

Mini Project Report

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Problem Statement

- (a) Design a photonic crystal based waveguide for on-chip applications at 450nm.*
- (b) How would you change the design parameter if the wavelength of operation is changed to 550nm? Show by numerical computations.*
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Code for computing the band diagram and the band gaps for a square lattice of dielectric rods (epsilon =12, radius of rods = 0.2)

```
import math
import meep as mp
from meep import mpb
import matplotlib
#matplotlib.use('Agg')

import matplotlib.pyplot as plt

# Determining the number of eigenstates computed at each k points.
num_bands=50

# k_points=Bloch wavevectors we want to compute the bands at.
# Setting it to the corners of irreducible Brillouin zone.
k_points=[mp.Vector3(),          # Gamma
           mp.Vector3(0.5),      # X
           mp.Vector3(0.5,0.5),  # M
           mp.Vector3()]         # Gamma

# Computing bands at a lot of intermediate points to get a continuous band structure
k_points=mp.interpolate(4,k_points)

# Setting up the size of the computational cells
geometry_lattice=mp.Lattice(size=mp.Vector3(1,1))

# Setting up geometric objects at the center of the lattice
geometry=[mp.Cylinder(0.2,material=mp.Medium(epsilon=12))]
geometry = mp.geometric_objects_lattice_duplicates(geometry_lattice, geometry)

# Setting the resolution
resolution=32
```

```
# Creating a ModeSolver object -
ms=mpb.ModeSolver(num_bands=num_bands,
                  k_points=k_points,
                  geometry=geometry,
                  geometry_lattice=geometry_lattice,
                  resolution=resolution)

# Printing and running
print("Square lattice of rods: TE bands")
ms.run_te()

# This outputs the z field components of the tm mode of the wave.
ms.run_tm(mpb.output_efield_z)

# This outputs the magnetic field z components for the te_modes,
# at the point X, and the energy density power (D power).
ms.run_te(mpb.output_at_kpoint(mp.Vector3(0.5),mpb.output_hfield_z,mpb.output_dpwr))

# sample points -

ms.run_te()
te_freqs = ms.all_freqs
te_gaps = ms.gap_list

ms.run_tm()
tm_freqs = ms.all_freqs
tm_gaps = ms.gap_list
print("tm gaps: ", tm_gaps)
print("freqs: ", tm_freqs)

tm_freqs = ms.all_freqs
tm_gaps = ms.gap_list
ms.run_te()
te_freqs = ms.all_freqs
te_gaps = ms.gap_list

import matplotlib.pyplot as plt
```

```

fig, ax = plt.subplots()
x = range(len(tm_freqs))
# Plot bands
# Scatter plot for multiple y values, see https://stackoverflow.com/a/34280815/2261298
for xz, tmz, tez in zip(x, tm_freqs, te_freqs):
    ax.scatter([xz]*len(tmz), tmz, color='blue')
    ax.scatter([xz]*len(tez), tez, color='red', facecolors='none')
ax.plot(tm_freqs, color='blue')
ax.plot(te_freqs, color='red')
ax.set_ylim([0, 1])
ax.set_xlim([x[0], x[-1]])

# Plot gaps
for gap in tm_gaps:
    if gap[0] > 1:
        ax.fill_between(x, gap[1], gap[2], color='blue', alpha=0.2)

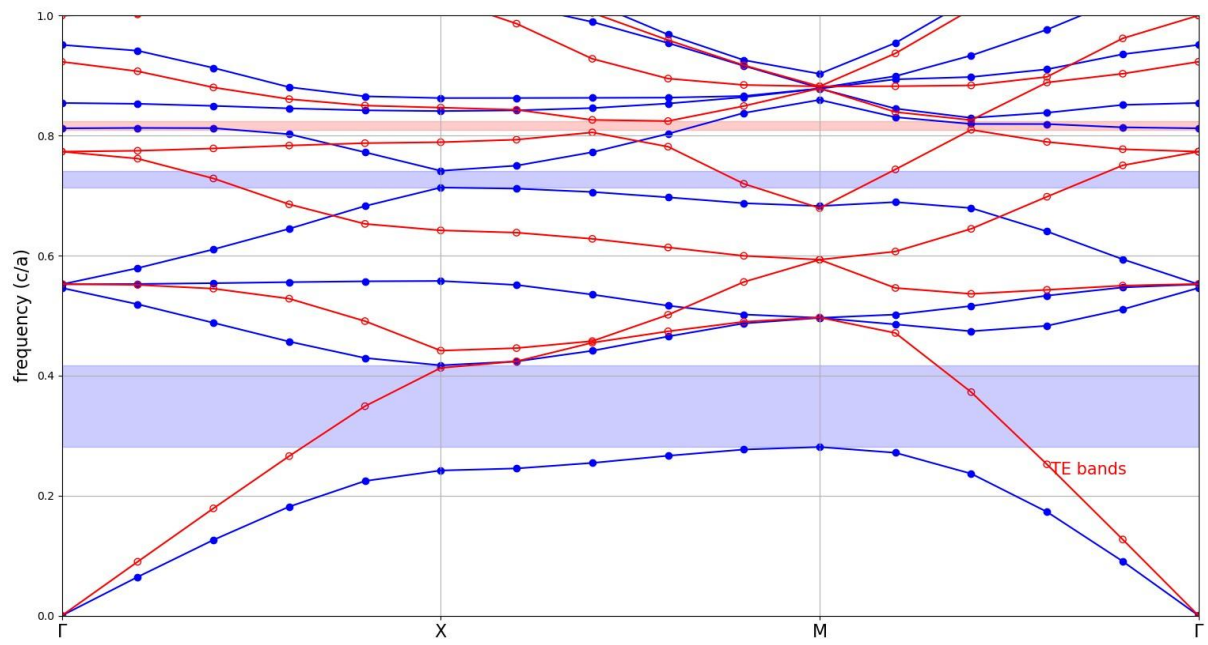
for gap in te_gaps:
    if gap[0] > 1:
        ax.fill_between(x, gap[1], gap[2], color='red', alpha=0.2)

# Plot labels
#ax.text(12, 0.04, 'TM bands', color='blue', size=15)
ax.text(13.05, 0.235, 'TE bands', color='red', size=15)

points_in_between = (len(tm_freqs) - 4) / 3
tick_locs = [i*points_in_between+i for i in range(4)]
tick_labs = ['Γ', 'X', 'M', 'T']
ax.set_xticks(tick_locs)
ax.set_xticklabels(tick_labs, size=16)
ax.set_ylabel('frequency (c/a)', size=16)
ax.grid(True)

plt.show()

```



Basic Band Diagram obtained without the line defects

We get a T.M. band gap from band 1 (0.280947955) to band 2 (0.417114251086).

The mid gap point is : [band 1 (0.280947) + band 2 (0.417114)] / 2 = 0.349035

```
harshmellow@LAPTOP-Q247CL28: /mnt/d
.0>
Band 87 range: 3.3663370723037094 at Vector3<0.09999999999999998, 0.09999999999999998, 0.0> to 3.39856934579999 at Vector3<0.5, 0.5, 0.0>
Band 88 range: 3.373257550224209 at Vector3<0.19999999999999996, 0.19999999999999996, 0.0> to 3.416628333283027 at Vector3<0.0, 0.0, 0.0>
Band 89 range: 3.386733617936092 at Vector3<0.5, 0.0, 0.0> to 3.4288125483279766 at Vector3<0.5, 0.2, 0.0>
Band 90 range: 3.413910987885264 at Vector3<0.5, 0.0, 0.0> to 3.446855038149843 at Vector3<0.4, 0.4, 0.0>
Band 91 range: 3.4068158925555627 at Vector3<0.30000000000000004, 0.0, 0.0> to 3.4875221389809714 at Vector3<0.5, 0.5, 0.0>
Band 92 range: 3.4454385059739234 at Vector3<0.3, 0.3, 0.0> to 3.552415628400321 at Vector3<0.5, 0.5, 0.0>
Band 93 range: 3.49853655972698 at Vector3<0.4, 0.0, 0.0> to 3.558776269597514 at Vector3<0.5, 0.5, 0.0>
Band 94 range: 3.5019614072187877 at Vector3<0.0, 0.0, 0.0> to 3.5650900748511933 at Vector3<0.5, 0.5, 0.0>
Band 95 range: 3.533776554740043 at Vector3<0.0, 0.0, 0.0> to 3.5769511276923884 at Vector3<0.5, 0.4, 0.0>
Band 96 range: 3.550316448989977 at Vector3<0.0, 0.0, 0.0> to 3.586176495051902 at Vector3<0.4, 0.4, 0.0>
Band 97 range: 3.5719790944148735 at Vector3<0.30000000000000004, 0.0, 0.0> to 3.602617699112341 at Vector3<0.5, 0.4, 0.0>
Band 98 range: 3.584414306359443 at Vector3<0.5, 0.30000000000000004, 0.0> to 3.6107226419768996 at Vector3<0.4, 0.4, 0.0>
Band 99 range: 3.595669851728043 at Vector3<0.5, 0.0, 0.0> to 3.6427616007653696 at Vector3<0.30000000000000004, 0.0, 0.0>
Band 100 range: 3.612271933014077 at Vector3<0.5, 0.2, 0.0> to 3.6644275705723777 at Vector3<0.30000000000000004, 0.0, 0.0>
Gap from band 1 (0.2809479552069594) to band 2 (0.4171142510865908), 39.01265378700949%
Gap from band 4 (0.7133951533557327) to band 5 (0.7413109663075104), 3.8380003458348084%
Gap from band 18 (1.527312708692077) to band 19 (1.5366744827927707), 0.6110844148899255%
total elapsed time for run: 18.509618282318115
done

Elapsed run time = 19.0582 s
(mp) harshmellow@LAPTOP-Q247CL28: /mnt/d$
```

Code creating a line defect waveguide - Simulating for 450 nm

```
import math
import meep as mp
from meep import mpb
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt

# Setting the lattice constant
a=0.15706372

# Setting the geometry lattice to be 20*lambda
geometry_lattice = mp.Lattice(size=mp.Vector3(11, 11))

# Setting our rods to be of radius 0.2*lattice_constant
geometry = [mp.Cylinder(0.2*a, material=mp.Medium(epsilon=12))]

# Duplicating our rods through the entire geometric lattice taken
geometry = mp.geometric_objects_lattice_duplicates(geometry_lattice, geometry)

# Creating a line defect through a for loop. This has the same radius=0.2*a.
for i in range(11):
    geometry.append(mp.Cylinder(0.2*a, center=mp.Vector3(1+i), material=mp.air))
    geometry.append(mp.Cylinder(0.2*a, center=mp.Vector3(-(1+i)), material=mp.air))

# Setting Resolution
resolution = 16

# Setting our k-points
k_points = [mp.Vector3(0.5, 0.5)]

# Setting number of eigenstates to calculate our at the wavevectors
num_bands = 100

# Interpolating the k-points
k_points=mp.interpolate(4,k_points)
```

```
# Creating a mode solver object
ms = mpb.ModeSolver(num_bands=num_bands,
                    k_points=k_points,
                    geometry=geometry,
                    geometry_lattice=geometry_lattice,
                    resolution=resolution)

ms.run_tm()

eps = ms.get_epsilon()
plt.imshow(eps.T, interpolation='spline36', cmap='binary')
plt.axis('off')
plt.savefig("pic.png")
```

Code creating a line defect waveguide - Simulating for 550 nm

```
import math
import meep as mp
from meep import mpb
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt

# Setting the lattice constant
a=0.19196678

# Setting the geometry lattice to be 20*lambda
geometry_lattice = mp.Lattice(size=mp.Vector3(11, 11))

# Setting our rods to be of radius 0.2*lattice_constant
geometry = [mp.Cylinder(0.2*a, material=mp.Medium(epsilon=12))]

# Duplicating our rods through the entire geometric lattice taken
geometry = mp.geometric_objects_lattice_duplicates(geometry_lattice, geometry)

# Creating a line defect through a for loop. This has the same radius=0.2*a.
for i in range(11):
    geometry.append(mp.Cylinder(0.2*a, center=mp.Vector3(1+i), material=mp.air))
    geometry.append(mp.Cylinder(0.2*a, center=mp.Vector3(-(1+i)), material=mp.air))

# Setting Resolution
resolution = 16

# Setting our k-points
k_points = [mp.Vector3(0.5, 0.5)]

# Setting number of eigenstates to calculate our at the wavevectors
num_bands = 100

# Interpolating the k-points
k_points=mp.interpolate(4,k_points)
```

```
# Creating a mode solver object
ms = mpb.ModeSolver(num_bands=num_bands,
                    k_points=k_points,
                    geometry=geometry,
                    geometry_lattice=geometry_lattice,
                    resolution=resolution)

ms.run_tm()

eps = ms.get_epsilon()
plt.imshow(eps.T, interpolation='spline36', cmap='binary')
plt.axis('off')
plt.savefig("pic.png")
```



This diagram shows the line defect formed by a missing row of rods in square lattice of radius $0.2a$ dielectric rods ($\epsilon=12$) in air.

For 450 nm obtained TM freqs after creating line defect

```
harshmellow@LAPTOP-Q247CL28: /mnt/d
iteration 19: trace = 2.490121533035302 (0.0285166% change)
iteration 21: trace = 2.489264176492278 (0.0147838% change)
iteration 23: trace = 2.488847852790378 (0.00609612% change)
iteration 25: trace = 2.488687671650991 (0.00263811% change)
iteration 27: trace = 2.488631097569828 (0.000774566% change)
iteration 29: trace = 2.488611532875827 (0.000331888% change)
iteration 31: trace = 2.488603340826857 (0.000105641% change)
iteration 33: trace = 2.488600447255414 (4.66979e-05% change)
iteration 35: trace = 2.488599498382557 (1.4523e-05% change)
Finished solving for bands 91 to 100 after 36 iterations.
Finished k-point with 39.9 mean iterations/band.
tmfreqs:, 1, 0.5, 0.5, 0, 0.0642824, 0.0632448, 0.0632459, 0.0634169, 0.0634404, 0.141341, 0.141401,
0.141413, 0.141472, 0.14177, 0.141787, 0.141829, 0.141872, 0.189627, 0.189767, 0.19023, 0.190264, 0
.227896, 0.227924, 0.228013, 0.228032, 0.228547, 0.228589, 0.228603, 0.228691, 0.260518, 0.260662, 0
.260686, 0.260771, 0.261339, 0.261419, 0.261414, 0.261447, 0.31579, 0.31582, 0.31585, 0.31601, 0.316
078, 0.316218, 0.316732, 0.316901, 0.316922, 0.31694, 0.317074, 0.31713, 0.340062, 0.340095, 0.34025
4, 0.340479, 0.341171, 0.341377, 0.341381, 0.341546, 0.384003, 0.384052, 0.384159, 0.384523, 0.38526
6, 0.385424, 0.385559, 0.385654, 0.404189, 0.404267, 0.404318, 0.404431, 0.405323, 0.405575, 0.40558
, 0.405786, 0.423412, 0.423437, 0.423436, 0.423656, 0.424727, 0.424824, 0.424839, 0.425146, 0.441613
, 0.441908, 0.443213, 0.443759, 0.459275, 0.45936, 0.459361, 0.459609, 0.460738, 0.460906, 0.461034,
0.461044, 0.484725, 0.486179, 0.486341, 0.486925, 0.500235, 0.501568, 0.50197, 0.501972, 0.501978,
0.501989, 0.502004, 0.503227
elapsed time for k point: 154.2304184436798
total elapsed time for run: 154.75793290138245
done
gaps []
epsilon: 1-9.59188, mean 1.03003, harm. mean 1.00695, 1.18519% > 1, 0.349459% "fill"

Elapsed run time = 155.2438 s
(mp) harshmellow@LAPTOP-Q247CL28:/mnt/d$
```

Observation -

The output clearly shows that there are new T.M. mode frequencies obtained within the band gaps. The band gaps were calculated before and the new frequencies are compared and highlighted. These defect modes lie within the band gap of 0.280947955 to 0.417114251086

Inference -

This is justifiable with the theory that if a line defect is created within the lattice then there should new frequencies obtained within the band gaps. This would mean that the light of 450 nm is able to propagate through the line defect and hence the working of the system is justified.

For 550 nm obtained TM freqs after creating line defect

```
harshmellow@LAPTOP-Q247CL28: /mnt/d
iteration 43: trace = 2.434679113954004 (9.74983e-05% change)
iteration 45: trace = 2.434673114820783 (0.00012783% change)
iteration 47: trace = 2.434665823269452 (0.000162033% change)
iteration 49: trace = 2.434655965060001 (0.000216192% change)
iteration 51: trace = 2.434647406379294 (0.000153883% change)
iteration 53: trace = 2.434642163215736 (8.87978e-05% change)
iteration 55: trace = 2.434640378933946 (2.7858e-05% change)
iteration 57: trace = 2.434639533932769 (1.49629e-05% change)
iteration 59: trace = 2.43463904688143 (8.74138e-06% change)
Finished solving for bands 91 to 100 after 59 iterations.
Finished k-point with 41.2 mean iterations/band.
tmfreqs:, 1, 0.5, 0.5, 0, 0.0642824, 0.0627048, 0.0627176, 0.0629818, 0.0629914, 0.140179, 0.140179,
0.140179, 0.14022, 0.140798, 0.140799, 0.140801, 0.140858, 0.188023, 0.188039, 0.188873, 0.188917,
0.22591, 0.225912, 0.225914, 0.22603, 0.226893, 0.226923, 0.226951, 0.226972, 0.25826, 0.258263, 0.2
58277, 0.258379, 0.259459, 0.259467, 0.259465, 0.259548, 0.312972, 0.312997, 0.312998, 0.313006, 0.3
13012, 0.313173, 0.31434, 0.314348, 0.314479, 0.314495, 0.314499, 0.314646, 0.336986, 0.337006, 0.33
701, 0.337016, 0.338607, 0.338616, 0.338593, 0.338809, 0.380369, 0.380375, 0.380381, 0.380461, 0.382
228, 0.382243, 0.382284, 0.382414, 0.400021, 0.400037, 0.400054, 0.400321, 0.401762, 0.401841, 0.402
182, 0.402187, 0.418888, 0.418905, 0.418916, 0.419134, 0.421055, 0.421155, 0.421194, 0.421195, 0.437
083, 0.437195, 0.439413, 0.439518, 0.4541, 0.454106, 0.454144, 0.454518, 0.456611, 0.45664, 0.456724
, 0.456724, 0.475215, 0.477497, 0.477598, 0.478155, 0.490146, 0.492583, 0.492727, 0.494573, 0.501894
, 0.501902, 0.501904, 0.50193
elapsed time for k point: 160.26310968399048
total elapsed time for run: 160.7604262828827
done
gaps []
epsilon: 1-11.4635, mean 1.04603, harm. mean 1.00923, 1.48765% > 1, 0.439862% "fill"
Elapsed run time = 161.2304 s
(mp) harshmellow@LAPTOP-Q247CL28: /mnt/d$
```

Observation -

The output clearly shows that there are new T.M. mode frequencies obtained within the band gaps. The band gaps were calculated before and the new frequencies are compared and highlighted. These defect modes lie within the band gap of 0.280947955 to 0.417114251086. The change in design parameter included the change in 'a', the lattice constant and correspondingly the radius of the rods.

Inference -

This is justifiable with the theory that if a line defect is created within the lattice then there should new frequencies obtained within the band gaps. This would mean that the light of 550 nm is able to propagate through the line defect and hence the working of the system is justified.

Theory -

The gap between the first and second TM bands from $\omega a/2\pi c = 0.280947955$ to $\omega a/2\pi c = 0.417114251086$. Expressed as a fraction of the midgap frequency $\omega a/2\pi c = 0.3490305$.

If we want our operating wavelength $\lambda = 2\pi c/\omega = 4.5 \mu\text{m}$ to lie at the middle of the gap, we need

$$\omega a/2\pi c = a/\lambda = a/4.5 \mu\text{m} = 0.3490305$$

$$\Rightarrow a = 0.15706372 \mu\text{m}.$$

Given the value of a , we calculate the rod radius $r = 0.2*a = 0.03141274 \mu\text{m}$, and the structure is completely determined.

Calculations supporting our system -

For 450 nm

The centre of the TM band gap from band 1 to band 2:

$$\frac{0.280947 + 0.417114}{2} = 0.3490305$$

Our operating frequency:

$$\begin{aligned}\lambda &= 450 \text{ nm} \\ &= 0.45 \mu\text{m}\end{aligned}$$

Rescaling our structure in accordance with our operating frequency:

$$\Rightarrow \frac{\omega a}{2\pi c} = \frac{a}{\lambda} = \frac{a}{0.45 \mu\text{m}} = 0.3490305$$

$$\Rightarrow a = 0.15706372$$

Also, the radius of rods: $0.2a$

$$= 0.2 \times 0.15706372$$

$$= 0.03141274$$

For 550 nm

The centre of the TM band gap from band 1 to band 2 :

$$\frac{0.280947 + 0.417114}{2} = 0.3490305$$

Our operating frequency :

$$\begin{aligned}\lambda &= 550 \text{ nm} \\ &= 0.55 \mu\text{m}\end{aligned}$$

Rescaling our structure in accordance with our operating frequency :

$$\Rightarrow \frac{\omega a}{2\pi c} = \frac{a}{\lambda} = \frac{a}{0.55 \mu\text{m}} = 0.3490305$$

$$\Rightarrow a = 0.19196678$$

Also, the radius of rods : $0.2a$

$$= 0.2 \times 0.19196678$$

$$= 0.03839336$$

The choice of geometry Lattice

we have chosen the geometry lattice to be 20λ

Here we have taken $\lambda = 550 \text{ nm}$
 $= 0.55 \mu\text{m}$

\Rightarrow our geometry lattice :

$$20 \times 0.55$$

$$\Rightarrow 11$$

\therefore geometry Lattice : 11×11

References

<https://mpb.readthedocs.io/>

<https://meep.readthedocs.io/en/latest/>

<http://ab-initio.mit.edu/book/photonic-crystals-book.pdf>