In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

Course Title:
Engineering Economics

3. Combining Factors and Spreadsheet Functions

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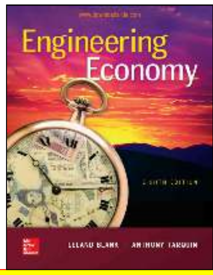
Learning Stage 1: The Fundamentals

- Chapter 1
  - Foundations of Engineering Economy
- Chapter 2
  - Factors: How Time and Interest Affect Money
- Chapter 3
  - Combining Factors and Spreadsheet Functions
- Chapter 4
  - Nominal and Effective Interest Rates



Chapter 3 of EE (BT) book 8th edition

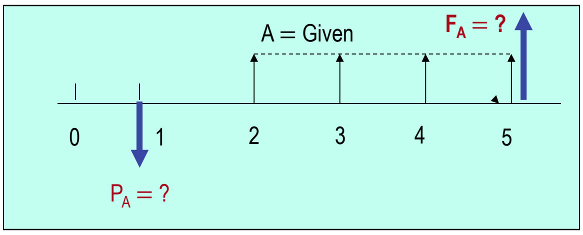
LEARNING OUTCOMES

- Purpose:
  - Use multiple factors and spreadsheet functions to find equivalent amounts for cash flows that have nonstandard placement.

1. Shifted uniform series
2. Shifted series and single cash flows
3. Shifted gradients

Shifted Uniform Series

- A shifted uniform series starts at a time other than period 1
- The cash flow diagram below is an example of a shifted series
  - Series starts in period 2, not period 1



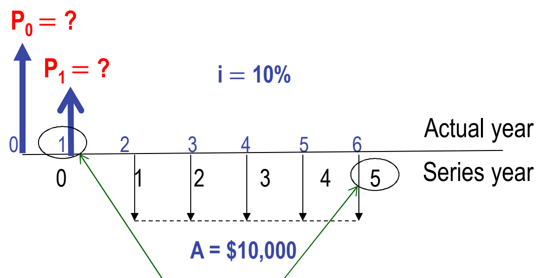
Shifted series usually require the use of multiple factors

Remember: When using P/A or A/P factor, P_A is always one year ahead of first A

When using F/A or A/F factor, F_A is in same year as last A

Example Using P/A Factor: Shifted Uniform Series

- The present worth of the cash flow shown below at i = 10% is:



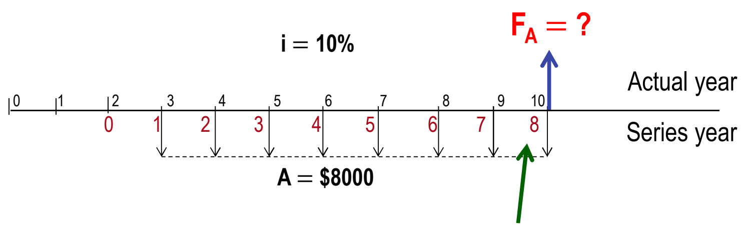
Solution: (1) Use P/A factor with n = 5 (for 5 arrows) to get $\mathbf{P_1}$ in year 1

(2) Use P/F factor with n = 1 to move P_1 back for $\mathbf{P_0}$ in year 0

$P_0 = P_1(P/F,10\%,1) = A(P/A,10\%,5)(P/F,10\%,1) = 10{,}000(3.7908)(0.9091) = \$34{,}462$

Example Using F/A Factor: Shifted Uniform Series

- How much money would be available in year 10 if $8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?



Solution: Re-number diagram to determine n = 8 (number of arrows)

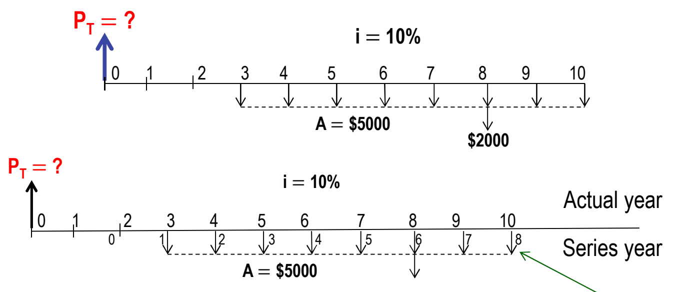
$$F_A = 8000(F/A,10\%,8)$$
$$= 8000(11.4359)$$
$$= \$91{,}487$$

Shifted Series and Random Single Amounts

- For cash flows that include uniform series and randomly placed single amounts:
  - Uniform series procedures are applied to the series amounts
  - Single amount formulas are applied to the one-time cash flows
  - The resulting values are then combined per the problem statement
  - The following slides illustrate the procedure

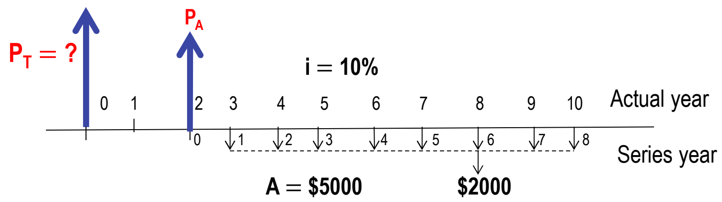
Example: Series and Random Single Amounts (1)

Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.



Solution: First, re-number cash flow diagram to get n for uniform series: n = 8

Example: Series and Random Single Amounts (2)

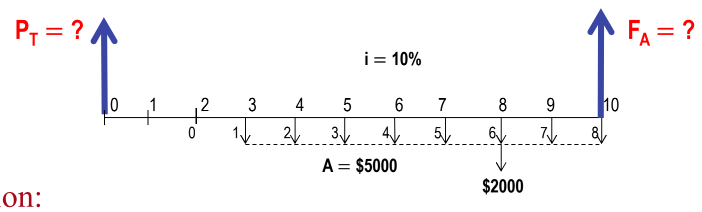


- Use P/A to get P_A in year 2: $P_A = 5000(P/A,10\%,8) = 5000(5.3349) = \$26,675$
- Move P_A back to year 0 using P/F: $P_0 = 26,675(P/F,10\%,2) = 26,675(0.8264) = \$22,044$
- Move \$2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$
- Now, add P_0 and P_{2000} to get P_T: $P_T = 22,044 + 933 = \$22,977$

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

- The same re-numbered diagram from the previous slide is used



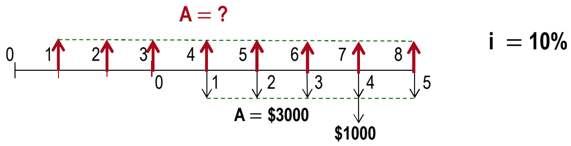
Solution:

- Use F/A to get F_A in actual year 10: $F_A = 5000(F/A,10\%,8) = 5000(11.4359) = \$57,180$
- Move F_A back to year 0 using P/F: $P_0 = 57,180(P/F,10\%,10) = 57,180(0.3855) = \$22,043$
- Move \$2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$
- Now, add two P values to get P_T: $P_T = 22,043 + 933 = \$22,976$ Same as before

As shown, there are usually multiple ways to work equivalency problems

Example: Series and Random Amounts

- Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



Approaches: 1. Convert all cash flows into P in year 0 and use A/P with n = 8

2. Find F in year 8 and use A/F with n = 8

Solution: Solve for F: F = 3000(F/A,10%,5) + 1000(F/P,10%,1)

= 3000(6.1051) + 1000(1.1000)

= $19,415

Find A: A = 19,415(A/F,10%,8)

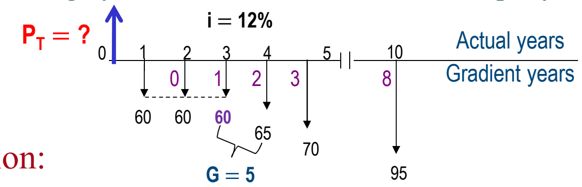
= 19,415(0.08744) = $1698

Shifted Arithmetic Gradients

- Shifted gradient begins
  - at a time other than between periods 1 & 2
- Present worth P_G is located 2 periods before gradient starts
- Must use multiple factors to find P_T in actual year 0
- To find equivalent A series,
  - find P_T at actual time 0 and apply (A/P, i, n)

Example: Shifted Arithmetic Gradient

- John expects the cost of a tractor part to increase by $5 per year beginning 4 years from now.
  - If the cost in years 1-3 is $60, determine the PW in year 0 of the cost through year 10 at an interest rate of 12% per year.



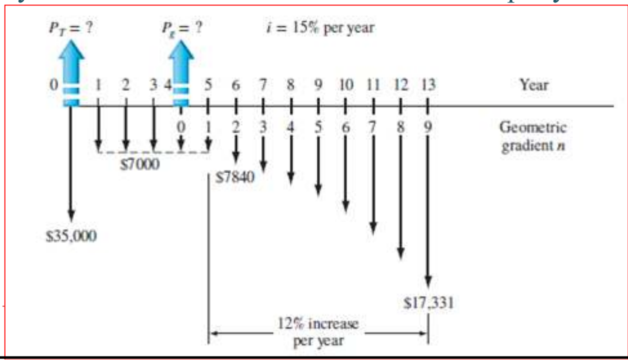
- Solution:
  - First find P_2 for G = $5 and base amount ($60) in actual year 2
    - $P_2 = 60(P/A,12\%,8) + 5(P/G,12\%,8) = \$370.41$
  - Next, move P_2 back to year 0, $P_0 = P_2(P/F,12\%,2) = \$295.29$
  - Next, find P_A for the $60 amounts of years 1 and 2
    - $P_A = 60(P/A,12\%,2) = \$101.41$
  - Finally, add P_0 and P_A to get P_T in year 0, $P_T = P_0 + P_A = \$396.70$

Shifted Geometric Gradients

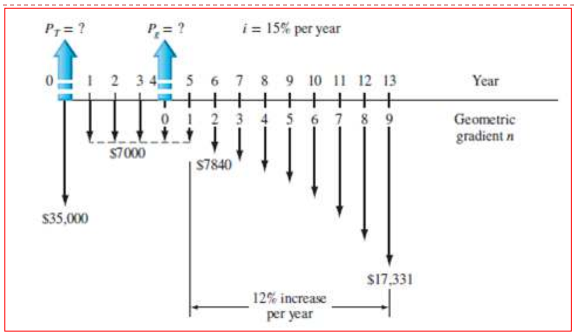
- Shifted gradient begins
  - at a time other than between periods 1 and 2
- Equation yields P_g for all cash flows (base amount A_1 is included)
  - Equation ($i \neq g$):
    - $\mathbf{P_g = A_1\{1 - [(1+g)/(1+i)]^n/(i-g)\}}$
- For negative gradient,
  - change signs on both g values
- There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient (1)

- Weirton signed a 5-year contract to purchase water treatment chemicals from a local distributor for $7000 per year.
  - When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years.
  - If an initial investment in storage tanks is $35,000, determine the PW in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient (2)



- Gradient starts between actual years 5 and 6; these are gradient years 1 & 2.
  - P_g is located in gradient year 0, which is actual year 4
    - $\mathbf{P_g = 7000\{1 - [(1 + 0.12)/(1 + 0.15)]^9/(0.15 - 0.12)\} = \$49,401}$
  - Move P_g and other cash flows to year 0 to calculate P_T
    - $\mathbf{P_T = 35,000 + 7000(P/A, 15\%, 4) + 49,401(P/F, 15\%, 4) = \ \$83,232}$

Negative Shifted Gradients

- For negative arithmetic gradients, change sign on G term from + to −
  - General equation for determining P:
    - P = present worth of base amount $- P_G$

Changed from + to −

- For negative geometric gradients, change signs on both g values

Changed from + to −

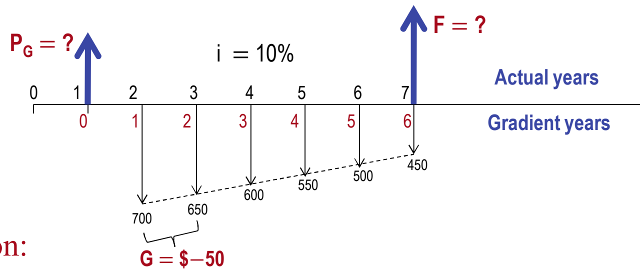
  - $P_g = A1\{1 - [(1 - g)/(1 + i)]^n/(i + g)\}$

Changed from − to +

- All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

- For the cash flows shown, find the FW in year 7 at i = 10% per year



- Solution:
  - Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2
  - P_G is located in gradient year 0 (actual year 1); base amount of \$700 is in gradient years 1-6

$P_G = 700(P/A, 10\%, 6) - 50(P/G, 10\%, 6) = 700(4.3553) - 50(9.6842) = \$2565$
$F = PG(F/P, 10\%, 6) = 2565(1.7716) = \$4544$

Summary of Important Points

- P for shifted uniform series is one period ahead of first A
  - n is equal to number of A values
- F for shifted uniform series is in same period as last A
  - n is equal to number of A values
- For gradients, first change equal to G or g occurs between gradient years 1 and 2
- For negative arithmetic gradients,
  - change sign on G from + to –
- For negative geometric gradients,
  - change sign on g from + to –

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