In the name of Allah

Amirkabir University of Technology
(Tehran Polytechnic)
Industrial Engineering Department

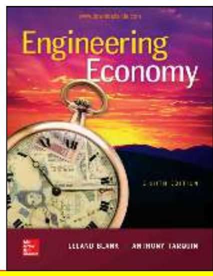
Course Title:
Engineering Economics

7. Rate of Return Analysis: One Project

By: Akbar Esfahanipour

Learning Stage 2: Basic Analysis Tools

- Chapter 5
  - Present Worth Analysis
- Chapter 6
  - Annual Worth Analysis
- Chapter 7
  - Rate of Return Analysis: One Project
- Chapter 8
  - Rate of Return Analysis: Multiple Alternatives
- Chapter 9
  - Benefit/Cost Analysis and Public Sector Economics



Chapter 7 of EE (BT) book 8th edition

LEARNING OUTCOMES

- Purpose:
  - Understand the meaning of rate of return and perform an ROR evaluation of a single project.

1. Understand meaning of ROR
2. Calculate ROR for cash flow series
3. Understand difficulties of ROR
4. Determine multiple ROR values
5. Calculate External ROR (EROR)
6. Calculate r and i for bonds

Interpretation of ROR

- Rate paid on unrecovered balance of borrowed money
  - such that final payment brings balance to exactly zero with interest considered.
- ROR equation can be written in terms of PW, AW, or FW
- Use trial and error solution by factor or spreadsheet
- Numerical value can range from −100% to infinity

ROR Calculation and Project Evaluation

- To determine ROR, find the i^* value in the relation
  - PW = 0 or AW = 0 or FW = 0
- Alternatively, a relation like the following finds i*
  - $PW_{outflow} = PW_{inflow}$

EVALUATION

- For evaluation, a project is economically viable if
  - $i^* \geq MARR$

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Finding ROR by Spreadsheet Function

Using the RATE function

= RATE(n, A, P, F)

P = $ − 200,000 A = $−15,000
n = 12 F = $435,000

Function is
= RATE(12, −15000, −200000,450000)

Display is i* = 1.9%

Using the IRR function

= IRR(first_cell, last_cell)

	A	B
1	Year	CF,$
2	0	-200,000
3	1	-15,000
4	2	-15,000
5	3	-15,000
6	4	-15,000
7	5	-15,000
8	6	-15,000
9	7	-15,000
10	8	-15,000
11	9	-15,000
12	10	-15,000
13	11	-15,000
14	12	435,000
15	**IRR function**	**1.9%**

= IRR(B2:B14)

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ROR Calculation Using PW, FW or AW Relation

- ROR is the unique i* rate at which
  - a PW, FW, or AW relation equals exactly 0
- Example: An investment of $20,000 in new equipment will generate income of $7000 per year for 3 years, at which time the machine can be sold for an estimated $8000.
  - If the company's MARR is 15% per year, should it buy the machine?

  Solution: The ROR equation, based on a PW relation, is:

  $$0 = -20{,}000 + 7000(P/A, i^*, 3) + 8000(P/F, i^*, 3)$$

  - Solve for i* by trial and error or spreadsheet: i* = 18.2% per year
  - Since i* > MARR = 15%, the company should buy the machine

Special Considerations for ROR

- May get multiple i* values
  - It will be discussed later
- i* assumes reinvestment of positive cash flows earn at i* rate
  - may be unrealistic
- Incremental analysis necessary for multiple alternative evaluations
  - It will be discussed later

Multiple ROR Values

- Multiple i* values may exist when there is more than one sign change in net cash flow (CF) series.
  - such CF series are called non-conventional CF

Type of Series	0	1	2	3	4	5	6	Number of Sign Changes
	Sign on Net Cash Flow by Year							
Conventional	−	+	+	+	+	+	+	1
Conventional	−	−	−	+	+	+	+	1
Conventional	+	+	+	+	+	−	−	1
Nonconventional	−	+	+	+	−	−	−	2
Nonconventional	+	+	−	−	−	+	+	2
Nonconventional	−	+	−	−	+	+	+	3

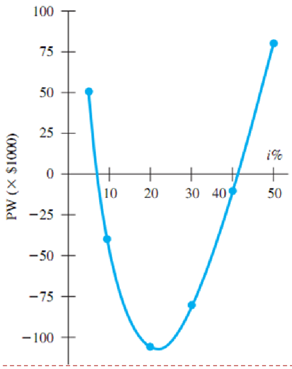
Examples of Conventional and Nonconventional Net Cash Flow for a 6-year Project

Multiple ROR Values

- Two tests for multiple i* values:
  - Descarte's rule of signs: total number of real i* values is ≤ the number of sign changes in net cash flow series.
    - This rule is derived from the fact that PW = 0, or FW = 0, or AW = 0 to find i* is an nth-order polynomial.
  - Norstrom's criterion: if the cumulative cash flow starts off negatively and has only one sign change, there is only one positive root.
    - Zero values in the series are neglected when applying this criterion.

Plot of PW for CF Series with Multiple ROR Values

Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	–500	S_1	+1500
2	–8100	S_2	–6600
3	+6800	S_3	+200



i%	5	10	20	30	40	50
PW ($1000)	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85

i* values at ~8% and ~41%

None of these rates can be used

Example: Multiple i* Values

- Determine the maximum number of i* values for this cash flow

Year	Expense	Income	Net cash flow	Cumulative CF
0	−12,000	n/a	−12,000	−12,000
1	−5,000	+3,000	−2,000	−14,000
2	−6,000	+9,000	+3,000	−11,000
3	−7,000	+15,000	+8,000	−3,000
4	−8,000	+16,000	+8,000	+5,000
5	−9,000	+8,000	−1,000	+4,000

Solution:

The sign on the net cash flow changes twice, indicating two possible i* values

The cumulative cash flow begins negatively with one sign change

Therefore, there is only one i* value (i* = 8.7%)

Multiple i* Values

- Assume there are two i* values for a particular cash flow series.

If the Results Are	What to Do
Both $i^* < 0$	Discard both values.
Both $i^* > 0$	Discard both values.
One $i^* > 0$; one $i^* < 0$	Use $i^* > 0$ as ROR.

- We need to change a cash flow to have a single i*

Removing Multiple i* Values

- Two new interest rates to consider:
  - Investment rate i_i – rate at which extra funds are invested external to the project
  - Borrowing rate i_b – rate at which funds are borrowed from an external source to provide funds to the project
- Two approaches to determine External ROR (EROR)
  - Modified ROR (MIRR)
  - Return on Invested Capital (ROIC)

Modified ROR Approach (MIRR)

- A Four step Procedure:
  - Determine PW in year 0 of all negative CF at i_b
  - Determine FW in year n of all positive CF at i_i
  - Calculate EROR = i' by FW = PW(F/P, i', n)
  - If i' ≥ MARR, project is economically justified

Example: EROR Using MIRR Method

- For the NCF shown below, find the EROR
  - by the MIRR method if MARR = 9%, i_b = 8.5%, and i_i = 12%

Year	0	1	2	3
NCF	+2000	−500	−8100	+6800

Solution: $PW_0 = -500(P/F, 8.5\%, 1) - 8100(P/F, 8.5\%, 2) = \$-7342$

$FW_3 = 2000(F/P, 12\%, 3) + 6800 = \$9610$

$PW_0(F/P, i', 3) + FW_3 = 0 \rightarrow -7342(1 + i')^3 + 9610 = 0$

i' = 0.939 (9.39%)

Since i' > MARR of 9%, project is justified

For calculating i' using spreadsheet: MIRR(first_cell:last_cell, i_b, i_i)

Return on Invested Capital Approach

- Measure of how effectively project uses funds that remain internal to project
- ROIC rate, i'', is determined using net-investment procedure
- Use a Three step Procedure
  - (1) Develop series of FW relations for each year t using:

$$\mathbf{F_t = F_{t-1}(1 + k) + NCF_t}$$

    - where: $\mathbf{k = i_i}$ if $\mathbf{F_{t-1} > 0}$ (extra funds available)
    - $\mathbf{k = i''}$ if $\mathbf{F_{t-1} < 0}$ (project uses all available funds)
  - (2) Set future worth relation for last year n equal to 0
    - i.e., $F_n = 0$ and solve for i''
  - (3) If i'' ≥ MARR, project is justified; otherwise, reject

ROIC Example

- For the NCF shown below, find the EROR by
  - the ROIC method if MARR = 9% and $i_i = 12\%$

Year	0	1	2	3
NCF	+2000	−500	− 8100	+6800

Solution:

Year 0: $F_0 = \$+2000$ — $F_0 > 0$; invest in year 1 at $i_i = 12\%$

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$ — $F_1 > 0$; invest in year 2 at $i_i = 12\%$

Year 2: $F_2 = 1740(1.12) - 8100 = \$-6151$ — $F_2 < 0$; use i'' for year 3

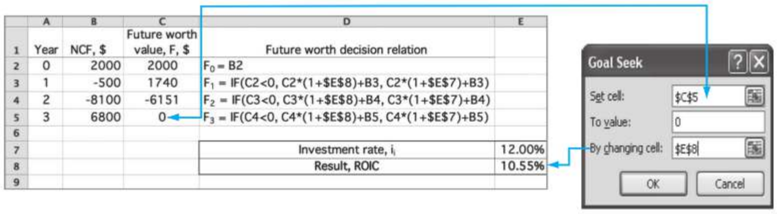
Year 3: $F_3 = -6151(1 + i'') + 6800$ — Set $F_3 = 0$ and solve for i''

$$-6151(1 + i'') + 6800 = 0 \rightarrow i'' = 10.55\%$$

Since i'' > MARR of 9%, project is justified

ROIC Example

- Using Goal Seek in Excel to solve $F_n = 0$ to find i”



Important Points to Remember

- About the computation of an EROR value
  - EROR values are dependent upon the selected investment and/or borrowing rates
  - Commonly, multiple i* rates, i’ from MIRR and i’’ from ROIC have different values
- In Modified ROR technique
  - When both the borrowing rate i_b and the investment rate i_i are exactly equal to any one of the multiple i* values,
    - the rate i' found by the MIRR function will equal the i* value.
- In ROIC technique
  - If the investment rate i_i is exactly equal to any one of the multiple i* values,
    - the rate found when the equation $F_n = 0$ is solved, will be i″ = i* value.

Important Points to Remember

- None of the details of the MIRR technique or the ROIC technique are necessary
  - if the PW or AW method of project evaluation is applied at a specific MARR.
- When the MARR is established, this is, in effect, fixing the i* value.
  - Therefore, a definitive economic decision can be made directly from the PW or AW value.

ROR of Bond Investment

- Bond is *IOU* (financing through debt, not equity) with face value (V), coupon rate (b), no. of payment periods/year (c), bond dividend (bond interest) (I), and maturity date (n). Amount paid for the bond is P.

$$\mathbf{I = Vb/c}$$

- General equation for i*:
  - $0 = -P + I(P/A, i*, n \times c) + V(P/F, i^*, n \times c)$
- Example: A $10,000 bond with 6% interest payable quarterly is purchased for $8000 and it matures in 5 years
  - What is the ROR (a) per quarter, (b) per year?

Solution: (a) I = 10,000(0.06)/4 = $150 per quarter

ROR equation is: $\mathbf{0 = -8000 + 150(P/A, i^*, 20) + 10,000(P/F, i^*, 20)}$

By trial and error or spreadsheet: i* = 2.8% per quarter

(b) Nominal i* per year = 2.8(4) = 11.2% per year

Effective i* per year = $(1 + 0.028)^4 - 1$ = 11.7% per year

Summary of Important Points

- ROR equations can be written
  - in terms of PW, FW, or AW & usually require trial & error solution
- i* assumes reinvestment of positive cash flows at i* rate
- More than 1 sign change in NCF may cause multiple i* values
- Descarte's rule of signs & Norstrom's criterion are useful
  - when multiple i* values are expected
- EROR can be calculated using MIRR or ROIC approach.
  - Assumptions about investment and borrowing rates is required.
- General ROR equation for bonds is

$$\mathbf{0 = -P + I(P/A, i^*, n \times c) + V(P/F, i^*, n \times c)}$$