In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

Course Title:
Engineering Economics

15. Sensitivity Analysis and Decision Making Under Risk

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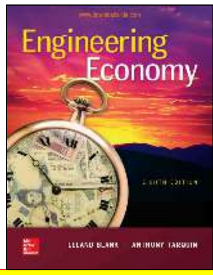
Learning Stage 3: Making Better Decisions

- Chapter 14
  - Effects of Inflation
- Chapter 15 *not covered in this course
  - Cost Estimation and Indirect Cost Allocation
- Chapter 16
  - Depreciation Methods
- Chapter 17
  - After-Tax Economic Analysis
- Chapter 18
  - Sensitivity Analysis and Staged Decisions
- Chapter 19
  - More on Variation and Decision Making under Risk



Chapter 18 & 19 of EE (BT) book 8th edition

LEARNING OUTCOMES

- Purpose:
  - Perform a sensitivity analysis of parameters; use expected values to evaluate staged funding options.

1. Explain sensitivity to parameter variation
2. Use three estimates for sensitivity analysis
3. Calculate expected value E(X)
4. Determine E(X) of cash flow series
5. Use decision trees for staged decisions

Parameters and Sensitivity Analysis

- Parameter:
  - A variable or factor for which an estimated or stated value is necessary.
- Sensitivity analysis
  - An analysis to determine how a measure of worth (e.g., PW, AW, ROR, B/C) changes when one or more parameters vary over a selected range of values.
- Procedure for sensitivity analysis:
  - Select parameter to analyze. Assume independence with other parameters
  - Select probable range and increment

Parameters and Sensitivity Analysis

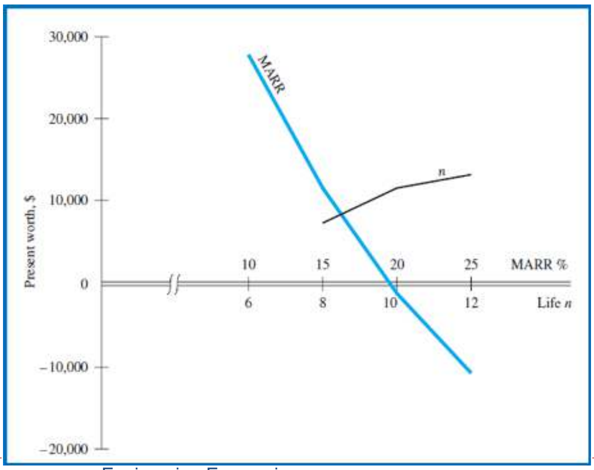
- Procedure for sensitivity analysis (cont'd):
  - Select measure of worth
  - Calculate measure of worth values
  - Interpret results. Graph measure vs. parameter for better understanding
- An Example:
  - Estimates for a new asset are a first cost of $80,000, zero salvage value, and $CFBT_t$ = $27,000 − 2000t.
    - MARR for the company varies over a wide range from 10% to 25% per year for different types of investments.
    - The economic life of similar machinery varies from 8 to 12 years. Evaluate the sensitivity of PW by varying
      - (a) MARR, while assuming a constant n value of 10 years
      - (b) n, while MARR is constant at 15% per year.

Parameters and Sensitivity Analysis

- Solution of example
  - PW = −80,000 + 25,000(P/A,10%,10) − 2000(P/G,10%,10) = $27,830

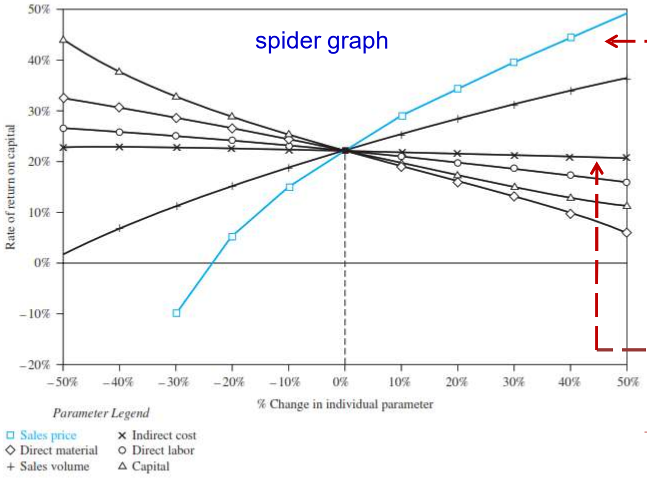
MARR, %	PW, $
10	27,830
15	11,512
20	−962
25	−10,711

n	PW, $
8	7,221
10	11,511
12	13,145



Sensitivity of Several Parameters

- When several parameters vary & analysis of each parameter is required
  - graph percentage change from the most likely estimate for each parameter vs. measure of worth



Plots with larger slopes (positive or negative) have a higher sensitivity with parameter variation e.g., sales price curve

Plots that are relatively flat have little sensitivity to parameter variation e.g., indirect cost curve

Use Data Tables in Excel to do this.

Three Estimate Sensitivity Analysis

- Applied when selecting one ME alternative from two or more
- For each parameter that warrants analysis, provide three estimates:
  - Pessimistic estimate P
  - Most likely estimate ML
  - Optimistic estimate O
- Calculate measure of worth for each alternative and 3 estimates and then select the best alternative

Notes
1. The pessimistic estimate may be the lowest for some parameters and the highest for others, e.g., low life estimates and high first cost estimates
2. When calculating the measure of worth, use ML estimate of a parameter as others varies. This is the independence assumption.

Example of three estimates sensitivity analysis

- An engineer is evaluating 3 alternatives for new equipment as follows.

Strategy		First Cost, $	Salvage Value S, $	AOC, $ per Year	Life n, Years
Alternative A					
Estimates	P	−20,000	0	−11,000	3
	ML	−20,000	0	−9,000	5
	O	−20,000	0	−5,000	8
Alternative B					
Estimates	P	−15,000	500	−4,000	2
	ML	−15,000	1,000	−3,500	4
	O	−15,000	2,000	−2,000	7
Alternative C					
Estimates	P	−30,000	3,000	−8,000	3
	ML	−30,000	3,000	−7,000	7
	O	−30,000	3,000	−3,500	9

P = pessimistic; ML = most likely; O = optimistic.

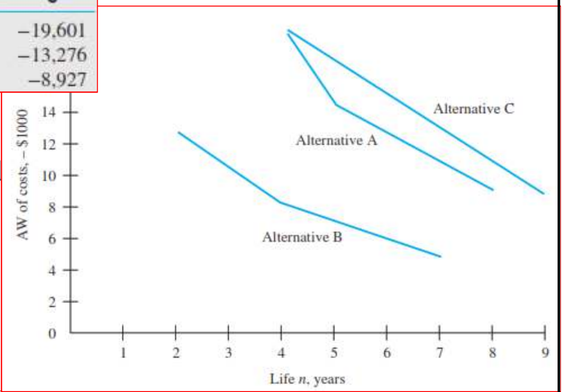
  - Perform a sensitivity analysis and determine the most economical alternative, using AW analysis at a MARR of 12% per year.

Example of three estimates sensitivity analysis

- Solution
  - Calculate AW for each alternative and each estimate then graph it.

Estimates	Alternative AW Values, $ A	B	C
P	−19,327	−12,640	−19,601
ML	−14,548	−8,229	−13,276
O	−9,026	−5,089	−8,927

Obviously, Alternative B is economically justified



Expected Value Calculations

- Expected Value
  - Long-run average observable if a project or activity is repeated many times
- Result is a point estimate based on anticipated outcomes and estimated probabilities

$$E(X) = \sum_{i=1}^{m} X_i P(X_i)$$

- Where: X_i = value of variable X for i = 1, …, m different values
- $P(X_i)$ = probability that a specific value of X will occur

In all probability statements, the sum is:

$$\sum_{i=1}^{m} P(X_i) = 1.0$$

When E(X) < 0, e.g., E(PW) = $−2550, a cash outflow is expected; the project is not expected to return the MARR used

Example: Probability and Expected Value

- Monthly M&O cost records over a 4-year period are shown in $200 ranges.
  - Determine the expected monthly cost for next year, if conditions remain constant.

Range,$, X	No. of months	Range,$, X	No. of months
100–300	4	700–900	6
300–500	12	900–1100	10
500–700	14	1100–1300	2

- Solution:

$$\begin{aligned}
P(X) &= \text{number of months}/48 \text{ months} \\
E(X) &= 200(4/48) + 400(12/48) + \cdots + 1200(2/48) \\
&= 1/48[200 \times 4 + 400 \times 12 + \cdots + 1200 \times 2] \\
&= 1/48[31{,}200] = \$650/\text{month}
\end{aligned}$$

Expected Value for Alternative Evaluation

- Two applications for Expected Value for estimates:
  1. Prepare information for use in an economic analysis
  2. Evaluate economic viability of fully formulated alternative

- Example: Second use for a complete alternative. Is the investment viable?

  P = $−5000 n = 3 years MARR = 15%

	Economic Condition		
	Receding (Prob. = 0.4)	Stable (Prob. = 0.4)	Expanding (Prob. = 0.2)
Year	Annual Cash Flow Estimates, $		
0	−5000	−5000	−5000
1	+2500	+2500	+2000
2	+2000	+2500	+3000
3	+1000	+2500	+3500

Example: Expected Value for Alternative Evaluation

	Economic Condition		
	Receding (Prob. = 0.4)	Stable (Prob. = 0.4)	Expanding (Prob. = 0.2)
Year	Annual Cash Flow Estimates, $		
0	−5000	−5000	−5000
1	+2500	+2500	+2000
2	+2000	+2500	+3000
3	+1000	+2500	+3500

Solution: Calculate PW value for each economic condition

$PW_R = -5000 + 2500(P/F, 15\%, 1) + 2000(P/F, 15\%, 2) + 1000(P/F, 15\%, 3)$

$= \$-656$ (cash outflow; not viable)

$PW_S = \$+708$ (cash inflow; viable)

$PW_E = \$+1309$ (cash inflow; viable)

Now, calculate expected value of PW estimates

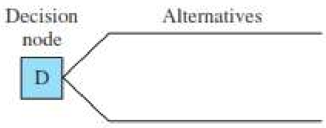
$$E(PW) = PW_R \times P(R) + PW_S \times P(S) + PW_E \times P(E)$$

$$= -656 \times 0.4 + 708 \times 0.4 + 1309 \times 0.2 = \$+283$$

On basis of $E(PW) > 0$ at 15% over 3 years, investment is viable

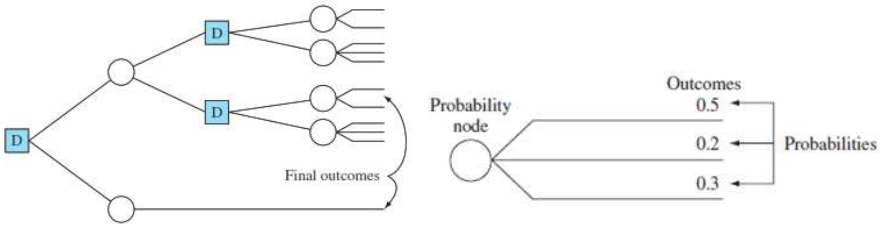
Decision Tree Characteristics

- Staged Decision
  - Alternative has multiple stages;
  - decision at one stage is important to next stage;
  - risk is an inherent element of the evaluation
- Decision Tree
  - Helps to make risk more explicit for staged decisions.



Decision Tree Characteristics

- A Decision Tree Includes:
  - More than one stage of selection
  - Selection of an alternative at one stage leads to another stage, e.g., node D
  - Expected results from a decision at each stage
  - Probability estimates for each outcome
  - Estimates of economic value (cost or revenue) for each outcome
  - Measure of worth as the selection criterion

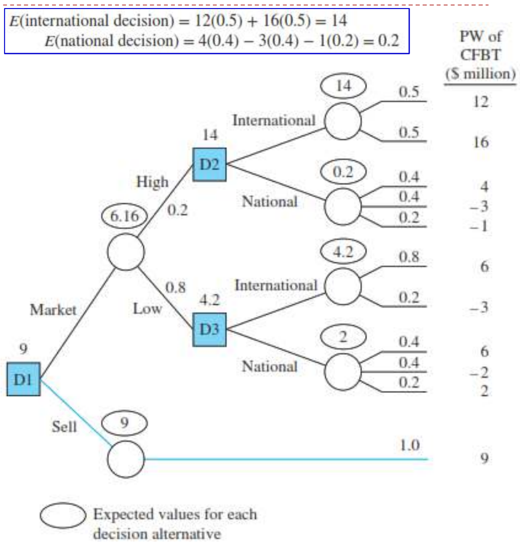


Solving a Decision Tree

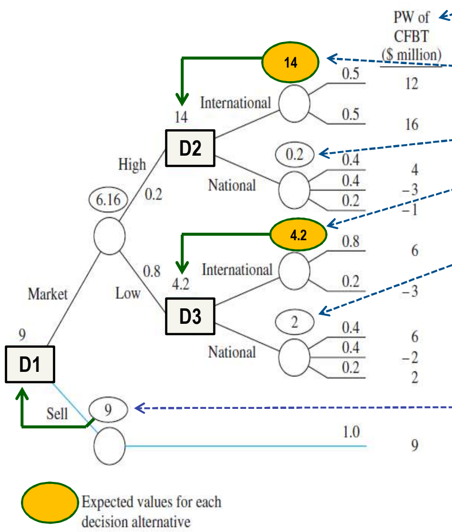
- Once the tree is developed, probabilities and economic information are estimated for each outcome branch, and the measure of worth is selected (usually PW),
  - use the following, starting at top right of tree:
- Procedure to solve a decision tree
  1. Determine PW for each outcome branch
  2. Calculate expected value for each alternative/decision node:
     $$E(\text{decision}) = \sum(\text{outcome estimate}) \times P(\text{outcome})$$
  3. At each decision node, select the best E(decision) value
  4. Continue moving to left to the tree's root to select the best alternative
  5. Trace the best decision path through the tree.

Example: Solving a Decision Tree

- A decision is needed to either market or sell a new invention.
  - If the product is marketed, the next decision is to take it international or national.
    - Assume the details of the outcome branches result in the decision tree.
  - Determine the best decision at the decision node D1.



Example: Solving a Decision Tree



1. PW of CFBT is estimated
2. PW for decision nodes

D2 $E(\text{int'l}) = 12(0.5) + 16(0.5) = 14$

$E(\text{nat'l}) = 4(0.4) - 3(0.4) - 1(0.2) = 0.2$

D3 $E(\text{int'l}) = 6(0.8) - 3(0.2) = 4.2$

$E(\text{nat'l}) = 6(0.4) - 2(0.4) + 2(0.2) = 2$

3. Decisions: 14 (int'l) @D2 and 4.2 (int'l) @D3
4. PW for decision node D1

D1 $E(\text{market}) = 14(0.2) + 4.2(0.8) = 6.16$

$E(\text{sell}) = 9(1.0) = 9$

Decision: 9 (sell)

5. Trace through tree; select D1 to sell at E(PW of CFBT) = $9 million

Summary of Important Points

- Sensitivity analysis
  - evaluates variation in parameters using a specific measure of worth (PW, ROR, B/C, etc.)
- in sensitivity analysis
  - Independence of parameters is assumed
- If E(PW) < 0,
  - an alternative is not expected to return the stated MARR, given the estimated probabilities
- Decision trees assist in making staged decisions
  - when risk is explicitly considered

LEARNING OUTCOMES

- Purpose:
  - Perform a sensitivity analysis of parameters; use expected values to evaluate staged funding options.

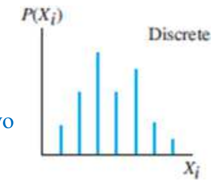
1. Understand decision making under risk
2. Construct probability distributions and cumulative distributions
3. Take random sample from a distribution
4. Estimate expected value and standard deviation from random sample
5. Use Monte Carlo sampling for simulation analysis to evaluate alternatives

Decision Making Classification

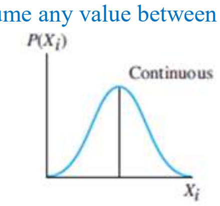
- Classifications of decision making:
  - certainty, risk, uncertainty
- Certainty
  - No variation in estimates
    - also called deterministic; no variation expected in a parameter
    - all estimates are single-valued
- Risk
  - Two or more values for variables with estimated chances for each
  - can use expected value/ simulation analysis
- Uncertainty
  - Two or more values for a variable, but chances of occurrence are not known or not estimated
    - states of nature are identified & approaches are relatively inconclusive

Decision Making Under Risk (1)

- Important elements
  - Random variable: Parameter that can take any one of several values
    - discrete – has several specific, isolated values



    - continuous – can assume any value between two



  - Probability
    - Chance (divided by 100) that variable will take on a specific value;
    - The range is $0 \leq P(X) \leq 1.0$

Decision Making Under Risk (2)

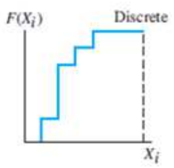
- More important elements
  - Probability distribution
    - Describes probability versus values of a variable

$$P(X_i) = \text{probability that } X \text{ equals } X_i$$

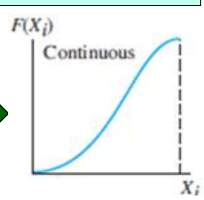
  - Cumulative distribution
    - Accumulation of probability over all values of variable up to and including a specific value

$$F(X_i) = \text{sum of all probabilities through the value } X_i = P(X \leq X_i)$$

cumulative discrete



cumulative continuous



Example: Continuous Distribution (1)

- Monthly cash flows were observed over a 3-year period:
  - Low: L = \$20,000 High: H = \$30,000
    Most frequent: \$28,000
  - Write and graph probability and cumulative distributions
- Solution:
  - Let C_2 represent monthly cash flow; all amounts in \$1000.
  - Probability follows triangular distribution with
    - mode M (most frequently observed value of C_2) and maximum probability at M = \$28.
    - This is a continuous variable with limits of \$20 and \$30
  - Probability distribution, probability at mode is

$$P(C_2 = M) = \mathbf{f(M)} = \frac{2}{H - L}$$

  - Cumulative distribution, cumulative probability at mode is

$$P(C_2 \leq M) = \mathbf{F(M)} = (M - L)/(H - L)$$

Example: Continuous Distribution (2)

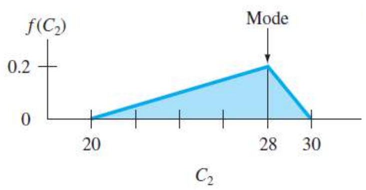
- Probability at mode M = \$28 is:

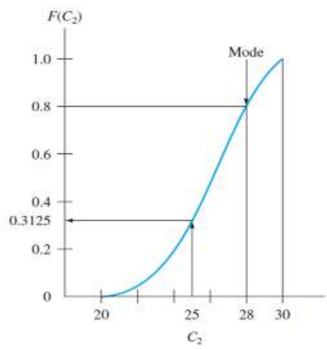
$$P(C_2 = 28) = \mathbf{f(28)} = 2/(30 - 20) = \mathbf{0.2}$$

- Cumulative probability at mode M = \$28 is:

$$P(C_2 \leq 28) = \mathbf{F(28)} = (28 - 20)/(30 - 20) = \mathbf{0.8}$$

The $f(C_2)$ distribution and $F(C_2)$ distribution are:





Random Sample

- Definition:
  - Selection in a random fashion of *n* values for a variable from a population with assumed or known probability distribution
- Variable values assumed to have same chance of occurring in sample and population
- Random numbers (RN) are taken
  - from either discrete or continuous distribution

Procedure to develop random sample (discrete or continuous)

1. Develop cumulative distribution F(X) from probability distribution
2. Assign RN values to F(X) scale in same proportion as probabilities
3. Determine scheme for selecting values from RN table for *n* values
4. Select the first number from RN table, enter F(X) scale, and record corresponding variable value. Repeat for *n* values.
5. Use *n* sample values for analysis and decision making under risk

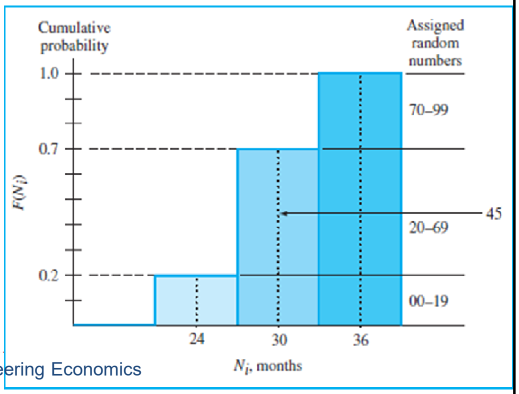
Example: Random Sample, Discrete Variable

- Develop a random sample of size $n = 15$ for the variable N, number of months, as described by the following probability distribution

$$P(N = N_i) = \begin{cases} 0.20 & N_1 = 24 \\ 0.50 & N_2 = 30 \\ 0.30 & N_3 = 36 \end{cases}$$

Solution:

This figure shows the cumulative probability distribution $F(N_i)$ with 20 numbers assigned to N_1 (i.e., 00-19), 20-69 to N_2, and 70-99 to N_3



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Example: Random Sample (1)

- Select 15 two-digit numbers from random number (RN) table
  - starting in column 1, row 3 and moving to the right

Random digits clustered into two-digit numbers

51	82	88	18	19	81	03	88	91	46	39	19	28	94	70	76	33	15	64	20	14	52
73	48	28	59	78	38	54	54	93	32	70	60	78	64	92	40	72	71	77	56	39	27
10	42	18	31	23	80	80	26	74	71	03	90	55	61	61	28	41	49	00	79	96	78
45	44	79	29	81	58	66	70	24	82	91	94	42	10	61	60	79	30	01	26	31	42
68	65	26	71	44	37	93	94	93	72	84	39	77	01	97	74	17	19	46	61	49	67
75	52	14	99	67	74	06	50	97	46	27	88	10	10	70	66	22	56	18	32	06	24

RN: 10 42 18 31 23 80 80 26 74 71 03 90 55 61 61

N: 24 30 24 30 30 36 36 30 36 36 24 36 30 30 30

Example: Random Sample (2)

- Use the 15 values to develop sample probabilities

Months, N	Times in sample	Sample probability	Population probability
24	3	**0.20**	0.20
30	7	**0.47**	0.50
36	5	**0.33**	0.30
	15		

- There is a Good agreement between sample probabilities and *actual* probabilities

Sample Estimate: Expected Value

- The expected value E(X) is the long-run expected average if the variable is sampled many, many times

Expected Value Measure and Estimate

	True Population Measure	True Population Measure	Sample Estimate	Sample Estimate
	Symbol	**Name**	**Symbol**	**Name**
Expected value	μ or E(X)	Mu or true mean	$\bar{X}$	Sample mean

Population : μ

Probability distribution : $E(X) = \sum X_i P(X_i)$

Sample : $\bar{X} = \dfrac{\text{sum of sample values}}{\text{sample size}} = \dfrac{\sum X_i}{n} = \dfrac{\sum f_i X_i}{n}$

Sample Estimate: Standard Deviation

- The standard deviation s or s(X) is the dispersion/spred of values about the expected value E(X) or the sample average $\overline{X}$

Standard Deviation Measure and Estimate

	True Population Measure	True Population Measure	Sample Estimate	Sample Estimate
	Symbol	**Name**	**Symbol**	**Name**
Standard deviation	σ or $\sqrt{\sigma^2}$	Sigma or true standard deviation	s or $\sqrt{s^2}$	Sample standard deviation

Population : $\sigma^2 = \mathrm{Var}(X)$ and $\sigma = \sqrt{\sigma^2} = \sqrt{\mathrm{Var}(X)}$

Probability distribution : $\mathrm{Var}(X) = \Sigma[X_i - E(X)]^2 P(X_i)$

Sample :

$$s^2 = \frac{\text{sum of (sample value} - \text{sample average)}^2}{\text{sample size} - 1}$$

$$= \frac{\Sigma(X_i - \bar{X})^2}{n-1}$$

$$s = \sqrt{s^2}$$

Fraction of Sample Values Between Two Limits

- Combine expected value and standard deviation to estimate
  - probability that a variable is between two limits (or is less than or more than a specified value)

- Example:
  - Find fraction of values (probability) between ±2s of E(Y) if sample statistics are:

    $E(Y) = 19.5$ km $\qquad s = 0.8$ km

  - Use sample results to determine fraction of sample values between $E(Y) \pm 2s = 19.5 - 2(0.8)$ and $19.5 + 2(0.8)$
  - The probability statement is:

$$P(\bar{Y} - 2s \leq \mathbf{Y} \leq \bar{Y} + 2s) = P(17.9 \leq \mathbf{Y} \leq 21.1)$$

Example: Mean and Standard Deviation (1)

- For the following nine air temperature readings, determine
  - (a) the sample mean, (b) the standard deviation, and (c) the percent of values within ±1 standard deviation of the mean:
    - 81, 86, 80, 91, 83, 83, 96, 85, 89
- Solution: (a) $\bar{X} = E(X) = (81 + 86 + \ldots + 85 + 89)/9$

$= 774/9$

$= \mathbf{86}$

(see next slide)

- Note: Spreadsheet functions for average and standard deviation:
  - =AVERAGE(X_1, X_2, …, X_n) or =AVERAGE(cell_1:cell_n)
  - =STDEV(X_1, X_2, …, X_n) or =STDEV(cell_1:cell_n)

Example: Mean and Standard Deviation (2)

(b)

Reading	Mean, $\bar{X}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
81	86	−5	25
86	86	0	0
80	86	−6	36
91	86	5	25
83	86	−3	9
83	86	−3	9
96	86	10	100
85	86	1	1
89	86	3	9
774	86	0	214

$s = \sqrt{214/(9-1)}$
$= 5.17$

(c) Range for $\pm 1s = 86 \pm 5.17 = 80.83$ to 91.17

Number of values in range = 7

or % of values in range = 7/9 = 77.8%

Simulation Analysis

- Simulation analysis uses random samples from the probability distribution of selected variables for alternative evaluation using a measure of worth
- Monte Carlo sampling: a commonly used simulation with the steps:
  1. Formulate alternatives: Set up alternatives and select measure of worth, such as PW, AW, ROR, or B/C
  2. Identify parameters with variation: Select parameters to be treated as random variables; estimate values for other parameters that are 'certain'
  3. Determine probability distributions: Determine discrete and continuous variables; describe probability distribution for each variable
  4. Conduct random sampling: Use sampling procedure discussed earlier
  5. Calculate measure of worth: Obtain n values of measure, i.e., PW
  6. Conduct measure of worth analysis: Construct probability distribution and calculate sample statistics, e.g., E(PW), s(PW), and E(PW) $\pm$ t $\times$ s(PW), where t = 1,2,3
  7. Draw conclusions: Decide which alternative is to be selected.

Simulation Analysis and Monte Carlo Sampling

- Independent variables
  - Simulation analysis assumes that all parameters and their distributions are independent of each other.
    This property is applied in step 5 of the Monte Carlo sampling procedure
- Standard distributions
  - Select relatively simple, standard distributions for spreadsheet (or manual) simulation. Set-up uses parameters such as expected value, standard deviation, and range
- Random number functions
  - For spreadsheet simulation, use RAND or RANDBETWEEN function to generate random sample
- Sample Size – Select sample size prior to performing simulation.
  - Spreadsheet simulations generate larger samples easily, e.g., n = 500 or 1000; manual simulation will have smaller n values.

Example 19.8 of the book

- There is an offer the two following systems.
  - As an incentive, the offer includes a guarantee of annual revenue for one of the systems for the first 5 years.
    - System 1. First cost is P = $12,000 for a set period of n = 7 years with no salvage value. No guarantee for annual net revenue is offered.
    - System 2. First cost is P = $8000, there is no salvage value, and there is a guaranteed annual net revenue of $1000 for each of the first 5 years, but after this period, there is no guarantee.
  - The equipment with updates may be useful up to 15 years, but the exact number is not known.
    - Cancellation anytime after the initial 5 years is allowed, with no penalty.

Example 19.8 of the book

- For either system, new versions of the equipment will be installed with no added costs.
  - If the MARR is 15% per year, use PW analysis to determine if neither, one, or both of the systems should be installed.
- Solution
  - Step 1. Formulation of alternatives.

$$\boxed{\begin{aligned} \mathrm{PW}_1 &= -P_1 + \mathrm{NCF}_1(P/A,15\%,n_1) \\ \mathrm{PW}_2 &= -P_2 + \mathrm{NCF}_G(P/A,15\%,5) \\ &\quad + \mathrm{NCF}_2(P/A,15\%,n_2-5)(P/F,15\%,5) \end{aligned}}$$

    - Where NCF: net cash flows (revenues), NCF_G: the guaranteed NCF of $1000 for system 2

Example 19.8 of the book

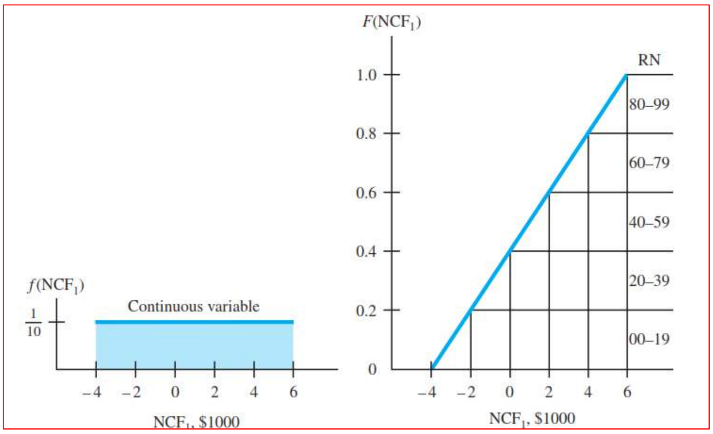
- Step 2. Parameters with variation
- System 1
  - Certainty. P_1 = \$12,000; n_1 = 7 years.
  - Variable. NCF_1 is a continuous variable, uniformly distributed (L = \$−4000 & H = \$6000) per year, because this is considered a high-risk venture.
- System 2
  - Certainty. P_2 = \$8000; NCF_G = \$1000 for first 5 years.
  - Variable. NCF_2: a discrete variable, uniformly distributed over L = \$1000 to H = \$6000 only in \$1000 increments, that is, \$1000, \$2000, etc.
  - Variable. n_2: a continuous variable that is uniformly distributed between L = 6 and H = 15 years.
- Updated relations

$$\begin{aligned} PW_1 &= -12{,}000 + NCF_1(P/A,15\%,7) \\ &= -12{,}000 + NCF_1(4.1604) \end{aligned}$$

$$\begin{aligned} PW_2 &= -8000 + 1000(P/A,15\%,5) \\ &\quad + NCF_2(P/A,15\%,n_2-5)(P/F,15\%,5) \\ &= -4648 + NCF_2(P/A,15\%,n_2-5)(0.4972) \end{aligned}$$

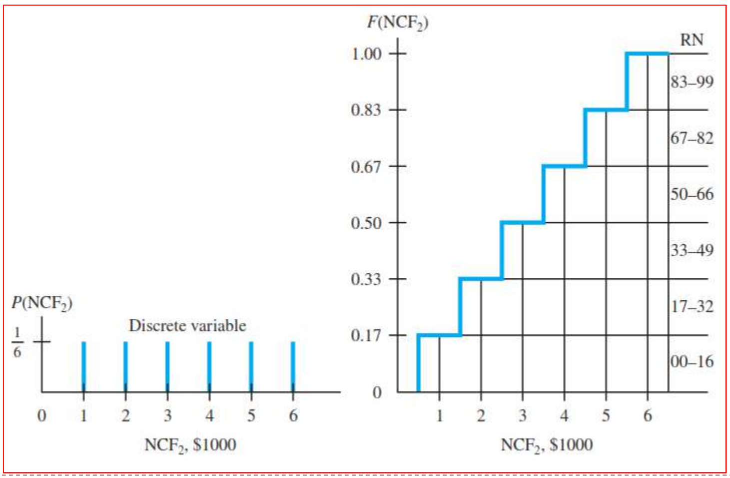
Example 19.8 of the book

- Step 3. Probability distributions for NCF_1



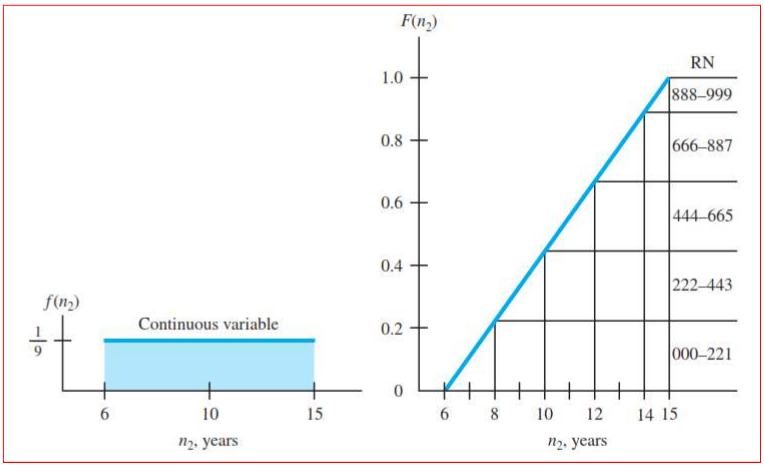
Example 19.8 of the book

- Step 3. Probability distributions for NCF_2



Example 19.8 of the book

- Step 3. Probability distributions for n



Example 19.8 of the book

- Step 4. Random sampling

NCF$_1$		NCF$_2$		n_2		
RN*	Value, $	RN†	Value, $	RN‡	Value, Year	Rounded§
18	−2200	10	1000	586	11.3	12
59	+2000	10	1000	379	9.4	10
31	−1100	77	5000	740	12.7	13
29	−900	42	3000	967	14.4	15
71	+3100	55	4000	144	7.3	8

(Random numbers table in slides 30)

*Randomly start with row 1, column 4 in Table 19–2.
†Start with row 6, column 14.
‡Start with row 4, column 6.
§The n_2 value is rounded up.

Example 19.8 of the book

- Step 5. Measure of worth calculation
  - With the five sample values in previous slide Table, calculate the PW values using relation in step 2

1. $PW_1 = -12{,}000 + (-2200)(4.1604) = \$-21{,}153$
2. $PW_1 = -12{,}000 + 2000(4.1604) = \$-3679$
3. $PW_1 = -12{,}000 + (-1100)(4.1604) = \$-16{,}576$
4. $PW_1 = -12{,}000 + (-900)(4.1604) = \$-15{,}744$
5. $PW_1 = -12{,}000 + 3100(4.1604) = \$+897$

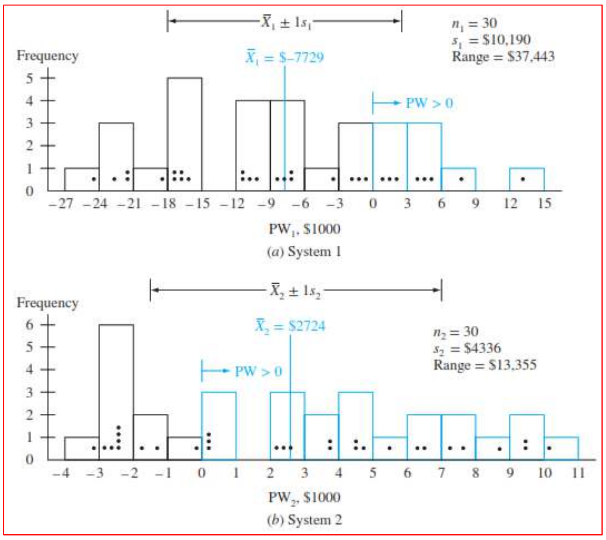
1. $PW_2 = -4648 + 1000(P/A,15\%,7)(0.4972) = \$-2579$
2. $PW_2 = -4648 + 1000(P/A,15\%,5)(0.4972) = \$-2981$
3. $PW_2 = -4648 + 5000(P/A,15\%,8)(0.4972) = \$+6507$
4. $PW_2 = -4648 + 3000(P/A,15\%,10)(0.4972) = \$+2838$
5. $PW_2 = -4648 + 4000(P/A,15\%,3)(0.4972) = \$-107$

  - Now, 25 more RNs are selected for each variable from the RN Table, and the PW values are calculated

Example 19.8 of the book

- Step 6. Measure of worth description

Probability distributions of simulated PW values for a sample of size 30



Example 19.8 of the book

- Step 6. Measure of worth description
  - PW_1. Sample values range from \$−24,481 to \$+12,962. The calculated measures of the 30 values are

$$\bar{X}_1 = \$-7729 \qquad s_1 = \$10,190$$

  - PW_2. Sample values range from \$−3031 to \$+10,324. The sample measures are

$$\bar{X}_2 = \$2724 \qquad s_2 = \$4336$$

- Step 7. Conclusions
  - Additional sample values will surely make the central tendency of the PW distributions more evident and may reduce the s values, which are quite large.
  - System 1. Do not accept this alternative since the sample indicates a probability of 0.27 (8 out of 30 values) having a positive PW.
  - System 2. Accept this alternative since it has a positive Mean and a probability of 67% (20 of the 30 PW values) having a positive PW.

Summary of Important Points

- Three classifications of decision making:
  - certainty, under risk, and uncertainty.
  - Virtually all decisions involve risk
- Decision making under risk uses
  - random variables (discrete or continuous) and their probability distribution
- Random sampling involves cumulative probability distributions and the use of a random number table or spreadsheet function
- Expected value E(X) and standard deviations are used routinely in making computations about risk and probability
- Simulation analysis using Monte Carlo sampling is a powerful technique to account for risk in economic analyses