In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

**Course Title:
Engineering Economics**

6. Annual Worth Analysis

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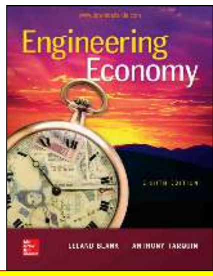
Learning Stage 2: Basic Analysis Tools

- Chapter 5
  - Present Worth Analysis
- Chapter 6
  - Annual Worth Analysis
- Chapter 7
  - Rate of Return Analysis: One Project
- Chapter 8
  - Rate of Return Analysis: Multiple Alternatives
- Chapter 9
  - Benefit/Cost Analysis and Public Sector Economics



Chapter 6 of EE (BT) book 8th edition

LEARNING OUTCOMES

- Purpose:
  - Utilize different annual worth techniques to evaluate and select alternatives.

1. Advantages of AW
2. Capital Recovery and AW values
3. AW analysis
4. Perpetual life
5. Life-Cycle Cost analysis

Advantages of AW Analysis

- AW calculated for only one life cycle
- Assumptions:
  - Services needed for at least the LCM of lives of alternatives
  - Selected alternative will be repeated in succeeding life cycles in same manner as for the first life cycle
  - All cash flows will be same in every life cycle
    - i.e., will change by only inflation or deflation rate

Alternatives usually have the following cash flow estimates

- Initial investment, P
  - First cost of an asset
- Salvage value, S
  - Estimated value of asset at end of useful life
- Annual amount, A
  - Cash flows associated with asset, such as annual operating cost (AOC), etc.

- Relationship between AW, PW and FW
  - $\mathbf{AW = PW(A/P, i\%, n) = FW(A/F, i\%, n)}$
    - n is years for equal-service comparison (value of LCM or specified study period)

Calculation of Annual Worth

- AW for one life cycle is the same for all life cycles!!

- Example: An asset has a first cost of $20,000, an annual operating cost of $8000 and a salvage value of $5000 after 3 years.
  - Calculate the AW for one and two life cycles at i = 10%

$$\mathbf{AW_{one} = -20,000(A/P, 10\%, 3) - 8000 + 5000(A/F, 10\%, 3)}$$
$$= \mathbf{\$ - 14,532}$$
$$\mathbf{AW_{two} = -20,000(A/P, 10\%, 6) - 8000}$$
$$\mathbf{-15,000(P/F, 10\%, 3)(A/P, 10\%, 6) + 5000(A/F, 10\%, 6)}$$
$$= \mathbf{\$ - 14,532}$$

Both AWs are the same

Capital Recovery and AW

- Capital recovery (CR)
  - the equivalent annual amount that an asset, process, or system must earn each year to just recover the first cost and a stated rate of return over its expected life.
    - Salvage value is considered when calculating CR.
    - $\mathbf{CR = -P(A/P, i\%, n) + S(A/F, i\%, n)}$
- Use previous example: note: AOC not included in CR
  - $\mathbf{CR = -20,000\left(\frac{A}{P}, 10\%, 3\right) + 5000\left(\frac{A}{F}, 10\%, 3\right) = \$ - 6532\ per\ year}$
- Now $\mathbf{AW = CR + A}$
  - $\mathbf{AW = -6532 - 8000 = \$ - 14,532}$

Same as the previous slide

Selection Guidelines for AW Analysis

- One alternative:
  - If AW = 0, the requested MARR is met or exceeded and the alternative is economically justified.
- Two or more alternatives:
  - Select the alternative with the AW that is numerically largest, that is, less negative or more positive.
    - This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.

ME Alternative Evaluation by AW

- Not necessary to use LCM for different life alternatives
- Example: A company is considering two machines.
  - Machine X has a first cost of \$30,000, AOC of \$18,000, and S of \$7000 after 4 years.
  - Machine Y will cost \$50,000 with an AOC of \$16,000 and S of \$9000 after 6 years.
  - Which machine should the company select at an interest rate of 12% per year?

Solution: $AW_X = -30{,}000(A/P,12\%,4) - 18{,}000 + 7{,}000(A/F,12\%,4) = \$-26{,}412$

$AW_Y = -50{,}000(A/P,12\%,6) - 16{,}000 + 9{,}000(A/F,12\%,6) = \$-27{,}052$

Select Machine X; it has the numerically larger AW value

AW of Permanent Investment

- Use A = Pi for AW of infinite life alternatives
- Find AW over one life cycle for finite life alternatives
- Example: Compare the alternatives below using AW and i = 10% per year

	C	D
First Cost, \$	−50,000	−250,000
Annual operating cost, \$/year	−20,000	−9,000
Salvage value, \$	5,000	75,000
Life, years	5	∞

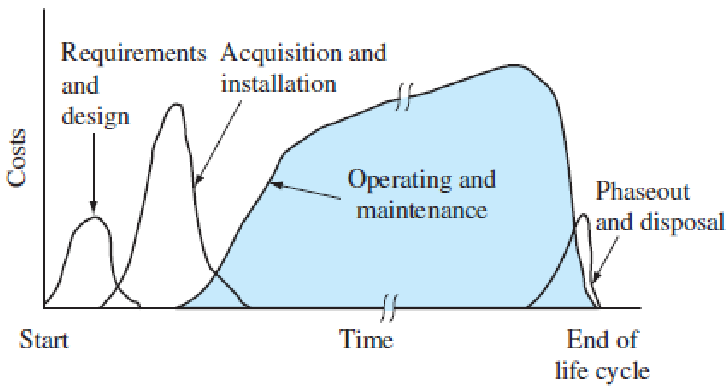
Solution: Find AW of C over 5 years and AW of D using relation $A = Pi$

$AW_C = -50{,}000(A/P,10\%,5) - 20{,}000 + 5{,}000(A/F,10\%,5) = \$-32{,}371$

$AW_D = Pi + AOC = -250{,}000(0.10) - 9{,}000 = \$-34{,}000$

Select alternative C

Typical Life-Cycle Cost Distribution by Phase



Typical Life-Cycle Cost Distribution by Phase

- LCC analysis includes all costs for entire life span, from concept to disposal.
- Best when large percentage of costs are O&M
- Includes phases of acquisition, operation, & phaseout
- Apply the AW method for
  - LCC analysis of 1 or more cost alternatives
- Use PW analysis
  - if there are revenues and other benefits considered

Example 6.7 LCC Analysis (1/4)

- In 1860s, GM, and PL Inc. both started in flour business in two different cities. During 2000-2010, GM purchased PL for more than $10 billion and integrated the product lines.
  - Food engineers, food designers, & food safety experts made many cost estimates as they determined the needs of consumers and the combined company's ability to technologically and safely produce and market new food products.
    - At this point only cost estimates have been addressed (no revenues or profits).
  - Assume that the major cost estimates below have been made based on a 6-month study about two new products that could have a 10-year life span for the GM.
    - Use LCC analysis at the industry MARR of 18% to determine the commitment size in AW terms.
      - Time is indicated in product-years. Since all estimates are for costs, they are not preceded by a minus sign.

Example 6.7 LCC Analysis (2/4)

Consumer habits study (year 0)	$0.5 million
Preliminary food product design (year 1)	0.9 million
Preliminary equipment/plant design (year 1)	0.5 million
Detail product designs and test marketing (years 1, 2)	1.5 million each year
Detail equipment/plant design (year 2)	1.0 million
Equipment acquisition (years 1 and 2)	$2.0 million each year
Current equipment upgrades (year 2)	1.75 million
New equipment purchases (years 4 and 8)	2.0 million (year 4) + 10% per purchase thereafter
Annual equipment operating cost (AOC) (years 3–10)	200,000 (year 3) + 4% per year thereafter
Marketing, year 2	$8.0 million
years 3–10	5.0 million (year 3) and −0.2 million per year thereafter
year 5 only	3.0 million extra
Human resources, 100 new employees for 2000 hours per year (years 3–10)	$20 per hour (year 3) + 5% per year
Phaseout and disposal (years 9 and 10)	$1.0 million each year

Example 6.7 LCC Analysis (3/4)

- Solution (cont'd):
  - For LCC analysis, first calculate PW by each phase and stage, add all PW values, then find AW over 10 years.
  - Acquisition phase:

Requirements definition: consumer study

$$PW = \$0.5$$

Preliminary design: product and equipment

$$PW = 1.4(P/F,18\%,1) = \$1.187$$

Detailed design: product and test marketing, and equipment

$$PW = 1.5(P/A,18\%,2) + 1.0(P/F,18\%,2) = \$3.067$$

  - Operation phase:

Construction and implementation: equipment and AOC

$$PW = 2.0(P/A,18\%,2) + 1.75(P/F,18\%,2) + 2.0(P/F,18\%,4) + 2.2(P/F,18\%,8)$$

$$+ 0.2\left[\frac{1-\left(\frac{1.04}{1.18}\right)^8}{0.14}\right](P/F,18\%,2) = \$6.512$$

Geometric Gradient with A1 = 0.2, g = 4% & i = 18%

Example 6.7 LCC Analysis (4/4)

- Solution:
  - Operation phase:

Use: marketing

$$PW = 8.0(P/F,18\%,2) + [5.0(P/A,18\%,8) - 0.2(P/G,18\%,8)](P/F,18\%,2) + 3.0(P/F,18\%,5)$$

$$= \$20.144$$

Use: human resources: (100 employees)(2000 h/yr)($20/h) = $4.0 million in year 3

$$PW = 4.0\left[\frac{1-\left(\frac{1.05}{1.18}\right)^8}{0.13}\right](P/F,18\%,2) = \$13.412$$

Geometric Gradient with A1 = 4, g = 5% & i = 18%

  - Phaseout phase: $PW = 1.0(P/A,18\%,2)(P/F,18\%,8) = \$0.416$
  - The sum of all PW of costs is PW = $45.238 million.
  - Finally, determine the AW over the expected 10-year life span.
    - AW = 45.238 million(A/P,18%,10) = $10.066 million per year
    - This is the LCC estimate of the equivalent annual commitment to the two proposed products.

Summary of Important Points

- AW method converts all cash flows to annual value at MARR
- Alternatives can be mutually exclusive, independent, revenue, or cost
- AW comparison is only one life cycle of each alternative
- For infinite life alternatives,
  - annualize initial cost as A = P(i)
- Life-cycle cost analysis
  - includes all costs over a project's life span