Chapter 1

1.1

Exercise 1A

1 **a** x^7 **b** $6x^5$ **c** $2p^2$ **d** $3x^{-2}$
  e k^5 **f** y^{10} **g** $5x^8$ **h** p^2
  i $2a^3$ **j** $2p^{-7}$ **k** $6a^{-9}$ **l** $3a^2b^{-2}$
  m $27x^8$ **n** $24x^{11}$ **o** $63a^{12}$ **p** $32y^6$
  q $4a^6$ **r** $6a^{12}$

2 **a** x^5 **b** x^{-2} **c** x^4
  d x^3 **e** x^5 **f** $12x^0 = 12$
  g $3x^{\frac{1}{2}}$ **h** $5x$ **i** $6\,x^{-1}$

3 **a** ± 5 **b** ± 9 **c** 3 **d** $\frac{1}{16}$
  e $\pm\frac{1}{3}$ **f** $\frac{1}{125}$ **g** 1 **h** ± 6
  i $\pm\frac{125}{64}$ **j** $\frac{9}{4}$ **k** $\frac{5}{6}$ **l** $\frac{64}{49}$

1.2

Exercise 1B

1 $2\sqrt{7}$ **2** $6\sqrt{2}$ **3** $5\sqrt{2}$
4 $4\sqrt{2}$ **5** $3\sqrt{10}$ **6** $\sqrt{3}$
7 $\sqrt{3}$ **8** $6\sqrt{5}$ **9** $7\sqrt{2}$
10 $12\sqrt{7}$ **11** $-3\sqrt{7}$ **12** $9\sqrt{5}$

13 $23\sqrt{5}$ **14** 2 **15** $19\sqrt{3}$
16 $\frac{\sqrt{5}}{5}$ **17** $\frac{\sqrt{11}}{11}$ **18** $\frac{\sqrt{2}}{2}$
19 $\frac{\sqrt{5}}{5}$ **20** $\frac{1}{2}$ **21** $\frac{1}{4}$
22 $\frac{\sqrt{13}}{13}$ **23** $\frac{1}{3}$

1.3

Exercise 1C

1 **a** $\log_4 256 = 4$ **b** $\log_3\left(\frac{1}{9}\right) = -2$
  c $\log_{10} 1\,000\,000 = 6$ **d** $\log_{11} 11 = 1$
2 **a** $2^4 = 16$ **b** $5^2 = 25$
  c $9^{\frac{1}{2}} = 3$ **d** $5^{-1} = 0.2$
  e $10^5 = 100\,000$
3 **a** 3 **b** 2 **c** 7 **d** 1
  e 6 **f** $\frac{1}{2}$ **g** -1 **h** 10

4 **a** 625 **b** 9 **c** 7 **d** 2
5 **a** 1.30 **b** 0.602 **c** 3.85 **d** -0.105
6 **a** 1.04 **b** 1.55 **c** -0.523 **d** 3.00

1.4

Exercise 1D

1 **a** $\log_2 21$ **b** $\log_2 9$ **c** $\log_5 80$
  d $\log_6\left(\frac{68}{81}\right)$ **e** $\log_{10} 120$
2 **a** $\log_2 8 = 3$ **b** $\log_6 36 = 2$
  c $\log_{12} 144 = 2$ **d** $\log_8 2 = \frac{1}{3}$
  e $\log_{10} 10 = 1$
3 **a** $3\log_a x + 4\log_a y + \log_a z$
  b $5\log_a x - 2\log_a y$
  c $2 + 2\log_a x$
  d $\log_a x + \frac{1}{2}\log_a y - \log_a z$
  e $\frac{1}{2} + \frac{1}{2}\log_a$

1.5

Exercise 1E

1 **a** 2.460 **b** 3.465 **c** 4.248
  d 0.458 **e** 0.774
2 **a** 1.27 **b** 2.09 **c** 0.721
  a $\frac{1}{2}, 512$ **e** $\frac{1}{16}, \frac{1}{4}$
3 **a** 6.23 **b** 2.10 **c** 0.431
  d 1.66
4 **a** $\frac{1}{2}, 512$ **b** $\frac{1}{16}, \frac{1}{4}$ **c** 2.52

1.6

Exercise 1F

1

NB $\left(\frac{1}{4}\right)^x = 4^{-x}$ so (c) is $y = 4^{-x}$
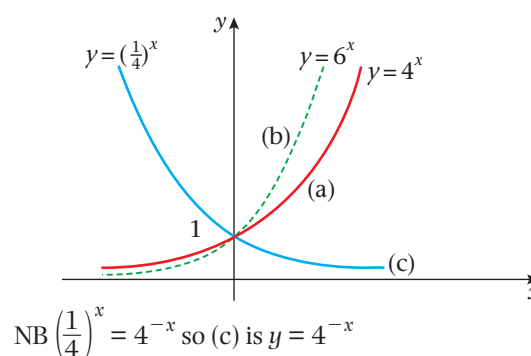
2
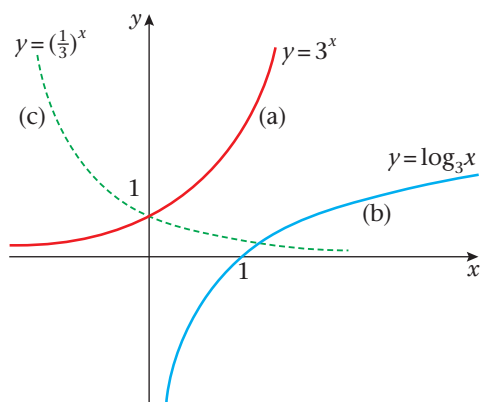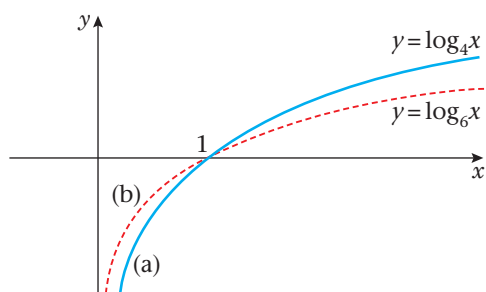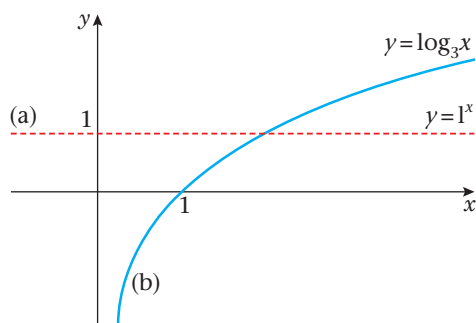

NB $y = \log_3 x$ is a reflection of $y = 3^x$ in the line $y = x$.

$y = \left(\frac{1}{3}\right)^x$ is $y = 3^{-x}$

3

4

  a $y = 1^x = 1$

  c $y = \log_3 x = 1$

       $\Rightarrow x = 3^1 = 3$

       So coordinates of intersection are $(3, 1)$

Exercise 1G

1 **a** y^8 **b** $6x^7$ **c** 32^x **d** $12b^9$

2 **a** $3x^6$ **b** ± 2 **c** $6x^2$ **d** $\frac{1}{2}x^{\frac{1}{3}}$

3 **a** $\frac{4}{9}$ **b** $\frac{3375}{4913}$

4 **a** $\frac{\sqrt{7}}{7}$ **b** $4\sqrt{5}$

5 **a** $\frac{\sqrt{3}}{3}$ **b** $\frac{15}{\sqrt{5}}$

6 **a** $2\log_d p + \log_d q$ **b** $\log_a p = 4, \log_d q = 1$

7 **a** $5^x = 80$

     So $x = \log_5 80$

$$\left(= \frac{\log_{10} 80}{\log_{10} 5} \right)$$

     i.e. $x = 2.72270\ldots$

            $= 2.72 \text{ 3sf}$

  b $7^x = 123$

     $\Rightarrow x = \log_7 123$

$$\left(= \frac{\log_{10} 123}{\log_{10} 7} \right)$$

     i.e. $x = 2.47297\ldots$

            $= 2.47 \text{ 3sf}$

8 **a** 9   **b** 12   **c** $\frac{1}{9}, 9$

9 $\frac{1}{3}, 9$

10 $-\frac{1}{3}, -2$

Chapter 2

2.1

Exercise 2A

1 $x(x + 4)$ **2** $2x(x + 3)$

3 $(x + 8)(x + 3)$ **4** $(x + 6)(x + 2)$

5 $(x + 8)(x - 5)$ **6** $(x - 8)(x - 2)$

7 $(x + 2)(x + 3)$ **8** $(x - 6)(x + 4)$

9 $(x - 5)(x + 2)$ **10** $(x + 5)(x - 4)$

11 $(2x + 1)(x + 2)$ **12** $(3x - 2)(x + 4)$

13 $(5x - 1)(x - 3)$ **14** $2(3x + 2)(x - 2)$

15 $(2x - 3)(x + 5)$ **16** $2(x^2 + 3)(x^2 + 4)$

17 $(x + 2)(x - 2)$ **18** $(x + 7)(x - 7)$

19 $(2x + 5)(2x - 5)$ **20** $(3x + 5y)(3x - 5y)$

21 $4(3x + 1)(3x - 1)$ **22** $2(x + 5)(x - 5)$

23 $2(3x - 2)(x - 1)$ **24** $3(5x - 1)(x + 3)$

2.2

Exercise 2B

1 $(x + 2)^2 - 4$ **2** $(x - 3)^2 - 9$

3 $(x - 8)^2 - 64$ **4** $(x + \frac{1}{2})^2 - \frac{1}{4}$

5 $(x - 7)^2 - 49$ **6** $2(x + 4)^2 - 32$

7 $3(x - 4)^2 - 48$ **8** $2(x - 1)^2 - 2$

9 $5(x + 2)^2 - 20$ **10** $2(x - \frac{5}{4})^2 - \frac{25}{8}$

11 $3(x + \frac{3}{2})^2 - \frac{27}{4}$ **12** $3(x - \frac{1}{6})^2 - \frac{1}{12}$

2.3

Exercise 2C

1 $x = 0$ or $x = 4$ **2** $x = 0$ or $x = 25$

3 $x = 0$ or $x = 2$ **4** $x = 0$ or $x = 6$

5 $x = -1$ or $x = -2$ **6** $x = -1$ or $x = -4$

7 $x = -5$ or $x = -2$ **8** $x = 3$ or $x = -2$

9 $x = 3$ or $x = 5$ **10** $x = 4$ or $x = 5$

11 $x = 6$ or $x = -1$ **12** $x = 6$ or $x = -2$

13 $x = -\frac{1}{2}$ or $x = -3$ **14** $x = -\frac{1}{3}$ or $x = \frac{3}{2}$

15 $x = -\frac{2}{3}$ or $x = \frac{3}{2}$ **16** $x = \frac{3}{2}$ or $x = \frac{5}{2}$

17 $x = \frac{1}{3}$ or $x = -2$ **18** $x = 3$ or $x = 0$

19 $x = 13$ or $x = 1$ **20** $x = 2$ or $x = -2$

21 $x = \pm\frac{\sqrt{5}}{3}$ **22** $x = 3 \pm \sqrt{13}$

23 $x = 1 \pm \frac{\sqrt{11}}{3}$ **24** $x = 1$ or $x = -\frac{7}{6}$

25 $x = -\frac{1}{2}$ or $x = \frac{7}{3}$ **26** $x = 0$ or $x = -\frac{11}{62}$

27 $x = -3 \pm 2\sqrt{2}$ **28** $x = -6 \pm \sqrt{33}$

29 $x = 5 \pm \sqrt{30}$ **30** $x = -2 \pm \sqrt{6}$

31 $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$ **32** $x = 1 \pm \frac{3}{2}\sqrt{2}$

33 $x = \frac{1}{8} \pm \frac{\sqrt{129}}{8}$ **34** No real roots

35 $x = -\frac{3}{2} \pm \frac{\sqrt{39}}{2}$ **36** $x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$

2.4

Exercise 2D

1 $b^2 - 4ac = (-2)^2 - 4 \times 1 = 0$ so equal roots
$x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0$ $\therefore \underline{x = 1}$

2 $b^2 - 4ac = (-2)^2 - 4(-1) = 8$ so two real roots
$x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$ or $\underline{2.41, -0.414}$ 3sf

3 $b^2 - 4ac = (-3)^2 - 4(-2) = 17$ so two real roots
$x = \frac{3 \pm \sqrt{17}}{2}$ $= \underline{3.56 \text{ or } -0.562}$ 3sf

4 $b^2 - 4ac = (-3)^2 - 4 \times 4 = 9 - 16 = -7$ so no real roots

5 $b^2 - 4ac = (1)^2 - 4 \times 2 \times (-2) = 17$ so two real roots
$x = \frac{-1 \pm \sqrt{17}}{4}$ $= \underline{0.781, -1.28}$ 3sf

6 $b^2 - 4ac = (-1)^2 - 4 \times 3 \times 3 = -35$ so no real roots

7
$$3x^2 = 7 - x$$
$$\Rightarrow 3x^2 + x - 7 = 0$$
$$b^2 - 4ac = 1^2 - 4 \times 3 \times (-7) = 1 + 84 = 85$$
so two real roots
$$x = \frac{-1 \pm \sqrt{85}}{6}$$
$$x = \underline{1.37 \text{ or } -1.70} \text{ 3sf}$$

8
$$2x^2 = x + 4$$
$$\Rightarrow 2x^2 - x - 4 = 0$$
$$b^2 - 4ac = (-1)^2 - 4 \times (2) \times (-4) = 33$$
so two real roots
$$x = \frac{1 \pm \sqrt{33}}{4}$$
$$x = \underline{1.69 \text{ or } -1.19} \text{ 3sf}$$

9 $-3 \pm \frac{\sqrt{5}}{2}, -0.38$ or -2.62

10 $\frac{+3 \pm \sqrt{17}}{2}, -0.56$ or 3.56

11 $-3 \pm \sqrt{3}, -1.27$ or -4.73

12 $\frac{5 \pm \sqrt{33}}{2}, 5.37$ or -0.37

13 $5 \pm \frac{\sqrt{31}}{3}, -3.52$ or 0.19

14 $\frac{1 \pm \sqrt{2}}{2}, 1.21$ or -0.21

15 $9 \pm \frac{\sqrt{53}}{14}, -0.12$ or -1.16

16 $\frac{-2 \pm \sqrt{19}}{5}, 0.47$ or -1.27

17 2 or $-\frac{1}{4}$

18 $\frac{-1 \pm \sqrt{78}}{11}, 0.71$ or -0.89

2.5

Exercise 2E

1
$$x^2 + 5x + 2 = 0$$
$$\alpha + \beta = -5$$
$$\alpha\beta = 2$$

a $2\alpha + 1 + 2\beta + 1 = 2(\alpha + \beta) + 2 = -10 + 2 = -8$
$(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1$
$= 8 - 10 + 1 = -1$
$\therefore$ new equation is $x^2 + 8x - 1 = 0$

b $\alpha\beta + \alpha^2\beta^2 = \alpha\beta(1 + \alpha\beta) = 2(1 + 2) = 6$
$(\alpha\beta)(\alpha^2\beta^2) = (\alpha\beta)^3 = 2^3 = 8$
$\therefore$ new equation is $x^2 - 6x + 8 = 0$

2 $\qquad x^2 + 6x + 1 = 0$
$\qquad\qquad \alpha + \beta = -6$
$\qquad\qquad \alpha\beta = 1$

a $\quad (\alpha + 3) + (\beta + 3) = (\alpha + \beta) + 6 = 0$
$\qquad (\alpha + 3)(\beta + 3) = \alpha\beta + 3(\alpha + \beta) + 9$
$\qquad\qquad = 1 - 18 + 9 = -8$
$\qquad \therefore$ new equation is $x^2 - 8 = 0$

b $\qquad\qquad \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$
$\qquad\qquad\qquad = \dfrac{36 - 2}{1} = 34$
$\qquad\qquad \dfrac{\alpha}{\beta} \times \dfrac{\beta}{\alpha} = 1$
$\qquad \therefore$ new equation is $x^2 - 34x + 1 = 0$

3 $\qquad\qquad x^2 - x + 3 = 0$
$\qquad\qquad \alpha + \beta = 1$
$\qquad\qquad \alpha\beta = 3$

a $\quad (\alpha + 2) + (\beta + 2) = (\alpha + \beta) + 4 = 1 + 4 = 5$
$\qquad (\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$
$\qquad\qquad = 3 + 2 + 4 = 9$
$\qquad \therefore$ new equation is $x^2 - 5x + 9 = 0$

b $\qquad\qquad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$\qquad\qquad\qquad = 1^2 - 6 = -5$
$\qquad\qquad \alpha^2 \times \beta^2 = (\alpha\beta)^2 = 3^2 = 9$
$\qquad \therefore$ new equation is $x^2 + 5x + 9 = 0$

4 $\qquad\qquad x^2 + x - 1 = 0$
$\qquad\qquad \alpha + \beta = -1$
$\qquad\qquad \alpha\beta = -1$

a $\qquad\qquad \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta + \alpha}{\alpha\beta} = \dfrac{-1}{-1} = 1$
$\qquad\qquad \dfrac{1}{\alpha} \times \dfrac{1}{\beta} = \dfrac{1}{\alpha\beta} = \dfrac{1}{-1} = -1$
$\qquad \therefore$ new equation is $x^2 - x - 1 = 0$

b $\qquad\qquad \dfrac{\alpha}{\alpha\beta} + \dfrac{\beta}{\alpha + \beta} = \dfrac{\alpha + \beta}{\alpha + \beta} = 1$
$\qquad\qquad \dfrac{\alpha}{\alpha + \beta} \times \dfrac{\beta}{\alpha + \beta} = \dfrac{\alpha\beta}{(\alpha + \beta)^2} = \dfrac{-1}{1} = -1$
$\qquad \therefore$ new equation is also $x^2 - x - 1 = 0$

Mixed Exercise 2F

1 **a** $x(3x + 4)$ $\qquad$ **b** $2y(2y + 5)$
$\quad$ **c** $x(x + y + y^2)$ $\qquad$ **d** $2xy(4y + 5x)$
2 **a** $(x + 1)(x + 2)$ $\qquad$ **b** $3x(x + 2)$
$\quad$ **c** $(x - 7)(x + 5)$ $\qquad$ **d** $(2x - 3)(x + 1)$
$\quad$ **e** $(5x + 2)(x - 3)$ $\qquad$ **f** $(1 - x)(6 + x)$
3 **a** $y = -1$ or -2 $\qquad$ **b** $x = \frac{2}{3}$ or -5

$\quad$ **c** $x = -\frac{1}{5}$ or 3 $\qquad$ **d** $5 \pm \dfrac{\sqrt{7}}{2}$

4 **a** $\dfrac{-5 \pm \sqrt{17}}{2}$, -0.44 or -4.56

$\quad$ **b** $2 \pm \sqrt{7}$, 4.65 or -0.65

$\quad$ **c** $\dfrac{-3 \pm \sqrt{29}}{10}$, 0.24 or -0.84

$\quad$ **d** $\dfrac{5 \pm \sqrt{73}}{6}$, 2.25 or -0.59

5 **a** $p = 3, q = 2, r = -7$ $\qquad$ **b** $-2 \pm \dfrac{\sqrt{7}}{3}$

6 ± 4

7 **a** $\alpha\beta = t$, $\alpha^2 + \beta^2 = 2t(2t - 1)$

$\quad$ **b** $t = 1 + \dfrac{\sqrt{577}}{2}$

$\quad$ **c** $x^2 - 2\sqrt{577}x + 1 = 0$

8 $\qquad\qquad 2x^2 - 7x + 3 = 0$
$\qquad\qquad x^2 - \tfrac{7}{2}x + \tfrac{3}{2} = 0$
$\qquad\qquad \alpha + \beta = \tfrac{7}{2}$
$\qquad\qquad \alpha\beta = \tfrac{3}{2}$

a $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \tfrac{49}{4} - 2 \times \tfrac{3}{2} = \tfrac{37}{4}$

b $\alpha - \beta = \sqrt{(\alpha - \beta)^2} = \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta}$
$\qquad\qquad = \sqrt{\tfrac{37}{4} - 2 \cdot \tfrac{3}{2}} = \sqrt{\tfrac{25}{4}} = \tfrac{5}{2}$

c $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)$
$\qquad\qquad = \tfrac{5}{2}\left(\tfrac{37}{4} + \tfrac{3}{2}\right) = \tfrac{5}{2} \times \tfrac{43}{4} = \tfrac{215}{8}$

9 $\qquad\qquad x^2 - 2tx + t = 0$

a $\qquad\qquad \alpha + \beta = 2t$
$\qquad\qquad \alpha\beta = t$
$\qquad\qquad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4t^2 - 2t$
$\qquad\qquad\qquad = 2t(2t - 1)$

b $\qquad\qquad \alpha + \beta = 2t$
$\qquad\qquad \alpha - \beta = 24$
$\qquad\qquad \Rightarrow 2\alpha = 2t + 24$
$\qquad\qquad\qquad \alpha = t + 12, \beta = t - 12$
$\qquad \therefore \alpha\beta = t \Rightarrow (t + 12)(t - 12) = t$
$\qquad$ i.e. $t^2 - 144 = t$ or $0 = t^2 - t - 144$
$\qquad \therefore t = \dfrac{1 \pm \sqrt{1 + 576}}{2} \therefore t = \dfrac{1 + \sqrt{577}}{2}$ $(t > 0)$

c $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{2t(2t - 1)}{t} = 2(2t - 1)$
$\qquad \dfrac{\alpha}{\beta} \times \dfrac{\beta}{\alpha} = 1$
$\qquad \therefore$ equation is $x^2 - 2(2t - 1)x + 1 = 0$
$\qquad\qquad$ or $x^2 - 2\sqrt{577}\,x + 1 = 0$

Chapter 3

3.1
Exercise 3A

1   **a** $x^2 + 5x + 3$ **b** $x^2 + x - 9$
    c $x^2 - 3x + 7$ **d** $x^2 - 3x + 2$
    e $x^2 - 3x - 2$

2   **a** $6x^2 + 3x + 2$ **b** $3x^2 + 2x - 2$
    c $2x^2 - 2x - 7$ **d** $-3x^2 + 5x - 7$
    e $-5x^2 + 3x + 5$

3   **a** $2x^3 + 5x^2 - 5x + 1 = (2x - 1)(x^2 + 3x - 1)$

$$\therefore \frac{2x^3 + 5x^2 - 5x + 1}{2x - 1} = x^2 + 3x - 1$$

    b $3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1)$

$$\therefore \frac{3x^3 + 2x^2 - 3x - 2}{(3x + 2)} = x^2 - 1$$

    c $6x^3 + x^2 - 7x + 2 = (3x - 1)(2x^2 + x - 2)$

$$\therefore \frac{6x^3 + x^2 - 7x + 2}{3x - 1} = 2x^2 + x - 2$$

    d $4x^3 + 4x^2 + 5x + 12 = (2x + 3)(2x^2 - x + 4)$

$$\therefore \frac{4x^3 + 4x^2 + 5x + 12}{2x + 3} = 2x^2 - x + 4$$

    e $2x^3 + 7x^2 + 7x + 2 = (2x + 1)(x^2 + 3x + 2)$

$$\therefore \frac{2x^3 + 7x^2 + 7x + 2}{2x - 1} = x^2 + 3x + 2$$

3.2 Exercise 3B

1 $(x - 1)(x + 3)(x + 4)$
2 $(x + 1)(x + 7)(x - 5)$
3 $(x - 5)(x - 4)(x + 2)$
4 $(x - 2)(2x - 1)(x + 4)$
5   **a** $(x + 1)(x - 5)(x - 6)$
    b $(x - 2)(x + 1)(x + 2)$
    c $(x - 5)(x + 3)(x - 2)$
6   **a** $(x - 1)(x + 3)(2x + 1)$
    b $(x - 3)(x - 5)(2x - 1)$
    c $(x + 1)(x + 2)(3x - 1)$
    d $(x + 2)(2x - 1)(3x + 1)$
    e $(x - 2)(2x - 5)(2x + 3)$
7   2
8 -16
9 $p = 3, q = 7$

3.3
Exercise 3C

1   **a**   27   **b** -6 **c**   0   **d**   1
2   27
3   18
4   30
6 -9
7 $8\frac{8}{27}$
8 $a = 5, b = -8$
9 $p = 8, q = 3$

3.4
Exercise 3D

1   **a** $x = 5, y = 6$ or $x = 6, y = 5$
    b $x = 0, y = 1$ or $x = \frac{4}{5}, y = \frac{3}{5}$
    c $x = -1, y = -3$ or $x = 1, y = 3$
    d $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ or $x = 6, y = 3$
    e $a = 1, b = 5$ or $a = 3, b = -1$
    f $u = 1\frac{1}{2}, v = 4$ or $u = 2, v = 3$
2 $(-11, -15)$ and $(3, -1)$
3 $(-1\frac{1}{6}, -4\frac{1}{2})$ and $(2, 5)$
4   **a** $x = -1\frac{1}{2}, y = 5\frac{3}{4}$ or $x = 3, y = -1$
    b $x = 3, y = \frac{1}{2}$ or $x = 6\frac{1}{3}, y = -2\frac{5}{6}$
5   **a** $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ or $x = 3 - \sqrt{13},$
      $y = -3 - \sqrt{13}$
    b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ or $x = 2 + 3\sqrt{5},$
      $y = 3 - 2\sqrt{5}$

3.5
Exercise 3E

1   **a** $x < 4$ **b** $x \geqslant 7$ **c** $x > 2\frac{1}{2}$
    d $x \leqslant -3$ **e** $x < 11$ **f** $x < 2\frac{3}{5}$
    g $x > -12$ **h** $x < 1$ **i** $x \leqslant ??$
    j   8   **k** $x > 1\frac{1}{7}$
2   **a** $x \geqslant 3$ **b** $x < 1$ **c** $x \leqslant -3\frac{1}{4}$
    d $x < 18$ **e** $x > 3$ **f** $x \geqslant 4\frac{2}{5}$
    g $x < 4$ **h** $x > -7$ **i** $x \leqslant -\frac{1}{2}$
    j $x \geqslant \frac{3}{4}$
3   **a** $x > 2\frac{1}{2}$ **b** $2 < x < 4$ **c** $2\frac{1}{2} < x < 3$
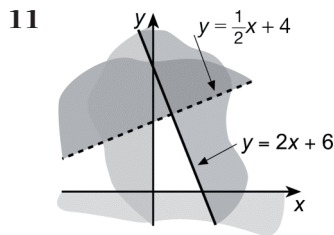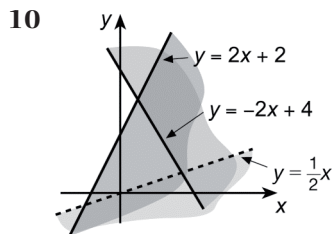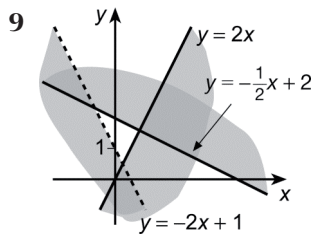   **d**   No values   **e** $x = 4$

3.6
Exercise 3F

1 **a** $3 < x < 8$ **b** $-4 < x < 3$
    c $x < -2, x > 5$ **d** $x \leq -4, x \geq -3$
    e $-\frac{1}{2} < x < 7$ **f** $x < -2, x > 2\frac{1}{2}$
    g $\frac{1}{2} < x < 1\frac{1}{2}$ **h** $x < \frac{1}{3}, x > 2$
    i $-3 < x < 3$ **j** $x < -2\frac{1}{2}, x > \frac{2}{3}$
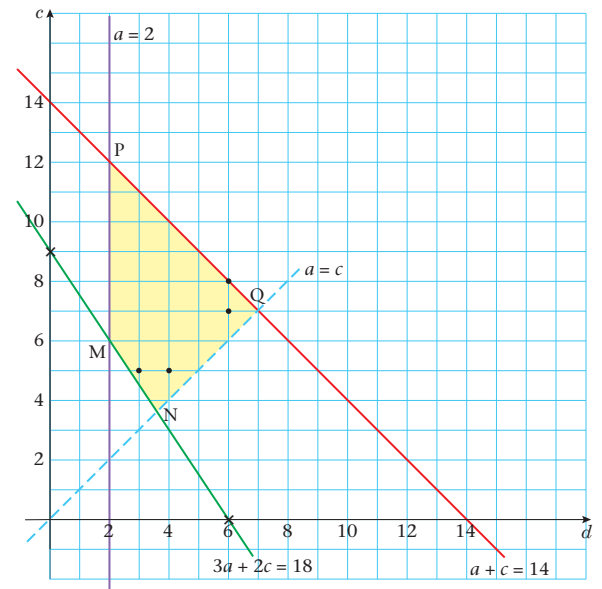   **k** $x < 0, x > 5$ **l** $-1\frac{1}{2} < x < 0$
2 **a** $-5 < x < 2$ **b** $x < -1, x > 1$
    c $\frac{1}{2} < x < 1$ **d** $-3 < x < \frac{1}{4}$

3.7
Exercise 3G

1 $-3 \leq x < 4$
2 $y < 2$ or $y \geq 5$
3 $2y + x \geq 10$ or $2y + x \leq 4$
4 $-2 \leq 2x - y \leq 2$
5 $4x + 3y \leq 12, y \geq 0$ and $y < 2x + 4$
6 $y \geq \leq \frac{3x}{4} - 3, y \leq 0$ and $y \geq -\frac{3x}{2} - 3$
7 $x \geq 0, y \geq 0, y < -\frac{3x}{2+9}$ and $y \leq -\frac{2x}{3+6}$
8 $y \geq 0, y \leq x + 2, y \leq 2x - 2$ and $y \leq 18 - 2x$

9

10

11

12
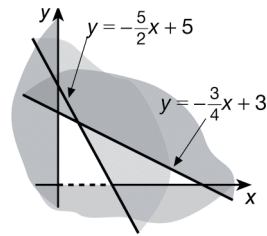
13 a Let a = no. of adults, and c = no. of children.
        $a + c \leq 14$ (no more than 14 passengers)
        $12a + 8c \geq 72$ (money raised must cover cost of £72)
        $c > a$ (more children than adults)
        $a \geq 2$ (at least 2 adults)

NB $12a + 8c \geq 72$ requires line $3a + 2c = 18$

b To find smallest sized group you need to consider points close to M and N
    M(2, 6) is 2 adults and 6 children
    Points close to N are (3, 5) and (4, 5)
    So the smallest sized group is 8: 2 adults and 6 children or 3 adults and 5 children.

c To find the maximum amount of money that can be made you need to consider points close to P and Q
    P(2, 12) raises $2 \times 12 + 12 \times 8 =$ £120
    Q(7, 7) is not in the region ($\because c > a$) but (6, 8) is on d
    (6, 8) raises $6 \times 12 + 8 \times 8 =$ £136
    So the maximum amount available for refreshments is £64 from taking 6 adults and 8 children

14

Let $\quad a$ = no. of machine A

$\qquad b$ = no. of machine B

$\quad 4a + 5b \qquad \leqslant 100 \qquad$ (floor area is 100m^2)

$\quad 2a + 3b \qquad \leqslant 40 \qquad$ (no. of operators is 40)

Profit would be P = $100a + 140b$

(this is parallel to $10a + 14b$)

The maximum profit wil run along RQ

Nearest to Point R (0, 13) will raise £1820

$\qquad\qquad$ Point (1, 12) will raise £1780

$\qquad\qquad$ Point Q (20, 0) will raise £2000

So maximum profit is from 20 Machine A

15 $\qquad$ Let $\qquad x$ = no. of ornament A

$\qquad\qquad\qquad y$ = no. of ornament B

$\qquad\qquad 3x + 2y \qquad \leqslant 48 \qquad$ (machine time is 48 h)

$\qquad\qquad 1.5x + 2.5y \leqslant 45 \qquad$ (craftsman's time is 45 h)

Profit P = $12x + 15y \quad$ (e.g. $12x + 15y = 120$ is drawn)

To find maximum profit drag the profit line towards the edges R, S, T. It will first cross at T, then R and finally S.

T (16, 0) gives a profit of $\qquad$ £192

R (0, 18) gives a profit of $\qquad$ £270

S is not a point giving whole numbers for x and y but the nearby points are (6, 14) and (7, 13)

(6, 14) gives a profit of $6 \times 12 + 14 \times 15 =$ £282

(7, 13) gives a profit of $7 \times 12 + 13 \times 15 =$ £279

So maxmimum profit is £282 from making 6 ornament A and 14 ornament B.

Mixed Exercise 3H

1 $x = 4, y = 3\frac{1}{2}$

2 (3, 1) and $(-2\frac{1}{5}, -1\frac{3}{5})$

3 $x = -1\frac{1}{2}, y - 2\frac{1}{4}$ and $x = 4, y = -\frac{1}{2}$

4 a $\quad x > 10\frac{1}{2}$ $\qquad\qquad$ **b** $\quad x < -2, x > 7$

5 $3 < x < 4$

6 a $\quad x < 2\frac{1}{2}$

$\quad$ **b** $\quad \frac{1}{2} < x < 5$

$\quad$ **c** $\quad \frac{1}{2} < x < 2\frac{1}{2}$

7 $x < 0, x > 1$

8 $A = 2, B = 4, C = -5$

9 $p = 1, q = 3$

10 $(x - 2)(x + 4)(2x - 1)$

11 7

12 $7\frac{1}{4}$

13 a $\quad p = 1, q = -15$ $\qquad$ **b** $\quad (x + 3)(2x - 5)$

14 a $\quad r = 3, s = 0$ $\qquad\qquad$ **b** $\quad 1\frac{13}{27}$

15 a $\quad (x - 1)(x + 5)(2x + 1)$ $\quad$ **b** $\quad -5, -\frac{1}{2}, 1$

16 -2

17 -18

18 $2, -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

19 $\frac{1}{2}, 3$

20 A: $y > 2x + 2, 4y + 3x + 12 > 0$
B: $y < 2x + 2, 4y + 3x + 12 > 0$
C: $y < 2x + 2, 4y + 3x + 12 < 0$
D: $y > 2x + 2, 4y + 3x + 12 < 0$

21 $\{y \geqslant 3 \text{ and } 2x + y \leqslant 6\}$

22

Chapter 4

4.1

Exercise 4A

1 a

b

c

d

e

f

g

h

i

j

2 a

b

c

d

e

f

g

h

i

j

3 a $y = x(x + 2)(x - 1)$

b $y = x(x + 4)(x + 1)$
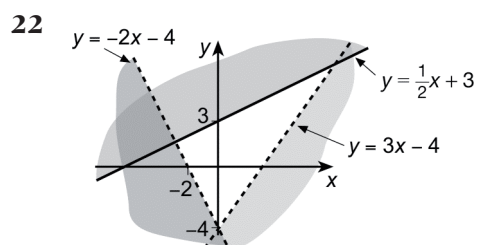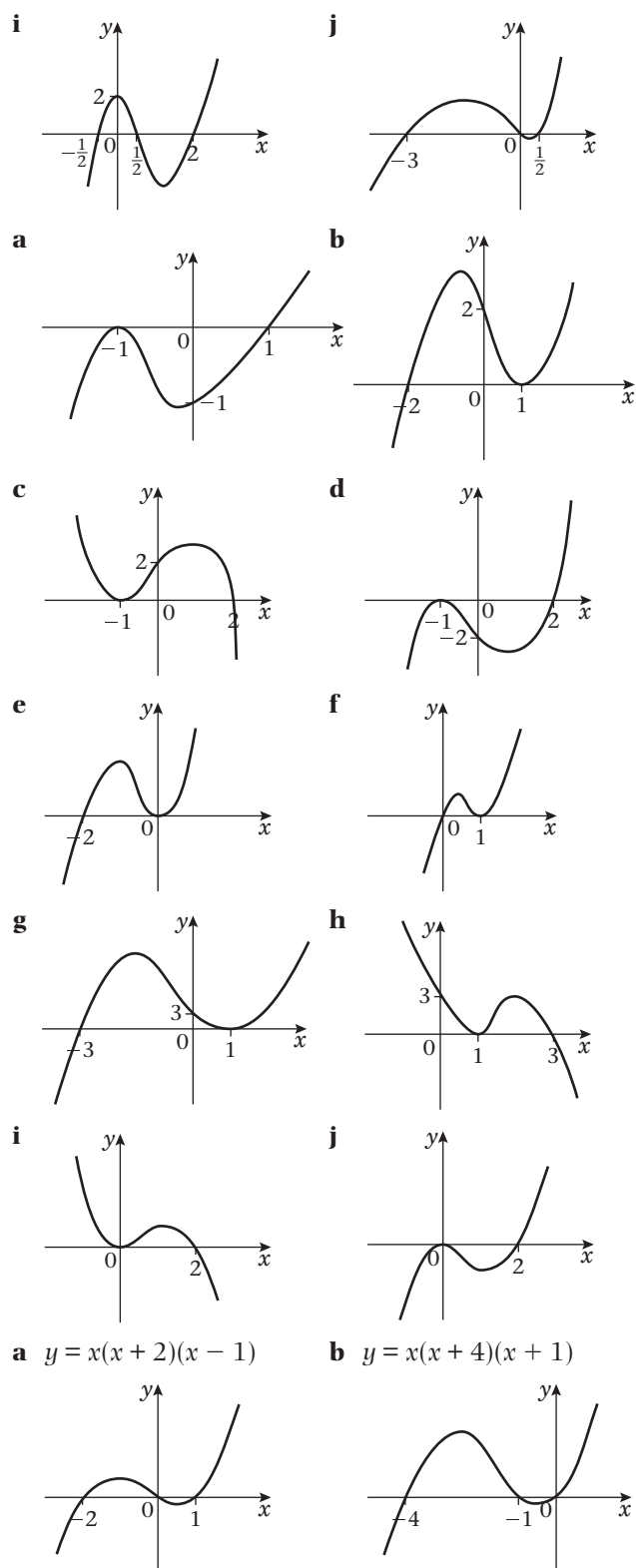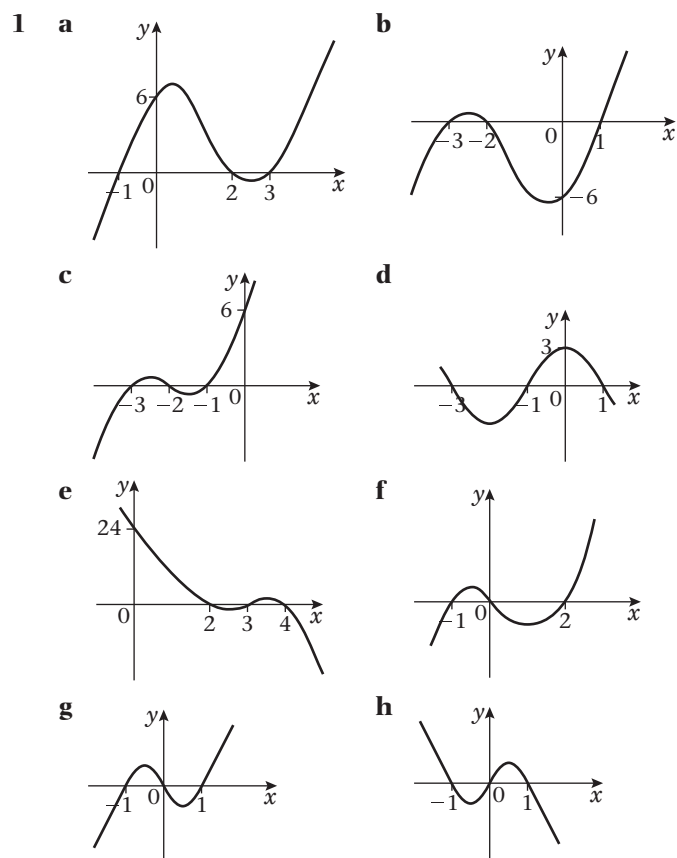
c $y = x(x+1)^2$
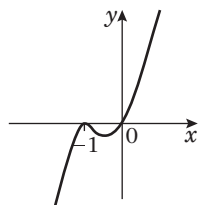
d $y = x(x+1)(3-x)$

e

e $y = x^2(x-1)$

f $y = x(1-x)(1+x)$
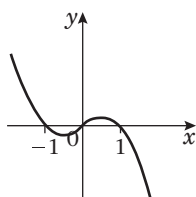
2 a

b

g $y = 3x(2x-1)(2x+1)$

h $y = x(x+1)(x-2)$

c

d

i $y = x(x-3)(x+3)$

j $y = x^2(x-9)$

e

4.2
Exercise 4B

1 a

b

4.3
Exercise 4C

1

$y = \frac{4}{x}$

$y = \frac{2}{x}$

c

d

2

$y = \frac{2}{x}$

$y = -\frac{2}{x}$

3

$y = -\frac{2}{x}$

$y = -\frac{4}{x}$

4

$y = \frac{8}{x}$

$y = \frac{3}{x}$

5

$y = -\frac{3}{x}$

$y = -\frac{8}{x}$
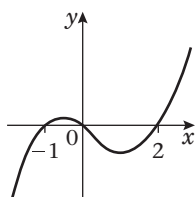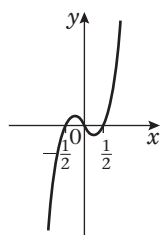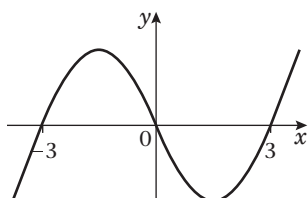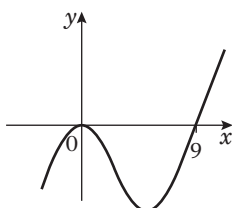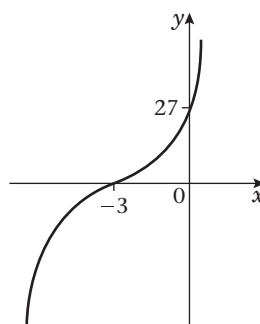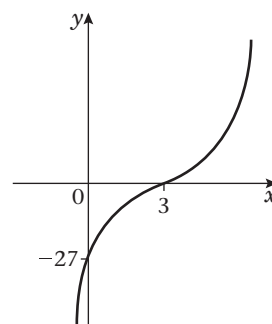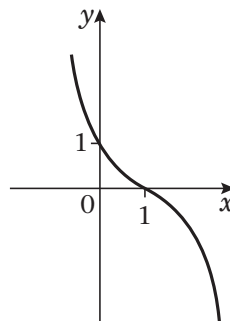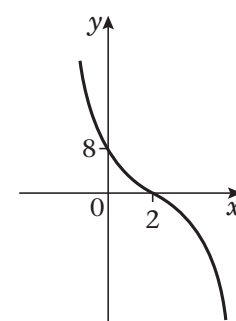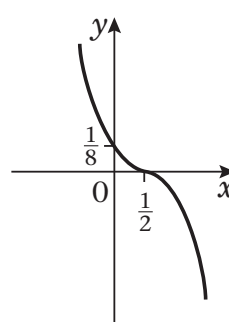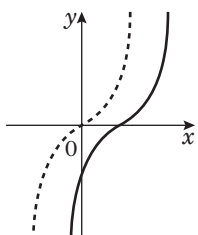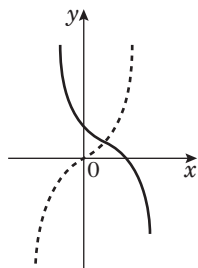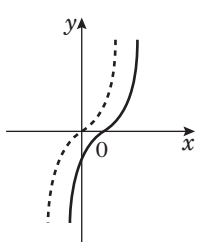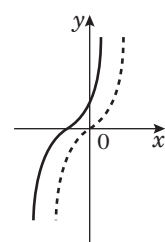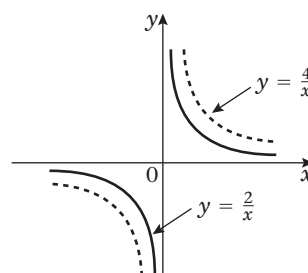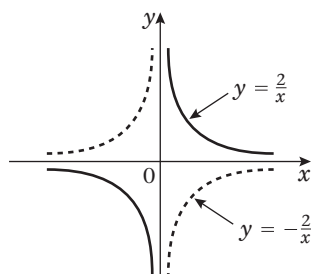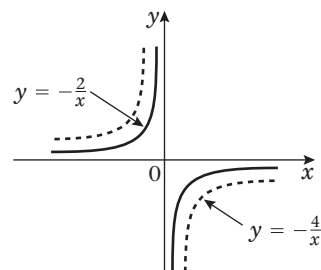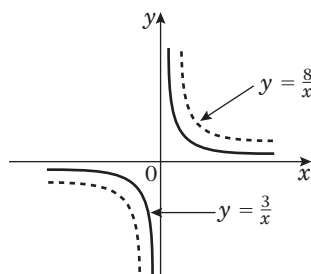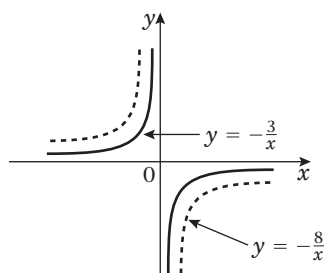
4.4
Exercise 4D

1 a i

$y = x^2$

$y = x(x^2 - 1)$

ii 3

iii $x^2 = x(x^2 - 1)$

b i
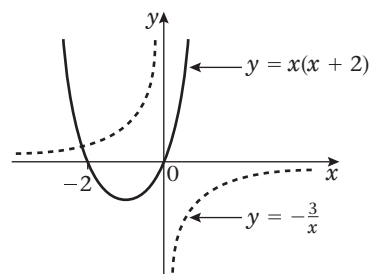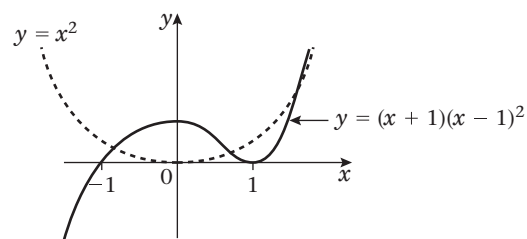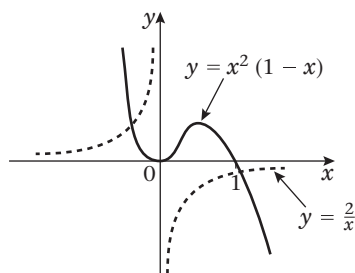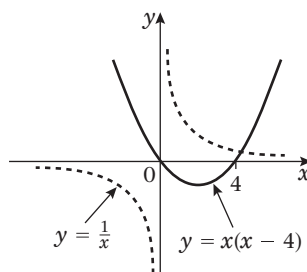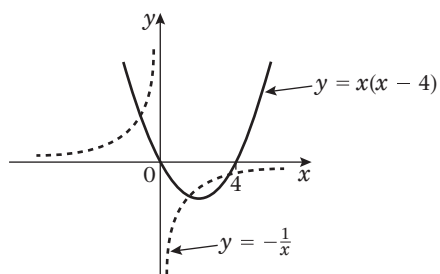
$y = x(x + 2)$

$y = -\frac{3}{x}$

ii 1

iii $x(x + 2) = -\dfrac{3}{x}$

c i

$y = x^2$

$y = (x + 1)(x - 1)^2$

ii 3

iii $x^2 = (x + 1)(x - 1)^2$

d i

$y = x^2 (1 - x)$

$y = \frac{2}{x}$

ii 2

iii $x^2(1 - x) = -\dfrac{2}{x}$

e i

$y = \frac{1}{x}$

$y = x(x - 4)$

ii 1

iii $x(x - 4) = \dfrac{1}{x}$

f **i**

$y = x(x - 4)$

$y = -\frac{1}{x}$

ii 3

iii $x(x - 4) = -\dfrac{1}{x}$

g **i**

$y = (x - 2)^3$

$y = x(x - 4)$

ii 1

iii $x(x - 4) = (x - 2)^3$

h **i**

$y = -\dfrac{2}{x}$

$y = -x^3$

ii 2

iii $-x^3 = -\dfrac{2}{x}$

i **i**

$y = x^2$

$y = -x^3$

ii 2

iii $-x^3 = x^2$

j **i**

$y = -x(x + 2)$

$y = -x^3$

ii 3

iii $-x^3 = -x(x + 2)$

2 a

$y = x^2(x - 4)$

$y = x(4 - x)$

b $(0, 0); (4, 0); (-1, -5)$

3 a

$y = x(1 + x)^3$

$y = x(2x + 5)$

b $(0, 0); (2, 18); (-2, -2)$

4 a

$y = (x - 1)(x + 1)$

$y = (x - 1)^3$

b $(0, -1); (1, 0); (3, 8)$

5 a

$y = x^2$

$y = -\dfrac{27}{x}$

b $(-3, 9)$

6 a

$y = x^2 - 2x$

$y = x(x - 2)(x - 3)$

b $(0, 0); (2, 0); (4, 8)$

7 a

$y = \frac{2}{x}$

$y = x^2(x - 3)$
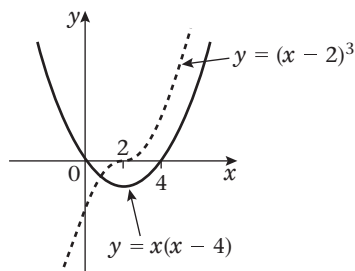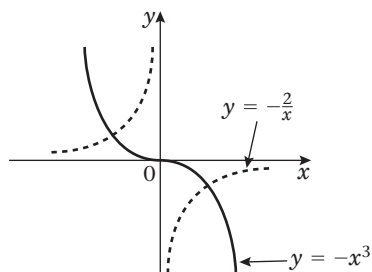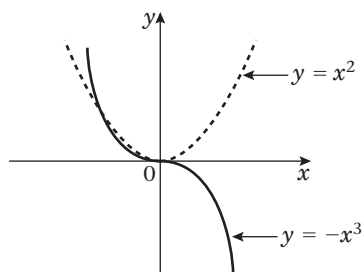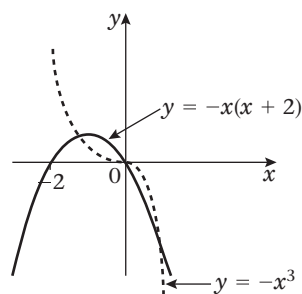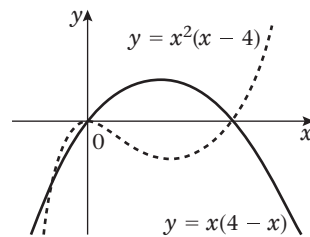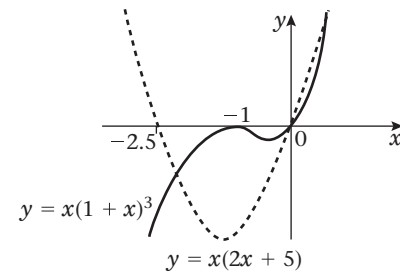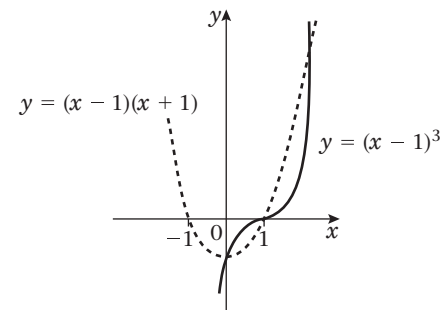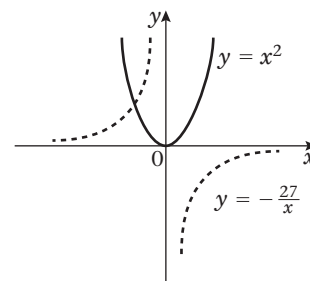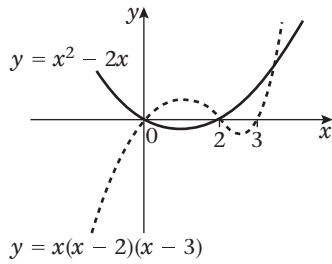
b Only 2 intersections

8 a

$y = 3x(x - 1)$

$y = (x + 1)^3$
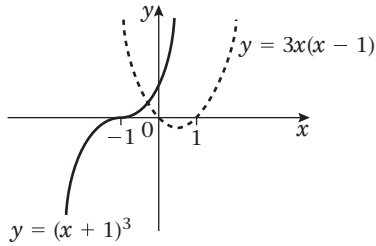
b Only 1 intersection

9 a

$y = \frac{1}{x}$

$y = -x(x - 1)^2$
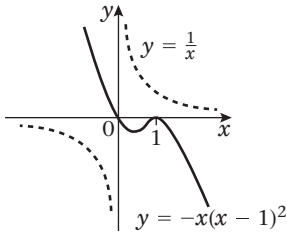
b Graphs do not intersect

10 a

$y = x(x - 2)^2$

$y = 1 - 4x^2$
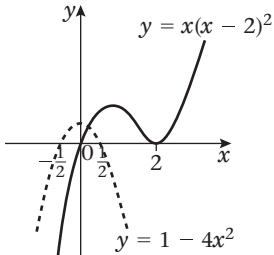
b 1, since graphs only cross once

11 a

$y = 6x$

$y = x^3 - 3x^2 - 4x$

b $(0, 0); (-2, -12); (5, 30)$

12 a

$y = 14x + 2$

$y = (x^2 - 1)(x - 2)$

b $(0, 2); (-3, -40); (5, 72)$

13 a

$y = (x - 2)(x + 2)^2$

$y = -x^2 - 8$
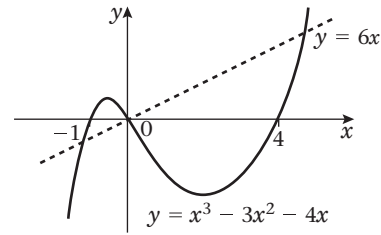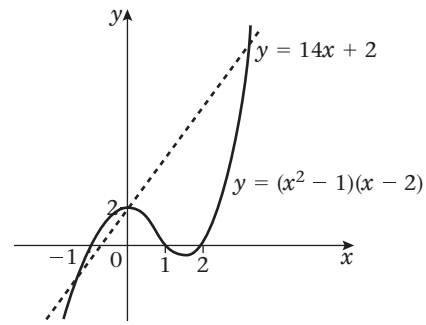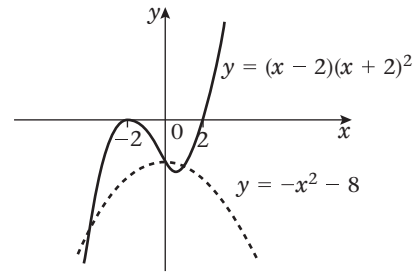
b $(0, -8); (1, -9); (-4, -24)$

4.5
Exercise 4E

1

2

3

4

5

6

7 $y = 3 + \dfrac{1}{x+1}$ Start with $y = \dfrac{1}{x}$
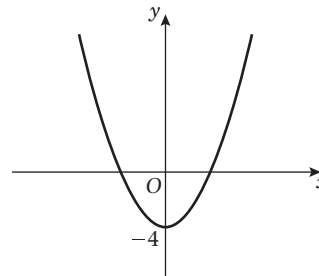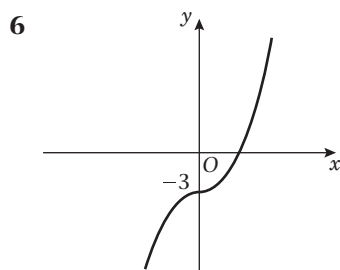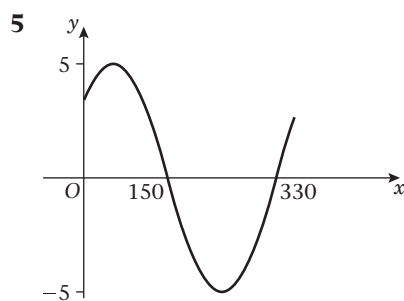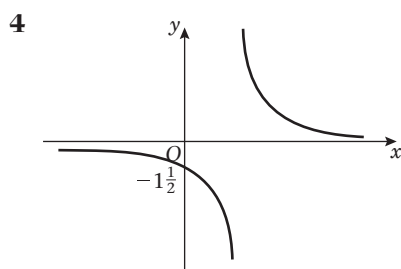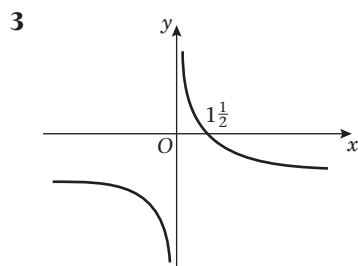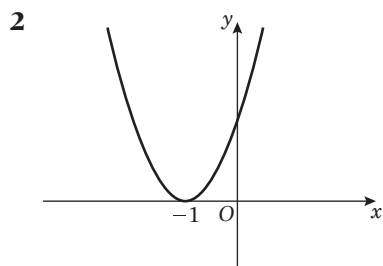
Transformation $f(x+1)$ gives
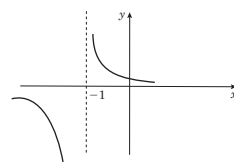
$$y = \dfrac{1}{x+1}$$

Finally transformation $f(x) + 3$ gives
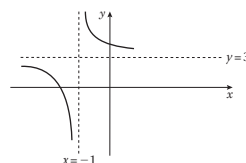
$$y = 3 + \dfrac{1}{x+1}$$

Asymptotes $x = -1$, $y = 3$

8 $y = \dfrac{1}{x-2} - 1$ Start with $y = \dfrac{1}{x}$

Transformation $f(x-2)$ gives
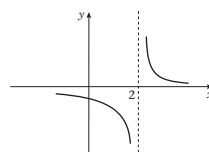
$$y = \dfrac{1}{x-2}$$

Finally $f(x) - 1$ gives
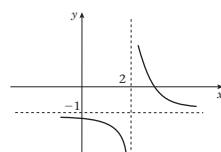
$$y = \dfrac{1}{x-2} - 1$$

Asymptotes $x = 2$, $y = -1$

9 $y = 2 + \dfrac{1}{x-1}$ Vertical asymptote is $x = 1$ (put denominator = 0)

Horizontal asymptote is $y = 2$ (let $x \Rightarrow \infty$)
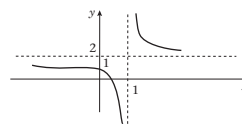
$x = 0$, $y = 1$

Asymptotes $x = 1$ and $y = 2$

10 $y = \dfrac{3 + 2x}{1 + x} = \dfrac{2(1+x) + 1}{1+x} = 2 + \dfrac{1}{1+x}$
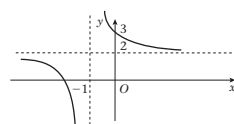
$x = 0$, $y = 3$
Asymptotes $x = -1$, $y = 2$

11 $y = 2 + \dfrac{3}{1-x}$ Vertical asymptote $x = 1$
Horizontal asymptote $y = 2$

Let $x = 0$ and $y = 2 + 3 = 5$
NB This is a series of
transformations of $y = -\dfrac{3}{x}$

12 $y = 5 - \dfrac{2}{3+x}$ Vertical asymptote $x = -3$
Horizontal asymptote $y = 5$

Let $x = 0$ and $y = 5 - \frac{2}{3} = 4\frac{1}{3}$
Again this is a series of
transformations of $y = -\dfrac{2}{x}$

13 $y = \dfrac{3}{2-x} - 4$ Vertical asymptote $x = 2$
Horizontal asymptote $y = -4$

Let $x = 0$ and $y = \frac{3}{2} - 4 = -2\frac{2}{3}$

This is a series of transformations
of $y = -\dfrac{3}{x}$

4.6
Exercise 4F

1 **a**

$y = e^x + 1$
$(0, 2)$
$y = 1$

b

$y = 4e^{-2x}$
$(0, 4)$

c

$y = 2e^x - 3$
$(0, -1)$
$y = -3$

d

4
$(0, 3)$
$y = 4 - e^x$

e

$y = 6 + 10e^{\frac{1}{2}x}$
16
6

f

$y = 100e^{-x} + 10$
110
10

2 **a**

$y = \ln(x + 1)$
-1
$(0, 0)$

b

$y = 2\ln(x)$
$x = 0$
$(1, 0)$

c

$x = 0$
$y = \ln(2x)$
$(\frac{1}{2}, 0)$

d $y = 3\ln(x - 2)$ $x > 2$

Start with $y = \ln x$

Use transformation $f(x - 2)$

Now use transformation $3f(x)$

$y = 3\ln(x - 2)$

e

$y = \ln(4 - x)$
$\ln(4)$
$x = 4$
$(3, 0)$
4

f

$y = 3 + \ln(x + 2)$
$x = 2$
$(0, 3 + \ln 2)$
-2
$(e^{-3} - 2, 0)$
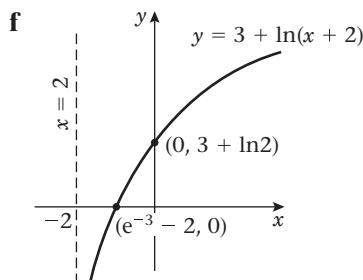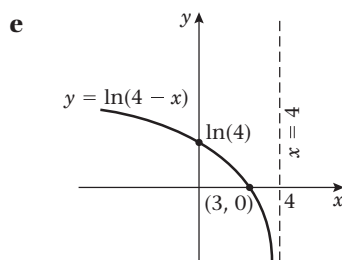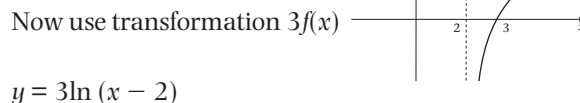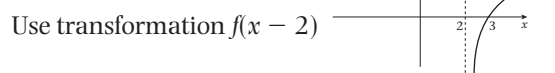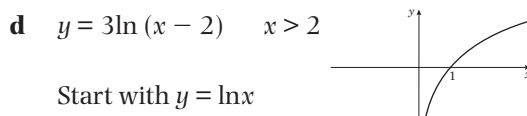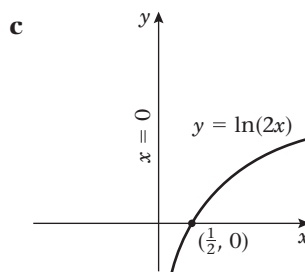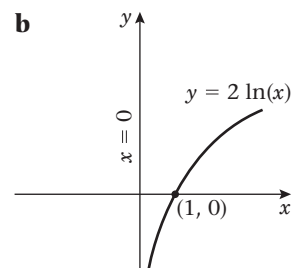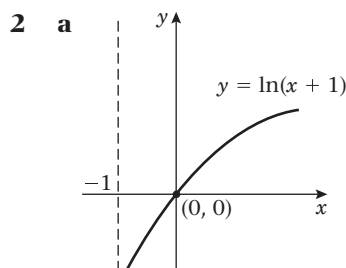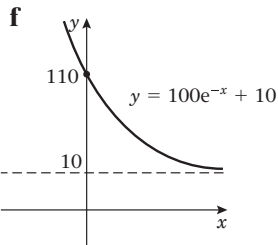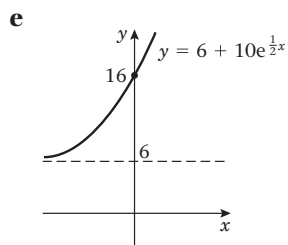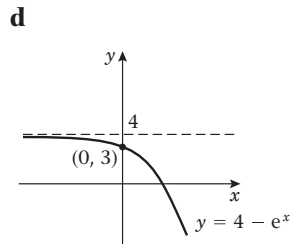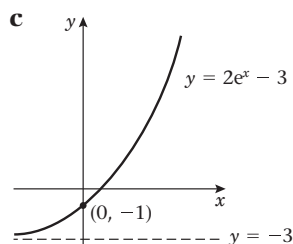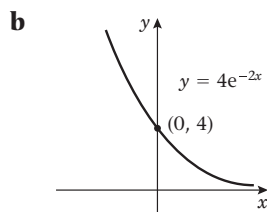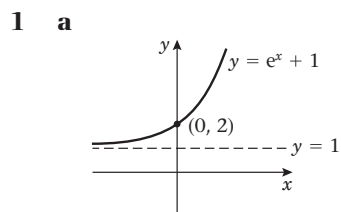
4.7

Exercise 4G

1 a $x = 1, y = 4.21; x = 5, y = 3.16$

b
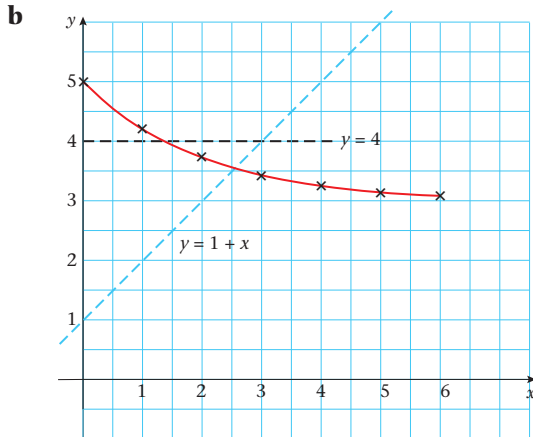
c $e^{-\frac{1}{2}x} = 0.5 \Rightarrow 3 + 2e^{-\frac{1}{2}x} = 3 + 2 \times 0.5 = 4$

Draw $y = 4$ and intersection is at ≈ 1.35

d $x = -2 \ln\left(\dfrac{x - 2}{2}\right)$

$\Rightarrow e^{-\frac{x}{2}} = \dfrac{x - 2}{2}$

$\Rightarrow 2e^{-\frac{x}{2}} = x - 2 \therefore 3 + 2e^{-\frac{1}{2}x} = 1 + x$

Draw $y = 1 + x$ and intersection at $x \approx 2.55$

2 a $x = 0, y = 2 + \frac{1}{3} = 2.33; x = 2.5, y = 6.06$

b
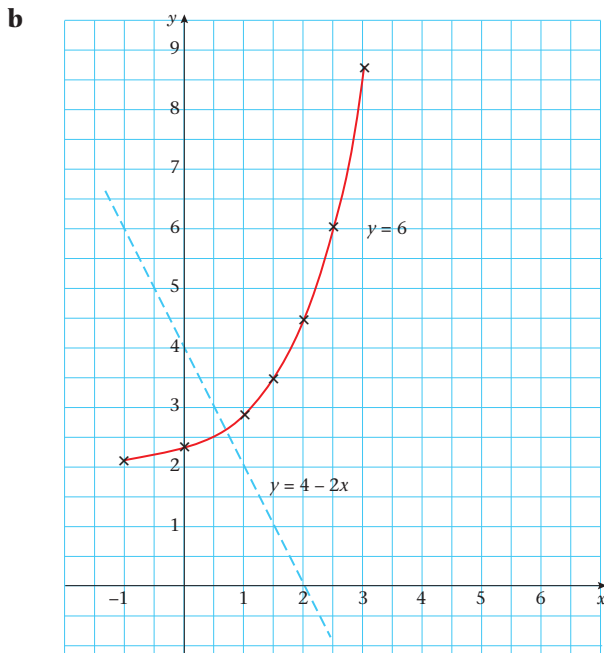
c $e^x = 12 \Rightarrow 2 + \frac{1}{3}e^x = 2 + \frac{1}{3} \times 12 = 6$

Draw line $y = 6$ and intersection is at ≈ 2.45

d $x = \ln(6 - 6x)$

$\Rightarrow e^x = 6 - 6x$

$\Rightarrow \frac{1}{3}e^x = 2 - 2x$

$\therefore 2 + \frac{1}{3}e^x = 4 - 2x$ so draw $y = 4 - 2x$

and intersection at ≈ 0.65 or 0.7 to 1sf

3 a $x = 1, y = 2 + \ln 1 = 2; x = 4, y = 3.39$

b
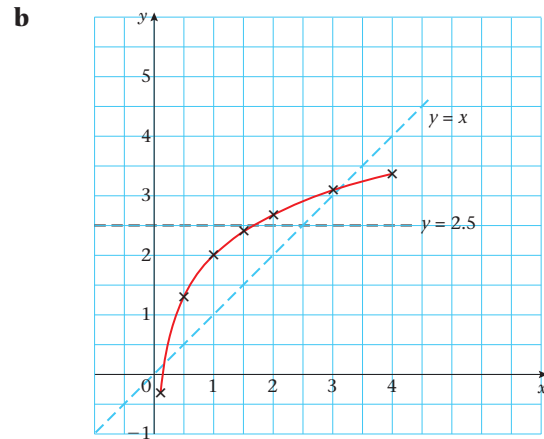
c $\ln x = 0.5$

$\Rightarrow 2 + \ln x = 2.5$ so draw $y = 2.5$

Intersection ≈ 1.60

d $x = e^{x - 2}$

$\Rightarrow \ln x = x - 2$

$\Rightarrow 2 + \ln x = x$ so draw $y = x$

Intersection ≈ 3.1 to 1sf

4 a $x = 30, y = 2.60; x = 75, y = 1.98$

b
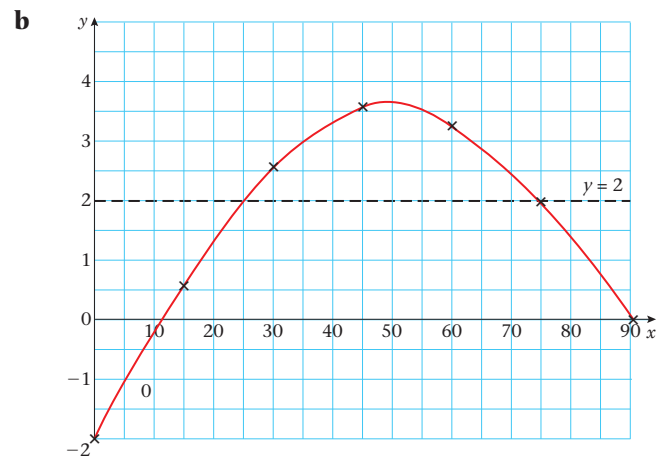
c $2 + 2\cos x = 5 \sin 2x$

$\Rightarrow 2 = 5 \sin 2x - 2 \cos x$

Draw $y = 2$

and intersections at ≈ 25.1 and 74.6

or $\quad$ 25 $\quad$ and $\quad$ 75 to 2sf

Mixed Exercise 4H

1 a

$y = x^2(x - 2)$

$2x - x^2$
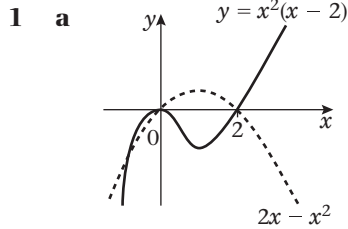
b $x = 0, -1, 2$; points $(0, 0), (2, 0), (-1, -3)$

2 a

$y = 1 + x$

$y = \frac{6}{x}$
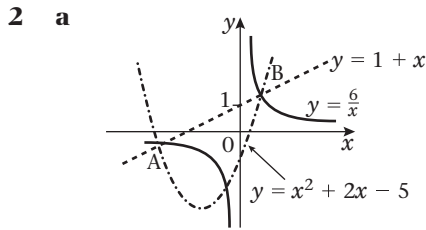
$y = x^2 + 2x - 5$

b $A(-3, -2)$ $B(2, 3)$

c $y = x^2 + 2x - 5$

3 a $y = x^2 - 2x - 3$
$= (x - 3)(x + 1)$
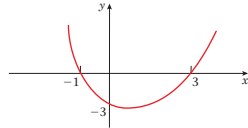
b $y = x^2 - 2x + 4$
$= (x - 1)^2 + 3$
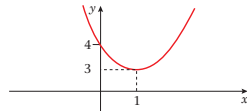
4 a $y = -(x^2 - 4x + 3)$
$= (3 - x)(x - 1)$
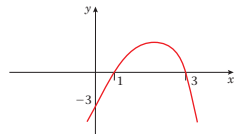
b $y = -(x^2 - 4x + 5)$
$= -\left[(x - 2)^2 + 1\right]$
$= -1 - (x - 2)^2$
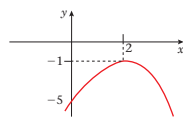
5 a $y = \frac{1}{2} e^x + 4$
$(x = 0, y = 4.5)$
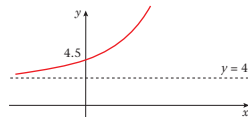
b $y = \ln(x + 1) + 2$
$(x = 0, y = 2)$
$y = 0 \Rightarrow \ln(x + 1) = -2$
$\Rightarrow x + 1 = e^{-2}$
$\therefore x = e^{-2} - 1 \approx -0.86$
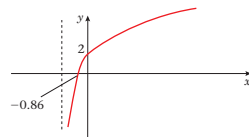
6 a $x = 5$ $y = 1 - \ln 4 = -0.39$
$x = 6$ $y = 1 - \ln 5 = -0.61$

b

$y = \frac{x}{4}$
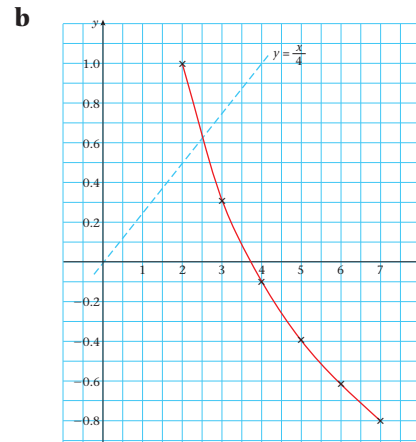
c $\ln(x - 1) = 0$
$\Rightarrow 1 - \ln(x - 1) = 1$ so draw $y = 1$
Intersection at $x = 2$

d $x = 1 + e^{1 + \frac{x}{4}}$

$\ln(x - 1) = 1 + \frac{x}{4}$

$\Rightarrow x - 1 = e^{1 - \frac{x}{4}}$

$\Rightarrow 1 - \ln(x - 1) = 1 - (1 + \frac{x}{4}) = \frac{x}{4}$ so draw $y = \frac{x}{4}$
and intersection at ≈ 2.5

Chapter 5

5.1

Exercise 5A

1 Arithmetic sequences are **a**, **b**, **c**, **h**, **l**

2 a $23, 2n + 3$ **b** $32, 3n + 2$
c $-3, 27 - 3n$ **d** $35, 4n - 5$
e $10x, nx$ **f** $a + 9d, a + (n - 1)d$

3 a £5800 **b** £$(3800 + 200m)$

4 a 22 **b** 40 **c** 39
d 46 **e** 18 **f** n

5.2

Exercise 5B

1 a $78, 4n - 2$ **b** $42, 2n + 2$
c $23, 83 - 3n$ **d** $39, 2n - 1$
e $-27, 33 - 3n$ **f** $59, 3n - 1$
g $39p, (2n - 1)p$ **h** $-71x, (9 - 4n)x$

2 a 30 **b** 29 **c** 32
d 31 **e** 221 **f** 77

3 $d = 6$

4 $a = 36, d = -3$, 14th term

5 24

6 $x = 5$; 25, 20, 15

7 $7 \times \frac{1}{2}, x = 8$

5.3
Exercise 5C

1 **a** 820 **b** 450 **c** -1140
d -294 **e** 1440 **f** 1425
g -1155 **h** $21(11x + 1)$
2 **a** 20 **b** 25 **c** 65
d 4 or 14 (2 answers)
3 2550
4 **i** £222 500 **ii** £347 500
5 1683, 32674
6 £9.03, 141 days
7 $d = -\frac{1}{2}, -5.5$
8 $a = 6, d = -2$

5.4
Exercise 5D

1 **a** $\displaystyle\sum_{r=1}^{10}(3r+1)$ **b** $\displaystyle\sum_{r=1}^{30}(3r-1)$

 c $\displaystyle\sum_{r=1}^{11}4(11-r)$ **d** $\displaystyle\sum_{r=1}^{16}6r$
2 **a** 45 **b** 210
 c 1010 **d** 112
3 19
4 49

5.5
Exercise 5E

1 **a** Geometric $r = 2$ **b** Not geometric
 c Not geometric **d** Geometric $r = 3$
 e Geometric $r = \frac{1}{2}$ **f** Geometric $r = -1$
 g Geometric $r = 1$ **h** Geometric $r = \frac{1}{4}$
2 **a** 135, 405, 1215 **b** $-32, 64, -128$
 c 7.5, 3.75, 1.875 **d** $\frac{1}{64}, \frac{1}{256}, \frac{1}{1024}$
 e p^3, p^4, p^5 **f** $-8x^4, 16x^5, -32x^6$
3 **a** $3\sqrt{3}$ **b** $9\sqrt{3}$

5.6
Exercise 5F

1 **a** $486, 39\,366, 2 \times 3^{n-1}$
 b $\frac{25}{8}, \frac{25}{128}, \frac{100}{2^{n-1}}$
 c $-32, -512, (-2)^{n-1}$
 d $1.610\,51, 2.357\,95, (1.1)^{n-1}$

2 10, 6250
3 $a = 1, r = 2$
4 $\pm\frac{1}{8}$
5 -6 (from $x = 0$), 4 (from $x = 10$)

5.7
Exercise 5G

1 **a** 255 **b** 63.938 (3 dp)
 c -728 **d** $546\frac{2}{3}$
 e 5460 **f** 19 680
 g 5.994 (3 dp) **h** 44.938 (3 dp)
2 $\frac{5}{4}, -\frac{9}{4}$
3 $2^{64} - 1 = 1.84\ x\ 10^{19}$
4 **a** £49 945.41 **b** £123 876.81
5 **a** 2.401 **b** 48.8234
6 19 terms
7 22 terms
8 26 days, 98.5 miles on the 25th day
9 25 years

5.8
Exercise 5H

1 **a** $\frac{10}{9}$ **b** Doesn't exist
 c $6\frac{2}{3}$ **d** Doesn't exist
 e Doesn't exist **f** $4\frac{1}{2}$
 g Doesn't exist **h** 90
 i $\frac{1}{1} - r$ if $|r| < |$ **j** $\frac{1}{1} + 2x$ if $|x| < \frac{1}{2}$
2 $\frac{2}{3}$
3 $-\frac{2}{3}$
4 20
5 $\frac{40}{3} = 13\frac{1}{3}$
6 $\frac{23}{99}$
7 4
8 40 m
9 $r < 0$ because $S_\infty < S_3, a = 12, r = -\frac{1}{2}$
10 $r = \pm\sqrt{\frac{2}{3}}$

Mixed Exercise 5I

1 **a** Add 6 to the previous term, i.e. $U_{n+1} = U_n + 6$
 (or $U_n = 6n - 1$)
 b Add 3 to the previous term, i.e. $U_{n+1} = U_n + 3$
 (or $U_n = 3n$)
 c Multiply the previous term by 3, i.e. $U_{n+1} = 3U_n$
 (or $U_n = 3^{n-1}$)

d Subtract 5 from the previous term,
i.e. $U_{n+1} = U_n - 5$ (or $U_n = 15 - 5n$)

e The square numbers ($U_n = n^2$)

f Multiply the previous term by 1.2, i.e. $U_{n+1} = 1.2U_n$
(or $U_n = (1.2)^{n-1}$)

Arithmetic sequences are:

a $a = 5, d = 6$

b $a = 3, d = 3$

c $a = 10, d = -5$

2 a 81   **b** 860

3 32

4 a £13 780   **b** £42 198

5 a $a = 25, d = -3$ **b** -3810

6 a 26 733   **b** 53 467

7 a 5   **b** 45

8 a $d = 5$ **b** 59

9 b $11k - \frac{9}{3}$ **c** 1.5   **d** 415

10 a Not geometric   **b** Geometric $r = 1.5$

b Geometric $r = \frac{1}{2}$ **c** Geometric $r = -2$

d Not geometric   **e** Geometric $r = 1$

11 a 0.8235 (4 dp), $10x (0.7)^{n-1}$

b $640, 5 \times 2^{n-1}$

c $-4, 4 \times (-1)^{n-1}$

d $\frac{3}{128}, 3 \times (-\frac{1}{2})^{n-1}$

12 a 4092   **b** 19.98 (2 dp)

c 50   **d** 3.33 (2 dp)

13 a 9   **b** $\frac{8}{3}$

c Doesn't converge   **d** $\frac{16}{3}$

14 b 60.72   **c** 182.25

d 3.16

15 b 200   **c** $333\frac{1}{3}$ **d** 8.95×10^{-4}

16 a 76, 60.8   **b** 0.876

c 367   **d** 380

17 a $1, \frac{1}{3}, -\frac{1}{9}$

18 a 0.8   **b** 10

c 50   **d** 0.189 (3sf)

19 a $-\frac{1}{2}$ **b** $\frac{3}{4}, -2$ **c** 14

Chapter 6

6.1

Exercise 6A

1 $1 + 8x + 28x^2 + 56x^3$

2 $1 - 12x + 60x^2 - 160x^3$

3 $1 + 5x + \frac{45}{4}x^2 + 15x^3$

4 $1 - 15x + 90x^2 - 270x^3$

5 a $p = 5$ **b** -10 **c** -80

6 $1 - 0.6x + 0.15x^2 - 0.02x^3$, 0.94148, accurate to 5 dp

7 a $-20x^3$ **b** $120x^3$ **c** $1140x^3$

8 $b = -2$

6.2

Exercise 6B

1 a $1 + 6x + 12x^2 + 8x3$, valid for all x

b $1 + x + x^2 + x^3$, $|x| < 1$

c $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$, $|x| < 1$

d $1 - 6x + 24x^2 - 80x^3$, $|x| < \frac{1}{2}$

e $1 - x - x^2 - \frac{5}{3}x^3$, $|x| < \frac{1}{3}$

f $1 - 15x + \frac{75}{2}x^2 + \frac{125}{2}x^3$, $|x| < \frac{1}{10}$

g $1 - x + \frac{5}{8}x^2 - \frac{5}{16}x^3$, $|x| < 4$

h $1 - 2x^2 + ...$, $|x| < \frac{\sqrt{2}}{2}$

2 $|x| < \frac{1}{2}$

3 $1 + \frac{3x}{2} - \frac{9}{8}x^2 + \frac{27}{16}x^3$, 10.148 891 88, accurate to 6 d.p.

4 $a = \pm 8, \pm 160x^3$

6 $1 - \frac{9x}{2} + \frac{27x^2}{8} + \frac{27x^3}{16}, x = 0.01$, 955.339(1875)

Mixed Exercise 6C

1 a $p = 16$ **b** 270   **c** -1890

2 a $1 - 20x + 180x^2 - 960x^3$

b $0.817 04, x = 0.01$

3 a $n = 8$ **b** $\frac{35}{8}$

4 a $1 + 24x + 264x^2 + 1760x^3$

b 1.268 16

c 1.268 241 795

d 0.006 45% (3 sf)

5 a $1 - 12x + 48x^2 - 64x^3$, all x

b $1 + 2x + 4x^2 + 8x^3$, $|x| < \frac{1}{2}$

c $1 - 2x + 6x^2 - 18x^3$, $|x| < \frac{1}{3}$

6 $1 - \frac{x}{4} - \frac{x^2}{32} - \frac{x^3}{128}$

7 $1 - \frac{3}{2}x^2 + \frac{27}{8}x^4 - \frac{135}{16}x^6$

8 $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}, \frac{1145}{512}$

9 a $n = -2, a = 3$

b -108

c $|x| < \frac{1}{3}$

Chapter 7

7.1
Exercise 7A

1
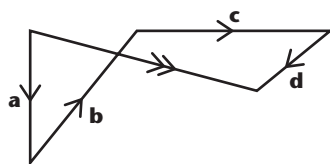
2 25

3 $\sqrt{569} \approx 23.9$

4 **a** $\mathbf{d} - \mathbf{a}$ **b** $\mathbf{a} + \mathbf{b} + \mathbf{c}$

  c $\mathbf{a} + \mathbf{b} - \mathbf{d}$ **d** $\mathbf{a} + \mathbf{b} + \mathbf{c} - \mathbf{d}$

7.2
Exercise 7B

1 **a** $2\mathbf{a} + 2\mathbf{b}$ **b** $\mathbf{a} + \mathbf{b}$ **c** $\mathbf{b} - \mathbf{a}$

2 **a** $\mathbf{b} - \frac{1}{2}\mathbf{a}$ **b** $\mathbf{b} - 3\mathbf{a}$

  c $\frac{3}{2}\mathbf{a} - \mathbf{b}$ **d** $2\mathbf{a} - \mathbf{b}$

3 **a** Yes ($\lambda\ 2$) **b** Yes ($\lambda\ 4$) **c** No

  d Yes ($\lambda\ -1$) **e** Yes ($\lambda\ -3$) **f** No

4 **a** $\lambda = \frac{1}{2}, \mu = -3$ **b** $\lambda = -2, \mu = 1$

  c $\lambda = \frac{1}{4}, \mu = 5$ **d** $\lambda = -2, \mu = -1$

  e $\lambda = 4, \mu = 8\frac{1}{2}$

5 **a** $\mathbf{b} - \mathbf{a}, \frac{5}{6}(\mathbf{b} - \mathbf{a}), \frac{1}{6}\mathbf{a} + \frac{5}{6}\mathbf{b}$

  b $-\frac{1}{6}\mathbf{a} + (\lambda - \frac{5}{6})\mathbf{b}$

  c $-\mu\mathbf{a} + (\mu - \lambda)\mathbf{b}$

  d $\lambda = \frac{1}{2}, \mu = \frac{1}{6}$

6 **a** $\mathbf{i} - \mathbf{a} + \mathbf{b}$ **ii** $\frac{2}{3}\mathbf{a} - \frac{7}{4}\mathbf{b}$

  b $(\frac{2}{3}\lambda - \mu)\mathbf{a} + (\frac{3}{4} - \frac{7}{4}\lambda + \mu)\mathbf{b} = 0$

  d $\frac{6}{13}$

  e $\frac{6}{13}\mathbf{a} + \frac{7}{13}\mathbf{b}$

  f $\frac{13}{10}$

7 **a** $-\mathbf{a} + \mathbf{b}$ **b** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

  c $\frac{3}{8}\mathbf{a}\ \frac{3}{8}\mathbf{b}$ **d** $-\frac{5}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$

  e $-\mathbf{a} + k\mathbf{b}$ **f** $5:3, k = 35$

8 **a** $\frac{1}{3}\mathbf{a}$

  b $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

  c $-\frac{1}{12}\mathbf{a} + \frac{3}{4}\mathbf{b}$

  d $2\mathbf{b}$

  e $-\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

  f $-\frac{1}{4}\mathbf{a} + \frac{9}{4}\mathbf{b}$

  g $1:3$

  h $-\frac{1}{3}\mathbf{a} + \mathbf{b}, -\mathbf{a} + 3\mathbf{b}, \overrightarrow{AG} = 3\overrightarrow{EB} \Rightarrow$ parallel

7.3
Exercise 7C

1 $\frac{5}{6}\mathbf{a} + \frac{1}{6}\mathbf{b}$

2 $-\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{c}$

3 $\overrightarrow{OC} = -2\mathbf{a} + 2\mathbf{b}, \overrightarrow{OD} = 3\mathbf{a} + 2\mathbf{b}, \overrightarrow{OE} = -2\mathbf{a} + \mathbf{b}$

7.4
Exercise 7D

1 **a** $\begin{pmatrix} 12 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ 16 \end{pmatrix}$ **c** $\begin{pmatrix} -21 \\ -29 \end{pmatrix}$

2 **a** $3\mathbf{i} - \mathbf{j}, 4\mathbf{i} + 5\mathbf{j}, -2\mathbf{i} + 6\mathbf{j}$

  b $\mathbf{i} + 6\mathbf{j}$

  c $-5\mathbf{i} + 7\mathbf{j}$

  d $\sqrt{40} = 2\sqrt{10}$

  e $\sqrt{37}$

  f $\sqrt{74}$

3 **a** $\frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **b** $\frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

  c $\frac{1}{25}\begin{pmatrix} -7 \\ 24 \end{pmatrix}$ **d** $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

4 -7 or -23

Mixed Exercise 7E

1 $m = 3, n = 1$

2 $m = -2, n = 5$

3 **a** $\overrightarrow{XM}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

  $\overrightarrow{XZ} = \begin{pmatrix} -10 \\ 6 \end{pmatrix}$

  b $v\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

  c $\begin{pmatrix} 8 \\ 0 \end{pmatrix} + w\begin{pmatrix} -10 \\ 6 \end{pmatrix}$

  d $v = \frac{2}{3}, w = \frac{1}{3}$

4 **a** $\overrightarrow{AC} = x + y; \overrightarrow{BE} = \frac{1}{3}y - x$

  b **i** $\overrightarrow{BF} = v(\frac{1}{3}y - x)$

    ii $\overrightarrow{AF} = x + \overrightarrow{BF} = x + v(\frac{1}{3}y - x)$

    iii $v = \frac{3}{4}$

5 $\mathbf{v} + \mathbf{w} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \sqrt{41}$

  $2\mathbf{v} - \mathbf{w} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \sqrt{29}$

  $\mathbf{v} - 2\mathbf{w} = \begin{pmatrix} 1 \\ -7 \end{pmatrix}, \sqrt{50}$

6 $p + q = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \sqrt{41}$

$3p + q = \begin{pmatrix} 9 \\ 2 \end{pmatrix}, \sqrt{85}$

$p - 3q = \begin{pmatrix} -7 \\ -16 \end{pmatrix}, \sqrt{305}$

7 a Chloe $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$; Leo $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$; Max $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

b Chloe: 74 km, 2.9 km/h
Leo: 41 km, 2.1 km/h
Max: 13 km, 1.2 km/h

Chapter 8

8.1

Exercise 8A

1 a -2 **b** -1 **c** 3 **d** $\frac{1}{3}$
  e $\frac{2}{3}$ **f** $\frac{5}{4}$ **g** $\frac{1}{2}$ **h** 2
  i $\frac{1}{2}$ **j** $\frac{1}{2}$ **k** -2 **l** $\frac{3}{2}$

2 a 4 **b** -5 **c** $-\frac{2}{3}$ **d** 0
  e $\frac{7}{5}$ **f** 2 **g** 2 **h** -2
  i -9 **j** -3 **k** $\frac{3}{2}$ **l** $-\frac{1}{2}$

3 a $4x - y + 3 = 0$ **b** $3x - y - 2 = 0$
  c $6x + y - 7 = 0$ **d** $4x - 5y - 30 = 0$
  e $5x - 3y + 6 = 0$ **f** $7x - 3y = 0$
  g $14x - 7y - 4 = 0$ **h** $27x + 9y - 2 = 0$
  i $18x + 3y + 2 = 0$ **j** $2x + 6y - 3 = 0$
  k $4x - 6y + 5 = 0$ **l** $6x - 10y + 5 = 0$

4 $y + 5x + 3$
5 $2x + 5y + 20 = 0$
6 $y + -\frac{1}{2}x + 7$
7 $y = \frac{2}{3}x$
8 $(3, 0)$
9 $(\frac{5}{3}, 0)$
10 $(0, 5), (-4, 0)$

8.2

Exercise 8B

1 a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{3}{5}$ **d** 2
  e -1 **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** 8
  i $\frac{2}{3}$ **j** -4 **k** $\frac{1}{3}$ **l** $\frac{1}{2}$
  m 1 **n** $\frac{q^2 - p^2}{q - p} = q + p$

2 7

3 12
4 $4\frac{1}{3}$
5 $2\frac{1}{4}$
6 $\frac{1}{4}$
7 26
8 -5

8.3

Exercise 8C

1 a $y = 2x + 1$ **b** $y = 3x + 7$
  c $y = -x - 3$ **d** $y = -4x - 11$
  e $y = \frac{1}{2}x + 12$ **f** $y = \frac{2}{3}x - 5$
  g $y = 2x$ **h** $y = -\frac{1}{2}x + 2b$

2 $y = 3x - 6$
3 $y = 2x + 8$
4 $2x - 3y + 24 = 0$
5 $-\frac{1}{5}$
6 $y = \frac{2}{5}x + 3$
7 $2x + 3y - 12 = 0$
8 $\frac{8}{5}$
9 $y = \frac{4}{3}x - 4$
10 $6x + 15y - 10 = 0$

8.4

Exercise 8D

1 a Perpendicular **b** Parallel
  c Neither **d** Perpendicular
  e Perpendicular **f** Parallel
  g Parallel **h** Perpendicular
  i Perpendicular **j** Parallel
  k Neither **l** Perpendicular

2 $y = -\frac{1}{3}x$
3 $4x - y + 15 = 0$
4 a $y = -2x + \frac{1}{2}$ **b** $y = \frac{1}{2}x$
  c $y = -x - 3$ **d** $y = \frac{1}{2}x - 8$
5 a $y = 3x + 11$ **b** $y = -\frac{1}{3}x + \frac{13}{3}$
  c $y = \frac{2}{3}x + 2$ **d** $y = -\frac{3}{2}x + \frac{17}{2}$
6 $3x + 2y - 5 = 0$
7 $7x - 4y + 2 = 0$

8.5
Exercise 8E

1 10	**2** 13	**3** 5
4 $\sqrt{5}$	**5** $2\sqrt{10}$	**6** $\sqrt{106}$
7 $\sqrt{113}$	**8** $a\sqrt{53}$	**9** $3b\sqrt{5}$
10 $5c$	**11** $d\sqrt{61}$	**12** $2e\sqrt{5}$

8.6
Exercise 8F

1 a $(0, 6); (4, 10)$ is ratio $3:1$
$$\left(\frac{3 \times 4 + 1 \times 0}{4}, \frac{3 \times 10 + 1 \times 6}{4}\right) = (3, 9)$$

b $(1, 5); (-2, 8)$ is ratio $1:2$
$$\left(\frac{2 \times 1 + 1 \times (-2)}{3}, \frac{(2 \times 5 + 1 \times 8)}{3}\right) = (0, 6)$$

c $(3, -7); (-2, 8)$ is ratio $3:2$
$$\left(\frac{2 \times 3 + 3x \times (-2)}{5}, \frac{2 \times (-7) + 3 \times 8}{5}\right) = (0, 2)$$

d $(-2, 5); (5, 2)$ is ratio $4:3$
$$\left(\frac{4 \times 5 + 3 \times (-2)}{7}, \frac{4 \times 2 + 3 \times 5}{7}\right) = (2, 3\tfrac{2}{7})$$

2 a $(4, 2); (6, 8)$ midpoint $\left(\frac{4 + 6}{2}, \frac{2 + 8}{2}\right)$
$= (5, 5)$

b $(0, 6); (12, 2)$ midpoint $\left(\frac{0 + 12}{2}, \frac{6 + 2}{2}\right)$
$= (6, 4)$

c $(2, 2); (-4, 6)$ midpoint $\left(\frac{2 - 4}{2}, \frac{2 + 6}{2}\right)$
$= (-1, 4)$

d $(-6, 4); (6, -4)$ midpoint $\left(\frac{-6 + 6}{2}, \frac{4 - 4}{2}\right)$
$= (0, 0)$

Mixed Exercise 8G

1 a $y = -3x + 14$ **b** $(0, 14)$
2 a $y = -\tfrac{1}{2}x + 4$ **b** $y = -\tfrac{1}{2}x + \tfrac{3}{2}, (1, 1)$
3 a $y = \tfrac{1}{7}x + \tfrac{12}{7}, y = -x + 12$ **b** $(9, 3)$
4 a $y = -\tfrac{5}{12}x + \tfrac{11}{6}$ **b** -22
5 a $y = \tfrac{3}{2}x - \tfrac{3}{2}$ **b** $(3, 3)$
6 $11x - 10y + 19 = 0$
7 a $y = -\tfrac{1}{2}x = 3$ **b** $y = \tfrac{1}{4}x + \tfrac{9}{4}$
8 a $y = \tfrac{3}{2}x - 2$ **b** $(4, 4)$ **c** 20
9 a $2x + y = 20$ **b** $y = \tfrac{1}{3}x + \tfrac{4}{3}$
10 a $\tfrac{1}{2}$ **b** 6 **c** $2x + y - 16 = 0$

11 a $(-1, 1); (4, 11)$ in ratio $3:2$
is $\left(\frac{2 \times (-1) + 3 \times 4}{5}, \frac{2 \times 1 + 3 \times 11}{5}\right)$
so $c = (2, 7)$

b $M_{AB} = \frac{11 - 1}{4 - -1} = \frac{10}{5} = 2 \therefore m_i = -\frac{1}{2}$
Equation of l is $y - 7 = -\frac{1}{2}(x - 2)$
$$2y - 14 = -x + 2$$
or $x + 2y - 16 = 0$

c $x = 0 \Rightarrow y = 8 \therefore D$ is $(0, 8)$
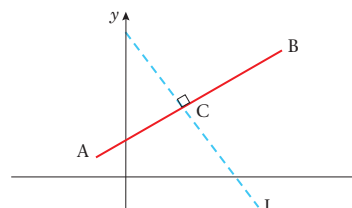
d ΔABD
$CD = \sqrt{2^2 + 1^2} = \sqrt{5}$
$AB = \sqrt{(4 - -1)^2 + (11 - 1)^2} = \sqrt{5^2 + 10^2} = 5\sqrt{5}$
.... Area of $\Delta ABD = \frac{1}{2} AB.CD = \frac{1}{2} \times \sqrt{5} \times 5\sqrt{5}$
$$= \frac{25}{2}$$

12 a $y = \tfrac{1}{3}x + \tfrac{1}{3}$

Chapter 9
9.1
Exercise 9A

1 $7x^6$	**2** $8x^7$	**3** $4x^3$
4 $\tfrac{1}{3}x^{-\frac{2}{3}}$	**5** $\tfrac{1}{4}x^{-\frac{4}{3}}$	**6** $\tfrac{1}{3}x^{-\frac{2}{3}}$
7 $-3x^{-4}$	**8** $-4x^{-5}$	**9** $-2x^{-3}$
10 $-5x^{-6}$	**11** $-\tfrac{1}{3}x^{-\frac{4}{3}}$	**12** $-\tfrac{1}{2}x^{-\frac{3}{2}}$
13 $-2x^{-3}$	**14** 1	**15** $3x^2$
16 $9x^8$	**17** $5x^4$	**18** $3x^2$

9.2
Exercise 9B

1 a $4x^3 - x^{-2}$ **b** $-x^{-3}$ **c** $-x^{-\frac{3}{2}}$
2 a 0 **b** $11\tfrac{1}{2}$
3 a $(2\tfrac{1}{2}, -6\tfrac{1}{4})$ **b** $(4, -4)$ and $(2, 0)$
   c $(16, -31)$ **d** $(\tfrac{1}{2}, 4), (-\tfrac{1}{2}, -4)$

4 a $x^{-\frac{1}{2}}$ **b** $-6x^{-3}$ **c** $-x^{-4}$

d $\frac{4}{3}x^3 - 2x^2$ **e** $-6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$

f $\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$ **g** $-3x^{-2}$ **h** $3 + 6x^{-2}$

i $5x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$ **j** $3x^2 - 2x + 2$

k $12x^3 + 18x^2$ **l** $24x - 8 + 2x^{-2}$

5 a 1 **b** $\frac{2}{9}$ **c** -4 **d** 4

9.3
Exercise 9C

1 a $y = e^{2x} \Rightarrow \dfrac{dy}{dx} = 2e^{2x}$

b $y = e^{-6x} \Rightarrow \dfrac{dy}{dx} = -6e^{-6x}$

c $y = e^x + 3x^2 \Rightarrow \dfrac{dy}{dx} = e^x + 6x$

d $y = \sin 2x \Rightarrow \dfrac{dy}{dx} = 2\cos 2x$

e $y = \cos 3x \Rightarrow \dfrac{dy}{dx} = -3\sin 3x$

f $y = 3\sin 4x + 4\cos 3x \Rightarrow \dfrac{dy}{dx} =$
$12\cos 4x - 12\sin 3x$

2 a $y = \sin 5x \Rightarrow \dfrac{dy}{dx} = 5\cos 5x$

b $y = 2\sin\frac{1}{2}x \Rightarrow \dfrac{dy}{dx} = 2 \times \frac{1}{2}\cos\frac{1}{2}x = \cos\frac{1}{2}x$

c $y = \sin 8x \Rightarrow \dfrac{dy}{dx} = 8\cos 8x$

d $y = 6\sin\frac{2}{3}x \Rightarrow \dfrac{dy}{dx} = 6 \times \frac{2}{3}\cos\frac{2}{3}x = 4\cos\frac{2}{3}x$

e $y = 2\cos x \Rightarrow \dfrac{dy}{dx} = -2\sin x$

f $y = 6\cos\frac{5}{6}x \Rightarrow \dfrac{dy}{dx} = 6 \times \frac{5}{6}\sin\frac{5}{6}x = -5\sin\frac{5}{6}x$

g $y = \cos 4x \Rightarrow \dfrac{dy}{dx} = -4\sin 4x$

h $y = 4\cos\left(\frac{x}{2}\right) \Rightarrow \dfrac{dy}{dx} = -4 \times \frac{1}{2}\sin\frac{x}{2} = -2\sin\frac{x}{2}$

3 $y = 2e^{-x} \Rightarrow \dfrac{dy}{dx} = -2e^{-x}$ @ $(0, 2)$

4 $y = 3\sin x \Rightarrow \dfrac{dy}{dx} = 3\cos x$ @ $\left(\frac{\pi}{3} = x\right)$

Remember x is in radians

$\therefore m = 3\cos\left(\frac{\pi}{3}\right) = \frac{3}{2}$ $\left[\cos\frac{\pi}{3} = \cos 60° = \frac{1}{2}\right]$

5 $y = 4\cos 2x \Rightarrow \dfrac{dy}{dx} = -8\sin 2x$ @ $\left(x = \frac{\pi}{4}\right)$

$\therefore m = -8\sin\left(2 \times \frac{\pi}{4}\right)$

$= -8\sin\frac{\pi}{4}$ $\left[\sin\frac{\pi}{2} = \sin 90° = 1\right]$

$= -8$

9.4
Exercise 9D

1 a $y = (1 + 2x)^4 \Rightarrow y' = 4(1 + 2x)^3 \times 2 = 8(1 + 2x)^3$

b $y = (1 + x^2)^3 \Rightarrow y' = 3(1 + x^2)^2 \times 2x = 6x(1 + x^2)^2$

c $y = (3 + 4x)^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}(3 + 4x)^{-\frac{1}{2}} \times 4 = \dfrac{2}{\sqrt{3 + 4x}}$

d $y = (x^2 + 2x)^3 \Rightarrow y' = 3(x^2 + 2x)^2 \times (2x + 2)$
$= 6(x + 1)(x^2 + 2x)^2$

2 a $y = 4e^{3x^2} \Rightarrow y' = 4e^{3x^2} \times 6x = 24x\,e^{3x^2}$
$8(1 + 2x)^3$

b $y = 9e^{3-x} \Rightarrow y' = 9e^{3-x} \times -1 = -9e^{3-x}$

c $y = e^{-6x} \Rightarrow y' = e^{-6x} \times -6 = -6e^{-6x}$

d $y = e^{x^2 + 2x} \Rightarrow y' = e^{x^2 + 2x} \times (2x + 2)$
$= 2(x + 1)\,e^{x^2 + 2x}$

3 a $y = \sin(2x + 1) \Rightarrow y' = \cos(2x + 1) \times 2 =$
$2\cos(2x + 1)$

b $y = \cos(2x^2 + 4) \Rightarrow y' = -\sin(2x^2 + 4) \times 4x \Rightarrow$
$-4x\sin(2x^2 + 4)$

c $y = \sin^3 x \Rightarrow y' = 3\sin^2 \times \cos x$
$3\sin^2 x \cos x$

d $y = \cos^2 2x \Rightarrow y' = 2\cos 2x \times (-\sin 2x) \times 2$
$= -4\sin 2x \cos 2x$

4 a $y = x(1 + 3x)^5 \Rightarrow y' = (1 + 3x)^5 + x.5(1 + 3x)^4 \times 3$
$= (1 + 3x)^4[1 + 3x + 15x]$
$= (1 + 3x)^4(1 + 18x)$

b $y = 2x(1 + 3x^2)^3 \Rightarrow y' = 2(1 + 3x)^3$
$+ 2x \times 3(1 + 3x^2)^3 \times 6x$
$= 2(1 + 3x^2)^2[1 + 18x^2]$

c $y = x^3(2x + 6)^4 \Rightarrow y' = 3x^2(2x + 6)^4$
$+ x^3 \times 4(2x + 6)^3 \times 2$
$= x^2(2x + 6)^3[6x + 18 + 8x]$
$= 2x^2(2x + 6)^3(7x + 9)$

5 a $y = xe^{2x} \Rightarrow y' = e^{2x} + x.2\,e^{2x} = e^{2x}(1 + 2x)$

b $y = (x^2 + 3)\,e^{-x} \Rightarrow y' = 2x \times e^{-x} + (x^2 + 3)(-e^{-x})$
$= e^{-x}(2x - x^2 - 3)$

c $y = (3x - 5)\,e^{x^2} \Rightarrow y' = 3 \times e^{x^2} + (3x - 5)e^{x^2} \times 2x$
$= e^{x^2}(6x^2 - 10x + 3)$

6 a $y = x\sin x \Rightarrow y' = \sin x + x\cos x$

b $y = \sin^{2x}\cos x$
$\Rightarrow y' = 2\sin x \cos x \times \cos x + \sin^2 x(-\sin x)$

$$y' = \sin x \,(2\cos^2 x - \sin^2 x)$$

c $\quad y = e^x \cos x \Rightarrow y' = e^x \cos x - e^x \sin x$
$$= e^x (\cos x - \sin x)$$

7 a $\quad y = \dfrac{5x}{x+1} \Rightarrow y' \quad = \dfrac{(x+1) \times 5 - 5x \times 1}{(x+1)^2}$
$$= \dfrac{5x + 5 - 5x}{(x+1)^2}$$
$$= \dfrac{5}{(x+1)^2}$$

b $\quad y = \dfrac{2x}{3x-2} \Rightarrow y' \quad = \dfrac{(3x-2) \times 2 - 2x \times 3}{(3x-2)^2}$
$$= \dfrac{6x - 4 - 6x}{(3x-2)^2}$$
$$= \dfrac{-4}{(3x-2)^2}$$

c $\quad y = \dfrac{3x^2}{(2x-1)^2} \Rightarrow y'$
$$= \dfrac{(2x-1)^2 \times 6x - 3x^2 \times 2(2x-1) \times 2}{(2x-1)^4}$$
$$= \dfrac{6x \,(2x-1)[2x - 1 - 2x]}{(2x-1)^4}$$
$$= \dfrac{-6x}{(2x-1)^3}$$

8 a $\quad y = \dfrac{x}{e^{2x}} \Rightarrow y' \quad = \dfrac{e^{2x} \times 1 - x \times 2e^{2x}}{e^{4x}} = \dfrac{1 - 2x}{e^{2x}}$

b $\quad y = \dfrac{e^x}{x+1} \Rightarrow y' \quad = \dfrac{(x+1)e^x - e^x \times 1}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$

c $\quad y = \dfrac{e^{x^2}}{x} \Rightarrow y' \quad = \dfrac{x \times 2xe^{x^2} - e^{x^2} \times 1}{x^2} = \dfrac{e^{x^2}(2x^2 - 1)}{x^2}$

9 a $\quad y = \dfrac{\sin x}{x} \Rightarrow y' \quad = \dfrac{x \cos x - \sin x \times 1}{x^2}$
$$= \dfrac{x \cos x - \sin x}{x^2}$$

b $\quad y = \dfrac{e^x}{\cos x} \Rightarrow y' \quad = \dfrac{\cos x \times e^x + e^x \sin x}{\cos^2 x}$
$$= \dfrac{e^x(\sin x + \cos x)}{\cos^2 x}$$

c $\quad y \dfrac{\sin^2 x}{e^2 x} \Rightarrow y' \quad = \dfrac{e^{2x} \times 2 \sin x \cos x - \sin^2 x \times 2e^{2x}}{e^{4x}}$
$$= \dfrac{2 \sin x e^{2x}\,(\cos x - \sin x)}{e^{4x}}$$
$$= \dfrac{2 \sin x(\cos x - \sin x)}{e^{2x}}$$

10 a $\quad y = x^2 \,(3x-1)^3 \Rightarrow y' = 2x\,(3x-1)^3$
$$+ x^2 \times 3\,(3x-1)^2 \times 3 \text{ when } x = 1$$
$$m = 2 \times 2^3 + 1 \times 9 \times 2^2 = 16 + 36 = 52$$

b $\quad y = (2x+3)e^{2x} \Rightarrow y' = 2(2x+3)e^{2x} + 2 \times e^{2x}$
$$\text{when } x = 0$$
$$m = 6e^0 + 2e^0 = 8$$

c $\quad y = 3 \sin^2 x \Rightarrow y' = 6 \sin x \cos x \text{ when } x = \dfrac{\pi}{4}$
$$m = 6 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = 3$$

d $\quad y = x \cos x \Rightarrow y' = \cos x - x \sin x \text{ when } x = \dfrac{\pi}{2}$
$$m = \cos \dfrac{\pi}{2} - \dfrac{\pi}{2} \sin \dfrac{\pi}{2} = 0 - \dfrac{\pi}{2} \times 1$$
$$= -\dfrac{\pi}{2}$$

9.5
Exercise 9E

1 a $\quad y + 3x - 6 = 0$
b $\quad 4y - 3x - 4 = 0$
c $\quad y = x$
2 a $\quad 7y + x - 48 = 0 \qquad$ **b** $\quad 17y + 2x - 212 = 0$
3 $\quad y = -8x + 10,\, 8y - x - 145 = 0$
4 $\quad y = 2ex - \dfrac{e}{2}$
5 $\quad y = \dfrac{1}{3}e$
6 $\quad y = x \sin x \quad \Rightarrow y' = \sin x + x \cos x @ (\pi, 0)$
$$\therefore m = \sin \pi + \pi \cos \pi = -\pi$$
$\therefore$ equation of tangent:
$$y - 0 = -\pi\,(x - \pi)$$
or $\qquad y = \pi^2 - x\pi$
7 $\quad y = 2 \cos^2 x \Rightarrow y' = 4 \cos x \,(-\sin x) @ (\tfrac{\pi}{4}, 1)$
$$\therefore \qquad m = -4.\dfrac{1}{\sqrt{2}}.\dfrac{1}{\sqrt{2}} = -2$$
normal has gradient $= \dfrac{1}{2}$
$\therefore$ equation of normal is:
$$y - 1 = \dfrac{1}{2}\left(x - \dfrac{\pi}{4}\right)$$
or $\qquad 8y - 8 = 4x - \pi$
or $\qquad 8y - 4x = 8 - \pi$

9.6
Exercise 9F

1 $\quad \dfrac{1}{4}x^4 + x^2 + c$
2 $\quad -2x^{-1} + 3x + c$

3 $2x^{\frac{5}{2}} - x^3 + c$

4 $\frac{4}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$

5 $x^4 + x^{-3} + rx + c$

6 $t^3 + t^{-1} + c$

7 $\frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$

8 $\frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$

9 $\dfrac{p}{5x^5} + 2tx - 3x^{-1} + c$

10 $\frac{p}{4}t^4 + q^2t + px^3t + c$

11 a $\frac{1}{2}x^4 + x3 + c$

   b $2x - \dfrac{3}{x} + c$

   c $\frac{4}{3}x^3 + 6x^2 + 9x + c$

   d $\frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$

   e $\frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

12 a $\int 2\sin 3x \,\mathrm{d}x \quad = -\frac{2}{3}\cos 3x + c$

   b $\int 3\mathrm{e}^{4x}\,\mathrm{d}x \quad = \frac{3}{4}\mathrm{e}^{4x} + c$

   c $\int 2\cos 3x \,\mathrm{d}x \quad = \frac{2}{3}\sin 3x + c$

   d $\int 2\mathrm{e}^{-x}\,\mathrm{d}x \quad = -2\mathrm{e}^{-x} + c$

13 a $5\mathrm{e}^x + 4\cos x + \dfrac{x^4}{2} + c$

   b $-2\cos x - 2\sin x + x^2 + C$

   c $5\mathrm{e}^x + 4\sin x + \dfrac{2}{x} + c$

   d $\mathrm{e}^x - \cos x + \sin x + C$

9.7

Exercise 9G

1 $10t$

2 $48 - 32t$

3 a $40 + 10t$ **b** 70 m/s

4 a $30 - 1t$ **b** 0 m/s

5 a $a = 32$

   b $v = 32t + 100$
$\Rightarrow s = 16t^2 + 100t + d$; when $t = 0, s = 0 \Rightarrow d = 0$
$\therefore \quad s = 16t^2 + 100t$

6 a $a = -32$

   b $v = 160 - 32t$
$\Rightarrow s = 160t - 16t^2 + d$;
$t = 0, s = 384 \Rightarrow d = 384$
$\therefore \quad s = 384 + 160t - 16t^2$

   c $s = 0 \Rightarrow 16(t^2 - 10t - 24) = 0$
i.e. $16(t - 12)(t + 2) = 0$
$\therefore$ passes through origin when $t = 12$

7 a $3t^2 + 8t - 5$ **b** $6t + 8$

   c $v = 6$ m/s, $a = 14$ m/s^2

8 a $v = 3t^2 - 4t + 3$ **b** $a = 6t - 4$

9 a $a = 2t + 10$ **b** $a = 14$ m/s^2

   c $v = t^2 + 10t + 5$
$\Rightarrow s = \frac{t^3}{3} + 5t^2 + 5t + d$; $s = 0$ when $t = 0 \Rightarrow d = 0$
$\therefore$ when $t = 2$ $s = \frac{8}{3} + 20 + 10 = 32\frac{2}{3}$

10 a $6 - 2t$ **b** 2 m/s

   c $v = 24 + 6t - t^2$
$\Rightarrow \quad s = 24t + 3t^2 - \dfrac{t^3}{3} + d$
$s = 100, t = 3 \Rightarrow 100 = 24 \times 3 + 3 \times 9 - \dfrac{27}{3} + d$
i.e. $100 = 72 + 27 - 9 + d \quad \therefore d = 10$
$\therefore \quad s = 10 + 24t + 3t^2 - \dfrac{t^3}{3}$

9.8

Exercise 9H

1 a -28 **b** -17 **c** $-\frac{1}{5}$

2 a 10 **b** 4 **c** 12.25

3 a $\left(-\frac{3}{4}, -\frac{9}{4}\right)$ **b** $\left(\frac{1}{2}, 9\frac{1}{4}\right)$

   c $\left(-\frac{1}{3}, 1\frac{5}{27}\right), (1, 0)$ **d** $(3, -18), \left(-\frac{1}{3}, \frac{14}{27}\right)$

   e $(1, 2), (-1, -2)$ **f** $(3, 27)$

5 $\left(\dfrac{3\pi}{8}, \dfrac{1}{\sqrt{2}}\,\mathrm{e}^{\frac{3\pi}{4}}\right)$ maximum, $\left(\dfrac{7\pi}{8}, -\dfrac{1}{\sqrt{2}}\,\mathrm{e}^{\frac{7\pi}{4}}\right)$ minimum

9.9

Exercise 9I

1 a 8 **b** $9\frac{3}{4}$

   c $19\frac{2}{3}$ **d** 21

   e $8\frac{5}{12}$

2 a i $2\ln 2$ **ii** 2π

   b i $\frac{8}{3}$ **ii** $\frac{64}{15}\pi$

3 a $A(1, 3), B(3, 3)$ **b** $1\frac{1}{3}$

4 $6\frac{2}{3}$

5 a $(2, 12)$ **b** $13\frac{1}{3}$

6 $3\frac{3}{8}$

9.10

Exercise 9J

1 $\frac{8}{9}\pi$

2 6π

3 $15\mathrm{e}^2$

4 $y = 5x^4 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} \quad = 20x^3$
$\therefore \delta y \approx 20x^3\,\delta x \quad = 20x^3 \times \dfrac{x}{200} \quad \left[0.5\% \text{ of } x = \dfrac{x}{200}\right]$

i.e. $\delta y = \dfrac{20x^4}{200} = \dfrac{x^4}{10}$

$\therefore$ % change in $y = \dfrac{\delta y}{y} \times 100 = \dfrac{x^4}{10 \times 5x^4} \times 100 = 2\%$

5 $y = 3x^2 \Rightarrow \dfrac{dy}{dx} = 6x$

$\therefore \delta y \approx 6x\,\delta x = 6x \times \dfrac{x}{100} = \dfrac{6x^2}{100}$

$\therefore$ % change in $y = \dfrac{\delta y}{y} \times 100 = \dfrac{6x^2}{3x^2 \times 100} \times 100$

$= 2\%$

6 For a sphere: $V = \dfrac{4}{3}\pi r^3 = \dfrac{dv}{dr} = 4\pi r^2$

$\delta r \approx 0.02$ cm

$\delta v \approx 4\pi r^2\,dr$

Use $r = 1 \Rightarrow$

$\delta v \approx 4\pi \times 0.02$

$= 0.25$ cm^3

Exercise 9K

1 $20\,\text{m} \times 40\,\text{m}$; $800\,\text{m}^2$

2 2000π cm^2

3 40 cm

4 $\dfrac{800}{4 + \pi}$ cm^2

5 $27\,216$ mm^2

Mixed Exercise 9L

1 a $x = 4, y = 20$

b $\dfrac{d^2y}{dx^2} = \dfrac{15}{8} > 0 \therefore$ minimum

2 $(1, -11)$ and $(\tfrac{7}{3}, -12\tfrac{5}{27})$

3 a $7\tfrac{31}{32}$

b $\dfrac{x^3}{3} - 2x - \dfrac{1}{x} - 2\tfrac{2}{3}$

c $f'(x) = \left(\dfrac{x-1}{x}\right)^2 > 0$ for all values of x

4 $(1, 4)$

5 a $y = 1 - \dfrac{x}{2} - \dfrac{\pi x}{4}$

c $\dfrac{2}{4 + \pi}$ m^2 $(0.280\,\text{m}^2)$

6 b $\dfrac{10}{3}$

c $\dfrac{d^2v}{dx^2} < 0 \therefore$ maximum

d $\dfrac{2300\pi}{27}$

e $22\tfrac{2}{9}\%$

7 a $\dfrac{250}{x^2} - 2x$

b $(5, 125)$

8 b $x = \pm2\sqrt{2}$, or $x = 0$

c $OP = 3$; $f''(x) > 0$ so minimum when $x = \pm2\sqrt{2}$ (maximum when $x = 0$

9 b A is $(-1, 0)$; B is $(\tfrac{5}{3}, 9\tfrac{13}{27})$

10 $3x^2 \cos 3x + 2x \sin 3x$

12 $\dfrac{x \cos x - \sin x}{x^2}$

13 b $y = 2x + 1$

14 a $-(x^3 - 2x)e^x + (3x^2 - 2)e^x$

15 $\dfrac{56\pi}{5}$

Chapter 10

10.1

Exercise 10A

1 a $9°$ **b** $12°$ **c** $75°$
d $90°$ **e** $140°$ **f** $210°$
g $225°$ **h** $270°$ **i** $540°$

2 a $26.4°$ **b** $57.3°$ **c** $65.0°$
d $99.2°$ **e** $143.2°$ **f** $179.9°$
g $200°$

3 a 0.479 **b** 0.156 **c** 1.74
d 0.909 **e** -0.897

4 a $\dfrac{2\pi}{45}$ **b** $\dfrac{\pi}{18}$ **c** $\dfrac{\pi}{8}$
d $\dfrac{\pi}{6}$ **e** $\dfrac{\pi}{4}$ **f** $\dfrac{\pi}{3}$
g $\dfrac{5\pi}{12}$ **h** $\dfrac{4\pi}{9}$ **i** $\dfrac{5\pi}{8}$
j $\dfrac{2\pi}{3}$ **k** $\dfrac{3\pi}{4}$ **l** $\dfrac{10\pi}{9}$
m $\dfrac{4\pi}{3}$ **n** $\dfrac{3\pi}{2}$ **o** $\dfrac{7\pi}{4}$
p $\dfrac{11\pi}{6}$

5 a 0.873 **b** 1.31 **c** 1.75
d 2.79 **e** 4.01 **f** 5.59

10.2

Exercise 10B

1 a **i** 2.7 **ii** 2.025 **iii** $7.5\pi\,(23.6)$
b **i** $16\tfrac{2}{3}$ **ii** 1.8 **iii** 3.6
c **i** $1\tfrac{1}{3}$ **ii** 0.8 **iii** 2

2 $\dfrac{10\pi}{3}$ cm

3 2π

4 $5\sqrt{2}$ cm

5 a 10.4 cm **b** $1\tfrac{1}{4}$

6 7.5

7 0.8

8 a $\dfrac{\pi}{3}$ **b** $\left(6 + \dfrac{4\pi}{3}\right)$ cm

9 6.8 cm

10 a $(R - r)$ cm **c** 2.43

10.3
Exercise 10C

1 a 19.2 cm^2 **b** $6.75\pi \text{ cm}^2$
   c $1.296\pi \text{ cm}^2$ **d** 38.3 cm^2
   e $5\frac{1}{3}\pi \text{ cm}^2$ **f** 5 cm^2
2 a 4.47 **b** 3.96
   c 1.98
3 12 cm^2
4 b 120 cm^2
5 $40\frac{2}{3} \text{ cm}$
6 a 12 **c** 1.48 cm^2
7 8.88 cm^2
8 a 1.75 cm^2 **b** 25.9 cm^2 **c** 25.9 cm^2
9 4.5 cm^2
10 b 28 cm
11 78.4 cm
12 b 28 cm

10.4
Exercise 10D

1 a $\dfrac{\sqrt{2}}{2}$ **b** $-\dfrac{\sqrt{3}}{2}$ **c** $-\dfrac{1}{2}$

   d $\dfrac{\sqrt{3}}{2}$ **e** $\dfrac{\sqrt{3}}{2}$ **f** $-\dfrac{1}{2}$

   g $\dfrac{1}{2}$ **h** $-\dfrac{\sqrt{2}}{2}$ **i** $\dfrac{-\sqrt{3}}{2}$

   j $-\dfrac{\sqrt{2}}{2}$ **k** -1 **l** -1

   m $\dfrac{\sqrt{3}}{3}$ **n** $-\sqrt{3}$ **o** $\sqrt{3}$

10.5
Exercise 10E

1 a $x = 84, y = 6.32$
   b $x = 13.5, y = 16.6$
   c $x = 85, y = 13.9$
   d $x = 80, y = 6.22$ (Isosceles $\triangle$)
   e $x = 6.27, y = 7.16$
   f $x = 4.49, y = 7.49$ (right-angled
2 a 48.1 **b** 45.6 **c** 14.8
   d 48.7 **e** 86.5 **f** 77.4
3 a $x = 74.6, y = 65.4$
      $x = 105, y = 34.6$
   b $x = 59.8, y = 48.4$
      $x = 120, y = 27.3$
   c $x = 56.8, y = 4.37$

      $x = 23.2, y = 2.06$
4 a 3.19 cm **b** $1.73 \text{ cm} (\sqrt{3} \text{ cm})$
   c 9.85 cm **d** 4.31 cm
   e 6.84 cm (isosceles)   **f** 9.80 cm
5 a $108(.2)°$ **b** $90°$ **c** $60°$
   d $52.6°$ **e** $137°$ **f** $72.2°$
6 a 23.7 cm^3 **b** 4.31 cm^3 **c** 20.2 cm^3
7 a $155°$ **b** 13.7 cm
8 a $x = 49.5$, area $= 1.37 \text{ cm}^2$
   b $x = 55.2$, area $= 10.6 \text{ cm}^2$
   c $x = 117$, area $= 6.66 \text{ cm}^2$
9 6.50 cm^2
10 a 36.1 cm^3 **b** 12.0 cm^3

10.6
Exercise 10F

1 a 11.7 cm **b** 14.2 cm **c** $34.4°$
   d $63.4°$
2 a 18.6 cm **b** 28.1 cm **c** $48.6°$
3 a 14.1 cm **b** 17.3 cm **c** $35.4°$
4 a 28.3 cm **b** 34.6 cm **c** $35.1°$
   c $19.5°$
5 a 4.47 m **b** 4.58 m **c** $29.2°$
   d $12.6°$ **e** $26.6°$
6 a 407 m **b** 402 m **c** $8.57°$
   c $13.3°$
7 a 43.3 cm **b** 68.7 cm **c** 81.2 cm
8 a 28.9 cm **b** 75.7 cm **c** $22.4°$
9 a 16.2 cm **b** $67.9°$ **c** 55.3 cm^2
   d $71.6°$
10 a 26.5 cm **b** $61.8°$ **c** 1530 cm^2
11 a $30.3°$ **b** $31.6°$ **c** $68.9°$
12 a $36.9°$ **b** 828 cm^2
13 a 15 m **b** $47.7°$ **c** €91 300
14 a $66.4°$ **b** $32.9°$
15 46.5 m
16 a $OW = 4290 \text{ m}$, $OS = 2760 \text{ m}$
   b $36.0°$ **c** 197 km/h

10.7
Exercise 10G

1 a $\dfrac{\sin^2 \theta}{2}$ **b** 5 **c** $-\cos^2 A$
   d $\cos \theta$ **e** $\tan x^0$ **f** $\tan 3A$
   g 4 **h** $\sin^2 \theta$ **i** 1
2 a LHS $= \sin^2 \theta + \cos^2 \theta + 2\sin\theta\cos\theta$
          $= 1 + 2\sin\theta\cos\theta$
          $= $ RHS

b LHS $= \dfrac{1 - \cos^2 \theta}{\cos \theta} = \dfrac{\sin^2 \theta}{\cos \theta} = \sin \theta \times \dfrac{\sin \theta}{\cos \theta}$

$= \sin \theta \tan \theta =$ RHS

c LHS $= \dfrac{\sin x°}{\cos x°} + \dfrac{\cos x°}{\sin x°} = \dfrac{\sin^2 x° + \cos^2 x°}{\sin x° \cos x°}$

$= \dfrac{1}{\sin x° \cos x°} =$ RHS

d LHS $= \cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1$

$= 2(1 - \sin^2 A) - 1 = 1 - 2 \sin^2 A =$ RHS

e LHS $= (4 \sin^2 \theta - 4 \sin \theta \cos \theta + \cos^2 \theta)$

$\quad + (\sin^2 \theta + 4 \sin \theta \cos \theta + 4 \cos^2 \theta)$

$= 5(\sin^2 \theta + \cos^2 \theta) = 5 =$ RHS

f LHS $= 2 - (\sin 2\theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$

$= 2(\sin^2 \theta + \cos 2\theta)$

$\quad - (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta)$

$= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$

$= (\sin \theta + \cos \theta)^2 =$ RHS

g LHS $= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$

$= \sin^2 x - \sin^2 y =$ RHS

3 a $\sin 35°$ **b** $\sin 35°$ **c** $\cos 210°$

   d $\tan 31°$ **e** $\cos \theta$ **f** $\cos 7\theta$

   g $\sin 3\theta$ **h** $\tan 5\theta$ **i** $\sin A$

   j $\cos 3x$

4 a 1 **b** 0 **c** $\dfrac{\sqrt{3}}{2}$

   d $\dfrac{\sqrt{2}}{2}$ **e** $\dfrac{\sqrt{2}}{2}$ **f** $-\dfrac{1}{2}$

   g $\sqrt{3}$ **h** $\dfrac{\sqrt{3}}{3}$ **i** 1

   j $\sqrt{2}$

5 a LHS $= \sin A \cos 60° + \cos A \sin 60°$

$\quad + \sin A \cos 60° - \cos A \sin 60°$

$= 2 \sin A \cos 60°$

$= 2 \sin A \left(\tfrac{1}{2}\right) = \sin A =$ R.H.S.

b LHS $= \dfrac{\cos A \cos B - \sin A \sin B}{\sin B \cos B} = \dfrac{\cos(A + B)}{\sin B \cos B}$

$=$ R.H.S.

c LHS $= \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}$

$= \dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}$

$= \tan x + \tan y =$ R.H.S.

d LHS $= \dfrac{\cos x \cos y - \sin x \sin y}{\sin x \sin y} + 1$

$= \cot x \cot y - 1 + 1 = \cot x \cot y =$ R.H.S.

e LHS $= \cos \theta \cos \dfrac{\pi}{3} - \sin \theta \sin \dfrac{\pi}{3} + \sqrt{3} \sin \theta$

$= \tfrac{1}{2} \cos \theta - \dfrac{\sqrt{3}}{2} \sin \theta + \sqrt{3} \sin \theta$

$= \tfrac{1}{2} \cos \theta - \dfrac{\sqrt{3}}{2} \sin \theta$

$= \sin \dfrac{\pi}{6} \cos \theta + \cos \dfrac{\pi}{6} \sin \theta$

$= \sin\left(\dfrac{\pi}{6} + \theta\right) =$ R.H.S.

10.8
Exercise 10H

1 a $270°$ **b** $60°, 240°$

   c $60°, 300°$ **d** $15°, 165°$

   e $140°, 220°$ **f** $135°, 315°$

   g $90°, 270°$ **h** $230°, 310°$

   i $45.6°, 134.4°$ **j** $135°, 225°$

2 a $-120, -60, 240, 300$ **b** $-171, -8.63$

   c $-144, 144$ **d** $-327, -32.9$

   e $150, 330, 510, 690$ **f** $251, 431$

3 a $-\pi, 0, \pi, 2\pi$ **b** $-\dfrac{4\pi}{3}, -\dfrac{2\pi}{3}, \dfrac{2\pi}{3}$

   c $-\dfrac{7\pi}{4}, -\dfrac{5\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ **d** $-0.14, 3.00, 6.14$

4 a $0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°$

   b $60°, 180°, 300°$

   c $22\tfrac{1}{2}°, 112\tfrac{1}{2}°, 202\tfrac{1}{2}°, 292\tfrac{1}{2}°$

   d $30°, 150°, 210°, 330°$

   e $300°$ **f** $225°, 315°$

   g $90°, 270°$ **h** $50°, 170°$

   i $165°, 345°$

5 a $-\dfrac{7\pi}{12}, -\dfrac{\pi}{12}$ **b** $1.48, 5.85$

10.9
Exercise 10I

1 a $30°, 210°$

   b $135°, 315°$

2 a $\pi, 2\pi$

   b $0.59, 3.73$

3 a $60°, 120°, 240°, 300°$

   b $0°, 180°, 199°, 341°, 360°$

   c $60°, 300°$

   d $30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$

   e $270°$

   f $0°, 18.4°, 180°, 198°, 360°$

   g $194°, 270°, 346°$

4 a $\dfrac{5\pi}{12}, \dfrac{11\pi}{12}, \dfrac{17\pi}{12}, \dfrac{23\pi}{12}$

   b $0.841, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 5.44$

   c $4.01, 5.41$

Mixed Exercise 10J

1 **a** $\cos^2 \theta \sin^2 \theta$

    b $\sin^4 3\theta$

    c 1

2 **a** 1 **b** $\tan y = \dfrac{4 + \tan x}{2 \tan x - 3}$

3 **a** $2 \sin 2\theta = \cos 2\theta \Rightarrow 2 \sin 2\theta \setminus \cos 2\theta = 1$

        $\Rightarrow 2 \tan 2\theta = 1 \Rightarrow \tan 2\theta = 0.5$

    b $13.3, 103.3, 193.3, 283.3$

4 **a** $225, 345$

    b $22.2, 67.8, 202.2, 247.8$

5 **a** $\dfrac{11\pi}{12}, \dfrac{23\pi}{12}$ **b** $\dfrac{2\pi}{3}, \dfrac{5\pi}{6}, \dfrac{5\pi}{3}, \dfrac{11\pi}{6}$

6 $0°, 131.8°, 228.2°$

7 $0, \pi, 2\pi$

8 **a** Max $1, \theta = 100°$; Min $= -1, \theta = 280°$

    b Max $1, \theta = 330°$; Min $= -1, \theta = 150°$

9 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\sqrt{3}\setminus 3$

    b $23.8°, 203.8°$

Review exercise

1 **a** $A = 5, B = -\frac{5}{2}, C = -28\frac{1}{4}$

    b $f_{\min} = -\frac{113}{4}, x = \frac{5}{2}$

2
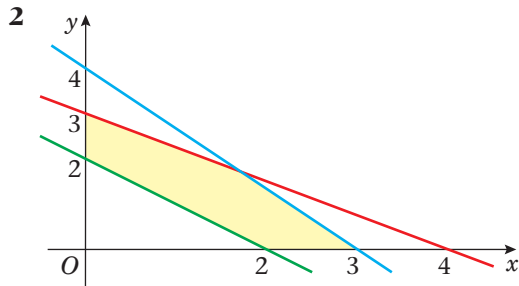
3 **a** 3 **b** $2\frac{1}{2}$ **c** $125\frac{1}{2}$

4 814

5 **a** –   **b** $P = 126.8°, Q = R = 26.6°$

6 **a** **i** $\frac{1}{4}\mathbf{b} - \mathbf{a}$ **ii** $\frac{1}{3}\mathbf{a} + \frac{1}{6}\mathbf{b}$ **iii** $\frac{1}{3}\mathbf{a} - \frac{5}{6}\mathbf{b}$

    b $\frac{2}{5}$ **c** $\frac{2}{3}$

7 **a** –   **b** $54.5°, 234.5°$

8 $4\frac{1}{2}$ m

9 $21.8°, 38.2°, 120°$

10 **a** $3.18, 6.69, 13.04$ **b** –

    c 2.3 **d** 0.6 (0.5 acceptable)

11 $x^6 - 18x^4 + 135x^2$

12 $-3 < p < 2$

13 –

14 **a** $(3.14), (5, 24)$ **b** $1\frac{1}{3}$

15 **a** –   **b** 12 cm   **c** $6\sqrt{3}$

    d $54.7°$ **e** $109.5°$

16 $\frac{1}{2}x\%$

17 **a–b** $(-3, -\frac{5}{3}), (2, \frac{5}{2})$ **c** $y = -\frac{6}{5}(x - 1)$

    d –

18 **a** $0.253, 2.89$ **b** $1.11, 2.68$

    c $1.91, 2.30$

19 **a** 625 **b** $2, 64$

    c $x = 2, y = 3$

20 –

21 $82.8°$

22 **a** $8\mathbf{i} - \mathbf{j}$ **b** –

23 $449\frac{2}{5}\pi$

24 **a** $\cos 2\theta = 2 \cos^2 \theta - 1$ **b** $\sin 2\theta = 2 \sin \theta \cos \theta$

    c –   **d** $0.767, 1.33, 2.86$ **e** 4

25 **a** -6 **b** 50 **c** 17

26 **a** $(2, 4), (5, 16)$ **b** $x \leqslant 2, x \geqslant 5$

27 **a** $-\mathbf{i} - 11\mathbf{j}$ **b** $\frac{13}{5}\mathbf{i} - \frac{2}{5}\mathbf{j}$

28 $73.9°$

29 **a** –   **b** $12\,791$

30 $15\,200$

31 **a** $r = \frac{1}{2}, r = -3$ **b** 10

32 $46.5°, 133.5°$

33 $\frac{1}{2}$

34 **a** $2y + x = 25$ **b** $(25, 0)$ **c** $(10, 0)$

35 **a** 4 m/s^2 **b** $25\frac{1}{3}$ m

36 **a** $4y = x + 23$ **b** $y = -4x + 26$

    c $(-3, 38)$ **d** $6\frac{16}{17}$

    e $12\frac{28}{51}$

37 **a** $-2p + q = 28, 3p + q = 18$

    b $-2, 24$

    c $(x + 2)(x - 3)(x - 4)$

    d –

    e $-\frac{12}{5}, 2$

    f $7\frac{7}{12}$

38 –

39 **a** $2y = 3x - 18$ **b** $3y = -2x + 51$

    c 156 **d** 216π

40 **a** $1 + \dfrac{x}{2p} - \dfrac{x^2}{8p^2} + \dfrac{x^3}{16p^3}$

    b $p = \pm\frac{1}{2}$

41 **a** –   **b** –   **c** 4.76

    d –   **e** 408

42 **a** –   **b** 4000

43 **a** $1 - 2x - 4x^2$ **b** 2.76132

    c 0.087% **d** $a = 1, b = -5, c = 8$

e $|x| < \frac{1}{6}$

44 $2y = x - 2$

45 $\pi[\frac{1}{4}e^8 + 4e^4 + 27\frac{3}{4}]$

46 a –   **b** –
    c 15, 75, 105, 165   **d** $\frac{3}{8}$

47 a **i** $y = 2$ **ii** $x = -1$
    b $(0, 3), (-\frac{3}{2}, 0)$
    c
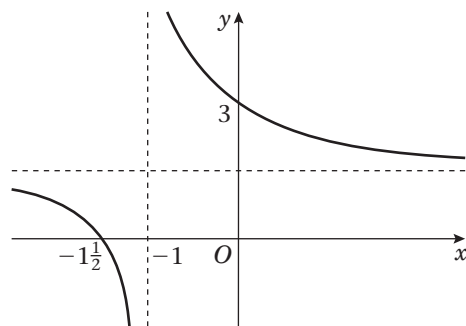
48 a $4\,\text{m/s}^2$ **b** 90 m

49 a $\frac{1}{2}\mathbf{b} - \frac{1}{3}\mathbf{a}$ **b** $2\mathbf{b} - \frac{4}{3}\mathbf{a}$ **c** –

50 a $\frac{dy}{dx} = 10x\cos 3x - 15x^2\sin 3x$

    b $\frac{dy}{dx} = \dfrac{3e^{3x}(x^2 + 3) - 2xe^{3x}}{(x^2 + 3)^2}$

51 0.212 m/s

52 a 1.39   **b** 28.7°

53 $p < -5, \ p > 2$

54 a 1, 3.75, 5.89, 6.92   **b** –
    c 0.79   **d** 2.1

55 a $A = -\frac{7}{2}, B = -\frac{9}{4}$ **b** $-\frac{9}{4}, x = \frac{7}{2}$
    c $(1, 4), (7, 10)$ **d** $(2, 0), (5, 0)$
    e –   **f** 24

56 $(-2, 1), (-1, 3)$

57 a **i** $\frac{p^2}{4} + 6$ **ii** 9   **b** $p = \pm 4$
    c $x^2 - 10x + 9 = 0$

58 a $-\frac{9}{11}, 5$ **b** 2
    c 4   **d** 16 380

59 a $\frac{dy}{dx} = 10xe^{2x} + 2(5x^2 - 2)e^{2x}$

    b $\frac{dy}{dx} = \dfrac{2x^3 - x^4 + 4x - 2}{(x - x^2)^2}$

60 ln 4

61 91.1°

62 $23\frac{2}{5}$

63 a $1 + \frac{x}{12} - \frac{x^2}{144}$ **b** $1 + \frac{x}{12} - \frac{x^2}{72}$

    c $|x| < 4$ **d** $1 + \frac{x}{6} - \frac{x^2}{72}$

    e 0.308

64 a $\cos 2A = 2\cos^2 A - 1$
    b $\sin 2A = 2\sin A \cos A$
    c –
    d 17.7°, 102.3°, 137.7°
    e $\frac{3\sqrt{3}}{8}$

65 $(6, -1), (1, 4)$

66 a $p^5 + 5p^4qx + 10p^3q^2x + 10p^2q^3x + 5pq^4x^4 + q^5x^5$
    b $p = \frac{6}{5}, q = \frac{12}{5}$ or $p = -2, q = 4$

67 a 3   **b** $q = 20$
    c $a = 2, b = 1$ **d** 9

68 a **i** $\sqrt{20}$ **ii** $\sqrt{40}$ **iii** $\sqrt{20}$
    b $\angle A = 90°, \angle B = \angle C = 45°$
    c $(5, 5)$ **d** $\sqrt{10}$

69 $\pm 60°$

70 67.4°

71 a 2   **b** $\log p$
    c $r = n - 1, s = n$ **d** –

72 a $2x^2 - 5x + 2 = 0$ **b** $x^2 - \frac{12}{p}x + \frac{12}{p} = 0$
    c $\frac{8}{3}$ **d** $\frac{3}{2}$

73 $-4 < p < 3$

Practice examination papers

Paper 1

1 80.4° or 99.6°

2 a –   **b** $p = -10, q = 33$

3 $20\,\text{cm}^2/\text{s}$

4 $x = 2 \ \ y = 3, \ \ x = 3 \ \ y = 2$

5 a $p = 6, q = -4$ **b** $5\mathbf{i} + \frac{4}{3}\mathbf{j}$

6 a $\left(\frac{1}{3}, 2\sqrt{\frac{5}{3}}\right)$ **b** $\frac{25}{3}\pi$

7 a 5   **b** $\frac{7r}{2} + \frac{3}{2}$ **c** $3\frac{1}{2}$ **d** 11

8 a $a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4$
       $+ 6ab^5x^5 + b^6x^6$
    b $a = 2 \ \ b = \frac{4}{3}, \ \ a = -2 \ \ b = -\frac{4}{3}$

9 a 5   **b** 28   **c** 9, 3

10 a $(2, 4)$ **b** $y = 4x - 4$
    c $y = 4$ **b** $8\,\text{units}^2$

11 a 11.0 cm   **b** 11.9 cm   **c** 40.1°
     d 101.4°   **e** 61.9°

Paper 2

1 $2e^{2x} \sin 3x + 3e^{2x} \cos 3x$

2 a $37.0°$ **b** $17.2 \, \text{cm}^2$

3 a i $y = 3$ **ii** $x = 2$ **b i** $(2\frac{2}{3}, 0)$ **ii** $(0, 4)$

c
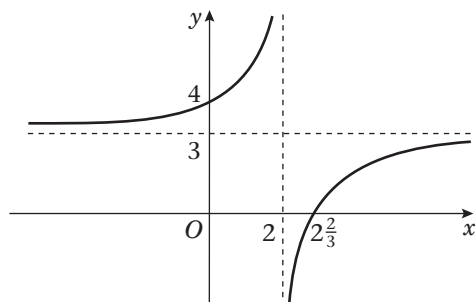
4 a

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1	0.649	-1.28	-4.52	-8.61	-12.8	-15.9	-15.9	-9.40

b graph drawn **c i** 1.9 **ii** 1.3

5 a $0, 3, 4$ **b** – **c** $11.8 \, \text{m}$

6 a $\pm\frac{1}{2}$ **b** $a = 120x$ **c** 4

7 a $(\ln 3, 36), (0, 4)$ **b** 32.02 **c** $82.2 \, \text{units}^2$

8 a – **b** 2.71 **c** – **d** 138

9 a – **b** -280 **c** 37

d $46\sqrt{37}$ **e** $9x^2 + 280 + 3 = 0$

10 a – **b i** $\dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$ **ii** $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

c $\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$

d $\sqrt{2} - 1$

e $\frac{20}{29}$