Single Neuron Modeling:Simplified models

Introduction

The Hodgkin-Huxley Model:

$$
\begin{aligned}
c\frac{\mathrm{d}V}{\mathrm{d}t} &= -\bar{g}_{\mathrm{Na}}m^3h\,(V-E_{\mathrm{Na}}) - \bar{g}_{\mathrm{K}}n^4\,(V-E_{\mathrm{K}}) - \bar{g}_{\mathrm{L}}\,(V-E_{\mathrm{L}}) + I_{\mathrm{ext}},\\
\frac{\mathrm{d}n}{\mathrm{d}t} &= \phi\,[\alpha_n(V)(1-n) - \beta_n(V)n]\\
\frac{\mathrm{d}m}{\mathrm{d}t} &= \phi\,[\alpha_m(V)(1-m) - \beta_m(V)m]\,,\\
\frac{\mathrm{d}h}{\mathrm{d}t} &= \phi\,[\alpha_h(V)(1-h) - \beta_h(V)h]\,,
\end{aligned}
$$

$$
\alpha_n(V) = \frac{0.01(V+55)}{1-\exp\left(-\frac{V+55}{10}\right)},\quad \beta_n(V) = 0.125\exp\left(-\frac{V+65}{80}\right),
$$

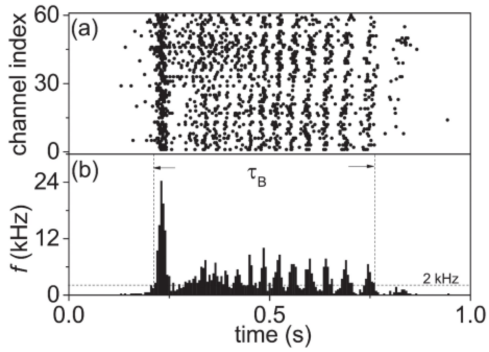
$$
\alpha_h(V) = 0.07\exp\left(-\frac{V+65}{20}\right),\quad \beta_h(V) = \frac{1}{\left(\exp\left(-\frac{V+35}{10}\right)+1\right)},
$$

$$
\alpha_m(V) = \frac{0.1\,(V+40)}{1-\exp\left(-(V+40)/10\right)},\quad \beta_m(V) = 4\exp\left(-(V+65)/18\right).
$$

$$
\phi = Q_{10}^{(T-T_{\mathrm{base}})/10}
$$

Weakness:

computationally expensive



Huang YT, et al. PLoS One. 2017

The Leaky Integrate-and-Fire(LIF) Neuron model

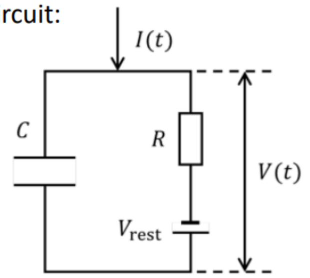
- The LIF neuron model

$$
\tau\frac{\mathrm{d}V}{\mathrm{d}t} = -(V-V_{\mathrm{rest}}) + RI(t)
$$

$$
\text{if } V > V_{\mathrm{th}},\quad V \leftarrow V_{\mathrm{reset}} \ \text{last}\ t_{\mathrm{ref}}
$$

Refractory period

Equivalent circuit:



Comparing to the HH model:

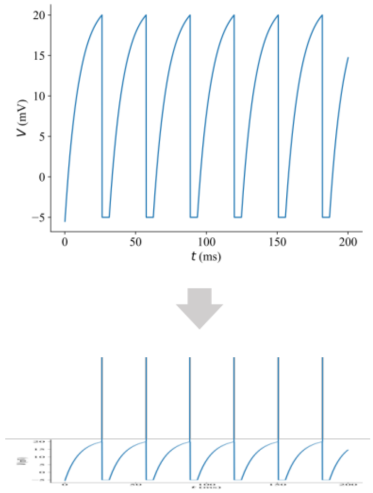
$$
c\frac{\mathrm{d}V}{\mathrm{d}t} = -\bar{g}_{\mathrm{Na}}m^3h\,(V-E_{\mathrm{Na}}) - \bar{g}_{\mathrm{K}}n^4\,(V-E_{\mathrm{K}}) - \bar{g}_{\mathrm{L}}\,(V-E_{\mathrm{L}}) + I_{\mathrm{ext}},
$$

- Refractory period(不应期，重置时间，膜电位不会改变)

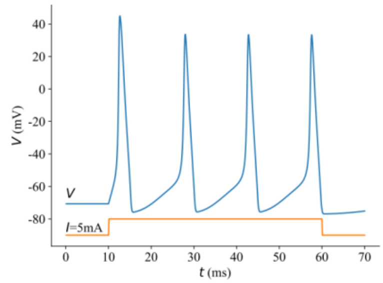
Given a constant current input:

$$\tau \frac{dV}{dt} = -(V - V_{rest}) + RI(t)$$

$$\text{if } V > V_{th}, \quad V \leftarrow V_{reset} \text{ last } t_{ref}$$



Comparing to the HH model:



- The dynamic features of the LIF model

General solution (constant input): $V(t) = V_{reset} + RI_c(1 - e^{-\frac{t-t_0}{\tau}})$

$$\tau \frac{dV}{dt} = -(V - V_{rest}) + RI(t)$$

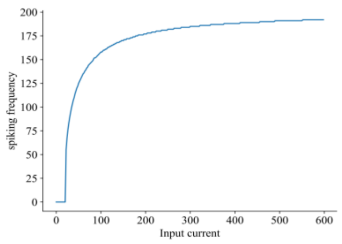
$$\text{if } V > V_{th}, \quad V \leftarrow V_{reset} \text{ last } t_{ref}$$

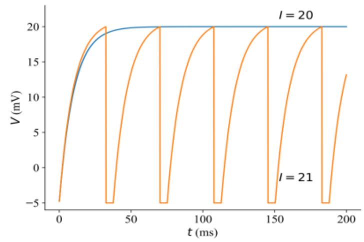
Firing frequency: $T = -\tau \ln\left(1 - \frac{V_{th} - V_{rest}}{RI_c}\right)$

$$f = \frac{1}{T + t_{ref}} = \frac{1}{t_{ref} - \tau \ln\left(1 - \frac{V_{th} - V_{rest}}{RI_c}\right)}$$

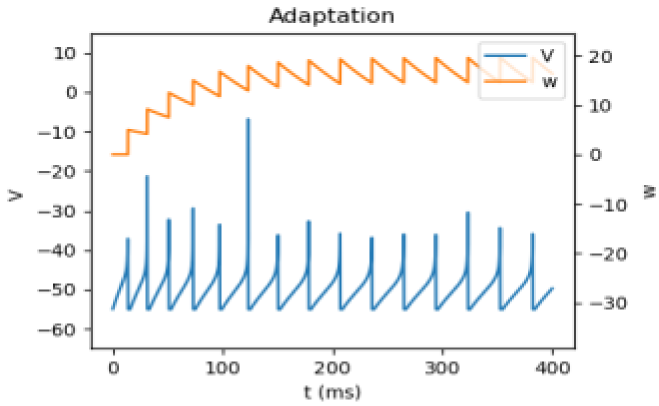
Rheobase current (minimal current):

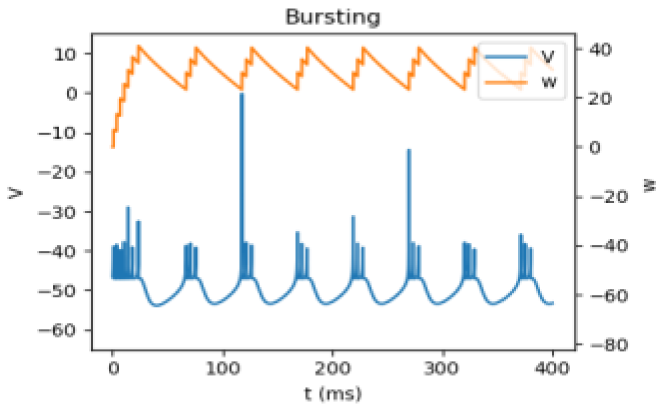
$$I_\theta = \frac{V_{th} - V_{reset}}{R}$$





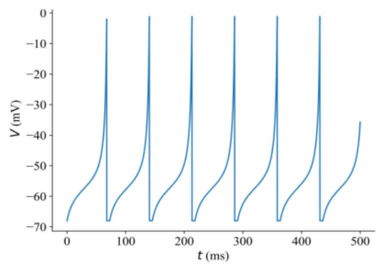
  - $V_{reset} + RI_c$要大于V_{th}才能持续放电
  - 可以求出I_c的最小值
- Strengths&weaknesses of the LIF model
  - Strengths:
    - Simple,high simulation Efficiency
    - Intuitive
    - Fits well the subthreshold membrane potential
  - Weakness
    - The shape of action potentials is over-simplified
    - has no memory of the spiking history
    - Cannot reproduce diverse firing patterns





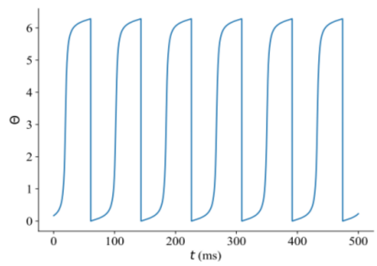
- Other Univariate neuron models
  - The Quadratic Integrate-and-Fire (QIF) model:

$$\tau \frac{\mathrm{d}V}{\mathrm{d}t} = a_0 (V - V_{\text{rest}})(V - V_c) + RI(t)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \ \text{ last } \ t_{\text{ref}}$$



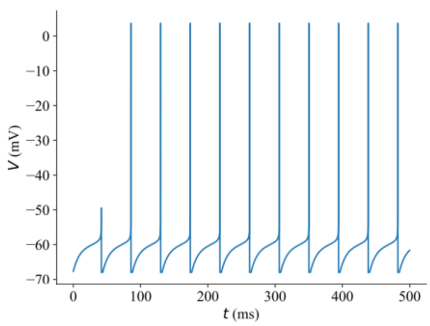
  - The Theta neuron model:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 1 - \cos\theta + (1 + \cos\theta)\,(\beta + I(t))$$



  - The Exponential Integrate-and-Fire (ExpIF) model:

$$\tau \frac{\mathrm{d}V}{\mathrm{d}t} = -(V - V_{\text{rest}}) + \Delta_T \mathrm{e}^{\frac{V - V_T}{\Delta_T}} + RI(t)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \ \text{ last } \ t_{\text{ref}}$$



The Adaptive Exponential Integrate-and-Fire (AdEx) Neuron model

- The AdEx neuron model

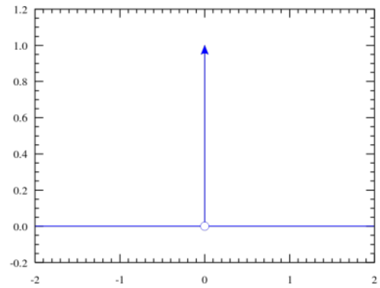
  Two variables:
  - V: membrane potential
  - w: adaptation variable

$$\tau_m \frac{\mathrm{d}V}{\mathrm{d}t} = -\left(V - V_{\text{rest}}\right) + \Delta_T \mathrm{e}^{\frac{V-V_T}{\Delta_T}} - Rw + RI(t)$$

$$\tau_w \frac{\mathrm{d}w}{\mathrm{d}t} = a\left(V - V_{\text{rest}}\right) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$

$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \text{ last } t_{\text{ref}}$$

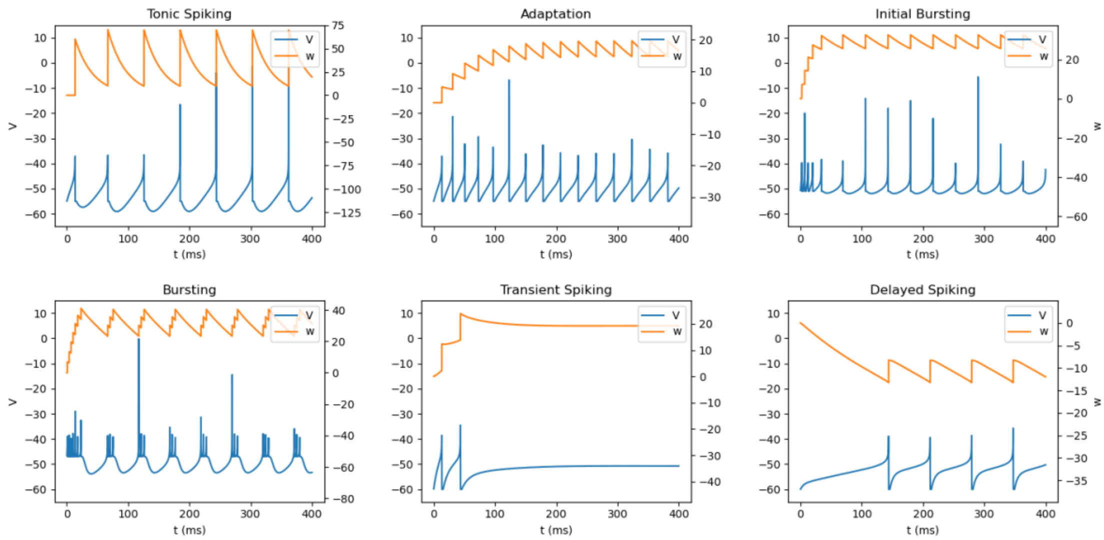
  - A larger w suppresses V from increasing
  - w decays exponentially while having a sudden increase when the neuron fires



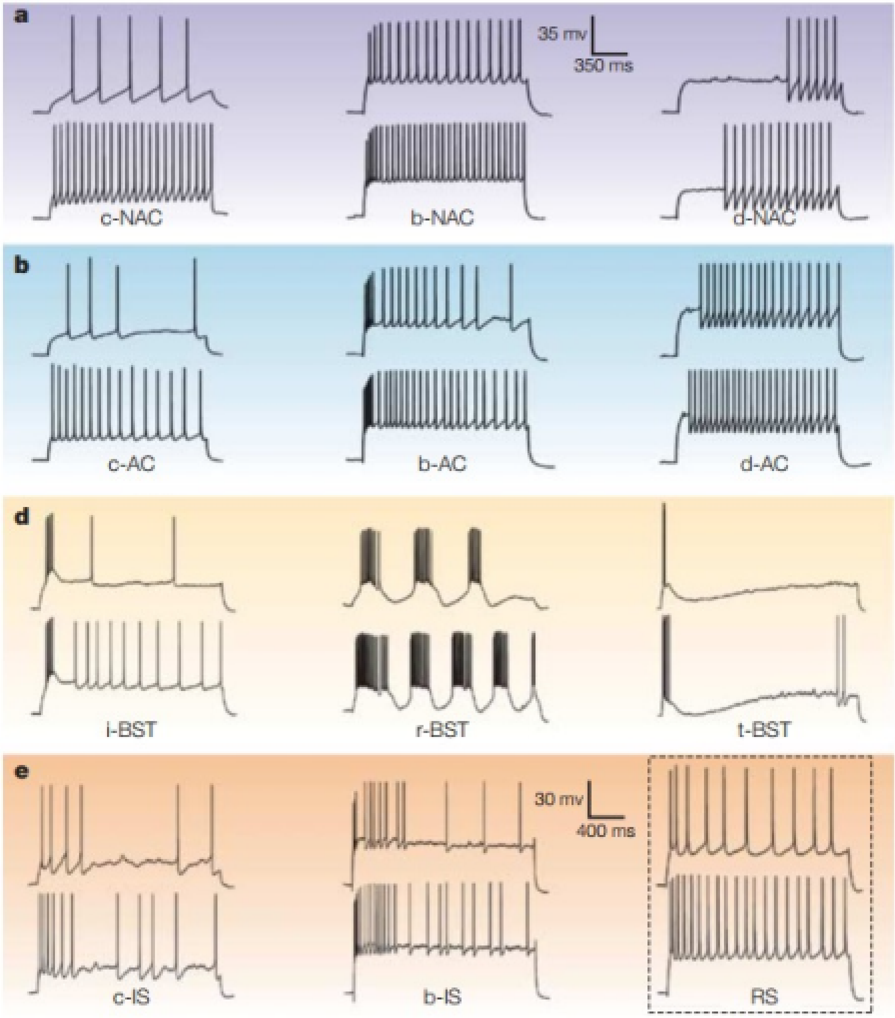
δ function

- 达到阈值之后w变为(w+b)
- firing patterns

  Firing patterns of the AdEx model:



- Categorization of firing patterns

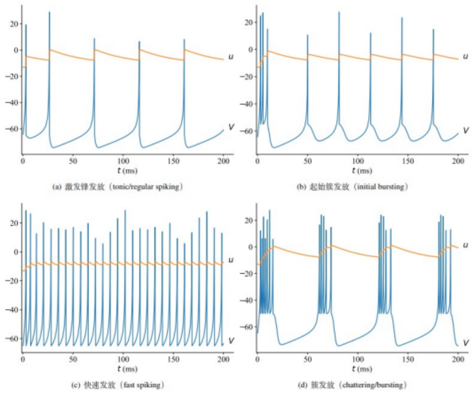


Markram H, et al. Nat Rev Neurosci. 2004

-
- Accoring to the steady-state firing time intervals
    - Tonic/regular spiking
    - Adapting
    - Bursting
    - Irregular spiking
- According to the intial-state features:
    - Tonic/classic spiking
    - initial burst
    - Delayed spiking
- Other multivarite neuron models
    - 从二次整合发放演化来的Izhikevich model

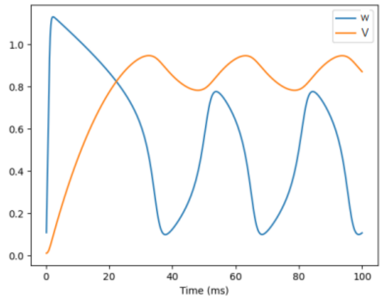
- The Izhikevich model:

$$
\begin{aligned}
\frac{\mathrm{d}V}{\mathrm{d}t} &= 0.04V^2 + 5V + 140 - u + I \\
\frac{\mathrm{d}u}{\mathrm{d}t} &= a\,(bV - u) \\
\text{if } V > \theta, &\quad V \leftarrow c,\ u \leftarrow u + d \ \text{ last } t_{\text{ref}}
\end{aligned}
$$



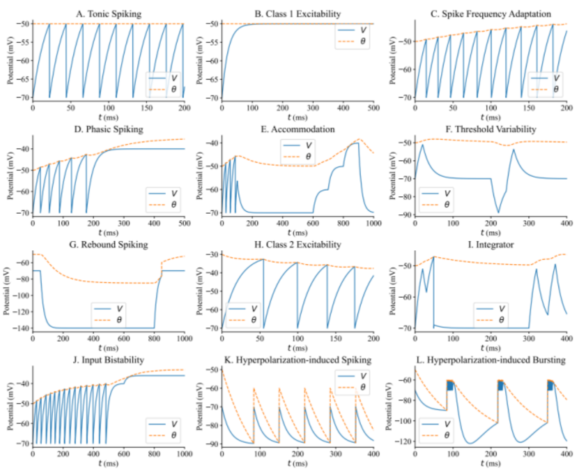
- 没有人为的重置(保持连续性)
    - The FitzHugh–Nagumo (FHN) model

$$
\begin{aligned}
\dot{v} &= v - \frac{v^3}{3} - w + RI_{\text{ext}} \\
\tau\dot{w} &= v + a - bw.
\end{aligned}
$$



- （n+2）个变量
    - The Generalized Integrate-and-Fire (GIF) model:

$$
\begin{aligned}
\tau\frac{\mathrm{d}V}{\mathrm{d}t} &= -(V - V_{\text{rest}}) + R\sum_j I_j + RI \\
\frac{\mathrm{d}\Theta}{\mathrm{d}t} &= a\,(V - V_{\text{rest}}) - b\,(\Theta - \Theta_{\infty}) \\
\frac{\mathrm{d}I_j}{\mathrm{d}t} &= -k_j I_j, \quad j = 1, 2, \ldots, n \\
\text{if } V > \Theta, &\quad I_j \leftarrow R_j I_j + A_j,\ V \leftarrow V_{\text{reset}},\ \Theta \leftarrow max\,(\Theta_{\text{reset}}, \Theta)
\end{aligned}
$$



- 对历史有记忆就是前段时间的状态对后面时间有影响

Dynamic analysis: phase-plane analysis

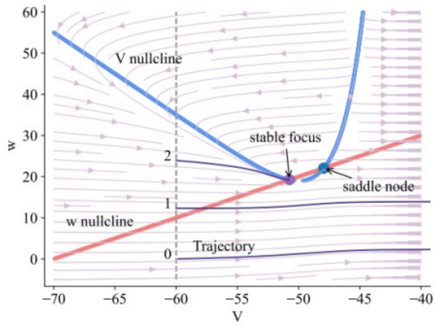
- Phase plane analysis
    - 对两个变量做相平面分析，分析动力学系统为什么会发生这样的行为(假设外部电流恒定，v和w都能确定v和w的梯度。零线一定是和其他线水平或垂直，一侧的方向一定是一致的)
    - 给初始值后，v和w会随着t变化，就可以画出轨迹(达到vreset即重置)

Analyzes the behavior of a dynamical system with (usually two) variables described by ordinary differential equations

$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$
$$\tau_w \frac{dw}{dt} = a(V - V_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{reset} \ \text{ last } \ t_{ref}$$

Elements:

- Nullclines: $dV/dt = 0;\ dw/dt = 0$
- Fixed points: $dV/dt = 0$ and $dw/dt = 0$
- The vector field
- The trajectory of variables



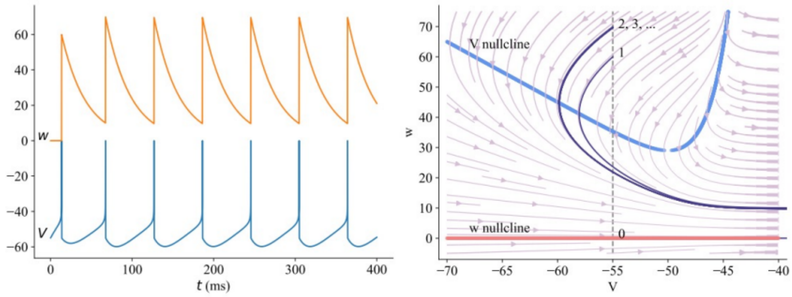
- Tonic spiking

1. Tonic spiking

$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$
$$\tau_w \frac{dw}{dt} = a(V - V_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{reset} \ \text{ last } \ t_{ref}$$

表 3.1: AdEx 模型各种发放形式对应的参数

发放形式	τ	τ_w	a	b	V_{reset}	I
激发锋发放	20	30	0	60	-55	65
适应	20	100	0	5	-55	65
起始簇发放	5	100	0.5	7	-51	65
簇发放	5	100	-0.5	7	-47	65
瞬时锋发放	10	100	1	10	-60	55
延迟发放	5	100	-1	5	-60	25



- Adaptation

2. Adaptation

$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$
$$\tau_w \frac{dw}{dt} = a(V - V_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$
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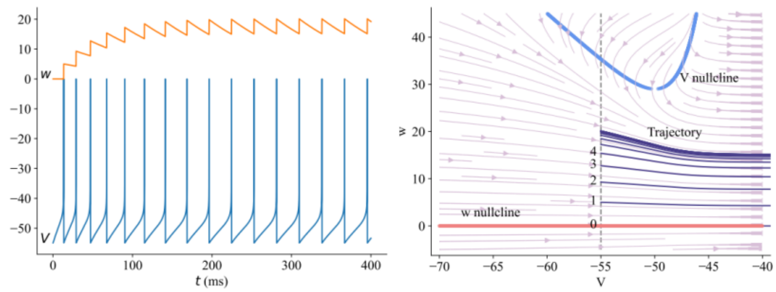

- Bursting

3. Bursting

$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V-V_T}{\Delta_T}} - Rw + RI(t)$$

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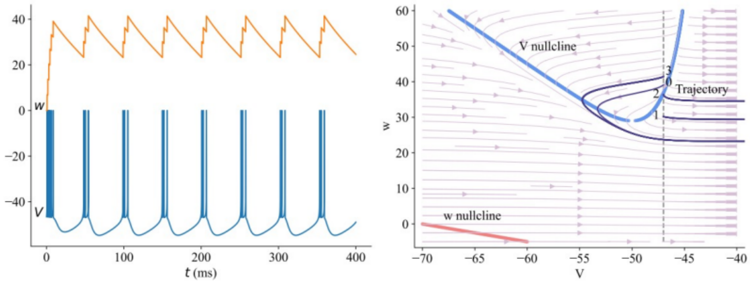

(a) 发放模式　　(b) 相平面分析

- Transient spiking(瞬态发放)

4. Transient spiking

$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V-V_T}{\Delta_T}} - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a(V - V_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$

$$\text{if } V > \theta, \quad V \leftarrow V_{reset} \text{ last } t_{ref}$$

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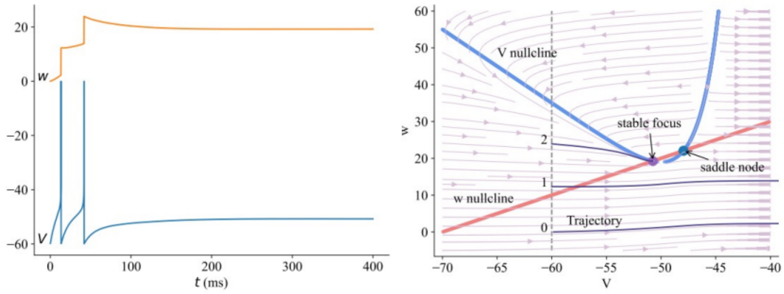


Dynamic analysis: bifurcation analysis

- Bifurcation analysis
  - 以一个参数为横轴，变化变量求得新的相图的固定点和鞍点、稳定焦点和不稳定焦点

Quantitative analysis of the existence and the properties of fixed points in a dynamical system with a changing parameter

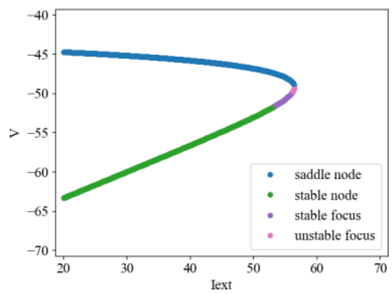
$$\tau_m \frac{dV}{dt} = -(V - V_{rest}) + \Delta_T e^{\frac{V-V_T}{\Delta_T}} - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a(V - V_{rest}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$

$$\text{if } V > \theta, \quad V \leftarrow V_{reset} \text{ last } t_{ref}$$

Elements:

- Lines of fixed points
- Stability properties of fixed points

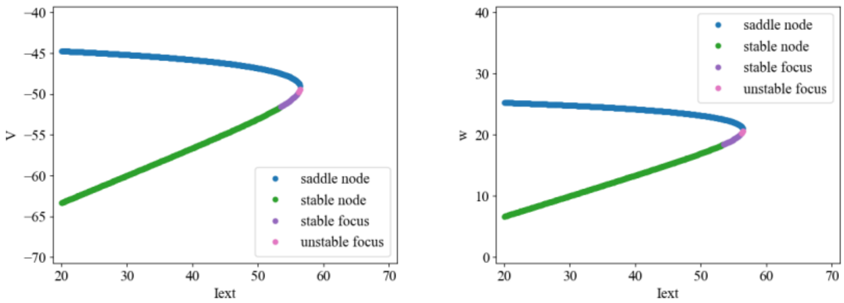


bifurcation analysis for 2 variables

Variables: V and w

Parameters: I_{ext}

$$\tau_m \frac{dV}{dt} = -(V - V_{\text{rest}}) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$
$$\tau_w \frac{dw}{dt} = a(V - V_{\text{rest}}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \text{ last } t_{\text{ref}}$$



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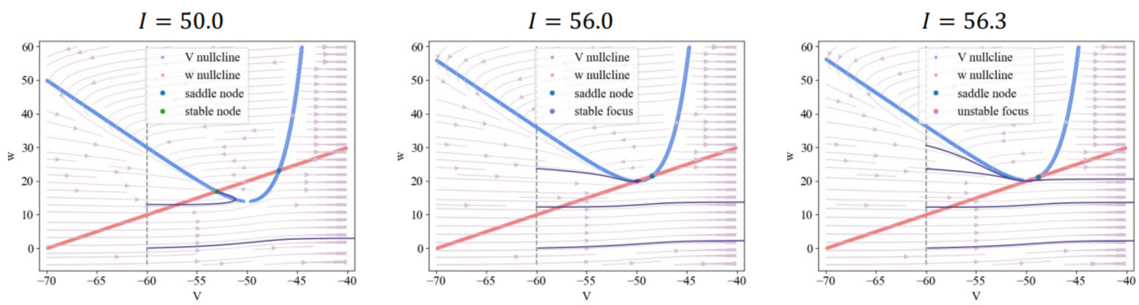
- 需要将两张图一起看，(以横轴坐标为统一标准)
- 观察stable focus和 unstable focus的区别（会产生突变）鞍点一侧吸引一侧排斥，unstable focus 两侧都是排斥

Subjects: two variables (V and w)

$$\tau_m \frac{dV}{dt} = -(V - V_{\text{rest}}) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$
$$\tau_w \frac{dw}{dt} = a(V - V_{\text{rest}}) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$
$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \text{ last } t_{\text{ref}}$$



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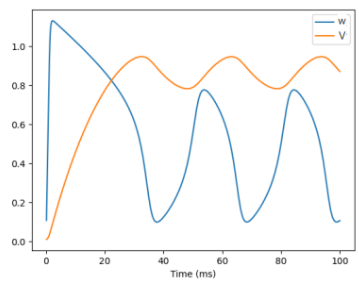
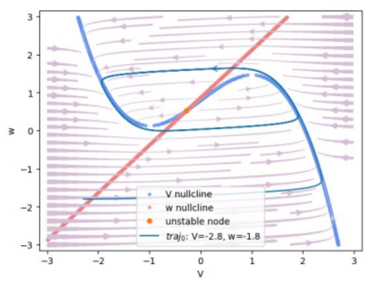
- Extended limit cycle(存在不稳定节点，相平面锁定曲线)

The FitzHugh–Nagumo (FHN) model

$$\dot{v} = v - \frac{v^3}{3} - w + RI_{\text{ext}}$$
$$\tau\dot{w} = v + a - bw.$$



This dynamical system, in certain conditions, exhibits a cyclic pattern of variable changes which can be visualized as a closed trajectory in the phase plane.



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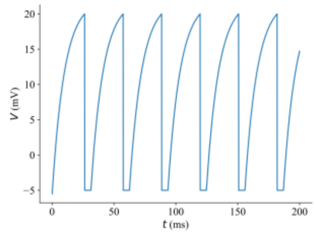
Summary

- Strengths &weaknesses of the LIF model

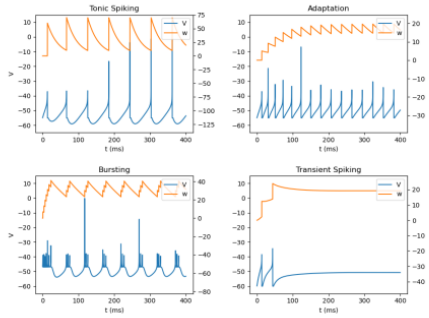
  - The Leaky Integrate-and-Fire (LIF) Neuron Model



$$\tau \frac{dV}{dt} = -(V - V_{\text{rest}}) + RI(t)$$

$$\text{if } V > V_{\text{th}}, \quad V \leftarrow V_{\text{reset}} \ \text{last} \ t_{\text{ref}}$$

  - The Adaptive Exponential Integrate-and-Fire (AdEx) Neuron Model



$$\tau_m \frac{dV}{dt} = -\left(V - V_{\text{rest}}\right) + \Delta_T e^{\frac{V - V_T}{\Delta_T}} - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\left(V - V_{\text{rest}}\right) - w + b\tau_w \sum_{t^{(f)}} \delta\left(t - t^{(f)}\right)$$

$$\text{if } V > \theta, \quad V \leftarrow V_{\text{reset}} \ \text{last} \ t_{\text{ref}}$$

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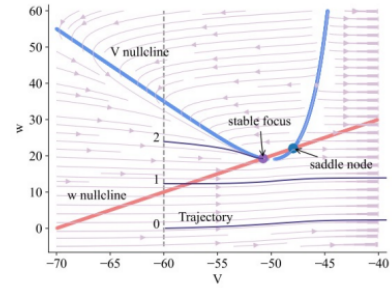
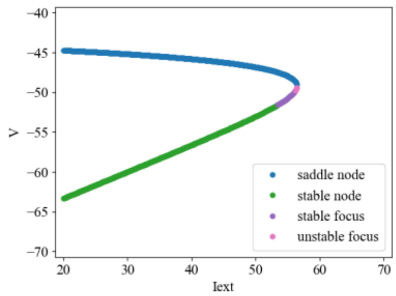
  - Dynamic analysis: phase-plane analysis



  - Dynamic analysis: bifurcation analysis



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