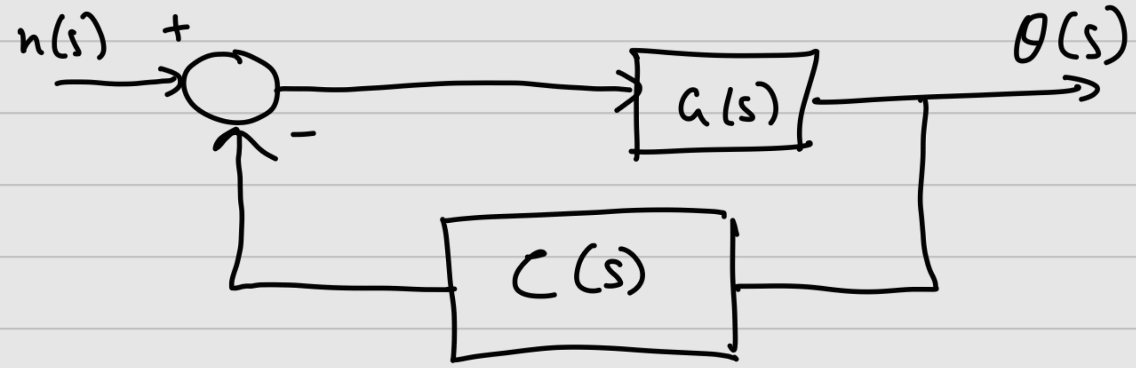


Q8)

For finding $n(s)$, we put

$\theta_d(s) = 0$

$\therefore$



$$\frac{\theta(s)}{n(s)} = \frac{G(s)}{1 + C(s)G(s)}$$

$x = 67028.47$

$$\frac{\theta(s)}{n(s)} = \frac{(s+23)(s+362)}{s^4 + s^3 - 1.19s^2 + 362s^3 + 362s^2 - 430.78s - x(s^2 + 103s + 1840)}$$

$$= \frac{(s^2+23s)(s+362)}{s^4 + 363s^3 + (360.81 + x)s^2 - (430.78 - 103x)s + 1840x}$$

$$= \left(\frac{s^2 + 3625 + 23s + 8326}{- \, - - \, \cdot \, \cdot} \right)$$

It is simulated in

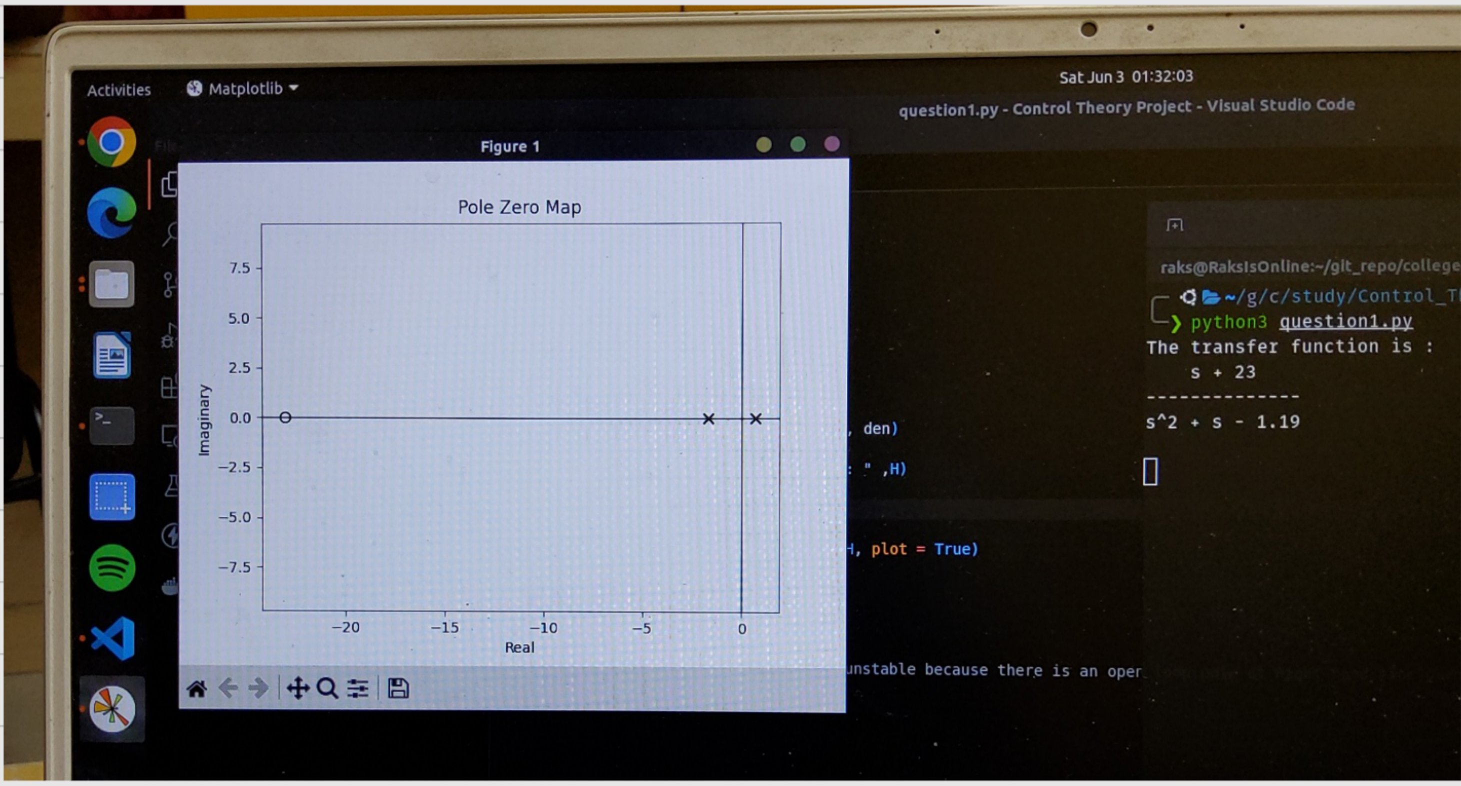
question8_1.py.

For θ_d input,

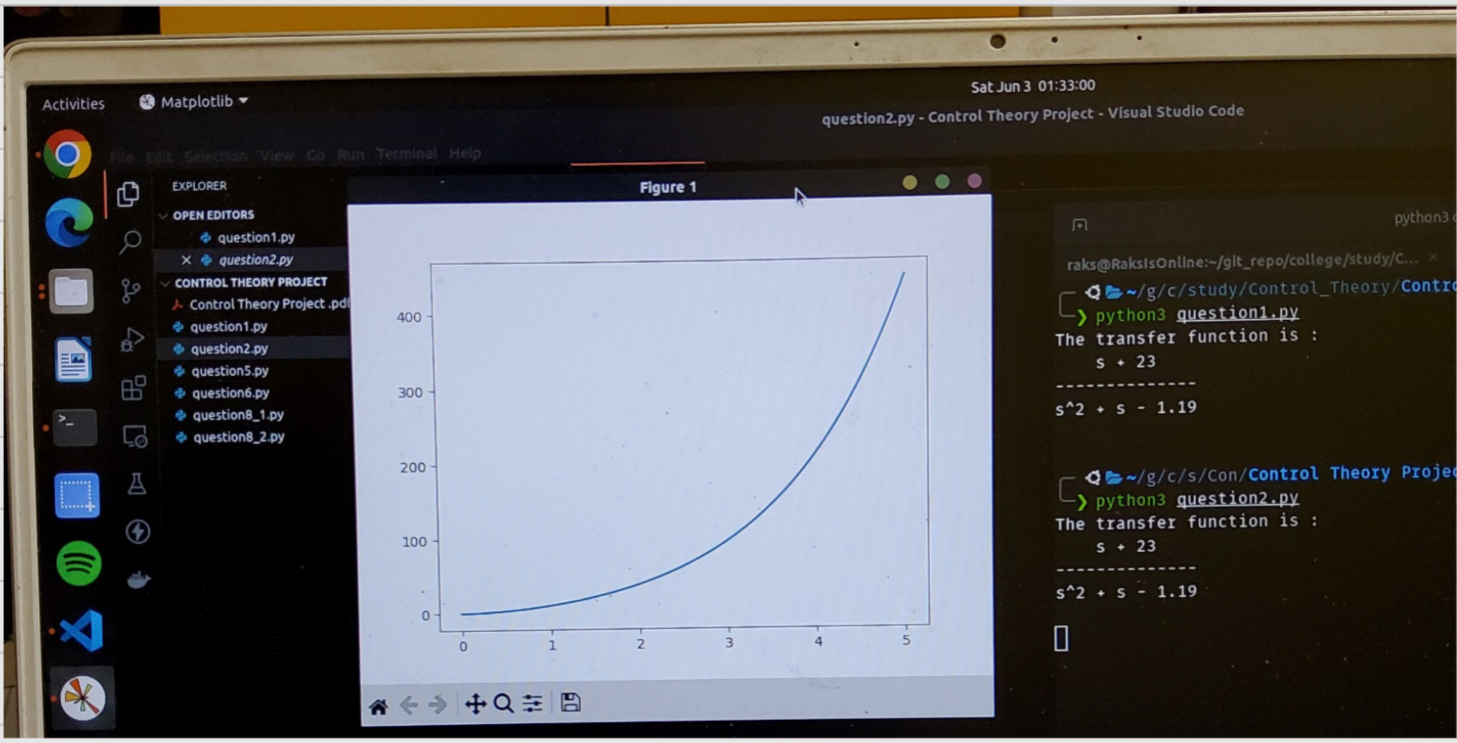
the graph is simulated

in question8_2.py.

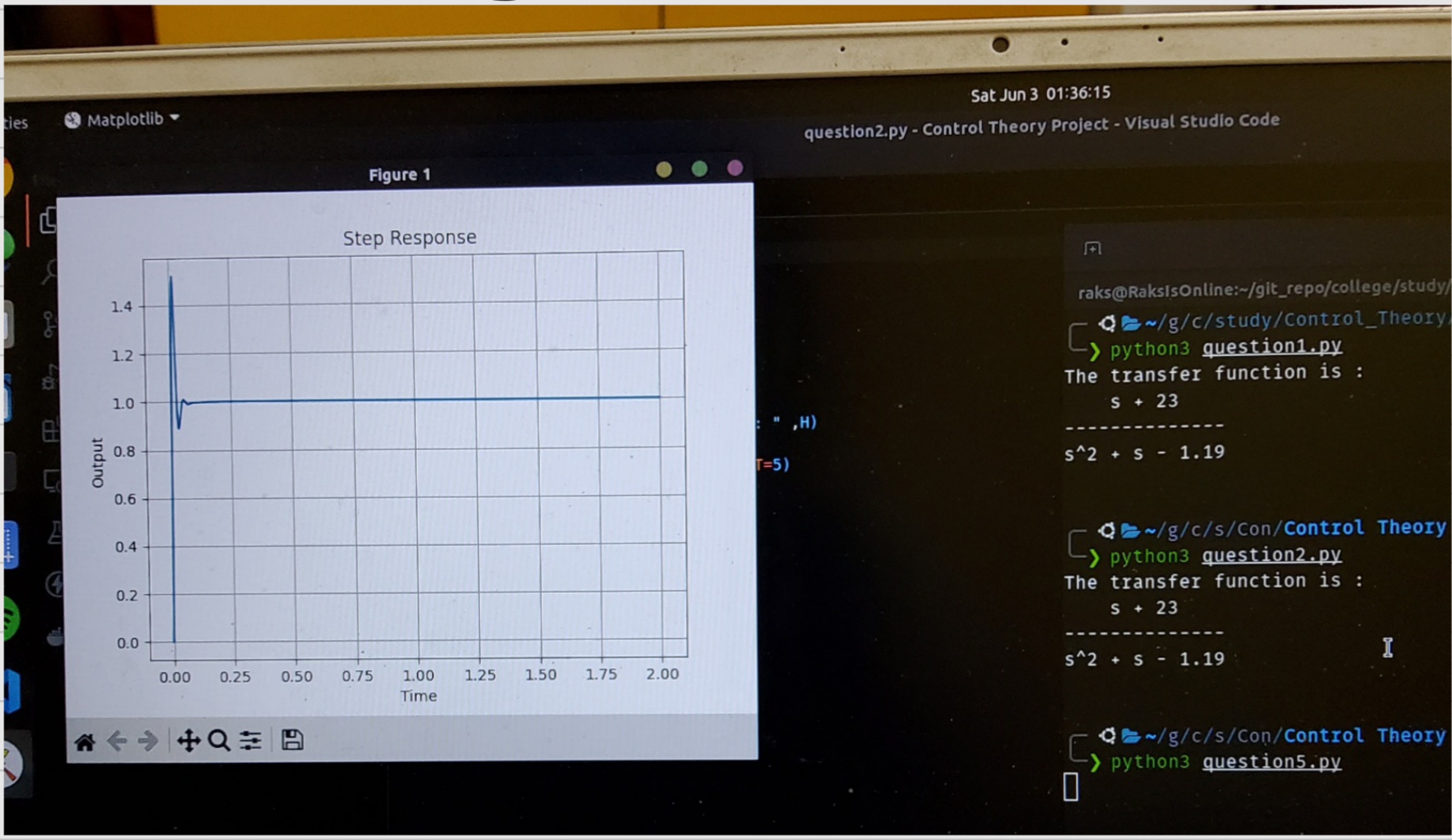
question1.py



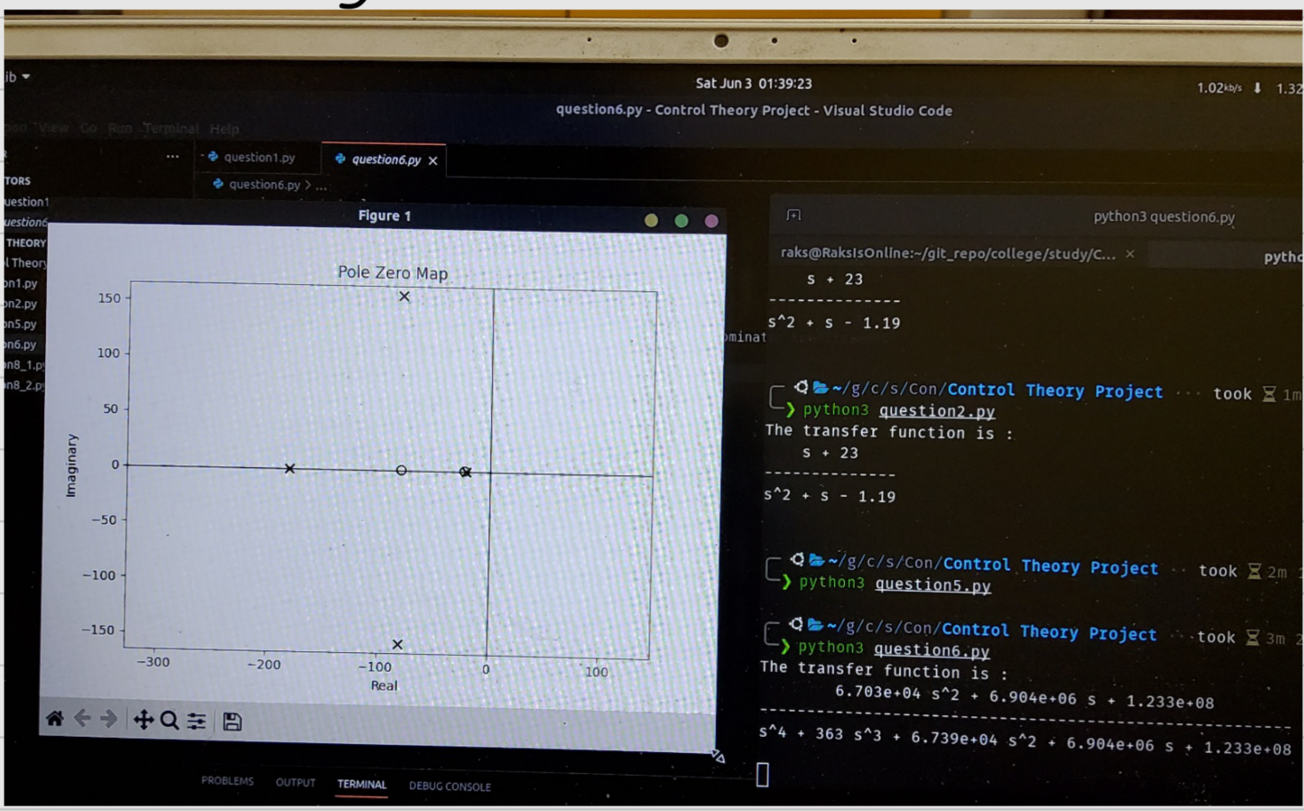
question2.py.



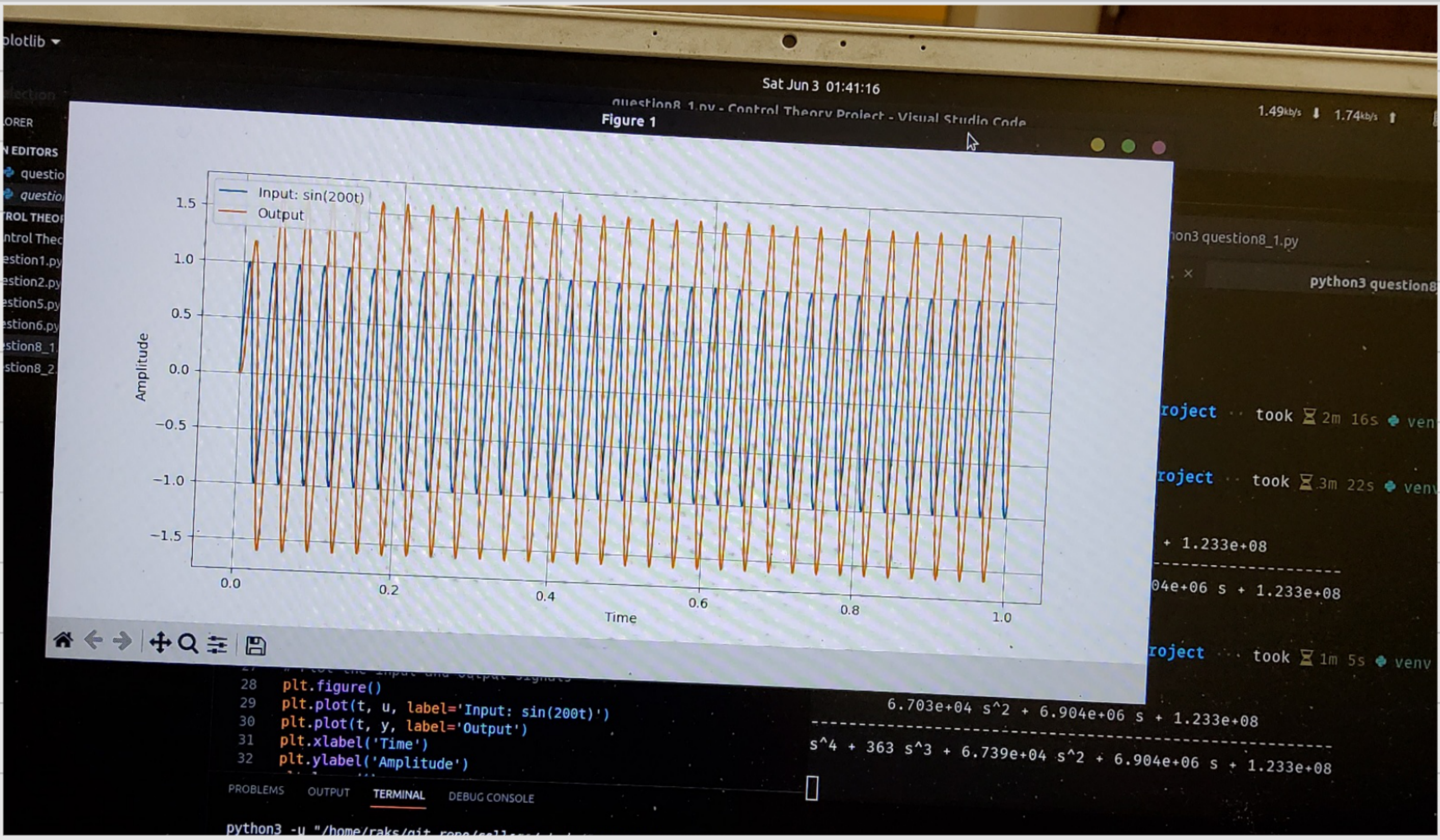
question5.py



question6.py



question8_1.py



question8_2.py