In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

Course Title:
Engineering Economics

2. Factors: How Time & Interest Affect Money

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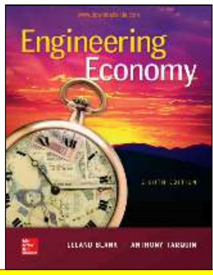
Learning Stage 1: The Fundamentals

- Chapter 1
  - Foundations of Engineering Economy
- Chapter 2
  - Factors: How Time and Interest Affect Money
- Chapter 3
  - Combining Factors and Spreadsheet Functions
- Chapter 4
  - Nominal and Effective Interest Rates



Chapter 2 of EE (BT) book 8th edition

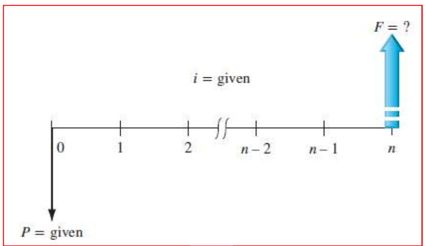
LEARNING OUTCOMES

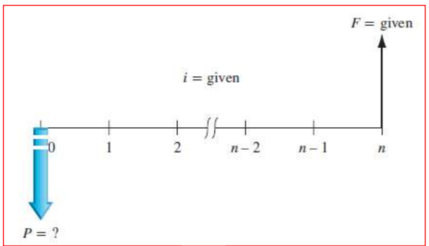
- Purpose:
  - Derive and use the engineering economy factors to account for the time value of money.

1. F/P and P/F Factors
2. P/A and A/P Factors
3. F/A and A/F Factors
4. Factor Values
5. Arithmetic Gradient
6. Geometric Gradient
7. Find i or n

Single Payment Factors (F/P and P/F)

Single payment factors involve only P and F. Cash flow diagrams are as follows:





Formulas are as follows:

$$F = P(1 + i)^n$$

$$P = F[1/(1 + i)^n]$$

Terms in parentheses or brackets are called *factors*. Values are in tables for i and n values

Factors are represented in standard factor notation such as (F/P, i, n),

where letter to left of slash is what is sought; letter to right represents what is given

F/P and P/F for Spreadsheets

- Future value F is calculated using FV function:

  = FV(i%,n,,P)

- Present value P is calculated using PV function:

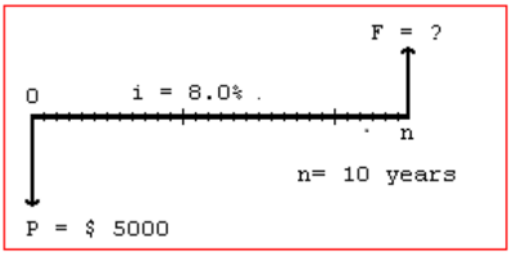
  = PV(i%,n,,F)

Note the use of double commas in each function

Example: Finding Future Value

- A person deposits $5000 into an account which pays interest at a rate of 8% per year.
  - The amount in the account after 10 years is:

The cash flow diagram is:



Solution:

F = P(F/P, i, n)

= 5000(F/P, 8%,10)

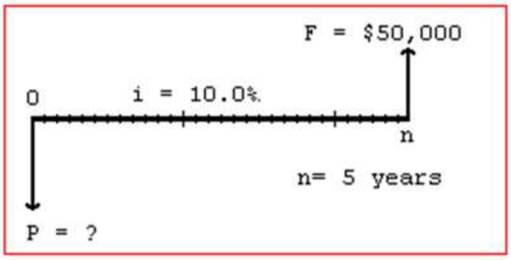
= 5000(2.1589)

= $10,794.50

Example: Finding Present Value

- A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now.
  - If the account will earn interest of 10% per year, the amount that must be deposited now is:

The cash flow diagrams is:



Solution:

$P = F(P/F,i,n)$

$= 50{,}000(P/F,10\%,5)$

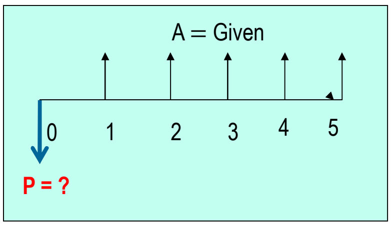
$= 50{,}000(0.6209)$

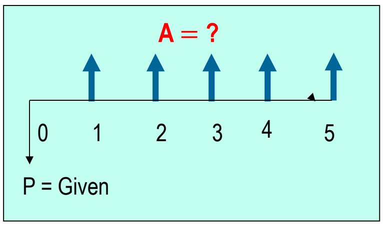
$= \$31{,}045$

Uniform Series Involving P/A and A/P

- The uniform series factors that involve P and A are derived as follows:
  - (1) Cash flow occurs in consecutive interest periods
  - (2) Cash flow amount is same in each interest period

The cash flow diagrams are:



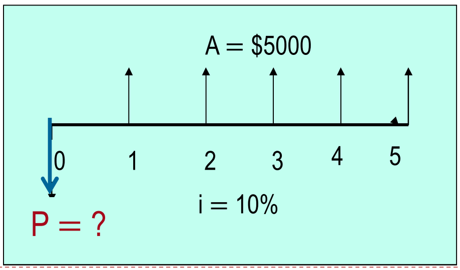


$P = A(P/A,i,n)$ ← Standard Factor Notation → $A = P(A/P,i,n)$

Note: P is one period Ahead of first A value

Example: Uniform Series Involving P/A

- A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra $5000 per year.
  - At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?



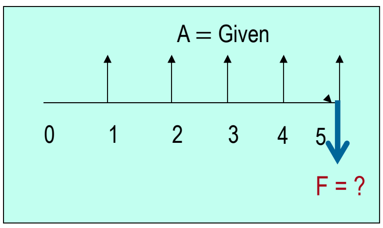
Solution:

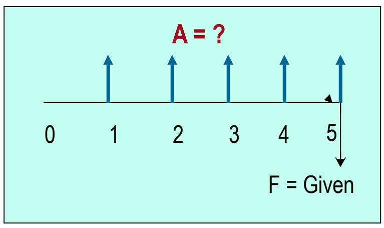
$$P = 5000(P/A, 10\%, 5)$$
$$= 5000(3.7908)$$
$$= \$18,954$$

Uniform Series Involving F/A and A/F

- The uniform series factors that involve F and A are derived as follows:
  - (1) Cash flow occurs in consecutive interest periods
  - (2) Last cash flow occurs in same period as F

Cash flow diagrams are:





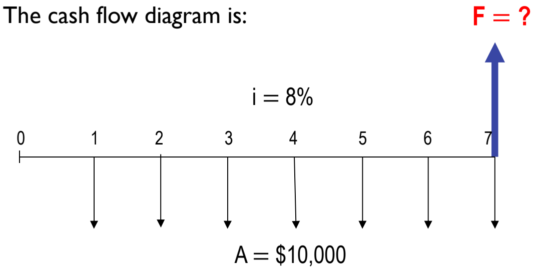
$F = A(F/A, i, n)$ ⟵ Standard Factor Notation ⟶ $A = F(A/F, i, n)$

Note: F takes place in the same period as last A

Example: Uniform Series Involving F/A

- An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year.
  - At an interest rate of 8% per year, how much will the savings amount to in 7 years?

The cash flow diagram is:



Solution:

$$F = 10{,}000(F/A,8\%,7)$$
$$= 10{,}000(8.9228)$$
$$= \$89{,}228$$

Factor Values for Untabulated i or n

- 3 ways to find factor values for untabulated i or n values
  - Use formula
  - Use spreadsheet function with corresponding P, F, or A value set to 1
  - Linearly interpolate in interest tables

Note that Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

Example: Untabulated i

- Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ⟵ OK

Spreadsheet: $= \mathrm{FV}(8.3\%, 10,, 1) = 2.2197$ ⟵ OK

Interpolation: 8% ------ 2.1589
8.3% ------ x
9% ------ 2.3674

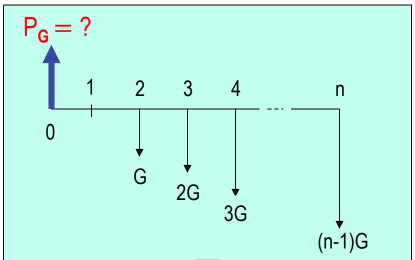
$$x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 - 2.1589]$$
$$= 2.2215$$ ⟵ (Too high)

Absolute Error $= 2.2215 - 2.2197 = 0.0018$

Arithmetic Gradients

- Arithmetic gradients change by the same amount each period

The cash flow diagram for the P_G of an arithmetic gradient is:



G starts between periods 1 and 2 (not between 0 and 1)

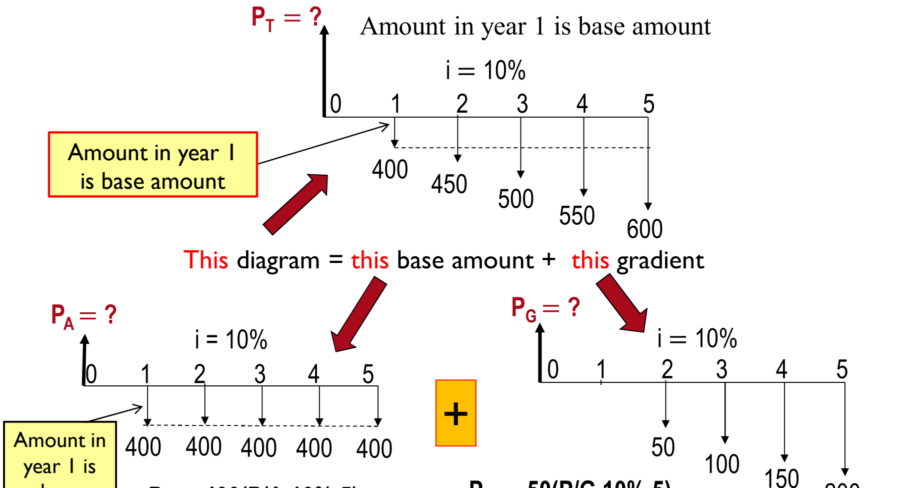
This is because cash flow in year 1 is usually not equal to G and is handled separately as a base amount

(shown on next slide)

Standard factor notation is $P_G = G(P/G, i, n)$

Note that P_G is located Two Periods Ahead of the first change that is equal to G

Typical Arithmetic Gradient Cash Flow

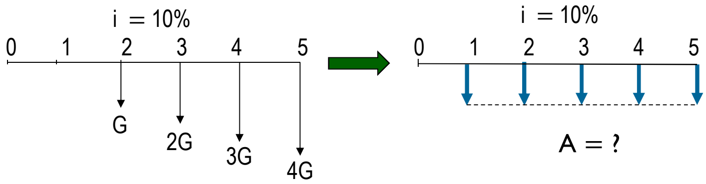


$P_A = 400(P/A,10\%,5)$ $P_G = 50(P/G,10\%,5)$

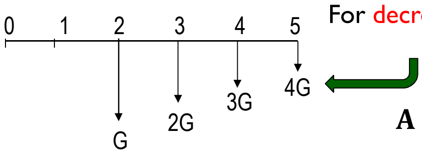
$$P_T = PA + PG = 400(P/A, 10\%, 5) + 50(P/G, 10\%, 5)$$

Converting Arithmetic Gradient to A

- Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)



General equation when base amount is involved is
$$\mathbf{A = base\ amount + G(A/G, i, n)}$$

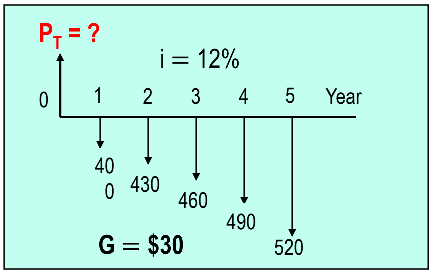


For decreasing gradients, change plus sign to minus

$$\mathbf{A = base\ amount - G(A/G, i, n)}$$

Example: Arithmetic Gradient

- The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year:



Solution:

$P_T = 400(P/A,12\%,5) + 30(P/G,12\%,5)$

$= 400(3.6048) + 30(6.3970)$

$= \$1,633.83$

The cash flow could also be converted into an A value as follows:

$A = 400 + 30(A/G,12\%,5)$

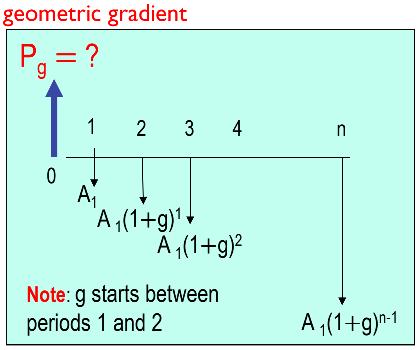
$= 400 + 30(1.7746)$

$= \$453.24$

Geometric Gradients

- Geometric gradients change by the same percentage each period

Cash flow diagram for present worth of geometric gradient



There are no tables for geometric factors

Use following equation for $g \neq i$:

$P_g = A_1\{1 - [(1+g)/(1+i)]^n\}/(i - g)$

where: A_1 = *cash flow in period 1*

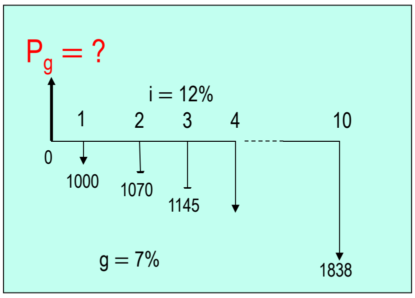
g = *rate of increase*

If $g = i$, $P_g = A_1 n/(1+i)$

Note: If g is negative, change signs in front of both g values

Example: Geometric Gradient

- Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10.
  - Use an interest rate of 12% per year.



Solution:

$$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$$
$$= \$7,333$$

Note that To find A, multiply P_g by (A/P,12%,10)

Unknown Interest Rate i

- Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)
  - Usually requires a trial & error solution or interpolation in interest tables
  - Procedure: Set up equation with all symbols involved & solve for i

Example: A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years.

The annual rate of return of the investment was:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P, i\%, 10) = 16{,}000$$
$$(A/P, i\%, 10) = 0.26667$$

Answer is (d)

- From A/P column at n =10 in the interest tables, i is between 22% & 24%

Unknown Recovery Period n

- Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)
  - Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables
  - Procedure: Set up equation with all symbols involved & solve for n

Example : A contractor purchased equipment for $60,000 that provided income of $8,000 per year.

At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years (b) 12 years (c) 15 years (d) 18 years

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P, 10\%, n) = 8{,}000$$
$$(A/P, 10\%, n) = 0.13333$$

Answer is (c)

From A/P column in i = 10% interest tables, n is between 14 and 15 years

Summary of Important Points

- In P/A and A/P factors, P is one period ahead of first A
- In F/A and A/F factors, F is in same period *as last* A
- To find untabulated factor values, best way is to use formula or spreadsheet
- For arithmetic gradients, gradient G starts between periods 1 and 2
- Arithmetic gradients have 2 parts, base amount (year 1) and gradient amount
- For geometric gradients, gradient g starts been periods 1 and 2
- In geometric gradient formula, A_1 is amount in period 1
- To find unknown i or n, set up equation involving all terms and solve for i or n