CHAPTER - 1

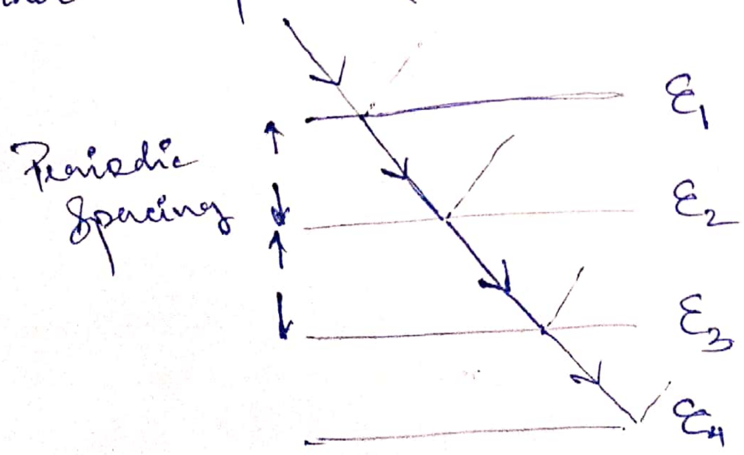
Photonic crystal - Optical analogue in which atoms and molecules are replaced by microscopic media with differing dielectric constants, and the periodic potential is replaced by a periodic dielectric function.(or, equivalently a periodic index of refraction).

Definition -
A low-loss periodic dielectric medium in which photonic band gaps. prevent light from propagating in certain directions with specified frequencies.

Metallic Cavity - Prevention of propagation of EM waves with frequencies below a certain threshold.

Metallic waveguide - Propagation only along its axis.
- Only for microwave regimes.
- Visible light is dissipated.
- Less frequency range.

Another example - Quarter wave stack. optical device.



- Different dielectric constants
- Partial reflections
- If ~~dielectric const~~ spacing is periodic, reflections interfere destructively and cancel out.
- Omnidirectional photonic crystal.

Complete Photonic Band Gap :-

Prevention of light source in a frequency range from

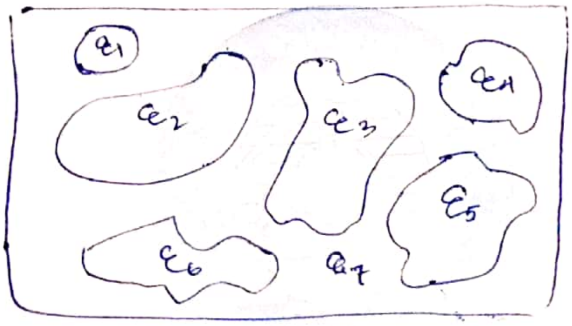
- any direction
- any polarization
- any source (despite near-normal incidence)

Exceptions - quasi-crystalline structures - non-periodic class of materials with complete photonic band gap. (Anderson Localization).

Maxwell's Equations -

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{D} = \rho \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$



$\vec{E}$ = Electric Field.
$\vec{H}$ = Magnetic Field.
$\vec{D}$ = Displacement Field.
$\vec{B}$ = Magnetic Induction Field.

A composite of macroscopic regions of homogeneous dielectric media.

→ ε → fn. of $\vec{r}$.
→ No sources of light ⇒ $J=0$ and $\rho=0$.
→ Isotropic.

Displacement field $\vec{D}$ is related to E as series -

$$\boxed{\frac{D_i}{\varepsilon_0} = \sum_j \varepsilon_{ij} E_j + \sum_{j,k} \chi_{i,j,k} E_j E_k + O(E^3)}$$

- Field strength small enough so higher terms are neglected.
- Macroscopic & Isotropic - so related by $\varepsilon(\vec{r})$.

$$\therefore \boxed{\vec{D}(\vec{r}) = \varepsilon_0 \underbrace{\varepsilon(\vec{r})}_{} E(\vec{r})}$$

→ purely real & +ve as transparent materials.

Also, we have $\vec{B}(\vec{r}) = \mu_0 \mu(\vec{r}) \vec{H}(\vec{r})$, for most dielectric materials $\mu(\vec{r}) = 1$ So, $\boxed{\vec{B}(\vec{r}) = \mu_0 \vec{H}(\vec{r})}$.

1. $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{H}(r,t) = 0}$
2. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \boxed{\vec{\nabla} \times \vec{E}(r,t) + \mu_0 \frac{\partial \vec{H}(r,t)}{\partial t} = 0}$
3. $\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \boxed{\vec{\nabla} \cdot [\varepsilon(\vec{r}) \vec{E}(\vec{r},t)] = 0}$
4. $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \boxed{\vec{\nabla} \times \vec{H} - \varepsilon_0 \varepsilon(\vec{r}) \frac{\partial \vec{E}(\vec{r},t)}{\partial t} = 0}$

<u>Mode profiles finding</u> — (Harmonic mode = spatial pattern times complex exponential)

$$H(r,t) = H(\vec{r}) e^{-i\omega t}$$
$$E(r,t) = E(\vec{r}) e^{-i\omega t}$$

$$\boxed{\begin{array}{l} \vec{\nabla} \cdot \vec{H}(\vec{r}) = 0 \\ \vec{\nabla} \cdot [\varepsilon(\vec{r}) \cdot \vec{E}(\vec{r})] = 0 \end{array}}$$

<u>Interpretation</u> — No sources & sinks of displacement and magnetic fields.

- <u>Transverse</u> $\underbrace{H(\vec{r})}_{\text{plane wave}} = \vec{a} e^{i(\vec{k}\cdot\vec{r})} \Rightarrow \vec{a} \cdot \vec{k} = 0$
- <u>transversality</u> conditions. $\boxed{(\vec{H} \cdot \vec{\omega}) = 0\ ? \rightarrow \text{meaning}}$

<u>Master Equation</u> —

$$\vec{\nabla} \times \vec{E} - i\omega\mu_0 \vec{H}(\vec{r}) = 0$$
$$\vec{\nabla} \times \vec{H} + i\omega \varepsilon_0 \varepsilon(\vec{r}) \vec{E}(\vec{r}) = 0.$$

- Substitute $E(\vec{r}) \rightarrow \boxed{\vec{\nabla} \times \left(\frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r})\right) = \left(\frac{\omega}{c}\right)^2 \vec{H}(\vec{r})}$

→ Master Equation.

Using this find $\boxed{\vec{E} = \frac{i}{\omega \varepsilon_0 \varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r})}$

$\hat{\Theta}$ = Master operator. (Linear operator)

$\rightarrow \quad \hat{\Theta}\vec{H}(\vec{r}) = \left(\frac{\omega}{c}\right)^2 \vec{H}(\vec{r})$

$\vec{H}(\vec{r})$ → eigenfunction/eigenvector

$\left(\frac{\omega}{c}\right)^2$ → eigenvalue

Eigenvalue problem.

$\vec{H}(\vec{r})$ - spatial patterns of harmonic modes.

Properties - 1) Real Eigenvalues. 2) Orthogonal. 3) Variational principle obtainable 4) Symmetry Properties.

Inner Product - $(F, G) \triangleq \int d^3\vec{r}\, F^*(\vec{r}) \cdot G(\vec{r})$

$(F, G) = (G, F)^*$

Normalization → $(F, F) = 1 \Rightarrow \boxed{F(\vec{r}) = \frac{F'(\vec{r})}{\sqrt{(F', F')}}}$,

Condition - $\boxed{(F', F') \neq 1}$

Hermithian Operator → $\boxed{(F, \hat{\Xi} G) = (\hat{\Xi} F, G)}$

Master operator $\hat{\Theta}$ is hermittian.

Proof: - $(F, \hat{\Theta} G) = \int d^3r\, \vec{F}^* \cdot \left(\vec{\nabla} \times \left(\frac{1}{\varepsilon}\vec{\nabla} \times \vec{G}\right)\right)$

$= \int d^3r\, (\vec{\nabla} \times \vec{F})^* \cdot \left(\frac{1}{\varepsilon}\vec{\nabla} \times \vec{G}\right)$

$= \int d^3r\, \left(\vec{\nabla} \times \frac{1}{\varepsilon}\vec{\nabla} \times \vec{F}\right)^* \cdot G$

$= \int d^3r\, (\hat{\Theta} F)^* \cdot G$

$= (\hat{\Theta} F, G)$

$\Rightarrow \boxed{(F, \hat{\Theta} G) = (\hat{\Theta} F, G)}$

- Surface terms periodic or decay at large distances.

$\boxed{\text{Eigenvalues of master operator are real nos}}$

Proof :-
$$\hat{\Theta} H(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r})$$
$$(H, \hat{\Theta} H) = \left(\frac{\omega}{c}\right)^2 (H, H)$$
$$\Rightarrow (H, \hat{\Theta} H)^* = \left(\frac{\omega^2}{c^2}\right)^* (H, H)^* = \left(\frac{\omega^2}{c^2}\right)^* (H, H)$$
$$\Rightarrow (\Theta H, H) = \left(\frac{\omega^2}{c^2}\right)^* (H, H) = \frac{\omega^2}{c^2} (H, H)$$
$$\Rightarrow \left(\frac{\omega^2}{c^2}\right)^* = \frac{\omega^2}{c^2} \quad \boxed{\Rightarrow (\omega^2)^* = \omega^2 \text{ or } \omega^2 \text{ is real}}$$

$\hat{\Theta}$ = Positive, semi-definite.

→ if $\omega_1 \neq \omega_2 \Rightarrow H_1(\vec{r})$ & $H_2(\vec{r})$ are orthogonal.
→ if $\omega_1 = \omega_2 \Rightarrow H_1(\vec{r})$ & $H_2(\vec{r})$ are degenerate.

Variational Theorem -

- Mode tends to concentrate its energy of electric-field in region of high dielectric constants.
- Smallest eigenvalue ω_0^2/c^2 corresponds to the field pattern that minimizes the functional →

$$U_f(\vec{H}) \triangleq \frac{(H, \hat{\Theta} H)}{(H, H)}$$

→ Rayleigh constant.

Gradient → Rate of change of the functional U_f w.r.t. H.
$= G$
$$\delta U_f = U_f(H + \delta H) - U_f(H) = [(\delta H, G) + (G, \delta H)]/2$$
$$G = \frac{2}{(H, H)}\left(\hat{\Theta} H - \left[\frac{(H, \hat{\Theta} H)}{(H, H)}\right] H\right)$$

At extremum $G = 0 \Rightarrow \vec{H}$ is an eigenvector of $\hat{\Theta}$
$\Rightarrow G = 0$.
→ $\hat{\Theta} H = \boxed{U_f(\vec{H})}\, \vec{H}$
→ constant.

U_f = Electromagnetic energy functional.

- Time averaged physical energy contribution →

$$U_E \triangleq \frac{\varepsilon_0}{4}\int d^3r\, \varepsilon(r)|E(r)|^2$$
$$U_H \triangleq \frac{\mu_0}{4}\int d^3r |H(r)|^2$$

$U_E = U_H$ as periodic shift b/w electric & magnetic fields.

Rate of energy transport/ (intensity) → Pyonting vector $\vec{S}$. - Depends on energy square.

$$\boxed{\vec{S} \triangleq \frac{1}{2} Re[E^* \times H]}$$

Time average flux of an EM ~~field~~ energy in the direction of S per unit time per unit area, time-harmonic field.

$\frac{\text{Energy Flux}}{\text{Energy density}}$ = Velocity of energy transport.

Scaling Properties -

$\varepsilon'(\vec{r}) = \varepsilon(\vec{r}/s)$, we take $r' = sr$ & $\nabla' = \nabla/s$.

so Master equation becomes →

$$\frac{s\nabla' \times \left(\frac{1}{\varepsilon(r'/s)} s\nabla' \times H(r'/s)\right)}{s^2} = \frac{\left(\frac{\omega}{c}\right)^2 H(r'/s)}{s^2}$$

$$= \nabla' \times \left(\frac{1}{\varepsilon'(r')} \nabla' \times H(r'/s)\right) = \left(\frac{\omega}{cs}\right)^2 H(r'/s)$$

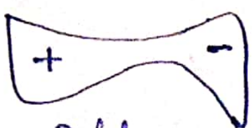
$\therefore$ $\boxed{H'(r') = H(r'/s), \quad \omega' = \frac{\omega}{s}}$ → Changes made for $\varepsilon'(r) = \varepsilon'(r/s)$

Mode Profile ↓ frequency

Symmetry to classify electromagnetic modes →

Even mode

$H(\vec{r}) = H(-\vec{r})$



Odd Mode

$H(r) = -H(r)$

If $H(r)$ is not belonging to a family of degenerate modes.

- Difficult to write down the exact boundary condition and solve the problem analytically.

$H(-r) = \alpha H(r)$, invert twice picking another factor α.

$\therefore H(r) = \alpha^2 H(r) \implies \boxed{\alpha = \pm 1}$

For inversion → Inversion operator O_I

- Inverts both vector f & its argument r.

$$\hat{O}_I f(r) = I f(Ir).$$

$$\hat{\Theta} = \hat{O}_I^{-1} \hat{\Theta} \hat{O}_I .$$

Cummutator Operator →

$$\hat{\Theta} = \hat{O}_I^{-1} \hat{\Theta} \hat{O}_I \Rightarrow \boxed{\hat{O}_I \hat{\Theta} - \hat{\Theta} \hat{O}_I = 0}$$

$$[\hat{A}, \hat{B}] \triangleq \hat{A}\hat{B} - \hat{B}\hat{A}$$

Condition of symmetricity → $[\hat{O}_I, \hat{\Theta}] = 0$

If symmetric $\Rightarrow [\hat{O}_I, \hat{\Theta}] H = 0 \Rightarrow \hat{O}_I(\hat{\Theta} H) - \hat{\Theta}(\hat{O}_I H) = 0.$

$\Rightarrow \boxed{\hat{\Theta}(\hat{O}_I H) = \frac{\omega^2}{c^2}(\hat{O}_I H)}$ If no degeneracy.

Conclusion - If $\hat{H}$ is a harmonic mode with frequency ω, then $\hat{O}_I H$ is also a mode with frequency ω.

If no degeneracy $\Rightarrow$ $\hat{O}_I \hat{H} = \alpha \hat{H}$ (as only one mode per freq.)

↳ eigenvalue problem, but $\alpha = \pm 1$ (eigenvalue)

Continuous Translational Symmetry -

A system with translational symmetry is unchanged by a translation through a displacement $\vec{d}$.

$$\boxed{\hat{T}_{\vec{d}}\, \varepsilon(\vec{r}) = \varepsilon(\vec{r} - \vec{d}) = \varepsilon(\vec{r})}$$

or

$$\boxed{[\hat{T}_d, \hat{\Theta}] = 0}$$

For z-direction - form of eigenfunction -

$$\hat{T}_d e^{ikz} = e^{ik(z-d)} = e^{-ikd} \cdot e^{ikz}$$

→ Homogeneous medium ⇒ Translational symmetry in all three directions.

Form - $H_k(\vec{r}) = H_0 e^{ik\vec{r}}$

→ Plane waves polarized in the direction H_0.

→ $k \cdot H_0 = 0$ (Transversality requirement).

→ From master equation - $\frac{\omega^2}{c^2} = \frac{|k|^2}{\varepsilon} \Rightarrow \boxed{\omega = \frac{c|k|}{\sqrt{\varepsilon}}}$

$\omega = \dfrac{c|k|}{\sqrt{\varepsilon}}$ → Dispersion Relation.

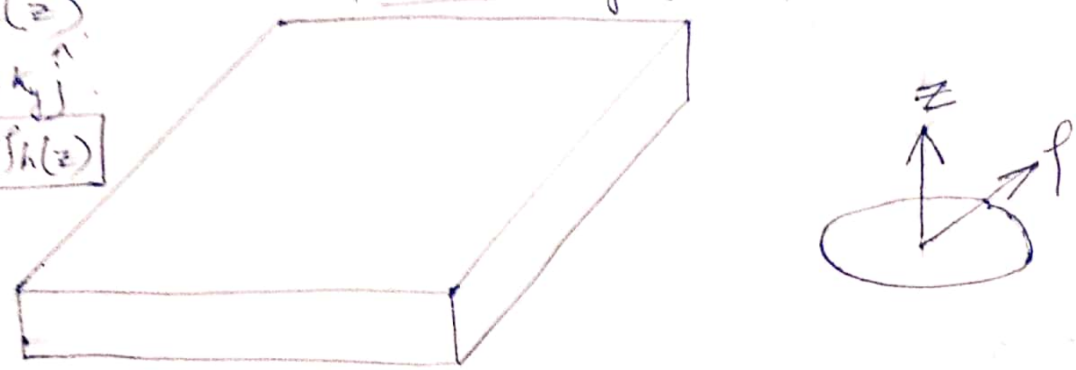
Example -

→ $\varepsilon(\vec{r}) = \varepsilon(z)$

→ $\vec{k} = k_x\hat{i} + k_y\hat{j}$

→ $\boxed{H_k(\vec{r}) = e^{ik\cdot\rho}h(z)}$

Plane sheet of glass.



- $h(z)$ cannot be determined exactly as no transversality in z direction, but $\vec{\nabla}\cdot H_k = 0 \Rightarrow \boxed{\vec{k}\cdot h = i\dfrac{\partial h_z}{\partial z}}$

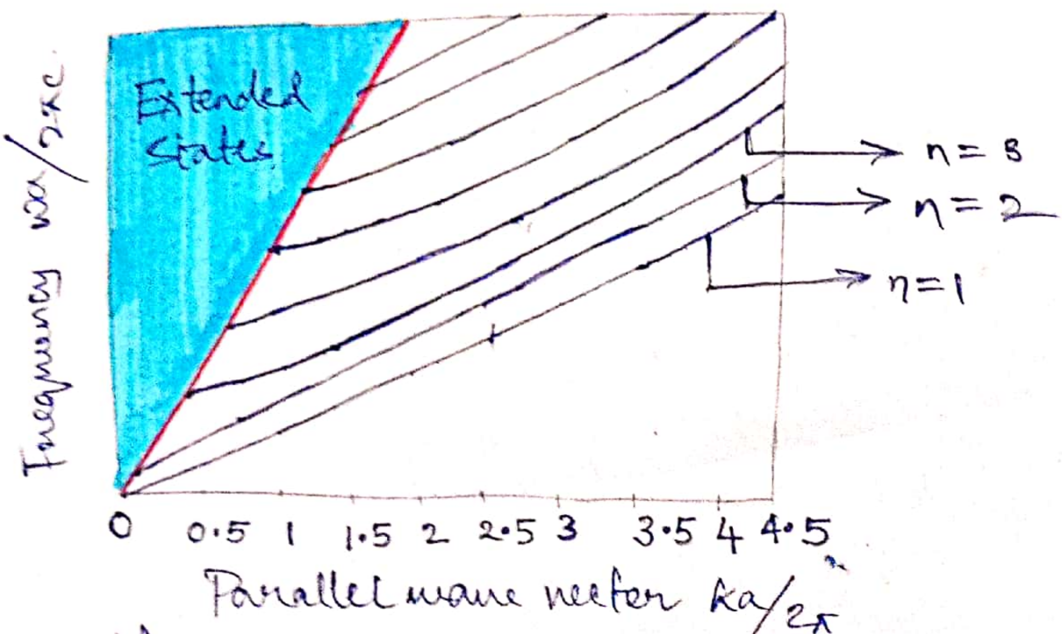
$h(z) =$ ~~phase~~ amp. determination

$e^{ik\cdot\rho} =$ ~~amph~~ phase determination.

Identifification of Plane Waves. -

$(\vec{k}, n)$ → $\vec{k}$ = wave vector, n = band number $\boxed{(n\uparrow \Rightarrow \omega\uparrow)}$

→ If degenerate modes are present then there also needs to be an index inclusion for degenerate modes with same n & $\vec{k}$.

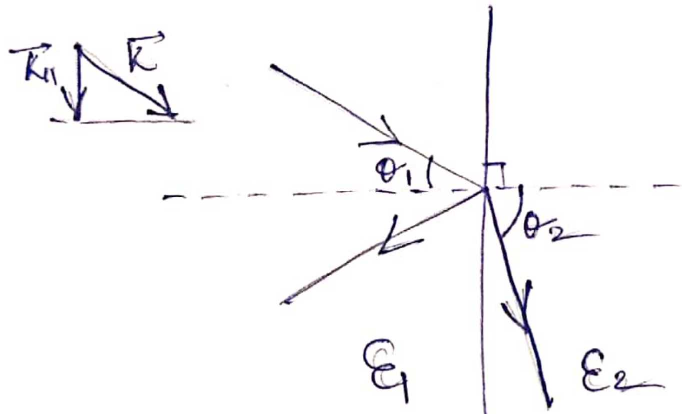


Harmonic modes frequencies.

- Blue Lines - Localized in glass.
- Shaded Blue - Continuum of states in air & glass
- Red = light line → $\omega = ck$.

→ Components of the wave vector $\vec{k}$ along the symmetry directions are conserved quantities.

Index Guiding —



For certain θ_1, no θ_2
⇒ Generalization of this result follows from translational symmetry which tells us that $k_{\parallel}$ is conserved.

Total Internal Reflection.

Snells law – $n_1 \sin\theta_1 = n_2 \sin\theta_2$, $n_i = \sqrt{\varepsilon_i}$

When $\theta_2 = 90°$, $\Rightarrow \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \theta_c$ = Critical Angle.

→ TIR is existent only for $n_1 > n_2$

Combination of two conservation Laws that follow from symmetry →

1) Conservation of frequency ω (from linearity and time-invariance of Maxwell's equations)
2) Conservation of $k_{\parallel}$ (from cont. translational sym).

$k_{\parallel} = |\vec{k}| \sin\theta$ & $|\vec{k}| = \frac{n\omega}{c}$

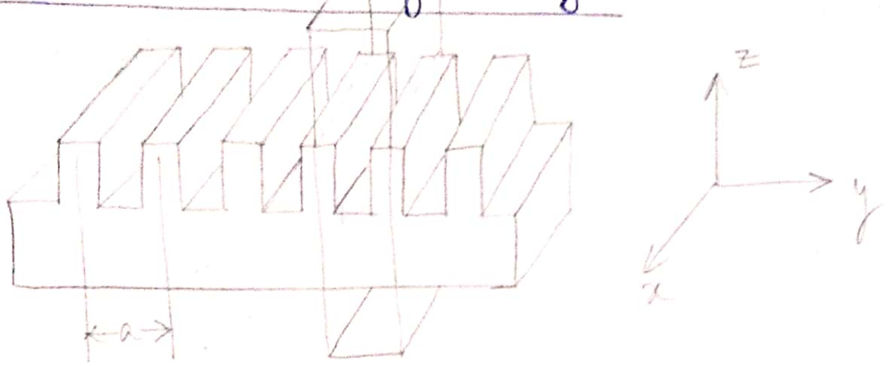
Snell's law → equal $k_{\parallel}$ on both sides of interface.

Explaination of ω vs. $k_{\parallel}$ diagram – (Imp.)

- Extended States → Superposition of plane waves with $\omega = c|k| = c\sqrt{k_{\parallel}^2 + k_{\perp}^2}$. $k_{\perp}$ = any value, hence spectrum is continuous. $\boxed{\text{(Light Cones)}}$. Solutions of Snell's Law.
- Solutions below light line $\omega = ck_{\parallel}$ →
  → $\varepsilon_{glass} > \varepsilon_{air} \Rightarrow \omega_{glass} < \omega_{air}$
  → $k_{\perp}^2 + k_{\parallel}^2 = \frac{\omega^2}{c^2} \Rightarrow k_{\perp} = \pm i\sqrt{k_{\parallel}^2 - \frac{\omega^2}{c^2}}$ in air, corresponding to fields that decay exponentially away from glass. (are evanescent) $\boxed{\text{Index-Guided Modes}}$
  → Guided bands for given $k_{\parallel}$ as localized in $\perp$ direction.

Scanned with CamScanner

Discrete Translational Symmetry –



→ Photonic crystals – Discrete Translational Symmetry

→ a = lattice constant

→ Basic step vector = Primitive lattice vector = $\vec{a} = a\hat{y}$.

→ $\varepsilon(r) = \varepsilon(r \pm a) = \varepsilon(r + \vec{R})$, $\vec{R} = l\vec{a}$, $l \in I$.

Box = Unit Cell.

Now, $[\hat{T}_{d\hat{x}}, \hat{\Theta}] = 0$ & $[\hat{T}_R, \hat{\Theta}] = 0$

Commutation with translation operators in $\hat{x}$ & $\hat{R}$ holds.

Determining Eigenfunctions –
(Plane Waves)

$$\hat{T}_{d\hat{x}} e^{ik_x x} = e^{ik_x(x-d)} = (e^{-ik_x d}) \cdot e^{ik_x x}$$
$$\hat{T}_R e^{ik_y y} = e^{ik_y(y-la)} = (e^{-ik_y(la)}) \cdot e^{ik_y y}$$

→ As k_y & $k_y + m\left(\frac{2\pi}{a}\right)$ give the same eigenvalues, they form a degenerate set with eigenvalue of $\hat{T}_R = e^{-i(k_y la)}$

→ $\vec{b} = b\hat{y}$ = reciprocal lattice vector.

Bloch's theorem, Bloch state, Floquent mode, Brillouin Zone →

Taking linear combination of degenerate vectors to get the form –

$$\vec{H}_{k_x,k_y}(\vec{r}) = e^{ik_x x} \sum_m C_{k_y,m}(z)\, e^{i(k_y + mb)y}$$
$$= e^{ik_x x} \cdot e^{ik_y y} \sum_m C_{k_y,m}(z)\, e^{imby}$$

$$\boxed{\vec{H}_{k_x,k_y}(\vec{r}) = e^{ik_x x} e^{ik_y y} \cdot u_{k_y}(y, z)}$$

Doubt – Why don't we consider $e^{ik_z z}$?

→ $u_{k_y}(y, z)$ is a periodic function in y &

$$u_{k_y}(y + la, z) = u_{k_y}(y, z).$$

→ Coefficients of c are determined by explicit solution.

Bloch's Theorem → $\vec{H}(\ldots, y, \ldots) \propto e^{ik_y y} \cdot \vec{u}_{k_y}(y, \ldots)$

Form of this = Bloch's State. (Solid-state Physics)
Floquet Mode. (In mechanics)

Mode frequency → $\omega(k_y) = \omega(k_y + mb)$ is periodic.
→ State with k_y & $k_y + mb$ is periodic.

Brillouin Zone – Non-redundant values of k_y → $\boxed{-\frac{\pi}{a} < k_y \leq \frac{\pi}{a}}$

For periodicity in all 3-dimensions –

→ Primitive lattice vectors $(\vec{a_1}, \vec{a_2}, \vec{a_3})$ span the space of lattice vectors.

$$\Rightarrow \boxed{\vec{R} = l\vec{a_1} + m\vec{a_2} + n\vec{a_3}}$$

→ Reciprocal lattice vectors of $(\vec{a_1}, \vec{a_2}, \vec{a_3}) = (\vec{b_1}, \vec{b_2}, \vec{b_3})$ so that $\boxed{\vec{a_i} \cdot \vec{b_j} = 2\pi \delta_{ij}}$. Form reciprocal lattice.

Bloch wave vector →

$\vec{K} = k_1\vec{b_1} + k_2\vec{b_2} + k_3\vec{b_3}$ where $\vec{K}$ lies in Brillouin zone.

3-D periodic systems are Bloch states.

Example – Crystal in which unit cell is rectangular box,

$$|k_i| \leq \frac{1}{2}.$$

Form – $H_K(\vec{r}) = e^{i\vec{K}\cdot\vec{r}}\, \vec{u}_K(\vec{r})$, $\vec{u}_K(\vec{r}) = \vec{u}_K(\vec{r} + \vec{R})$ for all lattice vectors $\vec{R}$.

Photonic Band Structures –

Finding $u_K(\vec{r})$ ⟶

$$\hat{\Theta} H_K = \left(\frac{\omega(\vec{K})}{c}\right)^2 H_K$$

$$\Rightarrow \vec{\nabla} \times \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times e^{i\vec{K}\cdot\vec{r}} u_K(\vec{r}) = \left(\frac{\omega(K)}{c}\right)^2 e^{i\vec{K}\cdot\vec{r}} u_K(\vec{r})$$

$$\Rightarrow (i\vec{K} + \vec{\nabla}) \times \left(\frac{1}{\varepsilon(\vec{r})} (i\vec{K} + \vec{\nabla}) \times \vec{u}_K(\vec{r})\right) = \left(\frac{\omega(K)}{c}\right)^2 u_K(\vec{r})$$

$$\Rightarrow \hat{\Theta}_K \vec{u}_K(\vec{r}) = \left(\frac{\omega(K)}{c}\right)^2 \vec{u}_K(\vec{r})$$

So, $\boxed{\hat{\Theta}_K \triangleq (i\vec{K} + \vec{\nabla}) \times \frac{1}{\varepsilon(\vec{r})} (i\vec{K} + \vec{\nabla}) \times}$

So, $\vec{u}_k(\vec{r})$ can be determined subject to

$$(i\vec{k}+\vec{\nabla})\cdot\vec{u}_k = 0 \quad \& \quad u_k(\vec{r}) = u_k(\vec{r}+\vec{R})$$

Band structure – Different modes obtained from varying k and then for a specific $\vec{k}$ form the frequency band for different n.

Rotational Symmetry and Irreducible Brillouin Zone –

Operator (3×3 Matrix) $R(\hat{n},\alpha)$ rotates vectors by an angle α about the $\hat{n}$ axis.

$$R(\hat{n},\alpha) = R.$$

Effect → Vector field rotates = $\hat{O}_R$, so →

$$\boxed{\hat{O}_R \cdot \vec{f}(\vec{r}) = R\vec{f}(R^{-1}\vec{r})}$$

If Rotation leaves system invariant, then $\boxed{[\hat{\Theta},\hat{O}_R]=0}$

OR $\hat{\Theta}(\hat{O}_R \vec{H}_{kn}) = \hat{O}_R(\hat{\Theta}\vec{H}_{kn}) = \left(\frac{\omega_n(k)}{c}\right)^2(\hat{O}_R\vec{H}_{kn})$

→ $\hat{O}_R\vec{H}_{kn}$ is none other than Bloch state with wave vector $R\vec{k}$.

Proof:- $\hat{T}_R(\hat{O}_R\vec{H}_{kn}) = \hat{O}_R(\hat{T}_{R^{-1}R}\vec{H}_{kn})$

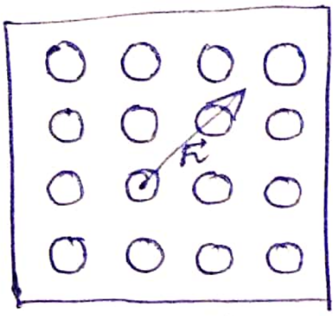
$$= \hat{O}_R(e^{-ik(R^{-1}R)}\vec{H}_{kn})$$

$$= \hat{O}_R(e^{-iR\vec{k}(\vec{R})}\vec{H}_{kn})$$

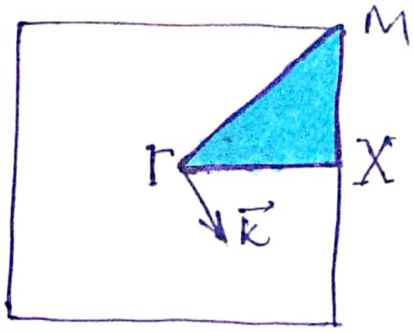
So $\boxed{\omega_n(R\vec{k}) = \omega_n(\vec{k})}$

Point Group – Collection of symmetry operations (rotations, reflections and inversions).

Irreducible Brillouin Zone – Smallest region inside Brillouin zone where $\omega_n(\vec{k})$ are not related by symmetry.



Real Lattice.



Brillouin Zone of Reciprocal Lattice.

Mirror Reflection Symmetry →

Two seperate eigenvalue equation for $\hat{\Theta}_k$, one for each polarization.

Case 1 – H_k is $\perp$ to the mirror plane, E_k is parallel

Case 2 – H_k is in plane, E_k is perpendicular.

The photonic crystal image is invariant under mirror reflections in the xz & yz planes.

$\hat{O}_{M_x}$ = Mirror Reflection Operator →

$$\boxed{\hat{O}_{M_x} \vec{f}(\vec{r}) = M_x \vec{f}(M_x \vec{r})}$$

$$\boxed{[\hat{\Theta}, \hat{O}_{M_x}] = 0}$$

→ $\vec{H}_k$ is just a Bloch state with reflected wave vector $M_x \vec{k}$.

$$\Rightarrow \hat{O}_{M_x} \vec{H}_k = e^{i\phi} \vec{H}_{M_x \vec{k}} = \pm H_k(\vec{r}) = M_x H_k(M_x \vec{r})$$

→ If $[\hat{\Theta}, \hat{O}_M] = 0 \Rightarrow M\vec{r} = \vec{r}$ & $M\vec{k} = \vec{k}$.

Even components → (H_x, H_y, E_z), = Transverse-Electric (TE modes)

Odd components → (E_x, E_y, H_z) = Transverse-Magnetic (TM modes)

Time Reversal Symmetry –

$$\omega_n(\vec{k}) = \omega_n(-\vec{k}).$$

Miller Indices – Traditional way to refer to a plane in a crystal lattice is the system of Miller Indices.