Assignment 2 EE 303

Objective:

-To generate and plot the probability density function (PDF) of following six distributions using MATLAB or OCTAVE using Normal (Gaussian) distribution as the base distribution.
-To verify obtainned PDF using theoritical plot of all distribution by (PDF) value of all six distributions.

1. Chi distribution
2. Chi-Squared distribution
3. Non-Central Chi-Squared distribution
4. Rice distribution
5. Gamma distribution
6. Nakagami-m distribution.

Methodology: (method to generate these six different PDFs using Gaussian distribution)

1.Chi distribution with degrees of freedom k.

Chi distribution is a continuous probability distribution. It is the distribution of the positive square root of the sum of squares of a set of independent random variables each following a standard normal distribution.

$$Y = \sqrt{\sum_{i=1}^{k} Z_i^2}$$

Here Z1,Z2,..Zn are independent, normally distributed random variables with mean 0 and standard deviation 1, Then Y is distributed according to the chi distribution. The chi distribution has one parameter k, which specifies the number of degrees of freedom. (i.e. the number of random variables).

In code:

Using z_i = randn(1,N) to obtain several(k) normally distributed random variables with mean 0 and standard deviation 1.
We can generate/transform to chi random variable(Y) by using these z random variables.
We can obtain Y by calculating square root of the sum of squares of k variables.
(Y = sqrt(Z_1^2 + Z_2^2 …. + Z_k^2)) and plotting its normalised histogram(Y) to get its plot for diff. k.

To verify:

-Then, we can compare the obtained simulated plot with theoritical PDF of chi distribution.

$$f(x;k) = \begin{cases} \dfrac{x^{k-1}e^{-x^2/2}}{2^{k/2-1}\Gamma\left(\frac{k}{2}\right)}, & x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

-Here, f(x,k) is PDF function of chi distribution with parameter k for k>0.
-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

2.Chi-Squared distribution with degrees of freedom k

The chi-squared distribution (also chi-square or χ^2-distribution) with *k* degrees of freedom is the distribution of a sum of the squares of *k* independent standard normal random variables.

$$Y = \sum_{i=1}^{k} Z_i^2$$

Here Z1,Z2,..Zn are independent, normally distributed random variables with mean 0 and standard deviation 1, Then Y is distributed according to the chi-square distribution. The chi-square distribution has one parameter k, which specifies the number of degrees of freedom.(i.e. the number of random variables).

In code:

Using z_i = randn(1,N) to obtain several(k) normally distributed random variables with mean 0 and standard deviation 1.
Similarly, we can generate/transform to chi-square random variable(Y) by using these z random variables.
We can obtain Y by calculating sum of squares of k variables.

($Y = (Z_1$^2 + Z_2^2 …. + Z_k^2)) and plotting its normalised histogram(Y) to get its plot for diff. k.

To verify: We can compare the obtained simulated plot with theoritical PDF of chi-square distribution.

$$f(x;k) = \begin{cases} \dfrac{x^{k-1}e^{-x^2/2}}{2^{k/2-1}\Gamma\left(\frac{k}{2}\right)}, & x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

-Here, f(x,k) is PDF function of chi-square distribution with parameter k for k>0.

-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

3. Non-Central Chi-Squared distribution with degrees of freedom k.

The noncentral chi-squared distribution (or noncentral chi-square distribution, noncentral χ^2-distribution) is a noncentral generalization of the chi-squared distribution.

Similar to chi-square it has:

$$Y = \sum_{i=1}^{k} Z_i^2$$

$$\lambda = \sum_{i=1}^{k} \mu_i^2$$

Here Z1,Z2,..Zn are independent, normally distributed random variables with mean u_i and standard deviation 1, Then Y is distributed according to the noncentral chi-squared distribution.

The noncentral chi-squared distribution has one parameter k, which specifies the number of degrees of freedom and λ which is related to the mean of the random variables. as sum of square of means of all Z_i.. λ is called the noncentrality parameter.

In code:

Using z_i = randn(1,N) to obtain several(k) normally distributed random variables with mean u and standard deviation 1 N(u_i,1). for $z_1,z_2…z_k$ = N(u_i,1).

Or we can take '1 random variable mean sqrt(λ) and variance 1 ' N(sq(λ),1) and 'others having mean 0 and variance 1' N(0,1)

for z_1 = N(sq(λ),1) and $z_2,z_3..z_k$ = N(0,1).

Similarly, we can generate/transform to noncentral chi-squared variable(Y) by using these z random variables.

We can obtain Y by calculating sum of squares of k variables.

($Y = (Z_1$^2 + Z_2^2 …. + Z_k^2)) and plotting its normalised histogram(Y) to get its plot for diff. k.

To verify:

-Then, we can compare the obtained plot with theoritical PDF of noncentral chi-squared distribution.

$$f_X(x;k,\lambda) = \sum_{i=0}^{\infty} \frac{e^{-\lambda/2}(\lambda/2)^i}{i!} f_{Y_{k+2i}}(x)$$

-Here, f(x,k) is PDF function of noncentral chi-squared distribution with parameter k for k>0 and for value λ.

-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

4. Rice distribution with parameter v.

The Rice distribution or Rician distribution is the probability distribution of the magnitude of a circularly-symmetric bivariate normal random variable, possibly with non-zero mean (noncentral).

$R \sim \text{Rice}(|\nu|, \sigma)$ if $R = \sqrt{X^2 + Y^2}$ where $X \sim N(\nu\cos\theta, \sigma^2)$ and $Y \sim N(\nu\sin\theta, \sigma^2)$

Here X, Y are independent, normally distributed random variables with mean vcos(θ) & vsin(θ) and standard deviation σ,

Where,v is distance between the reference point and the center of the bivariate distribution and θ is any real number.

Then R is distributed according to the rice distribution having parameter v and scale factor σ .

In code:

Using [X = randn(1,N)*σ + vcos(θ)] and [Y = randn(1,N)*σ + vsin(θ)] to obtain normally distributed random variables with mean vcos(θ) & vsin(θ) and standard deviation σ.

We can generate/transform to rice random variable(R) by using these X and Y random variables.
We can obtain R by calculating square root of sum of squares of X and Y variables.
(R = sqrt(X^2 + Y^2)) and plotting its normalised histogram(R) to get its plot for diff. values of v.

To verify:

-Then, we can compare the obtained simulated plot with theoritical PDF of rice distribution.

$$f(x \mid \nu, \sigma) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2+\nu^2)}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right)$$

-Here, f(x,k) is PDF function of rice distribution with parameter v for v>0 and for value σ>0

-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

5.Gamma distribution with parameters α and β.

The gamma distribution is a two-parameter family of continuous probability distributions with a shape parameter *k* and a scale parameter *θ*.
By parametrisation of a gamma distribution from normal distribution:

$$N^2(x;\sigma^2) = \mathrm{Gamma}(x; \frac{1}{2}, 2\sigma^2)$$

$$N^2_\Sigma(x;k,\sigma^2) = \mathrm{Gamma}(x; \frac{k}{2}, 2\sigma^2)$$

Here N is square of a normal random variable with mean 0. and standard deviation σ, Then gamma distribution can paramatrized for single squared normal distribution.
As, sum of two gammas (with the same scale parameter) equals another gamma, So gamma is equivalent to the sum of k square normal random variable having same σ.
having (alpha = k/2 and beta = 1/θ for θ = 2*σ)

In code:

Using z_i = randn(1,N) to obtain several(k) normally distributed random variables with mean 0 and standard deviation 1 N(0,1).
for $z_1, z_2 \ldots z_k$ = N(0,1).
We can generate/transform to gamma(Y) by using sum of square of these z random variables.
We can obtain Y by calculating chi-square random variable of z for k normal distributions , then scaling it by 2*σ.
Here,Gamma parmeter for using k value are 'alpha = k/2' and 'beta = 1 / 2*σ' (or 'θ = 2*σ')
Y = (Z_1^2 + Z_2^2 …. + Z_k^2) / 2*σ , and plotting its normalised histogram(Y) to get its plot for diff. alpha and beta value.

To verify:

-Then, we can compare the obtained simulated plot with theoritical PDF of gamma distribution.

$$f(x;\alpha,\beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$$

-Here, f(x,k) is PDF function of gamma distribution with parameter α and β for α,β > 0 .

-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

6.Nakagami-m distribution with parameters m and w.

Nakagami-*m* distribution is a probability distribution related to the gamma distribution. Nakagami-m distributions has two parameters: shape parameter m>1/2 and spread parameter w>0.

$$Y = \sqrt{\sum_{i=1}^{k} Z_i^2}$$

-Here Z1,Z2,..Zn are independent, normally distributed random variables with mean 0 and standard deviation 1, Then Y is distributed according to the chi distribution.

$$X = \sqrt{(\Omega/2m)}Y$$

-Nakagami distribution can be generated from the chi distribution Y with parameter k set to 2m and then following it by a scaling transformation of random variables by sqrt(w/(2m)) for k =2*m.

-So, Nakagami random variable X is generated by scaling transformation on a Chi distributed (Y) which is generated by normally distributed random variables N(0,1).

In code:

Using z_i = randn(1,N) to obtain several(k) normally distributed random variables with mean 0 and standard deviation 1 ~N(0,1).

for $z_1, z_2 \ldots z_k$ = N(0,1).

We can generate/transform to Chi~(Y) by using square root of sum of square of these z random variables.

We can obtain Nagakami-m (X) by calculating chi random variable Y for k = 2*m , then scaling it by sqrt(w/(2m)) for diferent values of m>0.5 such that (k = 2m is integer) and for w>0.

To verify:

-Then, we can compare the obtained simulated plot with theoritical PDF of nakagami-m distribution.

$$f(x;\, m, \Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right)$$

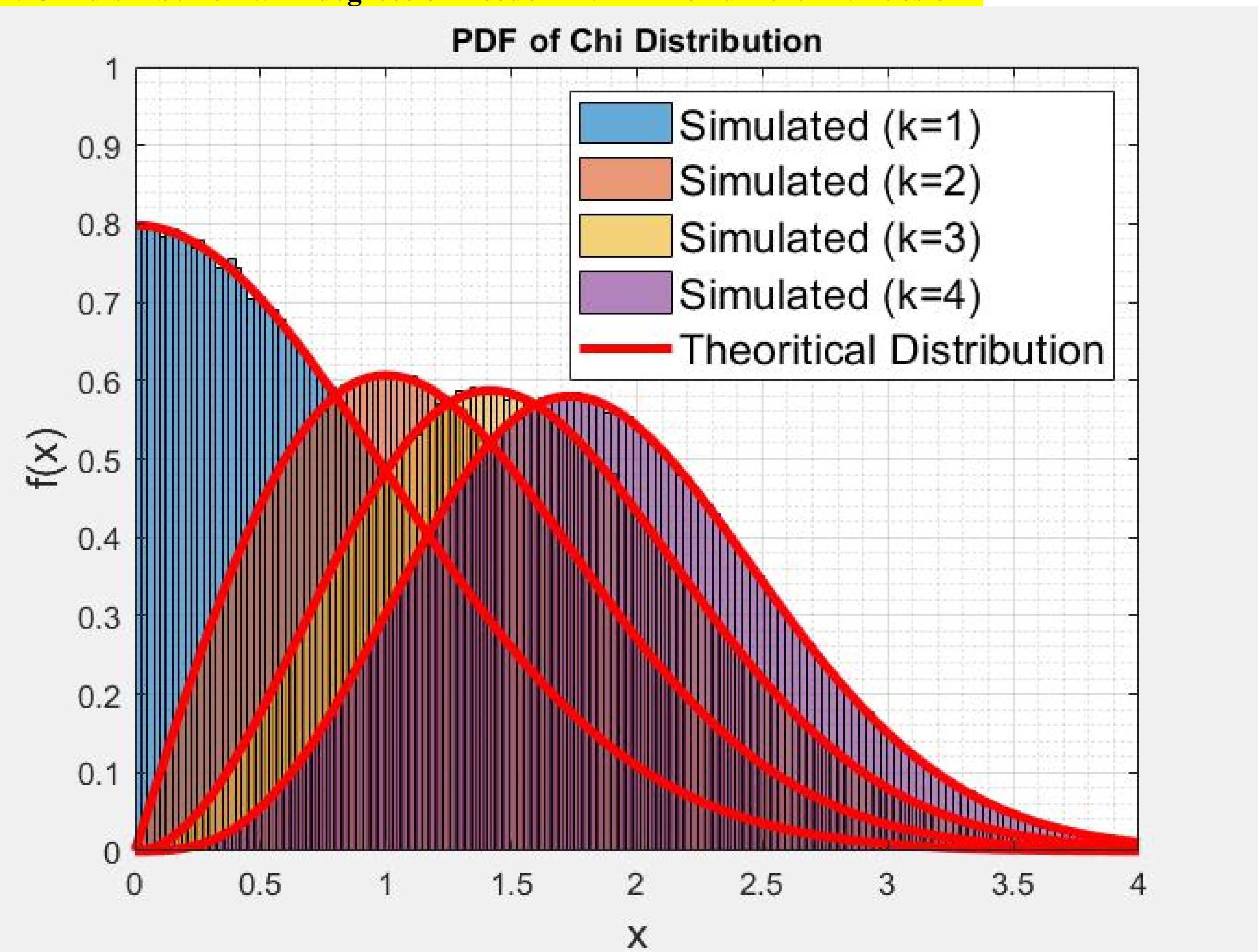
-Here, f(x,k) is PDF function of nagakami-m distribution with parameter m and Ω for m>=0.5 , Ω>0 .

-We can plot its distribution to compare the previous result with plot(x,f(x)) in matlab for x>0.

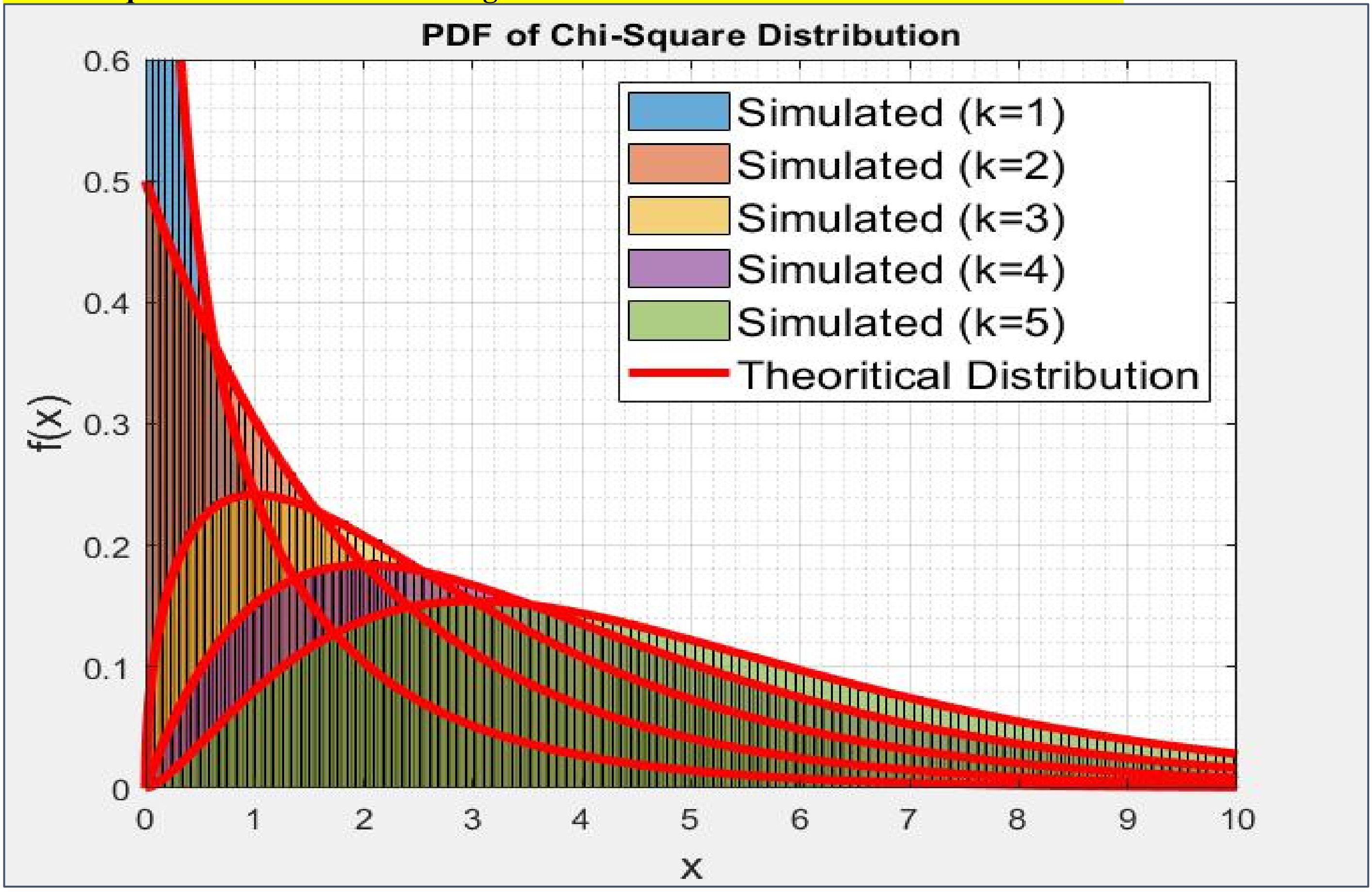
Results and Discussions:

-We have plotted the given six distribution generated using gaussian distribution and also plotted their theoritical PDF value of distribution, to verify them:

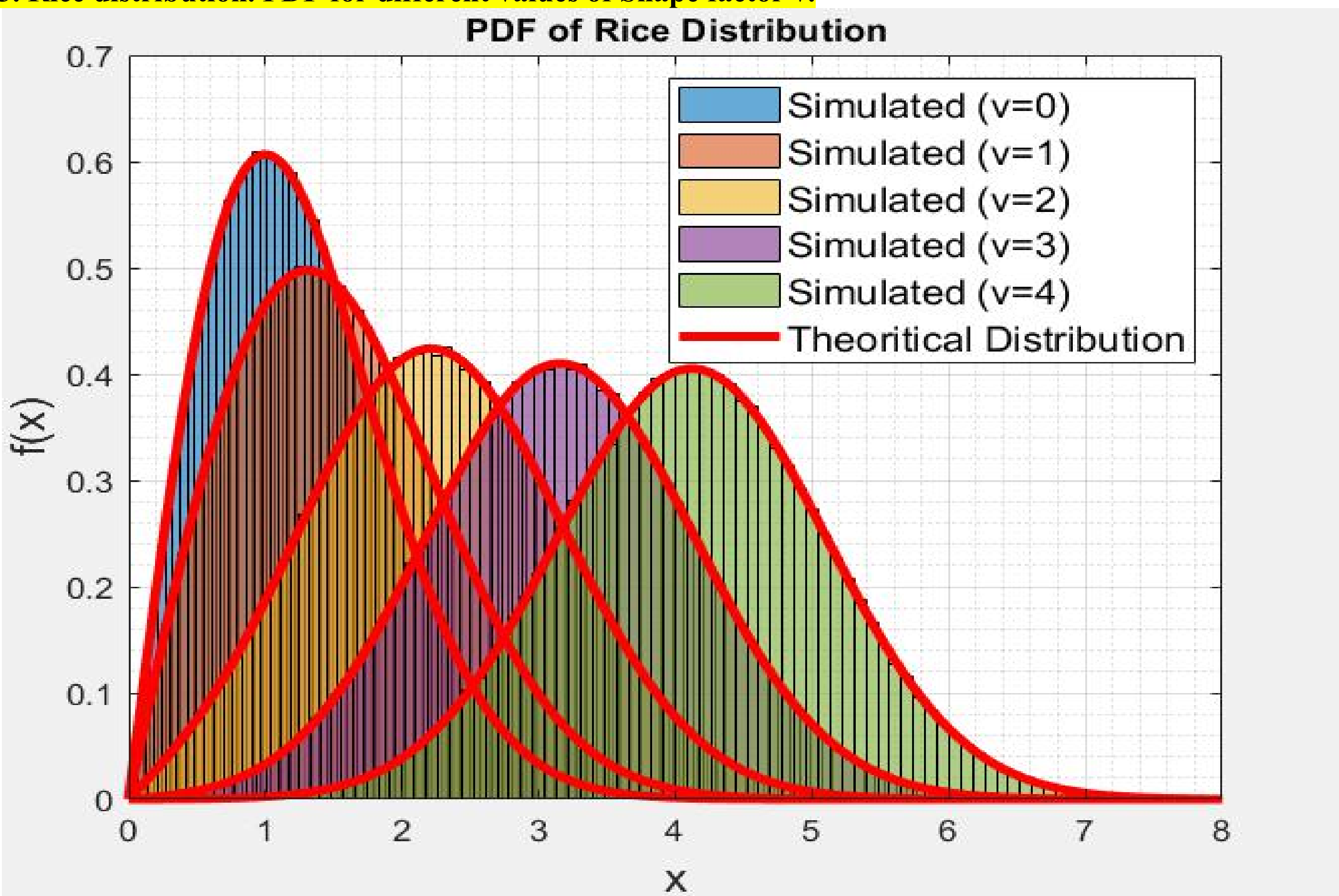
1. Chi distribution with degrees of freedom k. PDF for different values of k



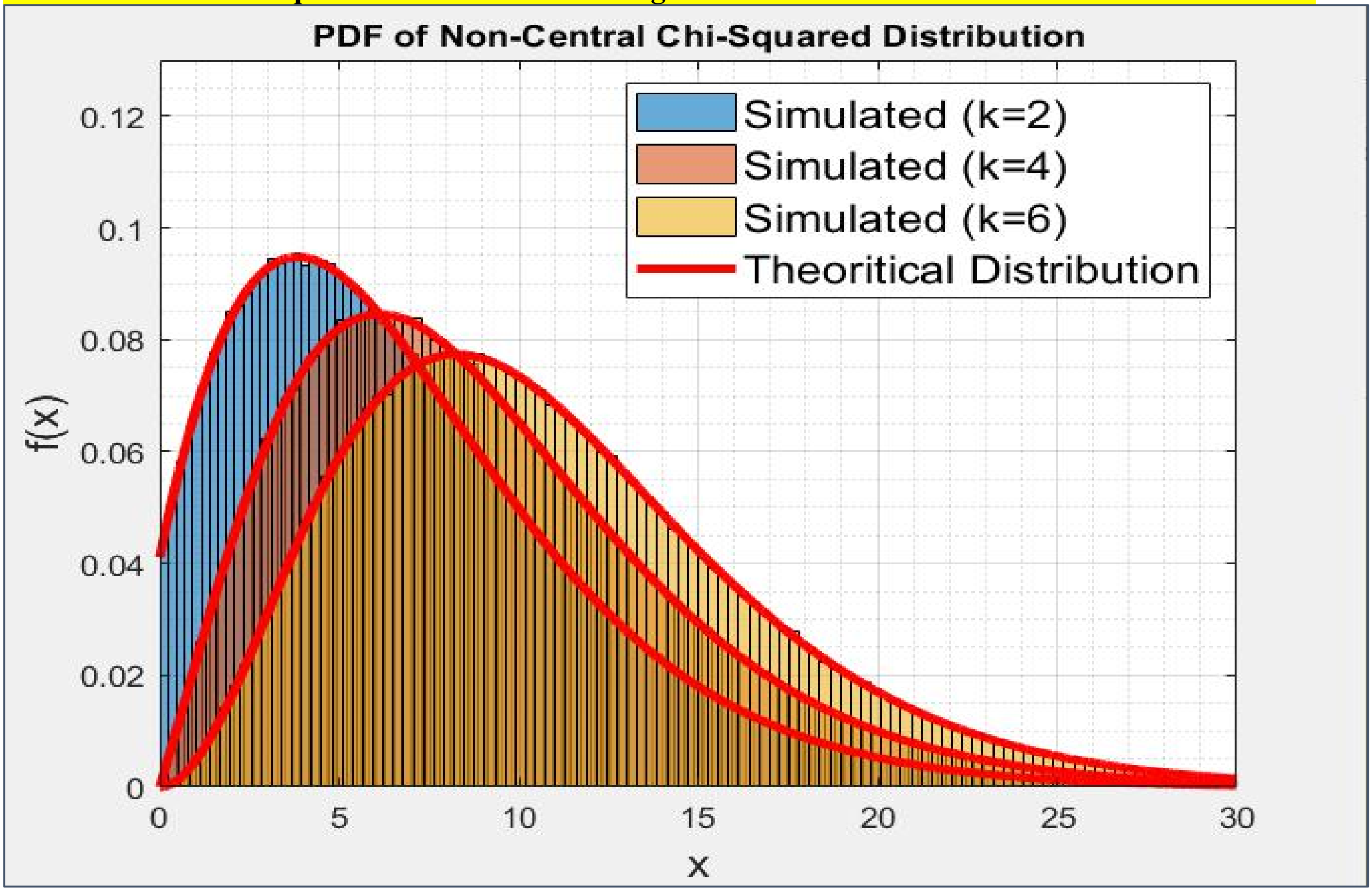
2. Chi-Squared distribution with degrees of freedom k. PDF for different values of k



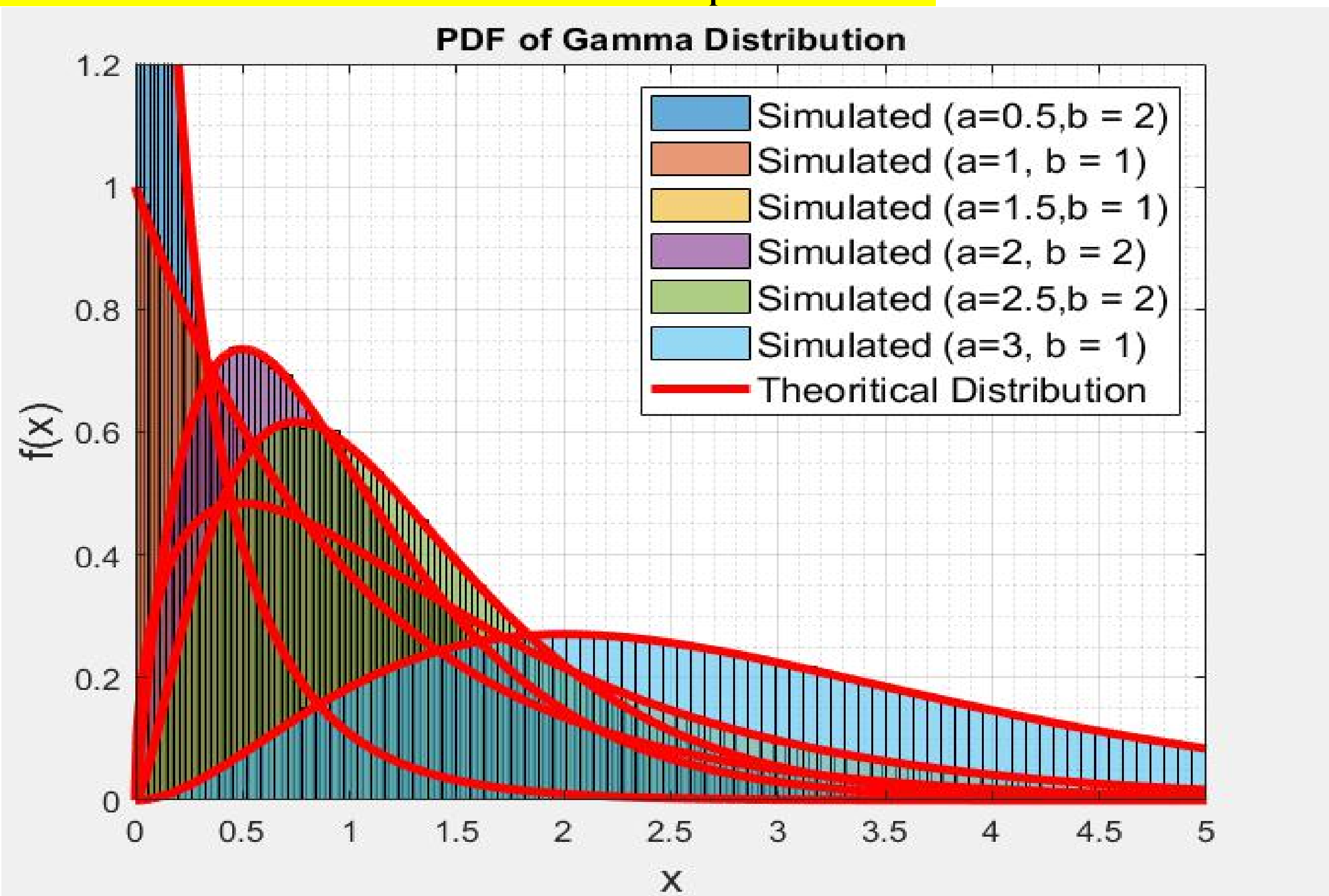
3. Rice distribution. PDF for different values of Shape factor v.



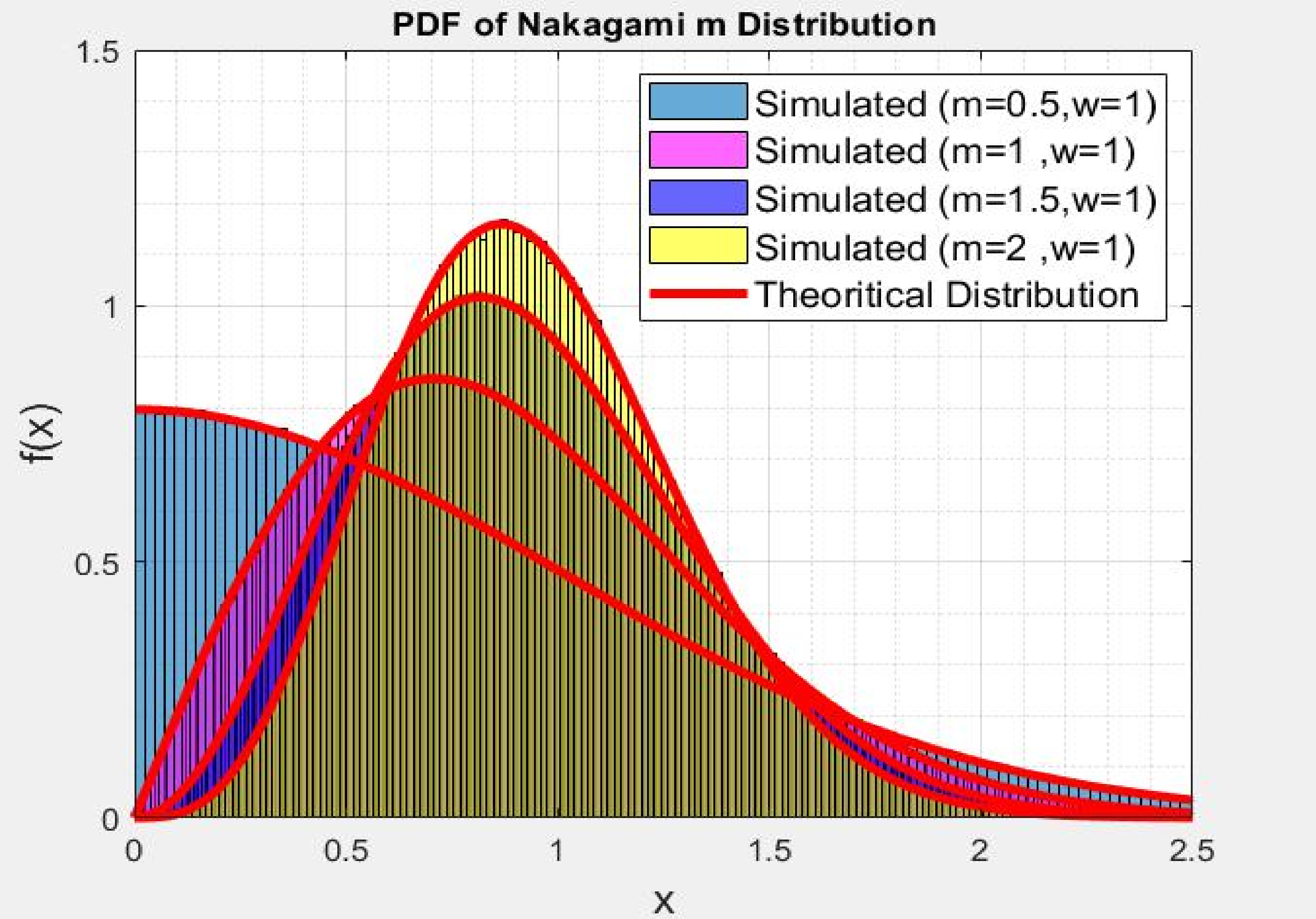
4. Non-Central Chi-Squared distribution with degrees of freedom k. PDF for different values of k.



5. Gamma distribution. PDF for different values of shape factor a and b.



6. Nakagami-m distribution. PDF for different values of shape factor m and w



Discussions:

-First we have take normal dist. **`Z = randn(1,N)`**

-Then we have transformed/generated it into required random variable **X** for each of six distribution.

-Plotting its plot for different values of changing **parameters** on same figure.

-Here we have used histogram and plot funtion of matlab to plot, the **generated** pdf of each of six distribution and to verify it **theoritically** using pdf function of each distribution by its formulas.

```
histogram(X,200,'Normalization','pdf'); %simulated/generated plot
plot(x,f(x),'r-','linewidth',3);         %theoritical plot
```

Concluding Remarks, and References:

-From the results obtained above , we clearly see that with the change in degrees of freedom or shape factor, the PDF plot changes for all six distributions.

-We have plotted the probability density function (PDF) of following six distributions using MATLAB using normal distribution as the base distribution.

-And verify this using theoritical plot of all distribution by (PDF) value of all six distributions in same plots.

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//Mahdy, Mervat. (2019). Performance Analysis of Selection Nakagami Distributions and Applications in Reliability.
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