Mean Squared Error

MSE ≈ MMSE

$\Sigma (\text{actual} - \text{predicted})^2$ ← MSE

$$E[(X - \hat{x})^2 | Y = y] = E[(X - g(y))^2 | Y = y]$$

let a be estimate of X

$$h(a) = E[(X-a)^2]$$
$$= E[X^2] - 2aE[X] + a^2$$
$$h'(a) = -2E[X] + 2a$$

$h'(a) = 0$ when $a = E[X]$

Mean Squared % Error

$$\Sigma \left(\frac{\text{actual} - \text{predicted}}{\text{actual}} \right)^2$$

...

$$h(a) = E\left[\left(\frac{X-a}{X}\right)^2\right]$$

$$1 - \frac{a}{X}$$

$$E\left[1 - 2\frac{a}{X} + \frac{a^2}{X^2}\right]$$

$$1 - 2E\left(\frac{a}{X}\right) + E\left(\frac{a^2}{X^2}\right)$$

$$1 - 2aE\left(\frac{1}{X}\right) + a^2 E\left(\frac{1}{X^2}\right)$$

$$h'(a) = -2E\left(\frac{1}{X}\right) + 2aE\left(\frac{1}{X^2}\right)$$

$$2E\left(\frac{1}{X}\right) = 2aE\left(\frac{1}{X^2}\right)$$

$$a = \frac{E\left(\frac{1}{X}\right)}{E(X^{-2})}$$

Absolute Error

$$h(a) = E[|X-a|]$$

$$= E[((X-a)^2)^{1/2}]$$

$$h'(a) = E\left[\tfrac{1}{2}((X-a)^2)^{-\frac{1}{2}} \cdot 2(X-a)\right]$$

$$= E\left[\tfrac{1}{2}\left(\frac{1}{|X-a|}\right) \cdot (2X-2a)\right]$$

$$= E\left[\frac{X}{|X-a|} - \frac{a}{|X-a|}\right]$$

... no way to solve for $h'(a) = 0$

Value of slope of absolute value at 0 cannot be determined because absolute value function is discontinuous.

To solve linear programming must be used. This applies to Absolute Percentage Error as well.