Credits – Mr W, Mr T, Mr R, Ms L

One day at dinner, I looked at my friend, Ms L, as she was eating her soup from a bowl, and thought to myself: "What is the best bowl that there can be?". I posed the question to her, and at this a great amount of conversation was had between the four of us present. Ms L, Mr R, Mr T and I discussed what the question meant, and we eventually decided on a more rigorous query.

Given a fixed amount of material to make a bowl to carry liquid in, what is the shape of the bowl that maximises the volume of liquid that can be carried?

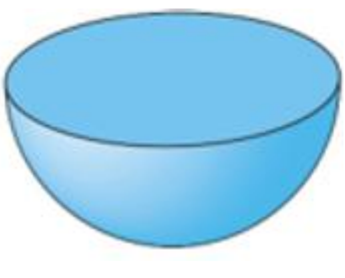
One key insight, proposed by Mr R, was that the top of the bowl would be flat, as the liquid would settle flat in the bowl, and so any material above the water line could be removed and placed somewhere else to expand the capacity of the bowl.

Another question is justifying why all the liquid must be together, and not stored in separate containers. If the liquid is stored in two separate parts, then there is some 'dry' portion where we can cut and separate the two containers. Then taking the container with the better surface to volume ratio, we could have used all the material to make a bowl of that shape, which would have been better than our original shape (even if the two original containers we separated out had identical shapes). (The maths for this is relatively trivial and left as an exercise for the reader)

With these two insights this allows us to use an argument Mr T proposed to suggest that the uncapped hemisphere is the optimal bowl shape.

Consider whatever shape we have that we think is the optimal shape for a bowl. We know that the liquid is stored together, so has one water line. This means it has one flat circumference upon which we can place a mirror, to reflect the shape in, to get a complete solid with no gaps. This solid has the same surface area to volume ratio as our bowl, as we have both doubled the amount of material and the volume inside.

We have now translated our problem to the question of what is the three dimensional solid with the best surface area to volume ratio. It has been proven that this is the sphere. Hence for an optimal bowl shape, we must get to a sphere when putting a mirror on top of it, hence we must have had an open hemisphere.



- The optimal bowl

Sadly this bowl is not stable. In fact, any force not applied directly through the initial point of contact of our bowl would cause some liquid to spill. This is clearly not ideal.

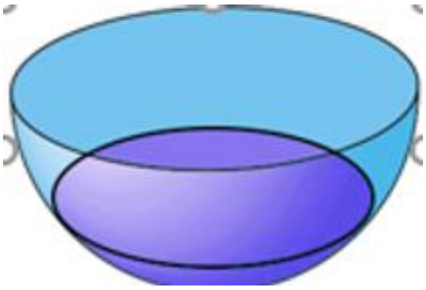
This naturally gave rise to the demand for a stable optimal bowl given some constraint. A circular base seems like quite a mild demand, and so this is what we impose:

Given a circular base for a bowl, what is the best shape (maximise volume of liquid carried), given a specified amount of extra material, for the bowl to take?

We pondered this for a while, guessing that it might be a truncated hemisphere, but struggling with the justification.

It was at this point that Mr W joined us, and so we asked him the question. As is usual he cut straight through the fog, giving us the solution to our conundrum.

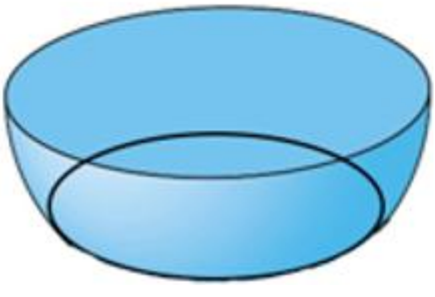
Suppose we had an optimal bowl shape with a circular base. Then we can take a hemispherical bowl and fix it below a certain height.



Purple is fixed, blue is changeable

Then we can adjust the top portion of the bowl so that it obtains the optimal bowl shape for a bowl with a circular base. But this would then mean that the new bowl we have just created would have a better general bowl shape than an open hemisphere. This is impossible, unless the best bowl we could have done was a truncated hemisphere (including the equator) to begin with.

Hence the optimal bowl with a circular base is a truncated hemisphere (including the equator).



- The optimal bowl with a circular base