How does the height and distance from the basket affect the optimal angle for the trajectory of a basketball shot?

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20 Pages

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1 Introduction

The "optimal angle" refers to the angle required to reach the basket for the lowest amount of initial velocity, thus meaning the **least amount of force required.** If people use the minimum amount of force on their shot, they are going to be better able to control other factors of the trajectory of the ball such as spin and direction.

As I play basketball, I wanted to explore an aspect of the mathematics behind the shot. I initially noticed that the trajectory of the ball was generally parabolic, so I wanted to experiment with modelling a basketball shot using mathematics.

In this exploration, I will calculate a theoretical optimal angle of release for a basketball, compare it to experimental data, and apply a one-sample Student's t-test to see whether experimental data matches theoretical data for one basketball player, and one non-basketball player at different starting heights.

This will allow me to see whether basketball and non-basketball players naturally shoot at the optimal angle. If they do, it will mean that they can have finer control over their shot.

2 Assumptions

- The ball is released from the top of the head, meaning that the starting height of the ball is equal to the height of the person.
- The height of a typical rim is 3.05 m [Hall, 2017] (10 ft) from the ground, which will be the end height for the ball.
- Air resistance is assumed to be negligible, so the ball is only affected by gravitational force, making the model a parabola.
- Gravitational force g in this case is assumed to be equal to 9.81 ms^{-2}.
- Angles will be measured in degrees, as the angle of a basketball shot is often referred to in degrees in the real world, so using degrees will allow for suitable angles without conversions.
- Acceleration stays constant when the ball moves in the air, which is a requirement for the equations of motion to hold true.

3 Projectile Motion

3.1 Explanation

The path of a basketball follows the motion of a projectile. A projectile when shot at an angle to a horizontal plane, under no air resistance, is only affected by gravitational force, and thus, the horizontal velocity (v_h) will remain equal to the initial horizontal velocity (u_h), while the vertical velocity (v_v) will not remain equal to the initial vertical velocity (u_v) as it keeps accelerating downwards at a rate equal to g. In terms of displacement, this means that the horizontal displacement (s_h) will increase by the same amount per unit time, while the vertical displacement (s_v) will initially increase, then will decrease, as seen in Figure 1, creating a parabola.

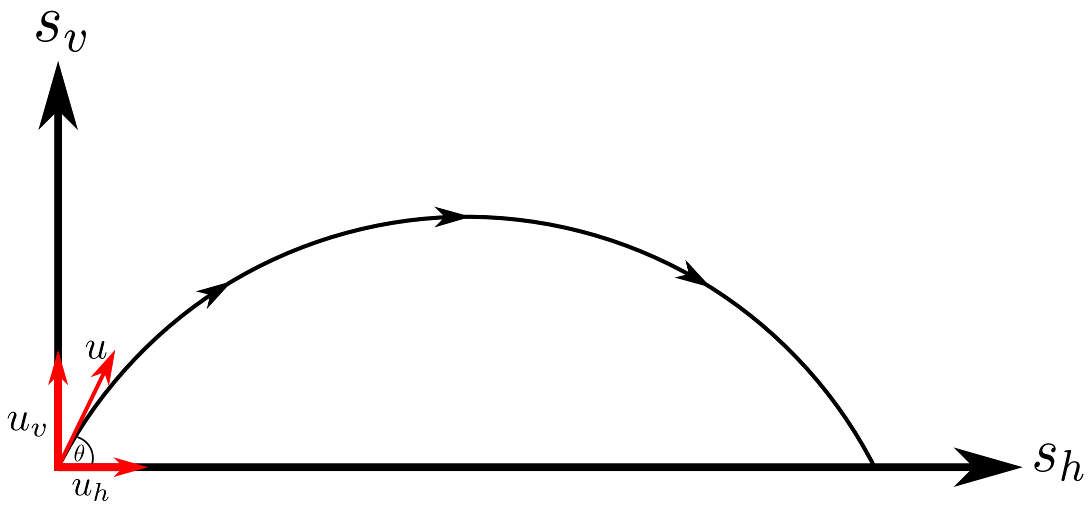

Figure 1: Parabola Explanation.

The initial velocity here (u) can be separated into two components — the vertical component (u_v), and the horizontal component (u_h) — which will allow us to calculate the separate components using equations of motion [Elert, 2020], then combine them later. We can see the horizontal velocity u_h stays constant, while the vertical velocity u_v first increases, then decreases after the turning point due to gravitational force.

However, as the basketball shot does not start from $s_v = 0$ due to the ball being shot from the chest of the shooter, and it does not end at $s_v = 0$ due to the ball needing to go through a 3.05 m high hoop, the model for our parabola should look more like the following:

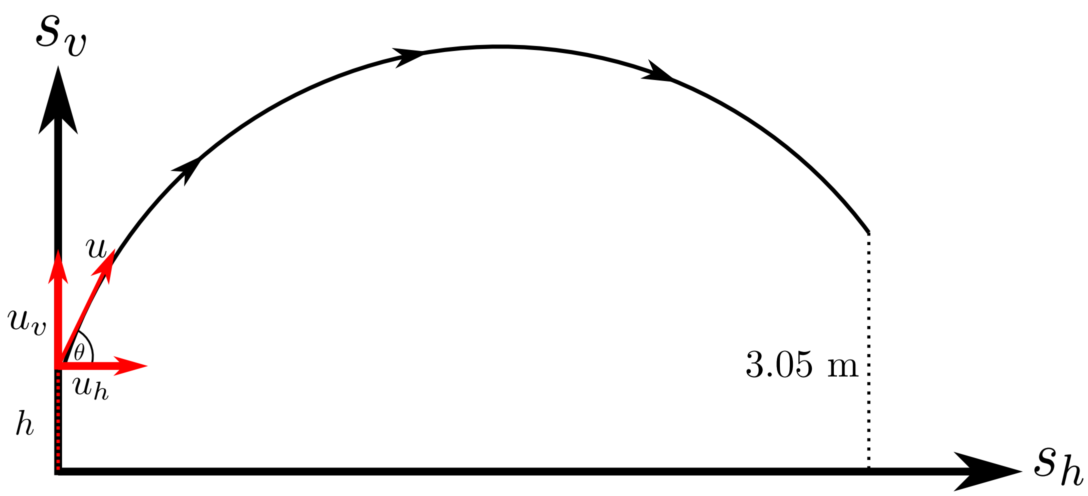

Figure 2: Adjusted Parabola.

In this adjusted figure, as the ball is released, the vertical displacement (s_v) is equal to the initial height of release h, and the ball's path ends when it hits the hoop.

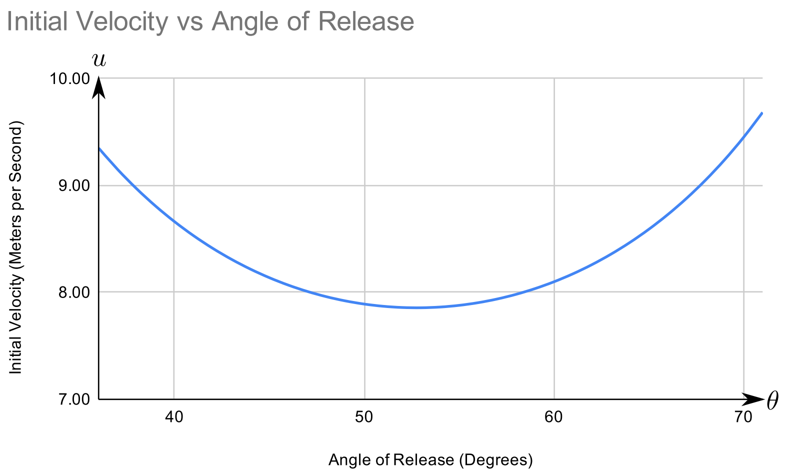

Figure 3: Graph showing the assumed relationship between angle of release and minimum velocity needed to reach the basket at a fixed distance.

In figure 3, we can see that shooting at a very sharp angle requires higher amounts of initial velocity, and shooting at a very shallow angle also requires a higher initial velocity when compared to the optimal angle — the minimum of the parabolic model. Using this assumption, to shoot with the lowest amount of force, and thus, the greatest amount of control will be to shoot at a specific angle. Calculations for this optimal angle will be derived below.

At a low angle, the initial velocity must be higher to compensate for the gravitational pull downwards. If the horizontal velocity were too slow, the ball would be pulled below the height of the rim before hitting the basket. Likewise, at a high angle, the ball would reach a higher apex, and thus, the vertical velocity would need to be high enough to reach said apex, meaning that the initial velocity would need to be higher.

3.2 Horizontal and Vertical Velocity

Separating the initial velocity u into its two components u_v and u_h will greatly simplify calculations compared to directly calculating with u, as the acceleration is different for both models.

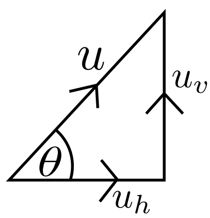

Figure 4: Simplified model.

As we know from trigonometric rules, we can use the model from Figure 4 to come up with the following equations:

$$\cos\theta = \frac{u_h}{u}$$
$$\sin\theta = \frac{u_v}{u}$$

These equations can then be used to be rearranged for the initial vertical and horizontal velocities:

$$u_h = u\cos\theta$$
$$u_v = u\sin\theta$$

3.2.1 Setup

Here we have two different equations of motion [Elert, 2020], one for displacement (s), and the other for velocity (v)

Displacement (Initial Velocity u, time t, acceleration a):

$$s = ut + \frac{1}{2}at^2$$

Velocity:

$$v = \frac{ds}{dt}[ut + \frac{1}{2}at^2]$$
$$v = u + at$$

The equations $u_h = u\cos\theta$ and $u_v = u\sin\theta$ both model the velocity at the beginning of the shot, meaning that it assumes that both the instantaneous acceleration (a) and time elapsed (t) is equal to 0.

3.2.2 Vertical Component

To find the vertical velocity (v_v), we can use equation $u + at$ in order to find the velocity at any given point during the trajectory of the shot.

In the vertical component, the ball constantly accelerates downwards due to gravitational force g, so within the equation of motion, a will be equal to -9.8 ms^{-2} assuming downwards is negative and upwards is positive.

Substituting the equation for u_v derived above into the equation, we get the following equation:

$$v_v = u\sin\theta - 9.8t$$

3.2.3 Horizontal Component

If air resistance is ignored in the horizontal component, the horizontal velocity (v_h) remains constant as explained in Section 3.1, so in other words, the horizontal acceleration (a_h) is 0, which means that if we substitute the equation for u_h into the equation for v, we will get the following equation:

$$v_h = u\cos\theta$$

3.3 Horizontal and Vertical Displacement

Once again, separating displacement into the horizontal and vertical components will greatly simplify calculations.

3.3.1 Vertical Component

Using the equation for displacement stated in section 3.2.1, if we substitute u_v, we get the equation $s_v = (u\sin\theta)t + \frac{1}{2}a_v t^2$. As the vertical component assumes that acceleration due to gravitational force (g) is constant at 9.8 ms^{-2}, we get the following equation:

$$s_v = (u\sin\theta)t + \frac{1}{2}gt^2$$
$$s_v = (u\sin\theta)t - 4.9t^2$$

3.3.2 Horizontal Component

Once again using the displacement equation found in section 3.2.1, if we substitute u_h instead, we get the equation $s_h = (u\cos\theta)t + \frac{a_h t^2}{2}$. As we know that the horizontal velocity is constant, the horizontal acceleration is equal to 0, which means that if we substitute it into the equation above, we get the following:

$$s_h = (u\cos\theta)t + \frac{1}{2} \times 0 \times t^2$$
$$s_h = (u\cos\theta)t$$

3.4 Issue and Solution

With the vertical displacement equation calculated earlier, it assumes that the ball follows the trajectory of Figure 1 rather than Figure 2, so we would need to adjust the equation found in section 3.3.1 to account for the starting height of release.

A solution for this issue would be to adjust the equation to account for the starting height of the shot. If we integrate v_v, we can get an equation that accounts for the starting vertical displacement of the ball.

$$s_v = \int (u \sin\theta - 9.8t)dt$$

$$s_v = (u \sin\theta)t - 4.9t^2 + c$$

Looking at Figure 2, we can see that the ball starts at (0,h), so adjusting the equation above, we get the following:

$$s_v = (u \sin\theta)t - 4.9t^2 + h$$

The same graph can also be modelled using transformations of graphs. If we let s_v be $s(t)$, we get $s(t) = (u \sin\theta)t - 4.9t^2$. As we know that to translate a graph by $\begin{pmatrix} 0 \\ h \end{pmatrix}$, we will need to add h to our function, giving us the function $s(t) + h = (u \sin\theta)t - 4.9t^2 + h$, leading to the same equation as the one derived above.

This equation more closely resembles Figure 2, and thus, is a more accurate representation of what happens in the real world.

As we assume the parabolic curve is simply translated upwards, the equation for horizontal displacement s_h is the same as the one found in section 3.3.2.

4 Calculation of Possible Release Angles of the Shot

As the trajectory of the basketball shot is assumed to be of a parabolic shape (section 2), using s_v as y, and s_h as x, we can use the general equation $s_v = as_h^2 + bs_h + c$ to model the ball's trajectory.

Using the equations we found for vertical and horizontal displacement, we have the following equations:

$$s_h = (u \cos\theta)t$$

$$s_v = (u \sin\theta)t - 4.9t^2$$

If we set the equation for s_v with s_h, we should be able to turn the equation to a general quadratic formula.

Rearranging s_h such that we solve for t, we get the equation $t = \frac{s_h}{u \cos\theta}$. If we substitute this equation into the equation for s_v above, we get the following equation:

$$s_v = (u\sin\theta)\left(\frac{s_h}{u\cos\theta}\right) - 4.9\left(\frac{s_h}{u\cos\theta}\right)^2 + h$$
$$s_v = \frac{s_h \times u\sin\theta}{u\cos\theta} - 4.9\left(\frac{s_h^2}{u^2\cos^2\theta}\right) + h$$

Because $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$, we can then write the equation in the form $s_v = as_h^2 + bs_h + c$:

$$s_v = -\frac{4.9}{u^2\cos^2\theta}s_h^2 + \tan\theta \times s_h + h$$

However, as the above equation has both $\cos\theta$ and $\tan\theta$, we would need to manipulate the equation such that there is only one trigonometric ratio to be able to solve for θ.

The following trigonometric identities will be used in order to manipulate the equation:

$$\sec^2\theta = \tan^2\theta + 1 \tag{1}$$
$$\sec\theta = \frac{1}{\cos\theta} \implies \sec^2\theta = \frac{1}{\cos^2\theta} \tag{2}$$

4.1 Manipulating the Equation for Vertical Displacement

Taking the $-\dfrac{4.9}{u^2\cos^2\theta}$ component from s_v, we can first apply identity (2), to get the following equation:

$$-\frac{4.9}{u^2}\sec^2\theta$$

Using identity (1), we then get:

$$-\frac{4.9}{u^2}(\tan^2\theta + 1)$$
$$-\frac{4.9\tan^2\theta}{u^2} - \frac{4.9}{u^2}$$

If we substitute that back into s_v, we get the following equation written in the quadratic form with respect to $\tan\theta$ as that is what we are looking to solve:

$$
\begin{aligned}
s_v &= \left(-\frac{4.9\tan^2\theta}{u^2} - \frac{4.9}{u^2}\right) s_h^2 + \tan\theta \times s_h + h \\
s_v &= -\frac{4.9 s_h^2}{u^2}\tan^2\theta + s_h \tan + h - \frac{4.9 s_h^2}{u^2}
\end{aligned}
$$

However, to use the quadratic equation to solve for θ, we will need the form $a\tan^2\theta + b\tan\theta + c = 0$, so thus, the equation needs to be manipulated to the following form:

$$
-\frac{4.9 s_h^2}{u^2}\tan^2\theta + s_h\tan\theta + h - \frac{4.9 s_h^2}{u^2} - s_v = 0
$$

4.2 Solving for the Angle

As the basketball hoop is situated 3.05 m (section 2) above the ground, and our initial starting height of 1.72 m already is part of the equation, s_v will be assumed to be equal to the height of the hoop for the following calculations.

$$
\begin{aligned}
h - s_v - \frac{4.9 s_h^2}{u^2} \\
h - 3.05 - \frac{4.9 s_h^2}{u^2} \\
-(3.05 - h) - \frac{4.9 s_h^2}{u^2}
\end{aligned}
$$

Thus, we have the following equation:

$$
-\frac{4.9 s_h^2}{u^2}\tan^2\theta + s_h\tan\theta - (3.05 - h) - \frac{4.9 s_h^2}{u^2} = 0
$$

We can use the quadratic equation to solve for $\tan\theta$.

Using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we can then substitute our values in the equation above.

$$
\tan\theta = \frac{-s_h \pm \sqrt{s_h^2 - 4\left(-\frac{4.9 s_h^2}{u^2}\right)\left(-(3.05 - h) - \frac{4.9 s_h^2}{u^2}\right)}}{2\left(\frac{-4.9 s_h^2}{u^2}\right)}
$$

Then, rearranging our equation to make solving for minimum initial velocity simpler, we can do the following:

$$\tan\theta = \frac{s_h \pm \sqrt{s_h^2 - 4s_h^2\left(-\frac{4.9}{u^2}\right)\left(-(3.05-h) - \frac{4.9s_h^2}{u^2}\right)}}{\frac{9.8s_h^2}{u^2}}$$

$$\tan\theta = \frac{s_h\left(1 \pm \sqrt{1 - 4\left(-\frac{4.9}{u^2}\right)\left(-(3.05-h) - \frac{4.9s_h^2}{u^2}\right)}\right)}{\frac{9.8s_h^2}{u^2}}$$

$$\tan\theta = \frac{u^2\left(1 \pm \sqrt{1 - 4\left(-\frac{4.9}{u^2}\right)\left(-(3.05-h) - \frac{4.9s_h^2}{u^2}\right)}\right)}{9.8s_h}$$

$$\tan\theta = \frac{u^2 \pm \sqrt{u^4 - \left(\frac{(59.78-19.6h)u^2}{u^4} + \frac{96.04s_h^2}{u^4}\right)u^4}}{9.8s_h}$$

$$\theta = \tan^{-1}\left(\frac{u^2 \pm \sqrt{u^4 - (59.78-19.6h)u^2 - 96.04s_h^2}}{9.8s_h}\right)$$

Now, we have an equation regarding the initial velocity, horizontal displacement (distance from the basket), and the angle of release ($\tan\theta$), meaning that with some rearrangement, we can figure out the optimal angle of release for a given distance, that provides the lowest initial velocity in order to provide an optimal angle of release.

4.3 Solving for Initial Velocity

Using Figure 4 in Section 3.2, as well as Pythagoras' theorem $a^2 + b^2 = c^2$, we can apply it to our model as the vertical and horizontal planes are perpendicular to each other, and thus, we can deduce that $u_h^2 + u_v^2 = u^2$.

5 Calculations from the Free-Throw Line for a Basketball Player

For all the calculations in this section, the initial release height h will be 1.79 m, my own height.

5.1 Theoretical Optimal Angle

The distance from the free-throw line to the basket is 4.57 m (15 ft), so if we substitute 4.57 for s_h in the equation above, we will be left with two variables θ and u.

$$\theta = \tan^{-1}\left(\frac{u^2 \pm \sqrt{u^4 - (59.78 - 19.6(1.79))u^2 - 96.04(4.57)^2}}{9.8(4.57)}\right)$$

$$\theta = \tan^{-1}\left(\frac{u^2 \pm \sqrt{u^4 - 24.70u^2 - 2005.786}}{44.786}\right)$$

We can set the radical $\sqrt{u^4 - 24.70u^2 - 2005.786}$ equal to 0 to find the minimum value of u.

The reason why the optimal angle can be found by setting the radical to 0 is because when the radical is negative due to u being too small, there will not be any real solutions. However, when u is large enough to make the radical a positive non-zero number, u will be greater, and thus, more force required.

Setting the radical to 0:

$$u^4 - 24.70u^2 - 2005.786 = 0$$
$$\therefore u^2 = 58.81 \text{ or } u^2 = -34.1$$

As initial velocity cannot be negative, as that would imply the ball flies backwards, we reject $u^2 = -34.1$, and as a result, $u^2 = 58.81 \implies u = 7.67$ ms^{-1}.

Using this value of u, we can then substitute our value into the equation to solve for θ, with the fact that the radical is 0 in mind:

$$\theta = \tan^{-1}\left(\frac{7.67^2}{44.786}\right)$$
$$\theta = 52.7^\circ$$

This means that in the real world, the angle of release from the free-throw line should tend towards 52.7°.

5.2 Real World Optimal Angle

I recorded myself shooting and making 30 shots from the free-throw line at a basketball court, and used Logger Pro[1] in order to mark the trajectory of a projectile on a video. With every shot, I used Logger Pro's graphing functions in order to find out the initial vertical and horizontal velocities of each shot, and then used those values and trigonometric identities to find the angle of release.

[1] https://www.vernier.com/product/logger-pro-3/

Figure 5: Example of the usage of Logger Pro on one of the shot attempts.

In this image, the yellow line show the axis s_h and s_v, the green line shows the distance from the basket to the shooter, and the blue line represents the trajectory of the ball in the air.

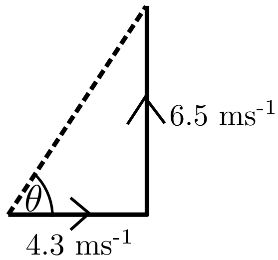

Figure 6: Model of initial velocities on a triangle, not drawn to scale.

In Figure 6, the initial velocities u_h and u_v were equal to 4.3 ms^{-1} and 6.5 ms^{-1} respectively, and using Pythagoras' theorem $a^2 + b^2 = c^2$, we can deduce that the initial velocity u is equal to $\sqrt{u_h^2 + u_v^2}$, which is equal to 7.79 ms^{-1}.

Additionally, using the rules of trigonometry, we can find the initial angle of release using the vertical and horizontal velocities by doing the following:

$$\tan\theta = \frac{6.5}{4.3}$$
$$\theta = \tan^{-1}\left(\frac{6.5}{4.3}\right)$$
$$\theta = 56.5°$$

Due to the central limit theorem stating that the distribution of sample means approximates a normal distribution as $n \to \infty$ as long as $n \geq 30$, I repeated this 29 more times for the other shot attempts in order for the t-test to more closely model a normally distributed curve, then calculated the mean angle of all results, as well as the standard deviation of the sample in order to run a t-test, assuming that the angle of release of a ball from a set distance is normally distributed.

Trial:	u_v (ms^{-1})	u_h (ms^{-1})	u (ms^{-1})	θ (°)
1	6.5	4.3	7.79	56.5
2	5.9	4.5	7.42	52.7
3	6.2	4.8	7.84	52.3
4	6.2	4.7	7.78	52.8
5	6.5	4.3	7.79	56.5
6	6.0	5.4	8.07	48.0
7	5.8	4.6	7.40	51.6
8	6.3	4.5	7.74	54.5
9	6.0	5.0	7.81	50.2
10	5.9	5.1	7.80	49.2
11	5.8	5.1	7.72	48.7
12	6.4	4.8	8.00	53.1
13	6.5	4.6	7.96	54.7
14	6.5	4.1	7.69	57.8
15	6.0	5.0	7.81	50.2

Table 1: A sample of the recorded results for the angle of release from the free-throw line.

In the sample above, the mean angle was measured to be 52.3°, with the sample standard deviation equal to 2.52. The result is quite close to the calculated theoretical angle, and with a non-negligible standard deviation, there is a very likely chance that the angle collected from the sample is part of the theoretical angle 52.7°.

It is worth noting that some values for u were actually below the theoretical minimum velocity calculated in section 5, which should not be possible in theory. The inconsistency in results here may be due to the method of collection of results, where graphing all points and calculating a quadratic equation for the motion of the ball, and then differentiating to find the gradient would potentially work better. Moreover, there may be a parallax error due to where the camera was placed, which may influence how the points are graphed — especially as Logger Pro does not take that into account. However, due to the amount of time it would take to record the angle of release for all shot attempts using this method, I decided to use only the vertical and horizontal velocities regarding the first two frames of the video, which may have been the reason why some initial velocities were lower than the theoretical minimum.

5.3 Comparison

If we assume that the angle of release is distributed normally with a mean equal to 52.7° from the free-throw line, as we do not know the population variance, we can apply a Student's t-test in order to model whether the angle of release from experimental data is likely to be the same as the one from a theoretical standpoint.

As calculated above, the mean angle of release was 52.3°.

Now, if we set a null hypothesis H_0 and alternative hypothesis H_1 to be $H_0 : \mu = 52.7°$, $H_1 : \mu \neq 52.7°$, we will be able to calculate perform a t-test with our results.

To calculate the one-sample t-value without a GDC, we can use the following expression, where μ_0 is the hypothesized value 52.7°, and $\overline{x}$ is the experimental mean for the angle of release:

$$t = \frac{\overline{x} - \mu_0}{\frac{S_n}{\sqrt{n}}}$$

Using the values that we derived from earlier, we can then calculate the t-value:

$$t = \frac{52.7 - 52.3}{\frac{2.52}{\sqrt{30}}}$$

$$t = 0.869$$

Now using this calculated t-value, we then need to find the critical t-value in order to make a comparison to see whether it would be suitable to consider our experimental results to be a part of the theoretical angle of release.

In order to find the critical t-value, we will need to first derive the degrees of freedom, which for a Student's t-test, would simply be the number of trials n minus one. As the critical t-value is different when the significance level is different, we will also need to set a certain significance level to test at. In this case, we will test at a 5% significance level.

In order to find the critical t-value, we can use a t-table [tdistributiontable.com, 2020] to find the result we need.

	Significance Level		
df	0.2	0.1	0.05
28	1.313	1.701	2.048
29	1.311	1.699	**2.045**
30	1.310	1.697	2.042

Table 2: T-Distribution table

In this case, the intersection of our degrees of freedom (df) **29** and the significance level 5% results in a critical t-value of **2.045**.

For calculated values less than the critical t-value, the result is considered not significant, and the opposite applies for when the calculated value is greater than the critical t-value.

As our calculated t-value 0.869 is less than the critical value 2.045, the result is not significant, and therefore, the null hypothesis H_0 is not rejected, and evidence suggests that the collected angle of release from experimental data is likely 52.7°.

We can also use a GDC in order to find the p-value and the t-value as alluded to earlier.

If we input the following values $\mu \neq \mu_0$, $\mu_0 = 52.7$, $\overline{x} = 52.3$, $S_n = 2.52$, $n = 30$ into the GDC[2], we find that the p-value is 0.391, and the t-value is 0.869 — the same as the one calculated by hand.

[2]The GDC used for this was the Casio fx-CG 20

Once again, as the p-value 0.391 is greater than our significance level of 0.05, H_0 is once again not rejected, and the result is not significant, once again reflecting the result of the hand calculated t-value.

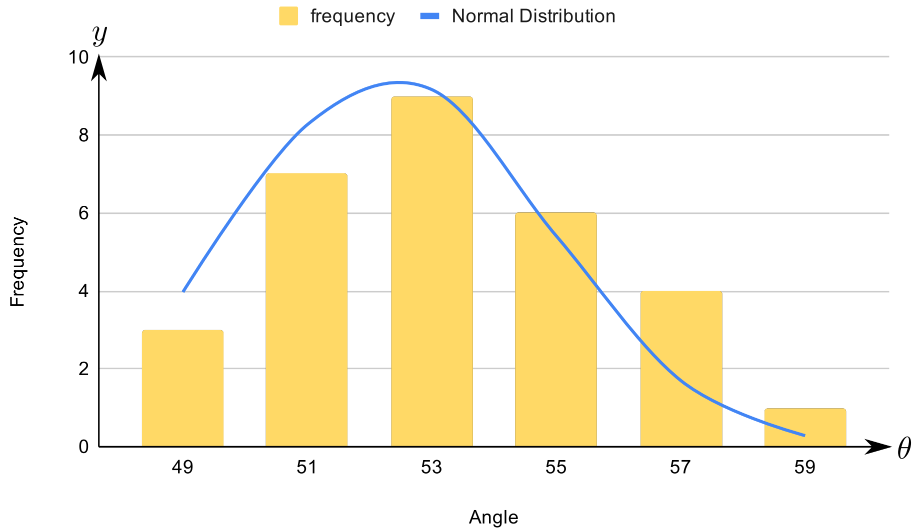

Figure 7: Frequency of angle from the free-throw line plotted alongside a t-distributed curve

The graph shows the frequency of the shot angle in six different ranges graphed alongside a normally distributed curve with mean 52.3° and standard deviation 2.52. Even if a t-distribution was used for the tests, it is an approximation for the normal distribution, and thus, would be similarly shaped to the normal distribution shown above.

Interestingly enough, the data had a positive skew. This may be because I am trying to force my shots up to hit the free-throw line, meaning I need to improve my physical conditioning, but this may also mean that I need to work more on shooting at a higher angle to give the ball a larger target to enter the basket and also to give me more control over my shot.

However, in general, we can conclude in this case that the angle of my shots are likely to be normally distributed with respect to the optimal angle of release.

6 Calculations from the Free-Throw Line for a Non-Basketball Player

I figured it would be interesting if a non-basketball player naturally tended towards the optimal angle when shooting the ball.

6.1 Theoretical Optimal Angle

Instead of doing this with an algebraic method, a graphical method can be used instead.

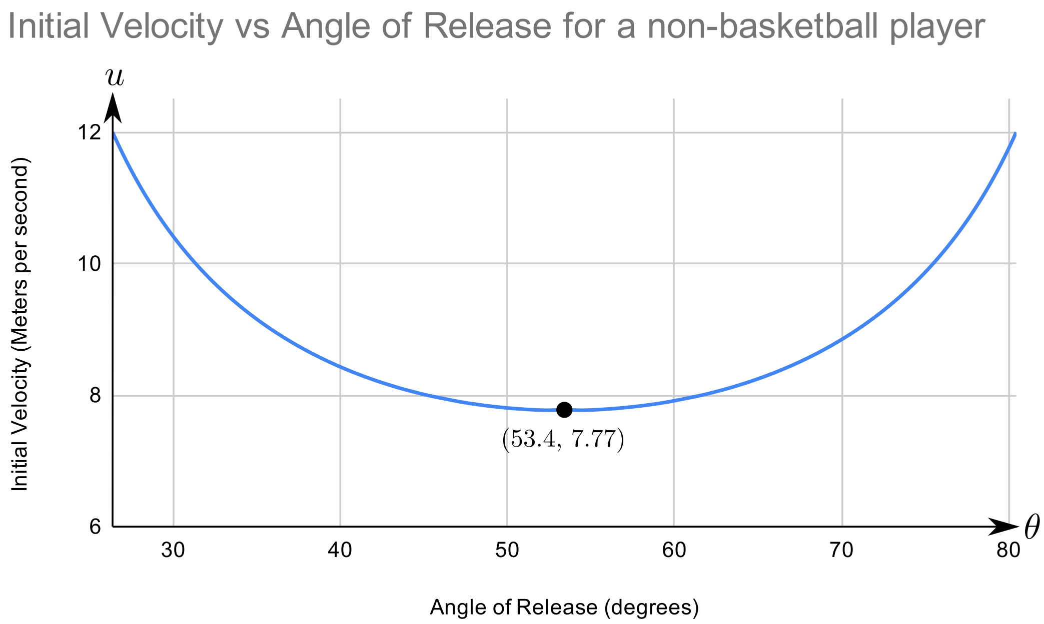

Figure 8: A graph showing the relationship between initial velocity and angle of release for a non-basketball player.

In the graph above, the minimum of the parabola is the angle of release needed to provide the lowest initial velocity possible. This minimum occurs at 53.4° with an initial velocity of 7.77 ms^{-1}.

Looking at the general shape of the graph, we can see that it matches our predictions in section 3.1. When the angle of release is lower or higher than the optimal angle, the initial velocity increases, meaning that when shooting outside the optimal angle, more force will be needed, and thus, less control can be put on the shot.

This minimum can also be proven algebraically:

$$\theta = \tan^{-1}\left(\frac{u^2 \pm \sqrt{u^4 - (59.78 - 19.6(1.67))u^2 - 96.04(4.57)^2}}{9.8(4.57)}\right)$$
$$\theta = \tan^{-1}\left(\frac{u^2 \pm \sqrt{u^4 - (27.05)u^2 - 2005.786}}{44.786}\right)$$

Then, setting the radical to 0, we get the following:

$$u^4 - 27.05u^2 - 2005.786 = 0$$
$$\therefore u^2 = 60.31 \text{ or } u^2 = -33.25$$

Once again, initial velocity cannot be negative, as that would imply the ball flies backwards, so we reject $u^2 = -33.25$, and as a result, $u^2 = 60.31 \implies u = 7.77$ ms^{-1}.

Using this value of u, we can then substitute our value into the equation to solve for θ

$$\theta = \tan^{-1}\left(\frac{7.77^2}{44.786}\right)$$

$$\theta = 53.4°$$

6.2 Real World Optimal Angle

Using the same method to calculate horizontal and vertical initial velocity, we get the results shown below. However, due to the non-basketball player being fatigued fairly quickly, I was only able to get 15 shot attempts. This may mean that the central limit theorem does not hold true, but for this case, we will assume it does.

Trial:	u_v (ms^{-1})	u_h (ms^{-1})	u (ms^{-1})	θ (°)
1	5.8	5.3	7.86	47.6
2	5.9	5.2	7.86	48.6
3	6.2	4.8	7.84	52.3
4	6.2	4.7	7.78	52.8
5	6.5	4.6	7.96	54.7
6	6.0	5.0	7.81	50.2
7	5.8	4.6	7.40	51.6
8	6.0	4.6	7.56	52.5
9	6.0	5.0	7.81	50.2
10	5.9	5.1	7.80	49.2
11	5.8	5.1	7.72	48.7
12	6.4	4.8	8.00	53.1
13	6.5	4.5	7.91	55.3
14	6.5	4.1	7.69	57.8
15	6.0	5.0	7.81	50.2

Table 3: Recorded results for the angle of release from the free-throw line by a non-basketball player.

Based on the results above, the mean angle θ is 51.9°, and the standard deviation of the sample is 2.71. The mean experimental angle is quite far from the theoretical optimal angle, so it is fairly unlikely that the experimental mean angle will be normally distributed with respect to the theoretical angle.

One again, we see a few instances of the initial velocity u being lower than the theoretical minimum 7.77 ms^{-1}, and this results from similar reasons to those stated in the section above.

6.3 Comparison

If we run a Student's t-test on this once again, this time directly using the GDC, we will once again be able to figure out whether the experimental angle is likely to be distributed about the theoretical angle 53.4°.

First, we will set a null hypothesis and alternative hypothesis to be $H_0 : \mu = 53.4°$, $H_1 : \mu \neq 53.4$ to reflect the results from the new height.

After inputting the following values $\mu \neq \mu_0$, $\mu_0 = 53.4$, $\overline{x} = 51.9$, $S_n = 2.71$, $n = 15$, we see that the t-value is -2.14, and the p-value is 0.05. As the t-value in this case is negative, we take the absolute value in order to compare it to the critical region.

	Significance Level		
df	0.2	0.1	0.05
13	1.35	1.771	2.16
14	1.345	1.761	**2.145**
15	1.341	1.753	2.131

Table 4: T-Distribution table

As the degrees of freedom for this test is 14, the t-value is 2.145.

In this case, as the absolute value of the calculated t-value above is 2.14, which is lower than 2.145, and the p-value is 0.05 is equal to the significance level 0.05 after rounding, so we accept the hypothesis H_0, and reject H_1, and as a result, evidence suggests that the non-basketball player tended to shoot at an angle of 53.4°.

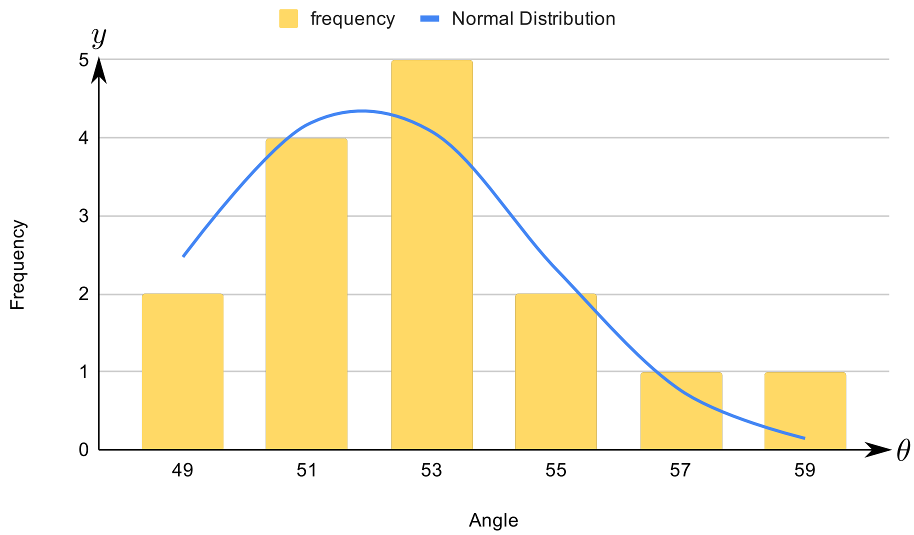

Figure 9: Frequency of angle from the halfway between the free-throw line and the basket plotted alongside a t-distributed curve.

When we look at the data plotted against a normal distribution based on the data, we can see that there is a positive skew.

This is most likely because the non-basketball player doesn't have enough strength to shoot the ball from the free-throw line to the basket, so they need to resort to using more strength, leading to a lower angle of release. However, if they did have enough strength, or the horizontal displacement was shortened between the point of release and the basket, the result may tend towards the optimal angle. Using a shorter distance such as half the distance between the free-throw line and the basket may help to reduce the positive skew of the angles collected.

7 Analysis

7.1 Evaluation

In a one-sample t-test, the population variance is not known, so it is interpreted from the sample. In this case, it is unknown whether the angle of a shot is normally distributed or not — however, given the variance in shooting angles during a real game situation, as well as the general variance in shooting angle with experimental data, it is likely that the angle of release is normally distributed, but not certain, as there are numerous variables that could influence a basketball shot.

Additionally, because the unbiased estimator (S^2_{n-1}) approaches the population variance σ^2 when $n \to \infty$, with more results, the t-test distribution would be closer to the true normally distributed model, and S^2_{n-1} will be closer to the population variance assuming that the angle of a basketball shot is normally distributed, meaning that with more shot attempts, we will be more confident in concluding whether my shot angle is normally distributed with a mean of the optimal angle.

Moreover, as the camera is situated on a tripod at a height below the basket, there is a potential parallax error that exists, which may cause the data plotted on Logger Pro to not reflect the true data in real life. Additionally, the action camera used has a wide-angle lens, which has the property of not keeping every line linearly scaled in the recording. While the camera software does have means to rectify this, it is by no means perfect, and thus, may have influenced the results collected, however, not likely by a significant degree to contradict results found. However, to improve this exploration, putting the camera at the same level as the basket should reduce the parallax error produced.

Furthermore, Some assumptions may not represent real life. For example, for the t-test to hold true, the assumption is that the angle of a basketball shot models a normal distribution, however, this is potentially not true.

Also, further explorations could consider the spin of the ball and how that may affect the path of the ball in the air, or the effect of air resistance on the ball.

In addition, the central limit theorem only concerns the mean of samples as the random variable, rather than the samples themselves. In that way, a 'good' experimental angle may not necessarily be normally distributed, but rather, skewed towards one side because the central limit theorem does not take that factor into account. The central limit theorem also tends to hold true only when $n \geq 30$, so more shot attempts by the non-basketball player may help to improve the reliability of the results found.''

Moreover, the result collected in section 6.3 is very close to the significance level, so there is a possibility that a Type II error may have occurred. If the non-basketball player were to have made more shots, the chances of the Type II error occurring will be lowered. This means that the result could possibly be contradictory to the hypothesis. A larger sample size and more attempts would be crucial to either prove or disprove this claim.

To improve this experiment, different basketball and non-basketball players could undergo the same test conditions to see whether the angle of release of the shot tends towards the optimal angle derived for each participant. With more participants, the reliability of the conclusions gained will increase, meaning that we can be more confident with rejecting or accepting the results from this investigation.

Additionally, the method used to collect the angle of release is potentially unreliable as it only uses two data points to figure out the horizontal and vertical initial velocities. This measure could be made more reliable using another data collection method.

7.2 Conclusion

Overall, I was able to derive an equation that finds the optimal angle of release at different starting distances and heights that represents the path of a basketball in the air. Along with that, I learnt how to interpret real world results by using a Student's t-test as well as plotting the data alongside a normal distribution. This allowed me to see that **both basketball and non-basketball players (to a lesser extent) naturally shoot at an angle near the optimal angle of release** to minimise the force needed to reach the basket. This means that naturally, players tend to shoot at an angle where they minimise the amount of force needed — whether because of more control in the case of basketball players, or less force in the case of non-basketball players.

The results found from this experiment could be useful for targeted shooting practice, as they could be used in order to help players practice getting their shot angle consistently distributed around the optimal angle. With this, players will be able to control their shots better with the lowest amount of force possible. With enough targeted practice, basketball players will be able to get their shots close to the optimal angle with their muscle memory, which will allow for more consistency while allowing for the maximum amount of control possible on the shot.

However, due to the many factors and variables going into a basketball shot, it is impractical in a real game situation to force the optimal angle of release for different distances, and thus, the way to ensure the best results possible with a shot would be to practice shooting in multiple different situations at an angle as close to the optimal angle to ensure consistency and full control over the shots.

8 References

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Appendix

Full table of results for the Real World Optimal Angle for a Basketball Player

Trial:	u_v (ms^{-1})	u_h (ms^{-1})	u (ms^{-1})	θ (°)
1	6.5	4.3	7.79	56.5
2	5.9	4.5	7.42	52.7
3	6.2	4.8	7.84	52.3
4	6.2	4.7	7.78	52.8
5	6.5	4.5	7.91	55.3
6	6.0	5.4	8.07	48.0
7	5.8	4.6	7.40	51.6
8	6.3	4.5	7.74	54.5
9	6.0	5.0	7.81	50.2
10	5.9	5.1	7.80	49.2
11	5.8	5.1	7.72	48.7
12	6.4	4.8	8.00	53.1
13	6.5	4.6	7.96	54.7
14	6.5	4.1	7.69	57.8
15	6.0	5.0	7.81	50.2
16	5.7	4.8	7.45	49.9
17	6.0	5.0	7.81	50.2
18	6.2	4.8	7.84	52.3
19	6.5	4.5	7.91	55.3
20	6.3	4.7	7.86	53.3
21	6.2	4.8	7.84	52.3
22	6.1	4.8	7.76	51.8
23	6.0	4.7	7.62	51.9
24	6.5	4.7	8.02	54.1
25	5.9	4.6	7.48	52.1
26	6.5	4.6	7.96	54.7
27	6.0	4.9	7.75	50.8
28	5.7	5.2	7.72	47.6
29	5.9	4.8	7.61	50.9
30	6.5	4.5	7.91	55.3