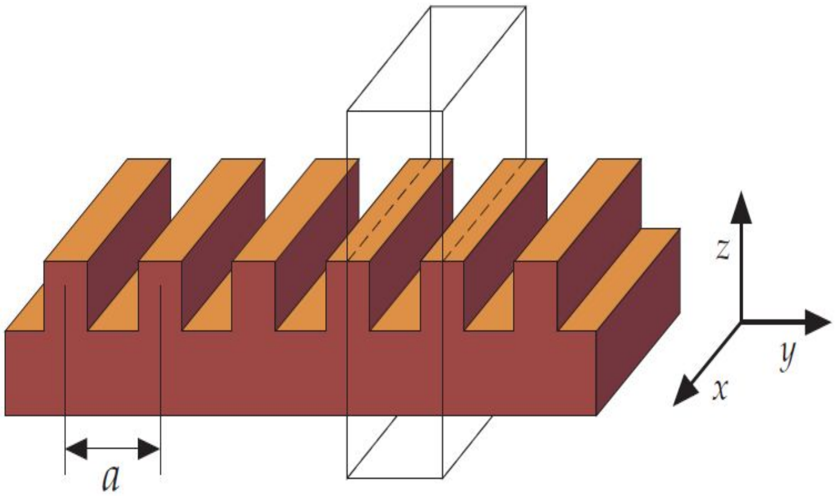
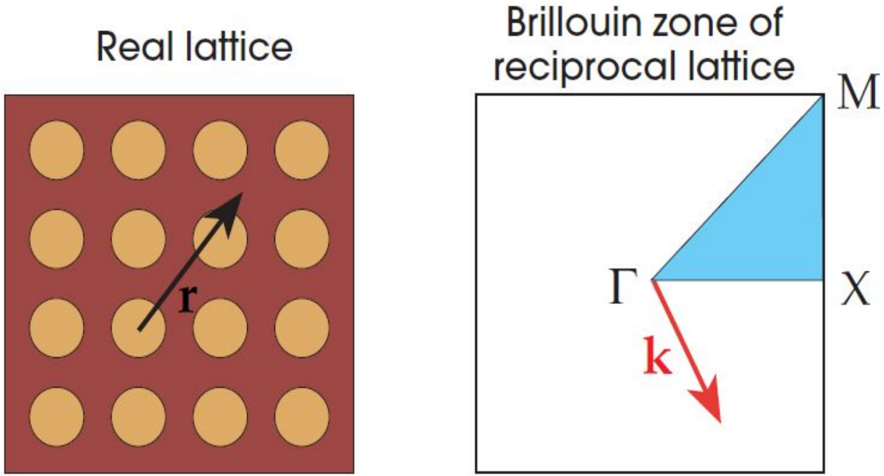




Indraprastha Institute of Information Technology, Delhi

B.TECH PROJECT PRESENTATION

High-resolution Sub-Wavelength Imaging Using Photonic Crystals

Advisor - Dr. Sayak Bhattacharya
Track of BTP - Research Track
Evaluation Committee Members -
Dr. Shobha Sundar Ram, Dr. Abhijit Mitra

Student - Rounak Saraf
Roll Number - 2018183

Acknowledgements

I would like to thank my Advisor, Dr. Sayak Bhattacharya who guided me throughout the course of the project. The pace and depth of the project was very appropriate to get a clear understanding of our steps to the project. He was a help in clearing all my doubts and questioning to solidify our understanding of the concepts and the software.

Also, I would like to thank my batchmate, Harsh Raj Sonkar, a student of the same research group who was there for moral support and for discussions as well.

Main Goal

Our main goal is to achieve sub-wavelength imaging using Photonic crystals. This is based on the angular spectrum representation.

The sub-wavelength particles emit evanescent waves. We tend to obtain these waves by the virtue of photonic crystals that are able to pick up these evanescent waves at their surface.

This is closely based on the phenomenon of solid state crystals that give rise to evanescent wave function of electrons at their surface.

Project Introduction

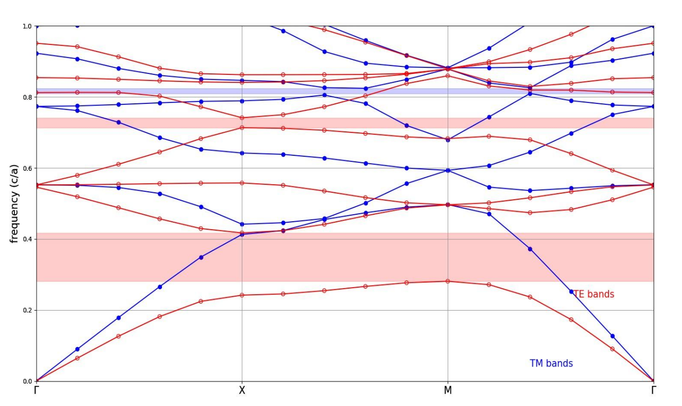
Softwares Used - Meep and MPB

Meep - Meep implements the finite-difference time-domain (FDTD) method for computational electromagnetics.

MPB - MPB is a software package to compute definite-frequency eigenstates of Maxwell's equations in periodic dielectric structures.

Steps taken towards our goal :-

- ➔ Understanding the theory of Photonic Crystals
- ➔ Understanding the working of the softwares
- ➔ Using MPB to plot the Transverse Electric and Transverse Magnetic bands with respect to the Photonic crystals
- ➔ Understanding effects of various parameters like resolution, interpolation etc.
- ➔ Using Meep to compute fields obtained by illuminating an object by a source



Theory of Photonic Crystals

Definition -

Optical Analogue in which atoms and molecules are replaced by microscopic media with differing dielectric constants and the periodic potential is replaced by a periodic dielectric function. (or, equivalently a periodic index of refraction).

Photonic Band Gap -

Prevention of light source in a frequency range from any direction, any source and any polarization. (despite near-normal incidence)

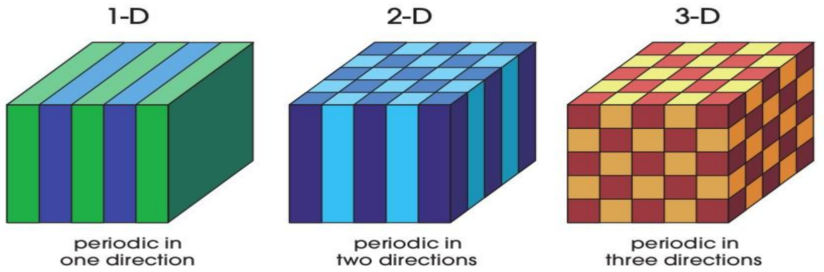


Figure 1: Simple examples of one-, two-, and three-dimensional photonic crystals. The different colors represent materials with different dielectric constants. The defining feature of a photonic crystal is the periodicity of dielectric material along one or more axes.

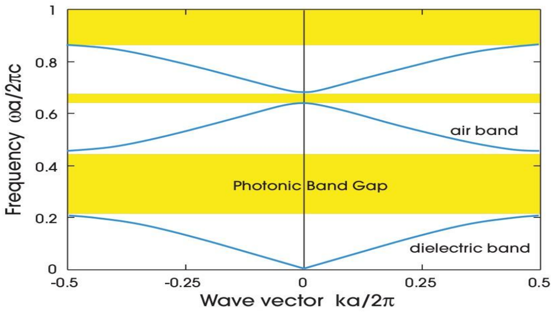


Figure 5: The photonic band structure of a multilayer film with lattice constant a and alternating layers of different widths. The width of the $\varepsilon = 13$ layer is $0.2a$, and the width of the $\varepsilon = 1$ layer is $0.8a$.

MAXWELL'S EQUATIONS AND BLOEMBERGEN RELATION

- A power series relates D to E (Bloembergen, 1965) as :
  $D_i/\varepsilon_0 = \Sigma_j \varepsilon_{ij} E_j + \Sigma_{jk} \chi_{ijk} E_j E_k + O(E^3)$

Maxwell equations as a linear Hermitian eigenvalue problem.

- $\nabla . B = 0$
- $\nabla . D = \rho$
- $\nabla \times E + \partial B/\partial t = 0$
- $\nabla \times H - \partial D/\partial t = J$

Where, E,H → macroscopic electric and magnetic field

D → displacement field

B → magnetic induction field

J → current density

- Assuming that dielectric materials-
  - Have small field strengths.
  - Macroscopic and isotropic.
  - No explicit frequency dependence.
  - Transparent ⇒ real and positive ε (r)

  $D(r) = \varepsilon_0 \varepsilon(r)\, E(r)$

  $B(r) = \mu_0 \mu(r)\, H(r)$

- Now, Maxwell equations :
  - $\nabla . H(r,t) = 0$
  - $\nabla . [\, \varepsilon(r) E(r,t) \,] = 0$
  - $\nabla \times E(r,t) + \mu_0 \, (\partial H(r,t) / \partial t) = 0$
  - $\nabla \times H(r,t) - \varepsilon_0 \varepsilon(r) \, (\partial E(r,t) / \partial t) = 0$

HARMONIC MODES AND MASTER EQUATION

- Harmonic Modes :
  - We separate time dependence from spatial dependence by expanding the fields into a set of Harmonic Modes / states of the system.
  - $H(r,t) = H(r)\,e^{-iwt}$ (spatial pattern or mode profile times a complex exponential)
  - $E(r,t) = E(r)\,e^{-iwt}$
  - For a given frequency,
    - $\nabla \cdot H(r) = 0$
    - $\nabla \cdot [\,\varepsilon(r)\,E(r)\,] = 0$

      ⇒ There are no point sources or sinks of displacement and magnetic fields in the medium.

- The other two Maxwell equations become -
  - $\nabla \times E(r) - iw.\mu_0 H(r) = 0$
  - $\nabla \times H(r) + iw.\varepsilon_0 \varepsilon(r) E(r) = 0$

$$\nabla \times (\,\varepsilon(r)^{-1}\,\nabla \times H(r)\,) = w^2/c^2 .\, H(r)$$

MASTER EQUATION

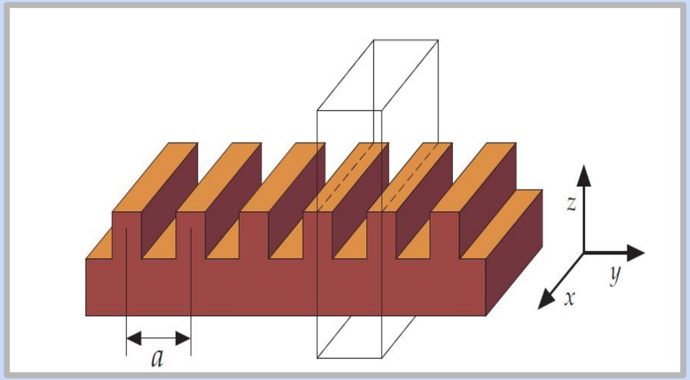
- Master equation : $\nabla \times (\,\varepsilon(r)^{-1}\,\nabla \times H(r)\,) = w^2/c^2 .\, H(r)$ can be written as:
  - Ⓗ $H(r) = w^2/c^2 .\, H(r)$,where Ⓗ $H(r) = \nabla \times (\,\varepsilon(r)^{-1}\,\nabla \times H(r)\,)$
  - Ⓗ is a linear operator.

We solve the fields as Eigenvalue problem.

DISCRETE TRANSLATIONAL SYMMETRY IN PHOTONIC CRYSTALS

Photonic Crystals have discrete translational symmetry.

- They are invariant under only distances that are a multiple of some fixed step length.
- In x-direction : continuous translational symmetry.
- In y-direction : discrete translational symmetry.
- Discrete symmetry : $\varepsilon(r) = \varepsilon(r \pm a)$. On repetition, $\varepsilon(r) = \varepsilon(r + R)$, where R = la, l is an integer.



- Due to translational symmetry, Ⓗ must commute with
  - All translational operators in x-direction.
  - All translational operators for lattice vectors $R = la\,\hat{y}$ in y-direction.
- We can identify the modes of Ⓗ as simultaneous eigenfunctions of both translational operators -
  - $\check{T}_{d\hat{x}}\, e^{i\,k_x x} = e^{i\,k_x (x-d)} = e^{-i\,k_x d}\, e^{i\,k_x x}$
  - $\check{T}_R\, e^{i\,k_y y} = e^{i\,k_y (y-la)} = e^{-i\,k_y la}\, e^{i\,k_y y}$
- All modes with wave vector of the form $k_y + m\,(2\pi/a)$ forms a degenerate set.
- Augmenting k_y by an integral multiple of $b = 2\pi/a$ leaves the state unchanged.

BLOCH'S THEOREM

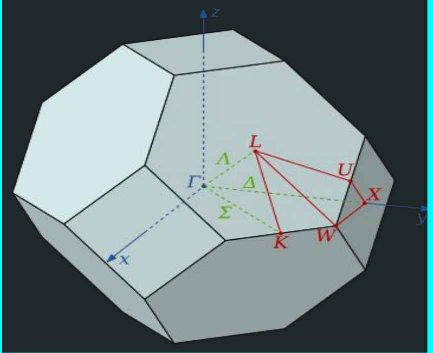
- Any linear combination of there degenerate eigenfunctions is itself an eigenfunction with same eigenvalue.
  - $H_{kx,ky}(r) = e^{i\,kx\,x} \Sigma_m c_{ky,m}(z)\, e^{i\,(ky+mb)\,y}$
  - $\Rightarrow\ e^{i\,kx\,x} \cdot e^{i\,ky\,y} \cdot u_{ky}(y,z)$
  - c's are the expansion coefficients to be determined by explicit solution.
- A discrete periodicity in y-direction leads to a y-dependence in H which is the product of plane wave and a y-periodic function.

$H(\ldots,y,\ldots) \propto e^{i\,ky\,y} \cdot u_{ky}(y,\ldots)$ --> Bloch's Theorem

- Bloch's state with wave vectors k_y differing by integral multiples of b= 2π/a are identical.
- Mode frequency must also be periodic in k_y :
  $w(k_y) = w(k_y + mb)$.
- We need only consider k_y to exist in the range $-\pi/a < k_y <= \pi/a$. This region of important, non-redundant values of k_y is Brillouin Zone.

BRILLOUIN ZONE

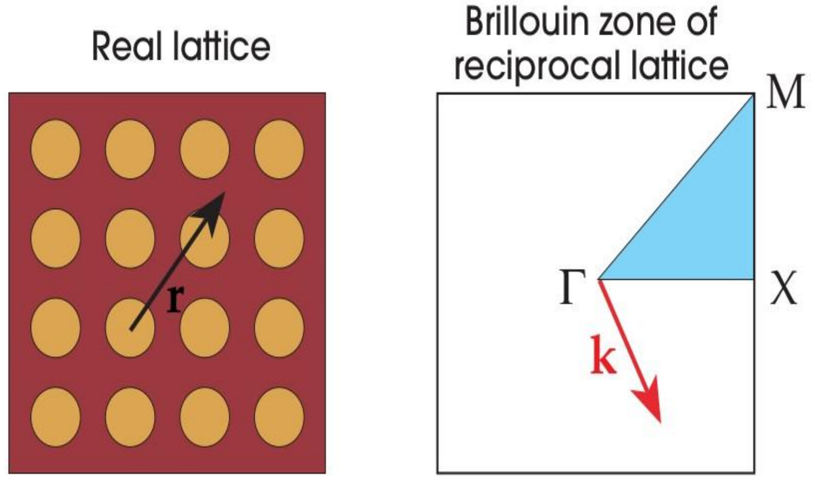
- Modes in Bloch form : $H_k(r)\, u_k(r) = e^{ikr}\ u_k(r+R)$
- A mode with wave vector k and a mode with wave vector k+G are the same mode if G is a reciprocal lattice vector.
- The one closest to k=0 is the First Brillouin Zone.



First Brillouin zone of FCC lattice, a truncated octahedron, showing symmetry labels for high symmetry lines and points

IRREDUCIBLE BRILLOUIN ZONE

The smallest region within the Brillouin zone for which w_n (k) are not related by symmetry is called the irreducible Brillouin zone.



Description of the left picture

A photonic crystal made using a square lattice. An arbitrary vector r is shown.

Description of the right picture

The Brillouin zone of the square lattice, centered at the origin (Γ).

An arbitrary wave vector k is shown. The irreducible zone is the light blue triangular wedge. The special points at the center, corner, and face are conventionally known as Γ , M , and X . The blue shaded region is the irreducible Brillouin zone, The rest of the Brillouin zone consists of redundant copies.

Plotting Band Structure Using MPB

Brief - MPB software is used to compute the band structure of a two-dimensional lattice of dielectric rods in air.

Steps for the code ->

- Determines the number of eigenstates computed at each k-points.
- Defines where the bands are computed at.
- Interpolating the k-points for obtaining a continuous band structure.
- Setting up geometric object, geometric lattice and computational cell.
- Simulating the code for obtaining TE and TM modes at the specified points.

Code for Square Lattice

```
import math
import meep as mp
from meep import mpb

# Determining the number of eigenstates computed at each k points.
num_bands=8

# k_points=Bloch wavevectors we want to compute the bands at.
# Setting it to the corners of irreducible Brillouin zone.
k_points=[mp.Vector3(),                    # Gamma
          mp.Vector3(0.5),                 # X
          mp.Vector3(0.5,0.5),             # M
          mp.Vector3()]                    # Gamma

# Computing bands at a lot of intermediate points to get a continuous band structure
k_points=mp.interpolate(4,k_points)

# Setting up geometric objects at the center of the lattice
geometry=[mp.Cylinder(0.2,material=mp.Medium(epsilon=12))]
```

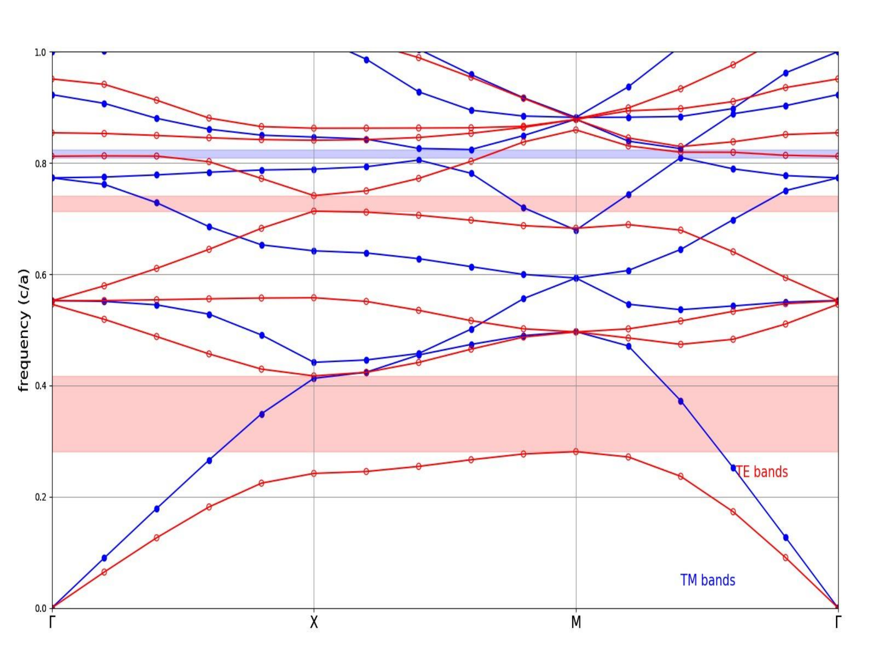
Code Continued

```
# Setting up the size of the computational cells
geometry_lattice=mp.Lattice(size=mp.Vector3(1,1))

# Setting the resolution
resolution=32

# Creating a ModeSolver object -
ms=mpb.ModeSolver(num_bands=num_bands,
                  k_points=k_points,
                  geometry=geometry,
                  geometry_lattice=geometry_lattice,
                  resolution=resolution)

# Printing and running
print_heading("Square lattice of rods: TE bands")
ms.run_te()
```

```
# This outputs the z field components of the tm mode of the wave.
ms.run_tm(mpb.output_efield_z)

# This outputs the magnetic field z components for the te_modes,
# at the point X, and the energy density power (D power).
ms.run_te(mpb.output_at_kpoint(mp.Vector3(0.5),
mpb.output_hfield_z, mpb.output_dpwr))

# sample points -
tm_freqs = ms.all_freqs
tm_gaps = ms.gap_list
ms.run_te()
te_freqs = ms.all_freqs
te_gaps = ms.gap_list

tm_freqs = ms.all_freqs
tm_gaps = ms.gap_list
ms.run_tm()
te_freqs = ms.all_freqs
te_gaps = ms.gap_list
```

Code For Plotting the Modes

```
import matplotlib.pyplot as plt

fig, ax = plt.subplots()
x = range(len(tm_freqs))

for xz, tmz, tez in zip(x, tm_freqs, te_freqs):
        ax.scatter([xz]*len(tmz), tmz, color='blue')
        ax.scatter([xz]*len(tez), tez, color='red', facecolors='none')
ax.plot(tm_freqs, color='blue')
ax.plot(te_freqs, color='red')
ax.set_ylim([0, 1])
ax.set_xlim([x[0], x[-1]])

for gap in tm_gaps:
        ax.fill_between(x, gap[1], gap[2], color='blue', alpha=0.2) if gap[0]>1

for gap in te_gaps:
        ax.fill_between(x, gap[1], gap[2], color='red', alpha=0.2) if gap[0]>1

ax.text(12, 0.04, 'TM bands', color='blue', size=15)
ax.text(13.05, 0.235, 'TE bands', color='red', size=15)

points_in_between = (len(tm_freqs) - 4) / 3
tick_locs = [i*points_in_between+i for i in range(4)]
tick_labs = ['Γ', 'X', 'M', 'Γ']
ax.set_xticks(tick_locs)
ax.set_xticklabels(tick_labs, size=16)
ax.set_ylabel('frequency (c/a)', size=16)
ax.grid(True)

plt.show()
```



The diagram shows the TM and TE bands when the geometric object is a cylinder of radius 0.2 units, of dielectric constant 12, The lattice chosen is a square lattice and the Modes are obtained at corners of the Irreducible Brillouin zone with the k-points being interpolated.

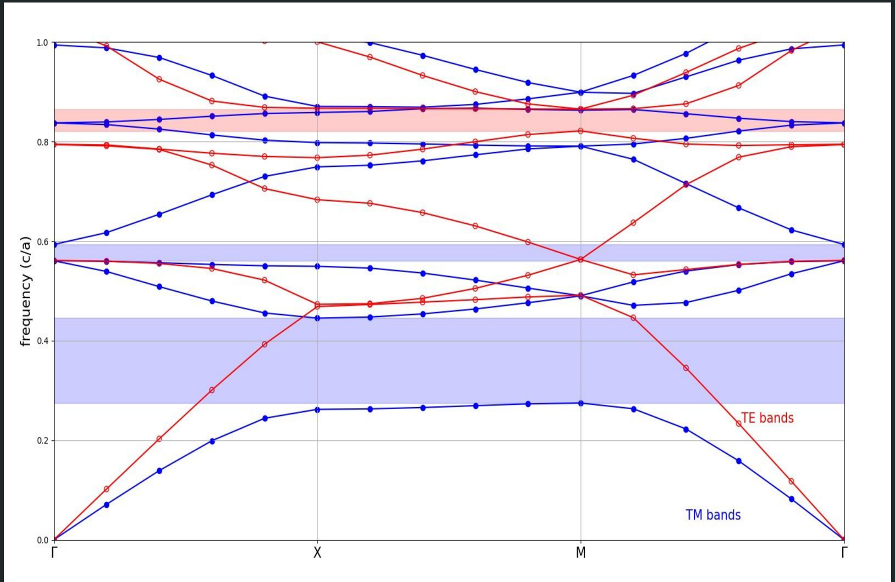
Code Changes For Triangular Lattice

The following were the changes in the code for a triangular lattice :-

```
geometry_lattice = mp.Lattice(size=mp.Vector3(1, 1),
basis1=mp.Vector3(math.sqrt(3)/2, 0.5),
basis2=mp.Vector3(math.sqrt(3)/2, -0.5))
```

```
k_points = [
        mp.Vector3(),              # Gamma
        mp.Vector3(y=0.5),         # M
        mp.Vector3(-1./3, 1./3),   # K
        mp.Vector3(),              # Gamma
]
```

```
ms.run_tm(mpb.output_at_kpoint(mp.Vector3(-1./3,
1./3), mpb.fix_efield_phase, mpb.output_efield_z))
```



The diagram shows the TM and TE bands when the geometric object is a cylinder of radius 0.2 units, of dielectric constant 12, The lattice chosen is a triangular lattice and the Modes are obtained at corners of the Irreducible Brillouin zone with the k-points being interpolated.

EXPERIMENTING WITH PARAMETERS

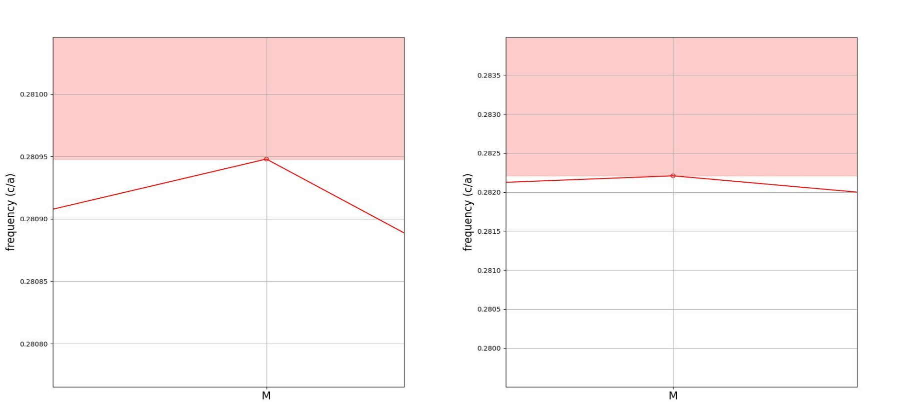
- ➔ We experimented with the output of the TE and TM modes for both the lattices by varying certain paramters.

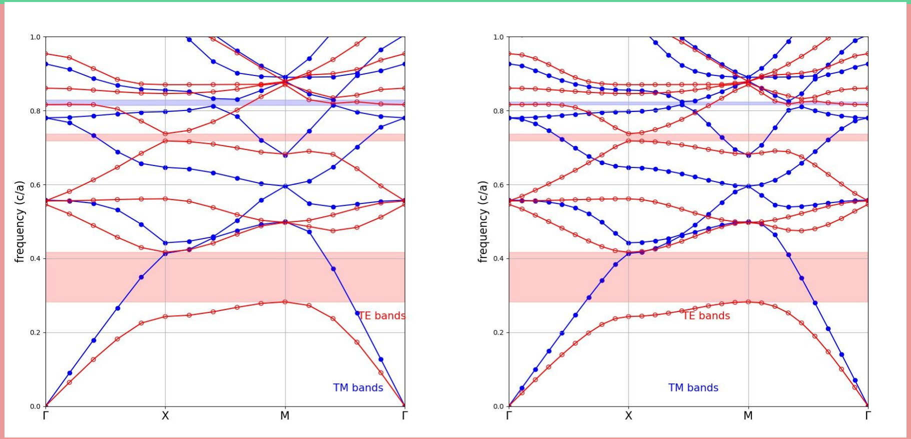
Parameters experimented with are :-

- ➔ Resolution - With the variation of resolution, the following changes in the outputs were observed -
  - Time Taken
  - Accuracy

The higher the resolution, the more the time taken and accuracy.

- ➔ K-points and interpolation - This simply gave us more points to plot and hence a more continuous and accurate graph and curves.



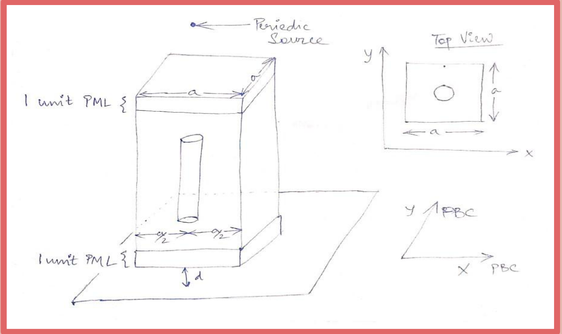


Illuminating Cylindrical Objects Using Meep

Basics

First Simulation - The First simulation involved a 1 unit block with its height along the y-direction. The source used is a continuous source of a constant frequency. PML Layers have been used at the boundary of the computational cell for implementing absorbing boundary conditions.

Second Simulation - The simulation involves illumination of a cylinder with a Gaussian or Periodic source placed above it. The spectra transmitted through the cylindrical object is observed at the computational cell below it. The diagram above explains it. PML and PBC have been used as boundary.

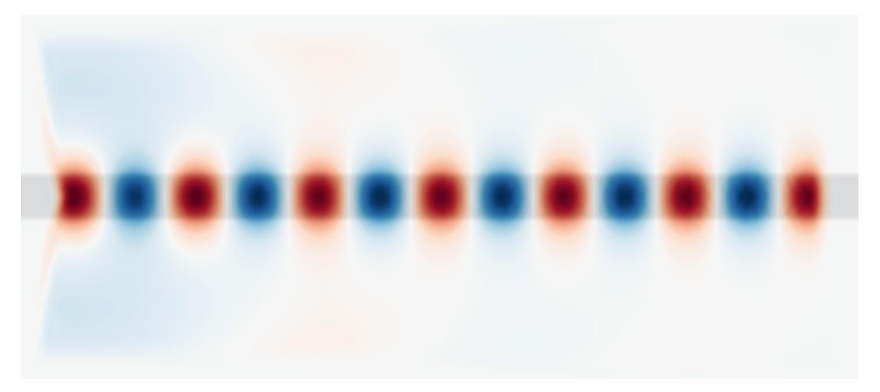


Code

```
import meep as mp

cell = mp.Vector3(16,8,0)
geometry = [mp.Block(mp.Vector3(mp.inf,1,mp.inf),
                     center=mp.Vector3(),
                     material=mp.Medium(epsilon=12))]

sources = [mp.Source(mp.ContinuousSource(frequency=0.15),
                     component=mp.Ez,
                     center=mp.Vector3(-7,0))]

pml_layers = [mp.PML(1.0)]
resolution = 10
sim = mp.Simulation(cell_size=cell,
                    boundary_layers=pml_layers,
                    geometry=geometry,
                    sources=sources,
                    resolution=resolution)

sim.run(until=200)
```

Code For Plotting

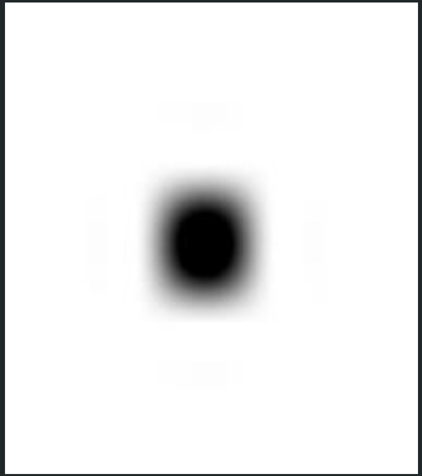
```
import matplotlib.pyplot as plt

eps_data = sim.get_array(center=mp.Vector3(),
size=cell, component=mp.Dielectric)
plt.figure()
plt.imshow(eps_data.transpose(),
interpolation='spline36', cmap='binary')
plt.axis('off')
plt.show()

ez_data = sim.get_array(center=mp.Vector3(),
size=cell, component=mp.Ez)
plt.figure()
plt.imshow(eps_data.transpose(),
interpolation='spline36', cmap='binary')
plt.imshow(ez_data.transpose(),
interpolation='spline36', cmap='RdBu', alpha=0.9)
plt.axis('off')
plt.show()
```



The first diagram depicts the 1 unit thick slab.
The second diagram depicts the fields that are traversed through when the continuous source is passed through the slab. The alternating red and blue fields are the positive and negative components of the wave. The fields are cut off due to PML layers at the boundary.

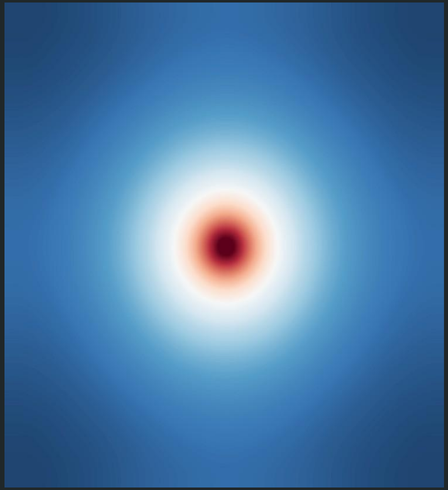
Illuminating cylindrical rod from top

```
from __future__ import division

import meep as mp

cell=mp.Vector3(4,4,0)

geometry_cylinder=[mp.Cylinder(center=mp.Vector3(0,0,0.2),height=1,radius=0.2,axis=mp.Vector3(0,0,1),material=mp.Medium(index=12))]
geometry=geometry_cylinder

k_point=mp.Vector3(0,0,0)
df=0.1
fcen=0.15
w=1
sources = [mp.Source(mp.GaussianSource(fcen,fwidth=df),
                     component=mp.Ez,
                     center=mp.Vector3(0,0,3),
                     size=mp.Vector3(0,w,0))]

pml_layers=[mp.PML(1.0)]
resolution=10

sim=mp.Simulation(cell_size=cell,geometry=geometry,k_point=k_point,sources=sources,resolution=resolution)

sim.run(until=200)
```



This shows the top view of the cylindrical object as observed before illumination from the source

This shows the spectra observed on the computation cell placed beneath the cylinder, at the opposite side of the source



FURTHER WORK AND REFERENCES

The next works include for the project include -

- Including a metal surface at the bottom of the cylinder and observe the change in the spectra obtained with and without the cylinder
- Subtract incident field from the resultant and calculate the reflected flux
- Remove noise from the obtained fluxes to get a clear spectra at the computational cell

- Photonic Crystals: Molding the flow of light second edition. John D. Joannopoulos, Steven G. Johnson, Joshua N. Winn, and Robert D. Meade
- Meep Software - https://meep.readthedocs.io/en/latest/
- MPB Software - https://mpb.readthedocs.io/en/latest/