In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

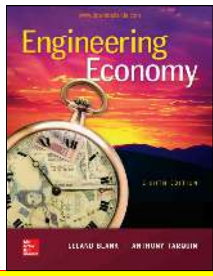
Course Title:
Engineering Economics

13. Depreciation Methods

By: Akbar Esfahanipour

Learning Stage 3: Making Better Decisions

- Chapter 14
  - Effects of Inflation
- Chapter 15 *not covered in this course
  - Cost Estimation and Indirect Cost Allocation
- Chapter 16
  - Depreciation Methods
- Chapter 17
  - After-Tax Economic Analysis
- Chapter 18
  - Sensitivity Analysis and Staged Decisions
- Chapter 19
  - More on Variation and Decision Making under Risk



Chapter 1۶ of EE (BT) book 8th edition

LEARNING OUTCOMES

- Purpose:
  - Use depreciation or depletion methods to reduce the book value of a capital investment in an asset or natural resource.

1. Understand basic terms of asset depreciation
2. Apply straight line and SYD methods of depreciation
3. Apply DB and DDB methods of depreciation; switch between DDB and SL methods
4. Apply MACRS method of depreciation
5. Apply Unit-of-Production (UOP) Depreciation
6. Explain depletion and apply cost depletion & percentage depletion methods

Depreciation Terminology

- Depreciation:
  - Book (noncash) method to represent decrease in value of a tangible asset over time
    - E.g., equipment, computers, vehicles, buildings, and machinery
    - depreciation amount is not an actual cash flow
- Depreciation is a tax-allowed deduction included
  - in tax calculations in virtually all industrialized countries.
  - depreciation lowers income taxes via
    - Taxes = (income − deductions) (tax rate)
- Two types of depreciation:
  - Book dep.: used for internal accounting to track value of assets or property over its life using any method
  - Tax dep.: used to determine taxes due based on tax laws
    - In USA only, it must be calculated using MACRS (Modified Accelerated Cost Recovery System)

Common Depreciation Terms

- First cost P or unadjusted basis B: Total installed cost of asset
- Book value BV_t: Remaining undepreciated capital investment in year t
- Recovery period n: Depreciable life of asset in years
- Market value MV: Amount realizable if asset is sold on an open market
- Salvage value S: Estimated trade-in or MV at end of asset's useful life
- Depreciation rate (recovery rate) d_t: Fraction of first cost or basis removed each year t
- Personal property: Possessions of company used to conduct business
  - two types of property: the income-producing and tangible possessions
- Real property: Real estate and all improvements (land is not depreciable)
- Half-year convention:
  - Assumes assets are placed in service or disposed of in midyear

Straight Line (SL) Depreciation

- Straight Line (SL) Depreciation
  - Book value decreases linearly with time

$$D_t = \frac{B - S}{n}$$

Where:
D_t = annual depreciation charge
t = year
B = first cost or unadjusted basis
S = salvage value
n = recovery period

$$BV_t = B - tD_t$$

Where: BV_t = book value after t years

SL depreciation rate is constant for each year: $d = d_t = 1/n$

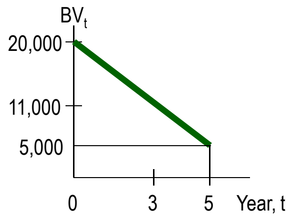
Example: SL Depreciation

- An argon gas processor has a first cost of \$20,000 with a \$5,000 salvage value after 5 years.
  - Find (a) D_3 and (b) BV_3 for year three. (c) Plot book value vs. time.

Solution: a) $\mathbf{D_3 = (B - S)/n}$
$= (20{,}000 - 5{,}000)/5$
$= \$3{,}000$

b) $\mathbf{BV_3 = B - tD_t}$
$= 20{,}000 - 3(3{,}000)$
$= \$11{,}000$

(c) Plot BV vs. time



Sum-of-Years-Digits (SYD) Depreciation

- The SYD method:
  - Depreciation in the first year is the largest and it will be decreased through the life of the asset.

$$D_t = \frac{\text{depreciable years remaining}}{\text{sum of years digits}}(\text{basis} - \text{salvage value})$$

$$D_t = \frac{n - t + 1}{\text{SUM}}(B - S)$$

  - where SUM is the sum of the digits 1 through n

$$\text{SUM} = \sum_{j=1}^{j=n} j = \frac{n(n+1)}{2}$$

$$BV_t = B - \frac{t(n - t/2 + 0.5)}{\text{SUM}}(B - S)$$

  - The rate of depreciation decreases each year as

$$d_t = \frac{n - t + 1}{\text{SUM}}$$

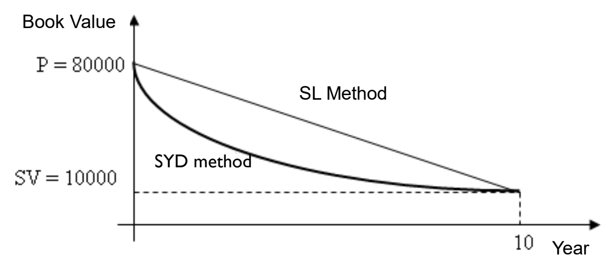
Example for SYD (1)

- Calculate the SYD depreciation charges and book values for an electro-optics equipment with B = $80,000, S = $10,000, and an 10-year recovery period

- Solution
  - The sum of the year's digits: SUM = 10*11/2 = 55,
  - D_1 = 10/55 * (80,000 – 10,000) = 12,727
  - D_2 = 9/55 * (80,000 – 10,000) = 11,455
  - D_3 = 8/55 * (80,000 – 10,000) = 8,909
  - …
  - D_{10} = 1/55 * (80,000 – 10,000) = 1,273

Example for SYD (2)

Year t	Depreciation D_t	Formula in Excel D_t	Book value BV_t
0			80,000
1	12,727	=SYD(80000;10000;10;**1**)	67,273
2	11,455	=SYD(80000;10000;10;**2**)	55,818
3	8,909	=SYD(80000;10000;10;**3**)	46,909
…			
10	1,273	=SYD(80000;10000;10;**10**)	10,000 = SV



Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

- It is determined by multiplying BV at beginning of year by fixed percentage d
  - Max rate for d is twice straight line rate (DDB), i.e., $d \le 2/n$
  - The rate for d in Declining Balance is $1/n$
  - Cannot depreciate below salvage value
- Depreciation for year t is obtained by:

$$D_t = dB(1-d)^{t-1} = dBV_{t-1}$$

Where: D_t = depreciation for year t
d = uniform depreciation rate (2/n for DDB)
B = first cost or unadjusted basis
BV_{t-1} = book value at end of previous year

- Book value for year t: $BV_t = B(1-d)^t$
- Implied d $= 1 - (S/B)^{1/n}$, if $BV_n = SV$

Example: Double Declining Balance (DDB)

- A depreciable construction truck has a first cost of $20,000 with a $4,000 salvage value after 5 years. Find
  - (a) depreciation, and
  - (b) book value after 3 years using DDB depreciation.
- Solution: (a) $d = 2/n = 2/5 = 0.4$

$$D_3 = dB(1-d)^{t-1}$$
$$= 0.4(20{,}000)(1-0.40)^{3-1} = \$2880$$

(b) $BV_3 = B(1-d)^t$
$$= 20{,}000(1-0.4)^3 = \$4320$$

Spreadsheet Functions for Depreciation

- Straight line function:
  - SLN(B, S, n)
- Sum of years digit
  - SYD(B, S, n, t)
- Declining balance function:
  - DB(B, S, n, t)
- Double declining balance function:
  - DDB(B, S, n, t, d)

Note: It is better to use the DDB function for DB and DDB depreciation.
DDB function checks for BV < S and is more accurate than the DB function.

Switching Between Depreciation Methods

- Switch between methods to maximize PW of depreciation

$$PW_D = \sum_{t=1}^{t=n} D_t \, (P/F, i\%, t)$$

  - A switch from DDB to SL in latter part of life is usually better
  - Can switch only one time during recovery period
- Procedure to switch from DDB to SL:
  - Each year t compute DDB and SL depreciation using the relations

$$D_{DDB} = d(BV_{t-1}) \qquad \text{and} \qquad D_{SL} = BV_{t-1} \,/\, (n - t + 1)$$

  - Select larger depreciation amount, i.e., $D_t = \max[D_{DDB}, D_{SL}]$
  - If required, calculate PW_D

Alternatively, use spreadsheet function VDB(B,S,n,start_t,end_t) to determine D_t

Example 16.3

- A company has purchased a new ore grading unit for $80,000.
  - The unit has an anticipated life of 10 years and a salvage value of $10,000.
  - Use the DB and DDB methods to compare the schedule of depreciation and book values for each year.
    - Solve by hand and by spreadsheet.
  - Solution: implied DB depreciation rate

$$\boxed{d = 1 - \left(\frac{10,000}{80,000}\right)^{1/10} = 0.1877}$$

  - Depreciation rate for DDB: 2/n = 0.2

Example 16.3

	Declining Balance, $		Double Declining Balance, $	
Year *t*	***D_t***	**BV_t**	***D_t***	**BV_t**
0	—	80,000	—	80,000
1	15,016	64,984	16,000	64,000
2	12,197	52,787	12,800	51,200
3	9,908	42,879	10,240	40,960
4	8,048	34,831	8,192	32,768
5	6,538	28,293	6,554	26,214
6	5,311	22,982	5,243	20,972
7	4,314	18,668	4,194	16,777
8	3,504	15,164	3,355	13,422
9	2,846	12,318	2,684	10,737
10	2,318	10,000	737	10,000

- Please note calculation of D_{10} and BV_{10} for the two methods:

D_{10} = 12,318*0.188 = 2,409
BV_{10} = 12,318 – 2,409 = 9,909 < SV

D_{10} = 10,737*0.2 = 2,074
BV_{10} = 10,737 – 2,074 = 8,663 < SV

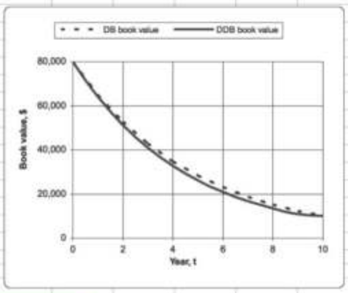
Example 16.3

	A	B	C	D	E
1					
2	First cost, B	$ 80,000			
3	Salvage, S	$ 10,000			
4	Rec'vry pd, n	10			
5	Depr. rate, d	0.1877476			
6		Declining balance (DB), $		Double declining (DDB), $	
7	Year, t	D(t)	DB book value	D(t)	DDB book value
8	0		80,000		80,000
9	1	15,020	64,980	16,000	64,000
10	2	12,200	52,780	12,800	51,200
11	3	9,909	42,871	10,240	40,960
12	4	8,049	34,822	8,192	32,768
13	5	6,538	28,284	6,554	26,214
14	6	5,310	22,974	5,243	20,972
15	7	4,313	18,661	4,194	16,777
16	8	3,503	15,157	3,355	13,422
17	9	2,846	12,311	2,684	10,737
18	10	2,311	10,000	737	10,000

= 1–(B3/B2)^0.1

= DDB(B$2,B$3,B$4,$A18,10*B5)

= DDB(B$2,B$3,B$4,$A18)



Factor for DB method = n * implied rate = 0.1877*10 = 1.877
Factor for DDB method =2.

Example for DDB when BV_n < SV (1)

- An 11-year life machine can be purchased $138,000 now with salvage value of $28,000.
  - Calculate depreciation charges and book values during its life using DDB method.

- Solution
  - $d = 2/n = 2/11 = 0.182$
  - $BV_{11} = P(1 - d)^{11} = 138000(1 - 0.182)^{11} = 15145 < 28000 = SV$
    - So we need to switch between the DDB and the SL method

Example for DDB when $BV_n < SV$ (2)

Year t	Depreciation D_t	Formula in Excel D_t	Book value BV_t
0			138,000
1	25,116	=DDB(138000;28000;11;**1**)	112,884
2	20,545	=DDB(138000;28000;11;**2**)	92,339
3	16,806	=DDB(138000;28000;11;**3**)	75,533
4	13,747	=DDB(138000;28000;11;**4**)	61,786
5	11,245	=DDB(138000;28000;11;**5**)	50,541
6	9,199	=DDB(138000;28000;11;**6**)	41,342
7	7,524	=DDB(138000;28000;11;**7**)	33,818
8	6,155	=DDB(138000;28000;11;**8**)	27,663 < 28,000
8	**5,818**		**28,000**
9	0		28,000
10	0		28,000
11	0		28,000

Example for DDB when $BV_n > SV$ (1)

- A 5-year life asset can be purchased $900 now with salvage value of $30.
  - Calculate depreciation charges and book values during its life using DDB method.
- Solution
  - $d = 2/n = 2/5 = 0.40$
  - $BV_5 = P\,(1 - d)^5 = 900\,(1 - 0.40)^5 = 70 > 30 = SV$
    - So we need to switch between the DDB and the SL method
    - Determine the switching time to maximize PW of depreciations

Example for DDB when $BV_n > SV$ (2)

540	D_{DDB} for year t	Book value BV_t	D_{SL} for year t
0		900	
1	360	540	174
2	216	324	127.5
3	130	194	98
4	78 → 82	116 → 112	82
5	46 → 82	70 → 30	86

=VDB(900;30;5;4;5;2;FALSE) = 82

- VDB(cost, salvage, life, start_period, end_period, [factor], [no_switch])
  - VDB: variable declining balance.
  - It returns depreciation of an asset for any partial periods, using the DDB method or some other method.
  - **No_switch** A logical value specifying whether to switch to SL method when $D_{SL} > D_{DDB}$.
    - If no_switch is FALSE or omitted, Excel switches to SL when required.

MACRS Depreciation (1)

- Required method to use for tax depreciation in USA only
- Originally developed to offer accelerated depreciation for economic growth

$$D_t = d_t B$$

Where: D_t = depreciation charge for year t
B = first cost or unadjusted basis
d_t = depreciation rate for year t (decimal)

- Get value for d_t from IRS table for MACRS rates

$$BV_t = B - \sum_{j=1}^{j=t} D_j$$

Where: D_j = depreciation in year j
$\sum D_j$ = all depreciation through year t

MACRS Depreciation (2)

- Always depreciates to zero;
  - i.e., no salvage value is considered
- Incorporates switching from DDB to SL depreciation
- Standardized recovery periods (n) are tabulated
  - n = 3, 5, 7, 10, 15, or 20 years for personal property (e.g., equipment/ vehicles)
  - n = 27.5 or 39 years for real property (e.g., rental property or structures)
- MACRS recovery time is always n+1 years;
  - half-year convention assumes purchase in midyear
- No special spreadsheet function;
  - We can use VDB for year t:

$$= \text{VDB}(B, 0, n, \text{MAX}(0, t-1.5), \text{MIN}(n, t-0.5), d)$$

where

B = unadjusted basis
0 = salvage value of $S = 0$
n = recovery period
$$d = \begin{cases} 2 & \text{if MACRS } n = 3, 5, 7, \text{ or } 10 \\ 1.5 & \text{if MACRS } n = 15 \text{ or } 20 \end{cases}$$

MACRS Depreciation (3)

Year	Depreciation Rate (%) for Each MACRS Recovery Period in Years					
	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 15$	$n = 20$
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.29
7			8.93	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.56	5.91	4.46
10				6.55	5.90	4.46
11				3.28	5.91	4.46
12					5.90	4.46
13					5.91	4.46
14					5.90	4.46
15					5.91	4.46
16					2.95	4.46
17–20						4.46
21						2.23

Depreciation Rates (d_t) Applied to the Basis B for the MACRS Method

Example: MACRS Depreciation

- A finishing machine has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Using MACRS, find (a) D and (b) BV for year 3.

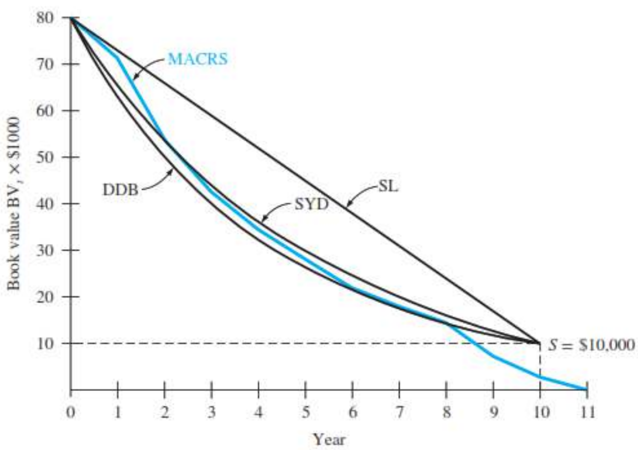
Solution: (a) From table, $d_3 = 19.20$

$$D_3 = 20{,}000(0.1920)$$
$$= \$3{,}840$$

(b) $BV_3 = 20{,}000 - 20{,}000(0.20 + 0.32 + 0.1920)$
$$= \$5{,}760$$

Note: Salvage value S = \$5,000 is not used by MACRS and $BV_6 = 0$

Comparison of Different depreciation methods



Comparison of book values for an \$80,000 asset with S = \$10,000 and n =10 years

Unit-of-Production (UOP) Depreciation

- Depreciation based on usage of equipment, not time
- Depreciation for year t obtained by relation

$$\mathbf{D_t} = \frac{\text{actual usage for year t}}{\text{expected total lifetime usage}}(\mathbf{B} - \mathbf{S})$$

- Example: A new mixer is expected to process 4 million yd^3 of concrete over 10-year life time.
  - Determine depreciation for year 1 when 400,000 yd^3 is processed. Cost of mixer was $175,000 with no salvage expected.
- Solution: $D_1 = \frac{400,000}{4,000,000}(175,000 - 0) = \$17,500$

Depletion Methods

- Depletion: book (noncash) method to represent decreasing value of natural resources
- Two methods: cost depletion (CD) & percentage depletion (PD)
- Cost depletion: Based on level of activity to remove a natural resource
  - Calculation: Multiply factor CD_t by amount of resource removed
    - Where: CD_t = first cost / resource capacity
  - Total depletion can not exceed first cost of the resource
- Percentage depletion: Based on gross income (GI) from a resource
  - Calculation: Multiply GI by standardized rate (%) from table
  - Annual depletion can not exceed 50% of company's taxable income (TI)
- Tax-allowed depletion amount for year t:

$$\text{Depletion} = \begin{cases} \max[CDA_t, PDA_t] & \text{if } PDA_t \leq 50\% \text{ of } TI_t \\ \max[CDA_t, 50\% \text{ of } TI_t] & \text{if } PDA_t < 50\% \text{ of } TI_t \end{cases}$$

Example: Cost and Percentage Depletion

- A mine purchased for \$3.5 million has a total expected yield of one million ounces of silver (i.e., total capacity of mine: 1 million ounces)
  - Determine the depletion charge in year 4 when 300,000 ounces are mined and sold for \$30 per ounce using
  - (a) cost depletion, (b) percentage depletion (c) Which is larger for year 4?
- Solution: Let depletion amounts equal CDA_4 and PDA_4
  - (a) Factor, CD_4 = 3,500,000/ 1,000,000 = \$3.50 per ounce
    $CDA_4 = 3.50(300{,}000) = \$1{,}050{,}000$
  - (b) Percentage depletion rate for silver mines is 0.15
    $PDA_4 = (0.15)(300{,}000)(30) = \$1{,}350{,}000$
  - (c) Claim percentage depletion amount, if it ≤ 50% of TI

Summary of Important Points

- Two types for depreciation: tax and book
- Classical methods are straight line and declining balance
- In USA only, MACRS method is required for tax depreciation
- Determine MACRS recovery period using either GDS or ADS
- Switching between methods is allowed;
  - MACRS switches automatically from DDB to SL to maximize write-off
- Depletion (instead of depreciation) used for natural resources
- Two methods of depletion:
  - cost (amount resource removed × CD_t factor) and percentage (gross income × tabulated %)