In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

Course Title:
Engineering Economics

5. Present Worth Analysis

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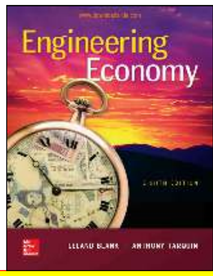
Learning Stage 2: Basic Analysis Tools

- Chapter 5
  - Present Worth Analysis
- Chapter 6
  - Annual Worth Analysis
- Chapter 7
  - Rate of Return Analysis: One Project
- Chapter 8
  - Rate of Return Analysis: Multiple Alternatives
- Chapter 9
  - Benefit/Cost Analysis and Public Sector Economics



Chapter 5 of EE (BT) book 8th edition

LEARNING OUTCOMES

- Purpose:
  - Utilize different present worth techniques to evaluate and select alternatives.

1. Formulate Alternatives
2. PW of equal-life alternatives
3. PW of different-life alternatives
4. Future Worth analysis
5. Capitalized Cost analysis

Formulating Alternatives

- Two types of economic proposals
  - Mutually Exclusive (ME) Alternatives: Only one can be selected; Compete against each other
  - Independent Projects: More than one can be selected; Compete only against DN
    - Do Nothing (DN) – An ME alternative or independent project to maintain the current approach; no new costs, revenues or savings
- Two types of cash flow estimates
  - Revenue: Alternatives include **both** estimates of costs (cash outflows) **and** revenues (cash inflows)
  - Cost: Alternatives include **only** costs;
    - revenues and savings assumed equal for all alternatives;
    - also called service alternatives

PW Analysis of Alternatives

- Convert all cash flows to PW using MARR
- Use costs by minus sign; receipts by plus sign

EVALUATION

- For one project, if PW > 0, it is justified
- For mutually exclusive alternatives, select one with numerically largest PW
- For independent projects, select all with PW > 0

Selection of Alternatives by PW

- Example: For the alternatives shown below, which should be selected if they are (a) mutually exclusive; (b) independent?

Project ID	Present Worth
A	$30,000
B	$12,500
C	$-4,000
D	$ 2,000

Solution: (a) Select numerically largest PW; alternative A

(b) Select all with PW > 0; projects A, B & D

Example: PW Evaluation of Equal-Life ME Alts.

- Example:
  - Alternative X has a first cost of $20,000, an operating cost of $9,000 per year, and a $5,000 salvage value after 5 years.
  - Alternative Y will cost $35,000 with an operating cost of $4,000 per year and a salvage value of $7,000 after 5 years.
  - At an MARR of 12% per year, which should be selected?

Solution: Find PW at MARR and select numerically larger PW value

$$PW_X = -20{,}000 - 9000(P/A, 12\%, 5) + 5000\left(\tfrac{P}{F}, 12\%, 5\right) = -\$49{,}606$$

$$PW_Y = -35{,}000 - 4000(P/A, 12\%, 5) + 7000(P/F, 12\%, 5) = -\$45{,}447$$

Select alternative Y

PW of Different-Life Alternatives

- Must compare alternatives for equal service
  - i.e., alternatives must end at the same time
- Two ways to compare equal service:
  - Least common multiple (LCM) of lives
  - Specified study period

Note: The LCM procedure is used unless otherwise specified

Assumptions of LCM approach

- Service provided
  - is needed over the LCM or more years
- Selected alternative
  - can be repeated over each life cycle of LCM in exactly the same manner
- Cash flow estimates are the same for each life cycle
  - i.e., change in exact accord with the inflation or deflation rate

Example: Different-Life Alternatives

- Compare the machines below using present worth analysis at i = 10% per year

	Machine A	Machine B
First cost, $	20,000	30,000
Annual cost, $/year	9000	7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: LCM = **6** years; repurchase A after 3 years

$$\mathbf{PW_A = -20,000 - 9000(P/A, 10\%, 6) - 16,000(P/F, 10\%, 3) + 4000(P/F, 10\%, 6) = \$ - 68,961}$$

$$\mathbf{PW_B = -30,000 - 7000\left(\tfrac{P}{A}, 10\%, 6\right) + 6000\left(\tfrac{P}{F}, 10\%, 6\right) = \$ - 57,100}$$

20,000 – 4,000 in year 3

Select alternative B

PW Evaluation Using a Study Period

- Once a study period is specified,
  - all cash flows after this time are ignored
- Salvage value is
  - the estimated market value at the end of study period
- Short study periods are often defined by management
  - when business goals are short-term
- Study periods are commonly used in equipment replacement analysis

Example: Study Period PW Evaluation

- Compare the alternatives below using present worth analysis
  - at i = 10% per year and a 3-year study period

	Machine A	Machine B
First cost, $	−20,000	− 30,000
Annual cost, $/year	− 9,000	− 7,000
Salvage/market value, $	4,000	6,000 (after 6 years) 10,000 (after 3 years)
Life, years	3	6

Solution: Study period = 3 years; disregard all estimates after 3 years

$$\mathbf{PW_A = -20{,}000 - 9000(P/A, 10\%, 3) + 4000(P/F, 10\%, 3) = \$-39{,}376}$$

$$\mathbf{PW_B = -30{,}000 - 7000(P/A, 10\%, 3) + 10{,}000(P/F, 10\%, 3) = \$-39{,}895}$$

Marginally, select A; different selection than for LCM = 6 years

Future Worth Analysis

- FW exactly like PW analysis, except calculate FW
- Must compare alternatives for equal service
  - i.e. alternatives must end at the same time
- Two ways to compare equal service:
  - Least common multiple (LCM) of lives
  - Specified study period

Note: The LCM procedure is used unless otherwise specified

FW of Different-Life Alternatives

- Compare the machines below using future worth analysis
  - at i = 10% per year

	Machine A	Machine B
First cost, $	−20,000	−30,000
Annual cost, $/year	−9000	−7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: LCM = 6 years; repurchase A after 3 years

$$FW_A = -20,000(F/P,10\%,6) - 9000(F/A,10\%,6) - 16,000(F/P,10\%,3) + 4000 = \$-122,168$$

$$FW_B = -30,000(F/P,10\%.6) - 7000(F/A,10\%,6) + 6000 = \$-101,157$$

Select B

Note: PW and FW methods will always result in same selection

Capitalized Cost (CC) Analysis

- CC refers to the PW of a project with a very long life,
  - i.e., PW as n becomes infinite

Basic equation is: $CC = P = \frac{A}{i}$

$$P = A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

  - "A" essentially represents the interest on a perpetual investment

- For example, in order to be able to withdraw \$50,000 per year forever at i = 10% per year,
  - the amount of capital required is 50,000/0.10 = \$500,000

- For finite life alternatives,
  - convert all cash flows into an A value over one life cycle and then divide by i.

Example: Capitalized Cost

- Compare the machines shown below on the basis of their capitalized cost. Use i = 10% per year

	Machine 1	Machine 2
First cost,\$	−20,000	− 100,000
Annual cost,\$/year	− 9000	− 7000
Salvage value, \$	4000	-----
Life, years	3	∞

Solution: Convert machine 1 cash flows into A and then divide by i

$$\mathbf{A_1 = -20,000(A/P,10\%,3) - 9000 + 4000(A/F,10\%,3) = \$ - 15,834}$$

$$\mathbf{CC_1 = -15,834/0.10 = \$ - 158,340}$$

$$\mathbf{CC_2 = -100,000 - 7000/0.10 = \$ - 170,000}$$

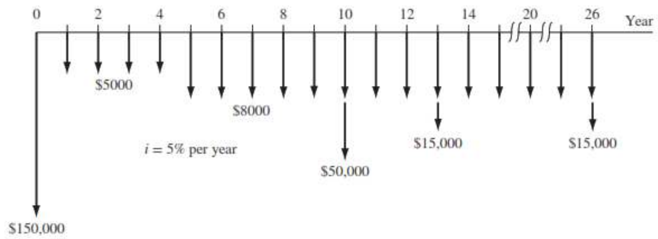
Select machine 1

Example (5.6) of Capitalized Cost

- A transportation company has just installed a new software to charge and track toll fees.
  - The director wants to know the total equivalent cost of all future costs incurred to purchase the software system.
  - If the new system will be used for the indefinite future, find the equivalent cost (a) now, a CC value, and (b) for each year hereafter, an AW value.
    - The system has an installed cost of **$150,000** and an additional cost of **$50,000 after 10 years**.
    - The annual software maintenance contract cost is **$5000** for the **first 4 years** and **$8000 thereafter**.
    - In addition, there is expected to be a recurring major upgrade cost of **$15,000 every 13 years**.
    - Assume that i = 5% per year for county funds.

Example (5.6) of Capitalized Cost (cont'd)

- Solution
  - a) Draw a cash flow diagram for two cycles
  - Find the PW of the nonrecurring costs of $150,000 now and $50,000 in year 10 at i = 5%. (i.e., CC_1)



$$\boxed{CC_1 = -150{,}000 - 50{,}000(P/F{,}5\%{,}10) = \$-180{,}695}$$

Example (5.6) of Capitalized Cost (cont'd)

- Convert the \$15,000 recurring cost to an A value over the first cycle of 13 years, and find the capitalized cost (CC_2) at 5% per year

$$A = -15{,}000(A/F,5\%,13) = \$-847$$
$$CC_2 = -847/0.05 = \$-16{,}940$$

- There are several ways to convert the annual software maintenance cost series to A and CC values.
  - A straightforward method is to, first, consider the \$−5000 an A series with a capitalized cost of

$$CC_3 = -5000/0.05 = \$-100{,}000$$

  - Second, convert the step-up maintenance cost series of \$−3000 to a capitalized cost CC_4 in year 4, and find the PW in year 0

$$CC_4 = \frac{-3000}{0.05}(P/F,5\%,4) = \$-49{,}362$$

Example (5.6) of Capitalized Cost (cont'd)

- The total capitalized cost CC_T for this company is the sum of the four-component CC values.

$$CC_T = -180{,}695 - 16{,}940 - 100{,}000 - 49{,}362 = \$-346{,}997$$

- b) the AW value forever:

$$AW = Pi = CC_T(i) = \$346{,}997(0.05) = \$17{,}350$$

- Correctly interpreted, this means that
  - the company have committed the equivalent of \$17,350 forever to operate and maintain the toll management software.

Summary of Important Points

- PW method converts all cash flows to present value at MARR
- Alternatives can be mutually exclusive or independent
- Cash flow estimates can be for revenue or cost alternatives
- PW comparison must always be made for equal service
- Equal service is achieved by using LCM or study period
- Capitalized cost is PW of project with infinite life;
  - $CC = P = A/i$