Experimental Average-Case Performance Evaluation of Online Algorithms for Routing and Wavelength Assignment and Throughput Maximization in WDM Optical Networks

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We investigate the problem of online routing and wavelength assignment and the related throughput maximization problem in wavelength division multiplexing optical networks. It is pointed out that these problems are highly inapproximable, that is, the competitive ratio of any algorithm is at least a polynomial. We evaluate the average-case performance of several online algorithms, which have no knowledge of future arriving connection requests when processing the current connection request. Our experimental results on a wide range of optical networks demonstrate that the average-case performance of these algorithms are very close to optimal.

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1. INTRODUCTION

Given wavelengths λ_1, λ_2, λ_3, ..., and a sequence of connection requests $\sigma = (r_1, r_2, \ldots, r_m)$ in a wavelength division multiplexing (WDM) network, where

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each connection request r_j is a source-destination pair $r_j = (s_j, d_j)$, $1 \le j \le m$, the *routing and wavelength assignment* (RWA) problem is to establish a lightpath p_j for each connection request r_j and assign a wavelength λ_{i_j} to each lightpath p_j, where $1 \le i_j \le k$, such that no two lightpaths, which share a common link, are assigned the same wavelength and that the number k of wavelengths used is minimized.

We also consider a related optimization problem of RWA, namely, the *throughput maximization* (TM) problem, in which we are given a fixed number k of wavelengths $\lambda_1, \lambda_2, \ldots, \lambda_k$, and a sequence σ of connection requests. The goal is to satisfy as many connection requests as possible by using the k wavelengths. It is clear that the decision version of the RWA problem can be reduced to the TM problem, i.e., the question whether k wavelengths is enough for σ is equivalent to the question whether the throughout is m when given k wavelengths. Hence, the RWA problem can be polynomially reduced to the TM problem. In particular, since the number of wavelengths to be minimized in the RWA problem does not exceed m, the RWA problem can be solved in $O(T(m) \log m)$ time by using any algorithm for the TM problem with time complexity $T(m)$ and the binary search method. However, it is not clear how to polynomially reduce the TM to the RWA problem.

Both the RWA and the TM problems contain two subproblems, namely, routing (finding a lightpath for each connection request) and coloring (assigning a wavelength to each lightpath). Each subproblem alone makes the RWA and TM problems NP-hard. When a lightpath is given for each connection request, the RWA problem becomes the *wavelength assignment* (WA) problem. It has been proved that the WA problem and the well known NP-hard graph coloring problem can be reduced to each other [Chlamtac et al. 1992]. Hence, the WA problem has high inapproximability; in particular, if NP $\neq$ ZPP, for any constant $\delta > 0$, no polynomial time WA algorithm can achieve approximation ratio $n^{1/2-\delta}$ or $m^{1-\delta}$ for m lightpaths in an n-node WDM network [Li 2002]. When there is only one wavelength, the TM problem is precisely the classical *maximum disjoint paths* (MDP) problem, that is, finding as many edge-disjoint paths as possible for a sequence σ of source–destination pairs. The MDP problem is also highly inapproximable; in particular, if P $\neq$ NP, for any constant $\delta > 0$, no polynomial time MDP algorithm can achieve approximation ratio $m^{1/2-\delta}$ for a WDM network with m edges [Guruswami et al. 1999]. Both the graph coloring and MDP problems are in Karp's original list of NP-hard problems and remain major open problems in the area of approximation algorithms for NP-hard optimization problems.

The RWA and TM problems have been extensively studied by many researchers in the last 10 years. Various heuristic methods have been proposed, such as genetic algorithms [Beckmann and Killat 1999], graph-theoretic modeling [Chen and Banerjee 1996], partition coloring [Li and Simha 2000], and integer linear program [Banerjee and Mukherjee 1996; Ramaswami and Sivarajan 1995]. A recent survey of various algorithms for the RWA problem can be found in Choi et al. [2000]. The reader is also referred to Ramaswami and Sivarajan [1998] for information on WDM optical networks.

In this paper, we consider online routing and wavelength assignment in WDM optical networks. In the online setting, connection requests arrive in the order of σ, one at a time. Upon the arrival of a connection request r_j, a lightpath p_j is established and its wavelength is assigned immediately without knowing the remaining connection requests $r_{j+1}, r_{j+2}, \ldots, r_m$, but only the past connection requests $r_1, r_2, \ldots, r_{j-1}$. Online RWA and TM algorithms are certainly very useful in real applications, since connection requests typically do not arrive at the same time and those arriving earlier should be processed before the entire sequence of requests is available.

It is not surprising that the online RWA and TM problems are highly inapproximable, since the offline RWA and TM problems already contain highly inapproximable graph coloring and disjoint paths problems as subproblems or special cases. Nevertheless, it is still possible that there exist effective approximation algorithms with excellent average-case performance. The main contribution of the paper is to develop several online RWA and TM algorithms and demonstrate by experimentation that the average-case competitive ratios of these algorithms are very close to optimal. It should be noticed that while existing work only compare heuristic algorithms with themselves, we are able to compare the performance of our algorithms with optimal solutions (actually, lower bounds for the optimal solutions).

The rest of the paper is organized as follows. In Section 2, we discuss the inapproximability of the online RWA and the TM problems. It is shown that for n-node WDM optical networks, there is no deterministic or randomized online RWA or TM algorithm that has a competitive ratio less than $n^{0.2075}$. In Section 3, we derive lower bounds for optimal solutions to the RWA problem. These lower bounds are to be compared with the solutions produced by our approximation algorithms. In Section 4, we describe several online RWA and TM algorithms, including first-fit, best-fit, densest-fit, and random-fit. In Section 5, we present results of extensive experiments to evaluate the average-case performance of our online algorithms on a wide range of WDM optical networks, including a mesh, four real, and three types of random networks. We conclude the paper in Section 6.

2. INAPPROXIMABILITY OF ONLINE RWA AND TM PROBLEMS

Let $\mathrm{ALG}(\sigma)$ denote the solution produced by algorithm ALG and $\mathrm{OPT}(\sigma)$ the optimal solution for an instance σ. For example, in the RWA problem, $\mathrm{ALG}(\sigma)$ denotes the number of wavelengths needed by algorithm ALG to establish lightpaths for the connection requests in σ and $\mathrm{OPT}(\sigma)$ denotes the minimum number of wavelengths needed to support the connection requests in σ. In the TM problem, $\mathrm{ALG}(\sigma)$ denotes the number of lightpaths established by algorithm ALG for the connection requests in σ by using the given number of wavelengths and $\mathrm{OPT}(\sigma)$ denotes the maximum number of lightpaths that can be established for the connection requests in σ. The *competitive ratio* of an online algorithm ALG is defined as

$$\sup_{\sigma}\left(\frac{\mathrm{ALG}(\sigma)}{\mathrm{OPT}(\sigma)}\right)$$

Table I. Results of the Online Path-Coloring Problem

Network	Lower Bound	Upper Bound
Linear arrays	3	3
Trees	$\Omega\left(\frac{\log n}{\log\log n}\right)$	$2\log n$
Meshes	$\Omega(\log n)$	$O(\log n)$
Brick-wall graphs	$n^{0.2075}$	N/A

for a minimization problem; and

$$\sup_{\sigma}\left(\frac{\mathrm{OPT}(\sigma)}{\mathrm{ALG}(\sigma)}\right)$$

for a maximization problem.

Algorithm ALG is said to be α-competitive, if for all σ,

$$\mathrm{ALG}(\sigma) \leq \alpha \cdot \mathrm{OPT}(\sigma)$$

for a minimization problem; and

$$\mathrm{ALG}(\sigma) \geq \frac{1}{\alpha} \cdot \mathrm{OPT}(\sigma)$$

for a maximization problem.

For a randomized algorithm, $\mathrm{ALG}(\sigma)$ is replaced by $E(\mathrm{ALG}(\sigma))$, where $E(\cdot)$ denotes the expectation of a random variable [Borodin and El-Yaniv 1998].

The RWA problem is also called *path-coloring* (PC) problem. Online path coloring has been studied extensively in the literature. It was shown that there is a three-competitive algorithm (called recursive greedy) for path coloring on linear array networks and no deterministic online algorithm is better than three-competitive [Kierstead and Trotter 1981]. For any n-node tree network, it was shown that both the classify-and-greedy-color algorithm [Bartal and Leonardi 1997] and the first-fit-coloring [Irani 1990] algorithm are $2\log n$-competitive. It was also proved in Bartal and Leonardi [1997] that any deterministic algorithm has competitive ratio at least $\Omega(\frac{\log n}{\log\log n})$ even for complete binary tree networks. Bartal and Leonardi also constructed the optimal $O(\log n)$-competitive algorithm for path coloring on $n \times n$ mesh networks. On brick-wall graphs, it was shown that any randomized algorithm is, at best, $n^{1-\log_4 3}$-competitive [Bartal and Leonardi 1996], where $1 - \log_4 3 = 0.2075187\ldots$. Table I summarizes the known results for the online PC problem.

The lower bound for brick-wall graphs implies the high inapproximability of the routing and wavelength assignment problem, that is, for arbitrary n-node WDM optical networks, there is no deterministic or randomized online routing and wavelength assignment algorithm that has a competitive ratio less than $n^{0.2075}$.

When there is only one wavelength, the TM problem becomes the MDP problem. It is a simple observation that any deterministic online algorithm for the MDP problem has competitive ratio at least $n - 1$, even on an n-node linear array network [Awerbuch et al. 1993]. Therefore, investigation has focused on randomized algorithms. Lower bounds for randomized algorithms for the MDP problem on linear array networks were established in Awerbuch et al. [1994a].

Table II. Results of the Online Maximum Disjoint Paths Problem

Network	Lower Bound	Upper Bound
Linear arrays	$\lfloor \frac{1}{2} \log n \rfloor$	$\lceil \log n \rceil$
Trees	N/A	$O(\log D)$
Meshes	$\frac{1}{2} \log n$	$O(\log n)$
Brick-wall graphs	$n^{0.2075}$	N/A

For tree networks with diameter D, several $O(\log D)$-competitive algorithms have been developed [Awerbuch et al. 1994a, 1994b; Leonardi et al. 1998]. The lower bound $\Omega(\log n)$ and the optimal $O(\log n)$ upper bound for randomized algorithms on $n \times n$ mesh networks are found in Awerbuch et al. [1994b] and Kleinberg and Tardos [1995], respectively. The randomized lower bound of $n^{0.2075}$ for brick-wall graphs is because of Bartal and Leonardi [1996]. Table II summarizes the known results for the online MDP problem.

The lower bound for brick-wall graphs implies the high inapproximability of the throughput maximization problem, that is, for arbitrary n-node WDM optical networks, there is no deterministic or randomized online throughput maximization algorithm that has a competitive ratio less than $n^{0.2075}$.

3. LOWER BOUNDS

The solutions produced by an approximation algorithm should be compared with optimal solutions. Unfortunately, it is infeasible to obtain optimal routing and wavelength assignment in reasonable amount of time even for moderate sized networks. In this section, we derive lower bounds for the minimum number of wavelengths required.

A *cutset* C of a connected graph (WDM network) is a set of $W(C)$ edges (optical links) $C = \{l_1, l_2, \ldots, l_{W(C)}\}$ whose removal results in disconnection of the network [Harary 1969], i.e., a partition of the network into two subnetworks with $n(C)$ and $n - n(C)$ nodes, respectively. For a sequence $\sigma = (r_1, r_2, \ldots, r_m)$ of connection requests, let $m(\sigma, C)$ denote the number of connection requests $r_j = (s_j, d_j)$ in σ such that s_j and d_j are in the two disjoint subnetworks separated by the cutset C. For each such r_j, the lightpath established for r_j must go through one of the $W(C)$ links $l_1, l_2, \ldots, l_{W(C)}$. Let L_l be the *load* on an optical link l, i.e., the number of lightpaths passing through l, and L be the maximum load on all optical links $l_1, l_2, \ldots, l_{W(C)}$, i.e., $L = \max_{1 \le i \le W(C)}(L_{l_i})$. The maximum load L on $l_1, l_2, \ldots, l_{W(C)}$ is at least

$$L \geq \frac{m(\sigma, C)}{W(C)}$$

Since $\mathrm{OPT}(\sigma) \geq L$ we obtain

$$\mathrm{OPT}(\sigma) \geq \frac{m(\sigma, C)}{W(C)}$$

The above lower bound is strengthened to

$$\mathrm{OPT}(\sigma) \geq \max_{C} \left(\frac{m(\sigma, C)}{W(C)} \right)$$

because C can be an arbitrary cutset. A cutset C, which maximizes the right side of the above inequality, is called a *limiting cutset* [Stern and Bala 2000]. The above lower bound is in a similar spirit to the classic Max-Flow Min-Cut Theorem [Ford and Fulkerson 1962], which states that the maximum possible flow between a pair of nodes is equal to the minimum capacity of all cutsets separating the two nodes. The above lower bound states that the minimum number of wavelengths required is at least the average load on the links of a limiting cutset.

The minimum size w of a cutset that results in an even partition of a network into two subnetworks of sizes $\lfloor n/2 \rfloor$ and $\lceil n/2 \rceil$ is called the *bisection width* of the network. By considering a cutset C with w links, we get a special lower bound for $\mathrm{OPT}(\sigma)$:

$$\mathrm{OPT}(\sigma) \geq \frac{m(\sigma, C)}{w}$$

The above discussion is summarized as the following theorem.

Lower Bound Theorem A. *For any WDM network and a sequence σ of connection requests, we have*

$$\mathrm{OPT}(\sigma) \geq \max_C \left(\frac{m(\sigma, C)}{W(C)} \right) \tag{1}$$

In particular, for a cutset C with $W(C)$ equal to the network's bisection width w, we have

$$\mathrm{OPT}(\sigma) \geq \frac{m(\sigma, C)}{w}$$

(Note: The above lower bound is valid for both online and offline RWA problems.)

A lower bound for the minimum number of wavelengths required can be translated directly to an upper bound for the maximum throughput. Let m' be fixed and σ' be any subsequence of σ with m' connection requests. If

$$\forall \sigma' : \max_C \left(\frac{m(\sigma', C)}{W(C)} \right) > k$$

then, the throughput is no more than m'. Unfortunately, the above upper bound for the maximum throughput needs exponential computation time and is practically useless. It is of great interest to find easy-to-calculate upper bounds for the TM problem.

Now we derive a lower bound for $E(\mathrm{OPT}(\sigma))$, where σ is a sequence of m random connection requests $r_1, r_2, \ldots, r_m$. We consider two models of random connection requests. In the *random drawing with replacement* model, each connection request $r_j = (s_j, d_j)$ is a source-destination pair drawn from the set of $n(n-1)/2$ possible pairs randomly with a uniform distribution. For such a randomly chosen connection request $r_j = (s_j, d_j)$, the probability that s_j and d_j are in the two separate parts of the network is

$$\frac{n(C)(n - n(C))}{n(n-1)/2}$$

Hence, for m independent random connection requests, the expected number of lightpaths passing through $l_1, l_2, \ldots, l_{W(C)}$ is

$$E(m(\sigma, C)) = \frac{n(C)(n - n(C))}{n(n-1)/2} m$$

In the *random drawing without replacement* model, the sequence σ contains m distinct connection requests $r_1, r_2, \ldots, r_m$. Therefore, the number $m(\sigma, C)$ of connection requests $r_j = (s_j, d_j)$ with s_j and d_j in the two separate parts of the network is a hypergeometric random variable, i.e.,

$$P\{m(\sigma, C) = i\} = \frac{\binom{n(C)(n-n(C))}{i}\binom{n(n-1)/2-n(C)(n-n(C))}{m-i}}{\binom{n(n-1)/2}{m}}$$

for all $0 \le i \le m$ [Feller 1968]. The expectation of $m(\sigma, C)$ is

$$E(m(\sigma, C)) = \frac{n(C)(n - n(C))m}{n(n-1)/2}$$

In both models, the expected maximum number of lightpaths passing through one of $l_1, l_2, \ldots, l_{W(C)}$ is at least

$$E(L) \ge \frac{E(m(\sigma, C))}{W(C)} = \frac{n(C)(n - n(C))}{n(n-1)/2} \frac{m}{W(C)}$$

Since $E(\mathrm{OPT}(\sigma)) \ge E(L)$, we have the following lower bound for $E(\mathrm{OPT}(\sigma))$:

$$E(\mathrm{OPT}(\sigma)) \ge \frac{n(C)(n - n(C))}{n(n-1)/2} \frac{m}{W(C)}$$

The above lower bound is strengthened to

$$E(\mathrm{OPT}(\sigma)) \ge \max_C \left(\frac{n(C)(n - n(C))}{n(n-1)/2} \frac{m}{W(C)} \right)$$

because C can be an arbitrary cutset. By considering a cutset C with $W(C)$ equal to the bisection width w, we get a special lower bound for $E(\mathrm{OPT}(\sigma))$:

$$E(\mathrm{OPT}(\sigma)) \ge \frac{\lfloor n/2 \rfloor \lceil n/2 \rceil}{n(n-1)/2} \frac{m}{w}$$

The above discussion is summarized as the following theorem.

Lower Bound Theorem B. *For any n-node WDM network and a sequence σ of m random connection requests, we have*

$$E(\mathrm{OPT}(\sigma)) \ge \max_C \left(\frac{n(C)(n - n(C))}{W(C)} \right) \frac{m}{n(n-1)/2} \tag{2}$$

In particular, if the network has bisection width w, we have

$$E(\mathrm{OPT}(\sigma)) \ge \frac{\lfloor n/2 \rfloor \lceil n/2 \rceil}{n(n-1)/2} \frac{m}{w} \approx \frac{m}{2w}$$

(Note: The above lower bound is valid for both online and offline RWA problems.)

Both Lower Bound Theorems A and B are applicable to the random drawing with/without replacement models.

4. ONLINE ALGORITHMS

While the known results on the worst-case performance of online PC and MDP problems are quite discouraging (i.e., the RWA and the TM problems have high inapproximability for arbitrary WDM networks), we take a different approach to attacking the online RWA and TM problems in this paper, that is, evaluating the average-case performance of (deterministic and randomized) online algorithms.

Let σ denote a sequence of m random connection requests $r_1, r_2, \ldots, r_m$. For such random input, both $\mathrm{ALG}(\sigma)$ and $\mathrm{OPT}(\sigma)$ become random variables. We also notice that ALG can be a randomized algorithm and a WDM network can be a random network. We define two *average-case* competitive ratios

$$\alpha(\mathrm{ALG}) = E\left(\frac{\mathrm{ALG}(\sigma)}{\mathrm{OPT}(\sigma)}\right)$$

and

$$\beta(\mathrm{ALG}) = \frac{E(\mathrm{ALG}(\sigma))}{E(\mathrm{OPT}(\sigma))}$$

where the expectations are taken over

—all sequences of m random connection requests;

—all random choices of algorithm ALG if it is a randomized algorithm;

—all samples of a random network.

The above three sources of randomness are independent of each other.

We will evaluate the average-case performance of several online algorithms for the RWA and the TM problems. All our algorithms visualize a WDM optical network $N = (V, E)$ as having separate copies, $N_1, N_2, N_3, \ldots$, one for each wavelength, such that all the connection requests routed on N_i use the wavelength λ_i and that lightpaths on the same copy N_i are edge-disjoint. Initially, there is only one copy N_1, and new copies will be introduced when necessary.

Assume that $N_1, N_2, \ldots, N_b$ are the current copies ever used. When processing a connection request r_j, an existing copy N_i is chosen to find a lightpath p_j for r_j and the lightpath p_j is assigned the wavelength λ_i. Then, the optical links occupied by p_j are deleted from N_i, so that these links cannot be used by later connection requests to prevent link overlapping.

Different algorithms use different strategies in identifying N_i. We will consider the following heuristics.

—*first-fit* (FF)—A shortest lightpath is sought in N_1 by using those optical links still not deleted. If there is no such a lightpath, a shortest lightpath is sought in $N_2, N_3, \ldots$, and so on, until a lightpath is found.

—*best-fit* (BF)—A shortest lightpath $p_{j,i}$ is sought in each of N_i, $1 \le i \le b$. Then, the shortest lightpath among $p_{j,1}, p_{j,2}, \ldots, p_{j,b}$ is chosen as p_j. (The BF algorithm is also called *shortest path with deletions* [Stern and Bala 2000].)

—*densest-fit* (DF)—A shortest lightpath is sought in N_i, which has the most optical links among $N_1, N_2, \ldots, N_b$. If such a lightpath cannot be established, a shortest lightpath is sought in the copy with the second most links, the copy with the third most links, ..., and so on, until a lightpath is found.

—*random-fit* (RF)—A shortest lightpath is sought in a randomly selected copy N_i, where N_i is chosen from all those copies which can provide shortest paths for r_j, say, $N_{i_1}, N_{i_2}, N_{i_3}, \ldots$, and each of these copies $N_{i_1}, N_{i_2}, N_{i_3}, \ldots$ are chosen with equal probability.

In all the above algorithms, a shortest lightpath is found by using the breadth-first search algorithm.

When no existing copy in $N_1, N_2, \ldots, N_b$ can provide a lightpath for r_j, a new copy N_{b+1} identical to N is initiated, so that a shortest lightpath p_j is established on N_{b+1} and assigned the wavelength λ_{b+1}. However, for the TM problem, the connection request is blocked (i.e., not satisfied and rejected) if b is already equal to k, the given number of wavelengths.

5. EXPERIMENTAL PERFORMANCE EVALUATION

Extensive experiments have been conducted to evaluate the average-case performance of the online algorithms presented in the last section for the RWA and the TM problems on a wide range of WDM optical networks.

5.1 The Methodology

In the experiments for the RWA problem (see Figure 1), for each combination of network N, algorithm ALG, and the number m of connection requests, we report $\bar{\alpha}$, $\bar{\beta}$, and $\overline{pl}$, whose meanings are explained as follows.

—The lower bound for OPT(σ) expressed in Equation (1) requires coverage of all cutsets C, which is certainly computationally infeasible. Hence, for each network N, there are $\eta(N)$ prechosen cutsets $C_1, C_2, \ldots, C_{\eta(N)}$ (the selection of the cutsets are based on particular networks), such that the lower bound for OPT(σ) in Equation (1) is simplified as

$$\tilde{lb} = \max_{1 \le i \le \eta(N)} \left(\frac{m(\sigma, C_i)}{W(C_i)} \right)$$

The above lower bound $\tilde{lb}$ is then used to be compared with ALG(σ). Thus, the following expectation

$$\bar{\alpha} = E\left(\frac{\mathrm{ALG}(\sigma)}{\tilde{lb}} \right)$$

is an over-estimation of α(ALG).

—The lower bound for E(OPT(σ)), expressed in Equation (2), also requires coverage of all cutsets C. For a particular network N, we can always choose a cutset C_1 which maximizes

$$\frac{n(C)(n - n(C))}{W(C)}$$

Experiment-RWA (N, ALG, m)
for ($j = 1$; $j \le s$; $j++$) **do** {
generate a sequence σ of m random connection requests using the random drawing without replacement model;
if N is a random network,
generate a connected sample of N;
calculate the lower bound for OPT(σ):

$$\tilde{lb}_j = \max_{1 \le i \le \eta(N)} \left(\frac{m(\sigma, C_i)}{W(C_i)} \right);$$

call algorithm ALG to find the number ALG_j of wavelengths used and the average path length pl_j;
let $\alpha_j = ALG_j / \tilde{lb}_j$;
if N is a random network,
calculate

$$lb_j = \frac{n(C_1)(n - n(C_1))}{W(C_1)} \cdot \frac{m}{n(n-1)/2},$$

where C_1 is the random cutset which cuts the unit square into upper and lower halves;
}
calculate

$$\bar{\alpha} = \frac{1}{s} \sum_{j=1}^{s} \alpha_j, \quad \delta_\alpha = \frac{2.575}{s\bar{\alpha}} \sqrt{\frac{1}{s-1} \left(s \sum_{j=1}^{s} \alpha_j^2 - \left(\sum_{j=1}^{s} \alpha_j \right)^2 \right)},$$

$$\overline{ALG} = \frac{1}{s} \sum_{j=1}^{s} ALG_j, \quad \delta_{ALG} = \frac{2.575}{s\overline{ALG}} \sqrt{\frac{1}{s-1} \left(s \sum_{j=1}^{s} ALG_j^2 - \left(\sum_{j=1}^{s} ALG_j \right)^2 \right)},$$

$$\overline{pl} = \frac{1}{s} \sum_{j=1}^{s} pl_j, \quad \delta_{pl} = \frac{2.575}{s \cdot \overline{pl}} \sqrt{\frac{1}{s-1} \left(s \sum_{j=1}^{s} pl_j^2 - \left(\sum_{j=1}^{s} pl_j \right)^2 \right)};$$

if N is not a random network with the designated C_1,
calculate

$$\bar{\beta} = \frac{\overline{ALG}}{\dfrac{n(C_1)(n - n(C_1))}{W(C_1)} \cdot \dfrac{m}{n(n-1)/2}};$$

else
calculate

$$\overline{lb} = \frac{1}{s} \sum_{j=1}^{s} lb_j, \quad \delta_{lb} = \frac{2.575}{s \cdot \overline{lb}} \sqrt{\frac{1}{s-1} \left(s \sum_{j=1}^{s} lb_j^2 - \left(\sum_{j=1}^{s} lb_j \right)^2 \right)},$$

and

$$\bar{\beta} = \frac{\overline{ALG}}{\overline{lb}};$$

report $\bar{\alpha}$, $\bar{\beta}$, $\overline{pl}$, and $\delta = \max(\delta_\alpha, \delta_{ALG}, \delta_{pl}, \delta_{lb})$.

Fig. 1. Description of an experiment for RWA.

Experiment-TM (N, ALG, m, k)
  for $(j = 1; j \le s; j++)$ **do** {
    generate a sequence σ of m random connection requests
    using the random drawing without replacement model;
    if N is a random network,
      generate a connected sample of N;
    call algorithm ALG to find

$$B_j = 1 - \frac{1}{m}(\text{the number of connection requests blocked}),$$

    when given k wavelengths;
  }
  calculate

$$\bar{B} = \frac{1}{s}\sum_{j=1}^{s} B_j, \quad \delta_B = \frac{2.575}{s\bar{B}}\sqrt{\frac{1}{s-1}\left(s\sum_{j=1}^{s} B_j^2 - \left(\sum_{j=1}^{s} B_j\right)^2\right)};$$

  report $\bar{B}$ and δ_B.

Fig. 2. Description of an experiment for TM.

Hence, the lower bound for $E(\text{OPT}(\sigma))$ in Equation (2) is simplified as

$$lb = \frac{n(C_1)(n - n(C_1))}{W(C_1)} \frac{m}{n(n-1)/2}$$

However, the following ratio

$$\bar{\beta} = \frac{E(\text{ALG}(\sigma))}{\frac{n(C_1)(n - n(C_1))}{W(C_1)} \frac{m}{n(n-1)/2}}$$

is still an overestimation of $\beta(\text{ALG})$. For a random network, the lower bound for $E(\text{OPT}(\sigma))$ in Equation (2) is modified as

$$\overline{lb} = E\left(\frac{n(C_1)(n - n(C_1))}{W(C_1)} \frac{m}{n(n-1)/2}\right)$$

where C_1 is the random cutset, which cuts the unit square into upper and lower halves geometrically, but is not necessarily a bisection of the network, and

$$\bar{\beta} = \frac{E(\text{ALG}(\sigma))}{\overline{lb}}$$

(See Section 5.2 for random network generation.)

—In addition to the number of wavelengths to be minimized, the average length $\overline{pl}$ of lightpaths should also be minimized, though this is a secondary optimization goal.

In the experiments for the TM problem (see Figure 2), for each combination of network N, algorithm ALG, the number m of connection requests, and the number k of available wavelengths, we report $\bar{B}$, which is (1−the expected blocking rate), i.e., the expected percentage of connection requests that are satisfied by using k wavelengths.

Our method for conducting the experiments follows the standard statistics [Freund 1981], as described in Figures 1 and 2, for the RWA and TM problems, respectively. Each experiment is repeated for sufficiently many times so that the 99% confidence interval is controlled under certain level. The 99% confidence interval is calculated as follows. For a random variable x with s observations $x_1, x_2, \ldots, x_s$, the mean of the samples is

$$\bar{x} = \frac{1}{s}\sum_{j=1}^{s} x_j$$

and the standard deviation is

$$S_x = \sqrt{\frac{1}{s(s-1)}\left(s\sum_{j=1}^{s} x_j^2 - \left(\sum_{j=1}^{s} x_j\right)^2\right)}$$

The 99% confidence interval to use $\bar{x}$ as an estimation of $E(x)$ is $\pm 100\delta_x\%$, where

$$\delta_x = 2.575 \cdot \frac{S_x}{\bar{x}\sqrt{s}} = \frac{2.575}{s\bar{x}}\sqrt{\frac{1}{s-1}\left(s\sum_{j=1}^{s} x_j^2 - \left(\sum_{j=1}^{s} x_j\right)^2\right)}$$

Of course, strictly speaking, because of the pseudo-random number generators and discrete and finite sample spaces, the 99% confidence intervals reported are approximate confidence intervals.

5.2 Optical Networks

Eight WDM optical networks are considered in our experiments, namely, a mesh network, four real networks, and three types of random networks:

—the 10×10 mesh network with $\eta = 2$ and C_1, C_2, shown in Figure 3;
—a 24-node ARPANET-like regional network [Zhang and Acampora 1995] with $\eta = 5$ and $C_1, \ldots, C_5$, shown in Figure 4;
—a 16-node NSFNET backbone [Banerjee and Mukherjee 2000] with $\eta = 2$ and C_1, C_2, shown in Figure 5;
—the 20-node European Optical Network (EON) [O' Mahony et al. 1995] with $\eta = 6$ and $C_1, \ldots, C_6$, shown in Figure 6;
—the 30-node UK Network [Appleby and Steward 1994] with $\eta = 6$ and $C_1, \ldots, C_6$, shown in Figure 7;
—100-node random-grid networks;
—50-node random-regular networks;
—50-node random-unit disk networks.

In Figures 3–7, the cutsets are arranged in decreasing order of

$$\frac{n(C_i)(n - n(C_i))}{W(C_i)}$$

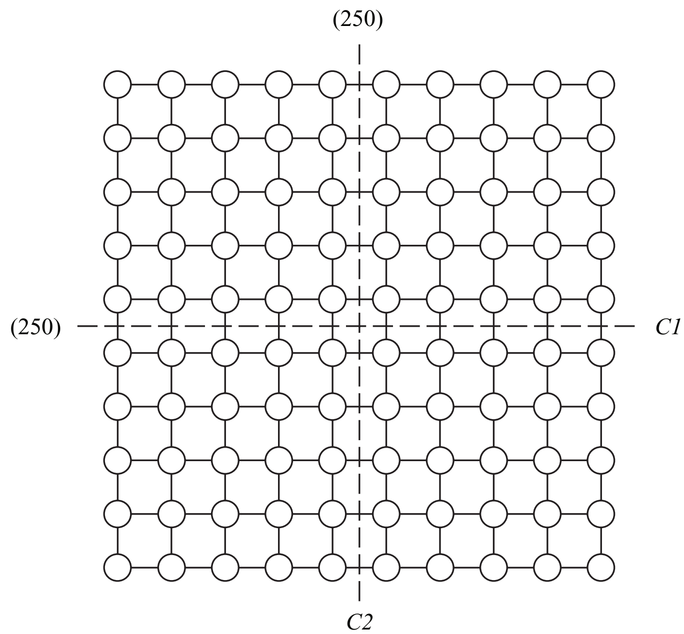

Fig. 3. A 10 × 10 mesh network.

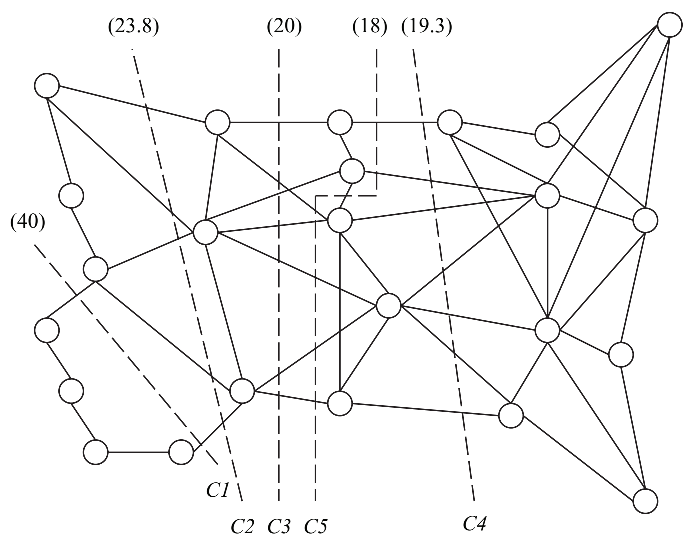

Fig. 4. A 24-node ARPANET-like network.

whose values are shown in the parentheses. The cutsets for random networks are described below.

Although a number of models are available in random graph theory, e.g., models A, B, and C in Palmer [1985], none of them is appropriate to model computer networks. We believe that a random network model should incorporate

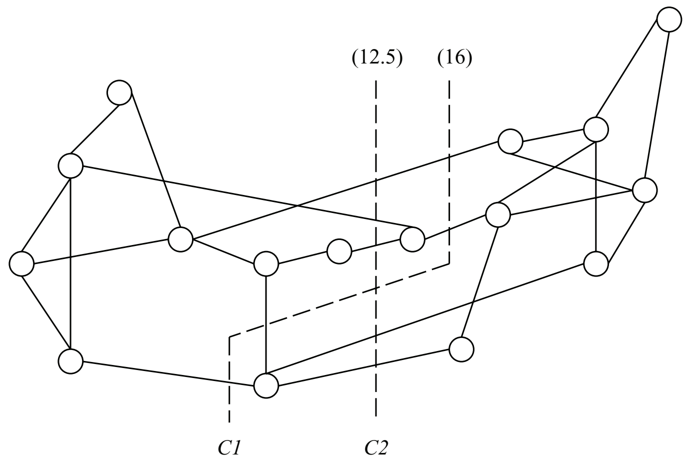

Fig. 5. A 16-node NSFNET backbone.

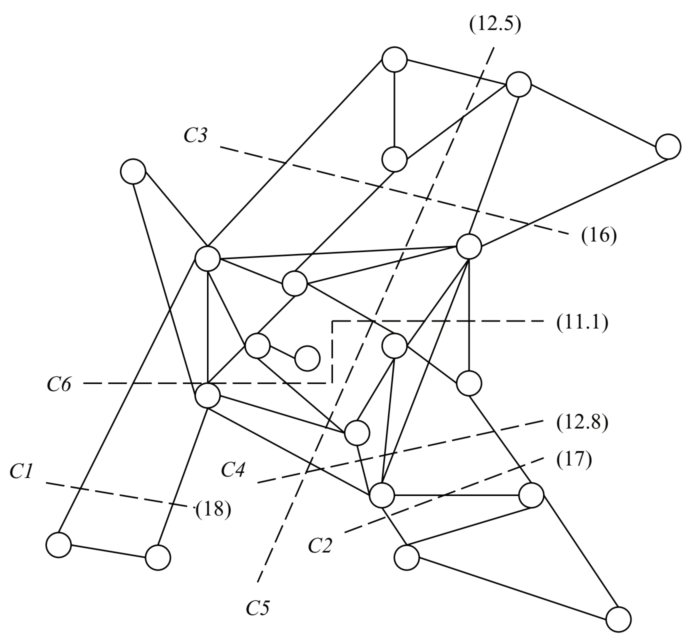

Fig. 6. The 20-node european optical network.

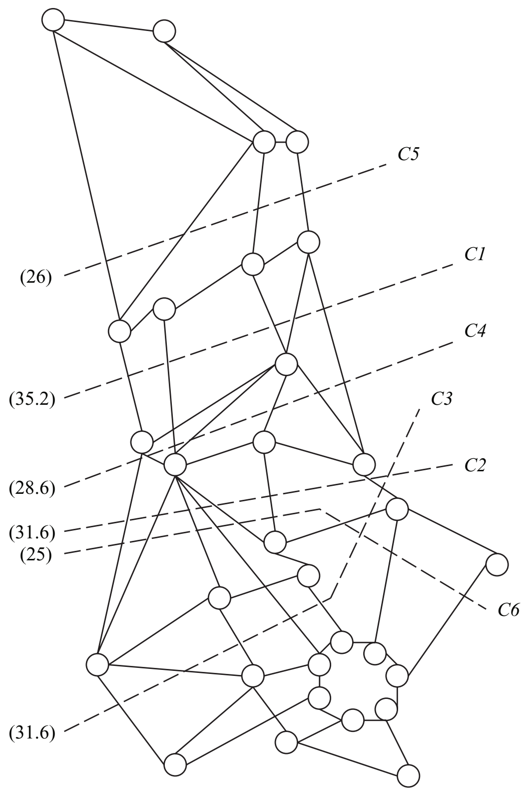

Fig. 7. The 30-node UK network.

link locality into consideration. In this research, we consider three types of random networks.

A *random-grid network* $N_q = (V, E)$ is a subnetwork of the mesh network and is generated as follows. In a $\sqrt{n} \times \sqrt{n}$ grid network, the n nodes in V are identical to the nodes in a $\sqrt{n} \times \sqrt{n}$ mesh network. Each link of the mesh network appears in a random grid network with probability q and is independent of the existence of other links, where $0 < q < 1$. Cutsets for random grid networks are the same as those for mesh networks.

A *random-regular network* $N_d = (V, E)$ is generated as follows. Let U be a unit square in the Euclidean plane. The n nodes $v_0, v_1, v_2, \ldots, v_{n-1}$ of V are chosen randomly and independently from U with a uniform distribution. For each node v_i, the d nearest nodes in V are made its neighbors, where $d \geq 1$. However, it is not guaranteed that v_i and v_j are in the set of d nearest neighbors of each other. The actual neighbors are selected in the following way. First, we make an order of the nodes, say, $(v_0, v_1, v_2, \ldots, v_{n-1})$. The degree of node v_i is $d_i = 0$ in the beginning. Then, for $0 \leq i \leq n-1$, assume that v_i already had d_i neighbors in $\{v_0, v_1, \ldots, v_{i-1}\}$. We choose the $d - d_i$ nearest neighbors of v_i from the nodes in $\{v_{i+1}, v_{i+2}, \ldots, v_{n-1}\}$, say, $v_{j_1}, v_{j_2}, \ldots, v_{j_{d-d_i}}$, whose numbers of neighbors are still less d, and increase each of $d_{j_1}, d_{j_2}, \ldots, d_{j_{d-d_i}}$ by 1.

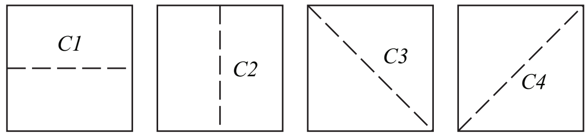

Fig. 8. Cutsets in a random network.

Table III. Characteristics of the Optical Networks

Network	n	$n(n-1)/2$	$n(C_1)$	$W(C_1)$	$n(C_1)(n-n(C_1))/W(C_1)$	lb
Mesh	100	4950	50	10	250	$m/19.8$
ARPANET	24	276	4	2	40	$m/6.9$
NSFNET	16	120	8	4	16	$m/7.5$
EON	20	190	2	2	18	$m/10.5$
UK network	30	435	8	5	35.2	$m/12.4$
R-GRID	100	4950	50	9	277.8	$m/17.8$
R-REGULAR	50	1225	25	38.8	16.1	$m/76$
R-DISK	50	1225	25	34.5	18.1	$m/67.6$

A *random-unit disk network* $N_r = (V, E)$ is generated as follows. The n nodes $v_0, v_1, v_2, \ldots, v_{n-1}$ of V are chosen randomly and independently from U, with a uniform distribution. Two nodes v_i and v_j are connected if, and only if, their distance is no longer than r, where $0 \le r \le 1/2$. The expected number of neighbors of a node is nq_r, where

$$q_r = \pi r^2 - \frac{8}{3}r^3 + \left(\frac{11}{3} - \pi\right)r^4$$

with $0 \le r \le 1/2$ [Li 2005].

Four cutsets are used for a random regular network and a random-unit disk network (Figure 8), each cuts the unit square in a different way.

In Table III, we give some quantitative characterization of the above optical networks. In particular, we give the lower bound lb for $E(\mathrm{OPT}(\sigma))$. For random networks, the value of $W(C_1)$ is obtained by simulation, where C_1 is the random cutset, which cuts the unit square into upper and lower halves. For each type of random networks, we produce 20,000 connected samples. The expected value of $W(C_1)$ is then calculated with the 99% confidence interval less than $\pm 0.8\%$.

5.3 Experimental Results

All the sequences of random connection requests are generated by using the random drawing without replacement model. We believe that similar conclusions can be drawn by using the random drawing with replacement model.

We only consider connected random networks, that is, a random network is regenerated if it is disconnected. The parameters q, d, and r of the three types of random networks are determined such that $q = 0.9$ and $d = nq_r = 10$. These parameter settings are to yield high connectedness of the random networks. To test the connectedness of the random networks with the above parameter settings, we generated 10,000 samples of each type of random networks. The numbers of connected samples of random-grid networks, random-regular networks, and random-unit disk networks are 9213, 9999, and 9495, respectively.

Table IVa. Experimental Data for RWA on the 10 × 10 Mesh Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
50	1.622	1.738	7.512	1.633	1.742	7.111	1.741	1.861	7.378	1.677	1.800	7.448
100	1.442	1.517	7.592	1.449	1.521	7.131	1.578	1.659	7.480	1.503	1.580	7.541
150	1.372	1.428	7.633	1.368	1.428	7.153	1.518	1.583	7.525	1.432	1.492	7.576
200	1.328	1.379	7.644	1.326	1.374	7.143	1.482	1.539	7.558	1.391	1.442	7.596
250	1.302	1.348	7.657	1.292	1.339	7.147	1.456	1.508	7.570	1.365	1.407	7.610
300	1.281	1.322	7.650	1.273	1.311	7.143	1.443	1.486	7.567	1.344	1.385	7.626
350	1.266	1.302	7.653	1.257	1.292	7.155	1.431	1.471	7.585	1.330	1.366	7.624
400	1.254	1.286	7.645	1.244	1.276	7.151	1.420	1.456	7.582	1.315	1.351	7.631
450	1.243	1.274	7.652	1.231	1.262	7.156	1.411	1.446	7.596	1.306	1.336	7.634
500	1.236	1.264	7.650	1.223	1.251	7.153	1.403	1.436	7.594	1.297	1.328	7.642

[a] 99% confidence interval ±0.741%.

Table IVb. Experimental Data for TM on the 10 × 10 Mesh Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.982	1.000	1.000	0.983	1.000	1.000	0.952	1.000	1.000	0.968	1.000	1.000
150	0.769	1.000	1.000	0.784	1.000	1.000	0.756	1.000	1.000	0.760	1.000	1.000
200	0.624	0.998	1.000	0.641	0.999	1.000	0.621	0.974	1.000	0.621	0.992	1.000
250	0.529	0.906	1.000	0.543	0.918	1.000	0.529	0.874	1.000	0.529	0.887	1.000
300	0.460	0.799	1.000	0.475	0.816	1.000	0.464	0.780	0.983	0.461	0.787	0.998
350	0.410	0.714	0.964	0.424	0.733	0.972	0.414	0.706	0.919	0.411	0.707	0.941
400	0.371	0.648	0.883	0.384	0.666	0.898	0.376	0.643	0.853	0.373	0.644	0.866
450	0.341	0.594	0.813	0.352	0.612	0.832	0.344	0.593	0.793	0.343	0.592	0.799
500	0.315	0.549	0.754	0.326	0.567	0.773	0.319	0.551	0.741	0.317	0.549	0.744

[a] 99% confidence interval ±0.257%.

In Tables IVb–XIb, the value of k is set to demonstrate changes of throughput. If k is too large, the throughput will always be 100%.

Each experiment is repeated for $s = 2000$ times and the 99% confidence interval is shown for each table, which is obtained from the maximum δ (see Figures 1 and 2) of all the experiments in a table. The 99% confidence interval is less than ±2%, with the exception of Table XIa for random-unit disk networks. It is noticed that the number of wavelengths used on random-unit disk networks has large variance. It has been observed that the probability distribution of the number of wavelengths used on random-unit disk networks has a long tail and the number of wavelengths may exceed, say, 256!

Our experimental data are displayed in Tables IV–XI for the eight WDM optical networks. Several observations are in order.

—All the four online algorithms exhibit excellent average-case performance on all the networks for the RWA problem, in the sense that for a wide range of m, both $\bar{\alpha}$ and $\bar{\beta}$ are very small (less than 2, except on random unit disk networks). In particular, as m increases, both $\bar{\alpha}$ and $\bar{\beta}$ decrease and approach 1. For the TM problem, high throughput can be achieved even for small k.

—The quality of $\bar{\alpha}$ and $\bar{\beta}$ depends on the quality of the lower bounds. We believe that the relatively large values of $\bar{\alpha}$ and $\bar{\beta}$ for the random-unit disk networks

Table Va. Experimental Data for RWA on a 24-Node ARPANET Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
20	1.239	1.211	3.207	1.238	1.217	3.013	1.267	1.229	3.086	1.257	1.225	3.129
40	1.120	1.098	3.244	1.114	1.099	2.984	1.136	1.123	3.109	1.124	1.107	3.173
60	1.073	1.071	3.270	1.070	1.060	2.958	1.092	1.086	3.115	1.076	1.067	3.176
80	1.051	1.049	3.265	1.047	1.048	2.945	1.068	1.065	3.115	1.055	1.053	3.189
100	1.040	1.042	3.285	1.038	1.038	2.935	1.053	1.051	3.112	1.044	1.042	3.196
120	1.032	1.027	3.277	1.032	1.033	2.920	1.044	1.046	3.108	1.035	1.038	3.197
140	1.026	1.026	3.282	1.028	1.029	2.922	1.039	1.040	3.111	1.030	1.029	3.197
160	1.023	1.020	3.281	1.023	1.020	2.908	1.034	1.035	3.107	1.025	1.028	3.203
180	1.020	1.022	3.284	1.020	1.016	2.904	1.030	1.028	3.106	1.023	1.025	3.203
200	1.018	1.014	3.282	1.019	1.019	2.906	1.027	1.028	3.108	1.020	1.020	3.199

[a] 99% confidence interval ±1.406%.

Table Vb. Experimental Data for TM on a 24-Node ARPANET Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$
20	0.953	1.000	1.000	0.958	1.000	1.000	0.953	1.000	1.000	0.953	1.000	1.000
40	0.793	0.974	1.000	0.800	0.972	1.000	0.781	0.972	1.000	0.783	0.973	1.000
60	0.629	0.898	0.980	0.642	0.898	0.980	0.620	0.893	0.978	0.622	0.894	0.981
80	0.515	0.829	0.927	0.530	0.834	0.929	0.514	0.809	0.925	0.515	0.814	0.927
100	0.441	0.741	0.883	0.456	0.758	0.882	0.439	0.722	0.876	0.438	0.726	0.879
120	0.385	0.661	0.843	0.400	0.682	0.846	0.385	0.647	0.821	0.385	0.650	0.827
140	0.344	0.595	0.789	0.359	0.620	0.804	0.343	0.587	0.764	0.343	0.589	0.768
160	0.312	0.544	0.729	0.325	0.570	0.753	0.311	0.539	0.709	0.312	0.539	0.712
180	0.285	0.501	0.676	0.299	0.528	0.703	0.286	0.498	0.662	0.285	0.498	0.663
200	0.263	0.465	0.631	0.276	0.491	0.658	0.264	0.464	0.619	0.264	0.464	0.621

[a] 99% confidence interval ±0.435%.

Table VIa. Experimental Data for RWA on a 16-Node NSFNET Backbone[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
10	1.714	1.734	2.714	1.721	1.745	2.652	1.774	1.804	2.706	1.742	1.758	2.698
20	1.498	1.471	2.736	1.512	1.487	2.653	1.572	1.557	2.743	1.531	1.521	2.754
30	1.367	1.360	2.779	1.376	1.355	2.652	1.467	1.447	2.745	1.417	1.401	2.758
40	1.303	1.294	2.770	1.300	1.288	2.661	1.396	1.387	2.762	1.343	1.338	2.772
50	1.257	1.248	2.772	1.251	1.242	2.656	1.367	1.355	2.771	1.300	1.293	2.766
60	1.221	1.218	2.779	1.218	1.215	2.659	1.334	1.329	2.767	1.268	1.262	2.776
70	1.196	1.194	2.777	1.193	1.191	2.662	1.305	1.304	2.771	1.242	1.237	2.769
80	1.177	1.173	2.778	1.175	1.173	2.663	1.288	1.284	2.770	1.222	1.218	2.777
90	1.162	1.159	2.780	1.157	1.153	2.658	1.276	1.273	2.774	1.206	1.204	2.779
100	1.149	1.146	2.779	1.145	1.142	2.663	1.263	1.259	2.770	1.190	1.189	2.778

[a] 99% confidence interval ±1.437%.

Table VIb. Experimental Data for TM on a 16-Node NSFNET Backbone[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	0.900	1.000	1.000	0.900	1.000	1.000	0.879	1.000	1.000	0.887	1.000	1.000
30	0.707	0.999	1.000	0.714	0.999	1.000	0.695	0.996	1.000	0.705	0.999	1.000
40	0.585	0.950	1.000	0.595	0.952	1.000	0.581	0.920	1.000	0.582	0.935	1.000
50	0.503	0.840	0.999	0.513	0.848	0.999	0.501	0.819	0.993	0.501	0.830	0.998
60	0.445	0.754	0.974	0.454	0.762	0.974	0.445	0.739	0.940	0.444	0.744	0.958
70	0.400	0.681	0.899	0.408	0.692	0.905	0.400	0.673	0.870	0.401	0.676	0.885
80	0.366	0.625	0.833	0.373	0.637	0.839	0.365	0.619	0.810	0.365	0.622	0.818
90	0.336	0.581	0.772	0.344	0.591	0.782	0.337	0.574	0.756	0.336	0.578	0.765
100	0.313	0.542	0.724	0.321	0.555	0.734	0.313	0.538	0.712	0.314	0.539	0.717

[a] 99% confidence interval ±0.549%.

Table VIIa. Experimental Data for RWA on the 20-Node European Optical Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
10	1.592	2.177	2.743	1.605	2.190	2.646	1.599	2.186	2.669	1.604	2.201	2.704
20	1.409	1.728	2.803	1.408	1.719	2.646	1.418	1.742	2.726	1.399	1.718	2.765
30	1.317	1.552	2.838	1.302	1.541	2.635	1.346	1.589	2.756	1.344	1.579	2.800
40	1.264	1.446	2.849	1.253	1.443	2.641	1.305	1.489	2.783	1.300	1.475	2.808
50	1.232	1.385	2.859	1.223	1.376	2.633	1.287	1.440	2.791	1.256	1.406	2.823
60	1.211	1.333	2.868	1.203	1.325	2.625	1.266	1.387	2.806	1.236	1.361	2.831
70	1.193	1.298	2.876	1.177	1.285	2.623	1.251	1.355	2.811	1.217	1.323	2.845
80	1.178	1.263	2.873	1.163	1.258	2.618	1.232	1.326	2.822	1.197	1.285	2.850
90	1.166	1.241	2.878	1.158	1.232	2.606	1.231	1.305	2.827	1.198	1.272	2.852
100	1.150	1.211	2.882	1.150	1.214	2.603	1.220	1.286	2.835	1.181	1.244	2.858

[a] 99% confidence interval ±1.792%.

Table VIIb. Experimental Data for TM on the 20-Node European Optical Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$
10	0.999	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000
20	0.982	1.000	1.000	0.981	1.000	1.000	0.980	1.000	1.000	0.980	1.000	1.000
30	0.905	0.999	1.000	0.909	0.999	1.000	0.891	0.999	1.000	0.895	0.999	1.000
40	0.780	0.996	1.000	0.789	0.996	1.000	0.768	0.996	1.000	0.770	0.996	1.000
50	0.673	0.984	1.000	0.682	0.986	1.000	0.663	0.979	1.000	0.666	0.981	1.000
60	0.587	0.952	0.999	0.601	0.953	0.999	0.586	0.930	0.999	0.585	0.941	0.999
70	0.524	0.892	0.996	0.538	0.901	0.997	0.523	0.866	0.995	0.523	0.876	0.996
80	0.473	0.822	0.989	0.488	0.839	0.989	0.475	0.801	0.980	0.474	0.809	0.986
90	0.435	0.761	0.972	0.448	0.779	0.974	0.435	0.746	0.948	0.435	0.749	0.960
100	0.402	0.705	0.937	0.416	0.725	0.942	0.403	0.695	0.906	0.402	0.699	0.919

[a] 99% confidence interval ±0.417%.

Table VIIIa. Experimental Data for RWA on the 30-Node UK Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
30	1.579	1.661	3.811	1.573	1.658	3.596	1.649	1.736	3.726	1.617	1.702	3.772
60	1.400	1.431	3.845	1.401	1.436	3.617	1.506	1.541	3.780	1.438	1.481	3.826
90	1.324	1.339	3.876	1.327	1.343	3.613	1.447	1.464	3.805	1.374	1.389	3.842
120	1.277	1.289	3.889	1.284	1.291	3.616	1.413	1.420	3.822	1.329	1.337	3.856
150	1.250	1.253	3.884	1.250	1.253	3.617	1.384	1.387	3.827	1.299	1.304	3.869
180	1.229	1.229	3.889	1.225	1.226	3.618	1.364	1.367	3.840	1.277	1.279	3.875
210	1.208	1.208	3.897	1.206	1.208	3.617	1.348	1.350	3.846	1.261	1.259	3.873
240	1.195	1.194	3.900	1.191	1.192	3.617	1.336	1.336	3.848	1.248	1.245	3.880
270	1.182	1.183	3.905	1.180	1.179	3.618	1.325	1.325	3.853	1.230	1.231	3.883
300	1.171	1.171	3.905	1.166	1.166	3.614	1.317	1.315	3.850	1.219	1.220	3.884

[a] 99% confidence interval ±1.085%.

Table VIIIb. Experimental Data for TM on the 30-Node UK Network[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=5$	$k=10$	$k=15$	$k=5$	$k=10$	$k=15$	$k=5$	$k=10$	$k=15$	$k=5$	$k=10$	$k=15$
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
60	0.869	1.000	1.000	0.874	1.000	1.000	0.845	1.000	1.000	0.855	1.000	1.000
90	0.662	0.997	1.000	0.676	0.997	1.000	0.657	0.986	1.000	0.658	0.995	1.000
120	0.535	0.907	1.000	0.551	0.914	1.000	0.538	0.876	1.000	0.536	0.891	1.000
150	0.455	0.782	0.994	0.470	0.800	0.994	0.458	0.770	0.968	0.457	0.774	0.986
180	0.399	0.688	0.926	0.413	0.706	0.932	0.402	0.683	0.889	0.401	0.684	0.906
210	0.356	0.616	0.839	0.369	0.635	0.852	0.361	0.616	0.815	0.359	0.614	0.824
240	0.323	0.559	0.763	0.336	0.579	0.781	0.327	0.563	0.750	0.326	0.559	0.755
270	0.297	0.513	0.702	0.309	0.533	0.721	0.302	0.519	0.696	0.299	0.515	0.697
300	0.275	0.476	0.651	0.286	0.494	0.670	0.279	0.481	0.649	0.278	0.478	0.648

[a] 99% confidence interval ±0.339%.

Table IXa. Experimental Data for RWA on 100-Node Random Grid Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
50	1.875	2.025	8.093	1.889	2.048	7.603	1.998	2.164	7.958	1.926	2.082	8.006
100	1.649	1.771	8.179	1.670	1.789	7.628	1.800	1.923	8.033	1.713	1.837	8.105
150	1.567	1.673	8.191	1.582	1.687	7.638	1.718	1.831	8.066	1.621	1.727	8.110
200	1.519	1.619	8.206	1.530	1.634	7.661	1.674	1.785	8.081	1.578	1.680	8.151
250	1.493	1.584	8.206	1.495	1.600	7.666	1.636	1.736	8.099	1.538	1.636	8.154
300	1.462	1.556	8.216	1.476	1.567	7.657	1.614	1.713	8.093	1.520	1.607	8.147
350	1.446	1.533	8.214	1.457	1.546	7.670	1.599	1.698	8.098	1.495	1.594	8.174
400	1.440	1.528	8.231	1.444	1.535	7.669	1.582	1.677	8.100	1.487	1.575	8.169
450	1.424	1.503	8.220	1.440	1.523	7.681	1.573	1.670	8.111	1.476	1.559	8.176
500	1.418	1.500	8.228	1.422	1.500	7.681	1.562	1.658	8.109	1.464	1.548	8.186

[a] 99% confidence interval ±0.965%.

Table IXb. Experimental Data for TM on 100-Node Random Grid Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$	$k=7$	$k=14$	$k=21$
50	0.999	1.000	1.000	0.999	1.000	1.000	0.998	1.000	1.000	0.999	1.000	1.000
100	0.791	1.000	1.000	0.803	1.000	1.000	0.783	1.000	1.000	0.787	1.000	1.000
150	0.554	0.982	1.000	0.568	0.981	1.000	0.558	0.964	1.000	0.559	0.974	1.000
200	0.433	0.832	0.997	0.442	0.847	0.997	0.437	0.822	0.993	0.437	0.827	0.995
250	0.362	0.686	0.960	0.367	0.701	0.964	0.363	0.688	0.939	0.362	0.687	0.953
300	0.312	0.586	0.857	0.315	0.597	0.866	0.312	0.589	0.844	0.313	0.587	0.845
350	0.275	0.512	0.746	0.279	0.523	0.760	0.276	0.517	0.744	0.276	0.516	0.746
400	0.248	0.458	0.662	0.250	0.467	0.679	0.247	0.461	0.666	0.247	0.461	0.663
450	0.226	0.418	0.597	0.228	0.424	0.610	0.225	0.419	0.603	0.226	0.418	0.601
500	0.208	0.383	0.547	0.209	0.389	0.559	0.207	0.384	0.550	0.208	0.383	0.550

[a] 99% confidence interval ±0.554%%.

Table Xa. Experimental Data for RWA on 50-Node Random Regular Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
50	1.654	1.672	2.850	1.630	1.648	2.794	1.638	1.659	2.794	1.653	1.671	2.821
100	1.669	1.894	2.973	1.677	1.911	2.811	1.728	1.965	2.849	1.687	1.929	2.919
150	1.535	1.745	3.026	1.571	1.786	2.815	1.598	1.818	2.885	1.560	1.788	2.968
200	1.487	1.671	3.055	1.504	1.706	2.827	1.551	1.776	2.898	1.512	1.714	2.993
250	1.449	1.646	3.076	1.470	1.657	2.823	1.517	1.715	2.912	1.465	1.658	3.014
300	1.423	1.615	3.092	1.438	1.631	2.831	1.487	1.697	2.924	1.443	1.647	3.022
350	1.407	1.614	3.101	1.431	1.619	2.829	1.470	1.669	2.934	1.409	1.608	3.034
400	1.396	1.575	3.105	1.404	1.589	2.828	1.461	1.653	2.934	1.405	1.610	3.044
450	1.380	1.556	3.115	1.404	1.601	2.828	1.452	1.639	2.937	1.388	1.573	3.049
500	1.358	1.544	3.120	1.391	1.584	2.824	1.434	1.632	2.942	1.380	1.559	3.059

[a] 99% confidence interval ±1.706%.

Table Xb. Experimental Data for TM on 50-Node Random Regular Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	0.999	1.000	1.000	0.998	1.000	1.000	0.998	1.000	1.000	0.998	1.000	1.000
150	0.967	1.000	1.000	0.966	1.000	1.000	0.960	1.000	1.000	0.964	1.000	1.000
200	0.854	0.998	1.000	0.854	0.999	1.000	0.845	0.999	1.000	0.849	0.999	1.000
250	0.738	0.994	1.000	0.745	0.992	1.000	0.737	0.991	0.999	0.738	0.993	1.000
300	0.655	0.974	0.999	0.661	0.969	0.999	0.655	0.966	0.999	0.654	0.970	0.999
350	0.589	0.925	0.997	0.599	0.925	0.995	0.591	0.914	0.995	0.590	0.922	0.996
400	0.538	0.863	0.989	0.548	0.865	0.988	0.540	0.854	0.986	0.536	0.858	0.989
450	0.496	0.799	0.973	0.508	0.806	0.972	0.498	0.796	0.968	0.494	0.799	0.974
500	0.460	0.748	0.947	0.472	0.755	0.944	0.462	0.749	0.939	0.460	0.747	0.946

[a] 99% confidence interval ±0.476%.

Table XIa. Experimental Data for RWA on 50-Node Random Unit Disk Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$	$\bar{\alpha}$	$\bar{\beta}$	$\overline{pl}$
50	2.407	2.600	3.247	2.396	2.593	3.008	2.382	2.587	3.048	2.335	2.575	3.136
100	2.443	2.887	3.370	2.515	3.033	3.000	2.518	2.977	3.105	2.489	2.918	3.195
150	2.370	2.825	3.404	2.434	2.881	2.995	2.474	2.895	3.105	2.435	2.959	3.190
200	2.357	2.732	3.436	2.374	2.827	2.995	2.345	2.768	3.125	2.357	2.780	3.216
250	2.328	2.769	3.449	2.375	2.784	2.999	2.399	2.810	3.131	2.312	2.738	3.220
300	2.293	2.744	3.455	2.376	2.850	2.980	2.328	2.745	3.143	2.389	2.786	3.231
350	2.294	2.737	3.472	2.323	2.757	2.972	2.377	2.771	3.132	2.315	2.810	3.232
400	2.266	2.714	3.466	2.274	2.705	2.966	2.336	2.783	3.131	2.253	2.684	3.237
450	2.229	2.670	3.466	2.300	2.719	2.994	2.312	2.791	3.141	2.251	2.686	3.236
500	2.291	2.697	3.482	2.234	2.685	2.967	2.308	2.699	3.134	2.206	2.670	3.228

[a] 99% confidence interval ±5.123%.

Table XIb. Experimental Data for TM on 50-Node Random Unit Disk Networks[a]

	First-Fit			Best-Fit			Densest-Fit			Random-Fit		
m	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$	$k=3$	$k=6$	$k=9$
50	0.981	0.997	0.999	0.979	0.996	0.998	0.981	0.997	0.999	0.978	0.996	0.999
100	0.892	0.981	0.994	0.887	0.984	0.993	0.887	0.983	0.993	0.889	0.982	0.993
150	0.763	0.950	0.984	0.768	0.948	0.983	0.767	0.947	0.983	0.762	0.947	0.983
200	0.658	0.898	0.962	0.659	0.896	0.961	0.654	0.899	0.962	0.654	0.901	0.963
250	0.570	0.833	0.937	0.575	0.836	0.930	0.567	0.834	0.936	0.573	0.840	0.933
300	0.510	0.771	0.898	0.511	0.773	0.903	0.512	0.770	0.900	0.504	0.770	0.901
350	0.457	0.712	0.859	0.464	0.712	0.862	0.458	0.715	0.856	0.458	0.717	0.857
400	0.417	0.661	0.821	0.423	0.666	0.817	0.416	0.666	0.815	0.418	0.662	0.814
450	0.382	0.619	0.772	0.389	0.621	0.780	0.382	0.615	0.774	0.383	0.615	0.781
500	0.357	0.578	0.730	0.362	0.585	0.735	0.356	0.576	0.735	0.358	0.580	0.737

99% confidence interval ±1.062%.

are because of our inability to find tighter lower bounds. Those data in Table XIa obtained from loose lower bounds do not accurately reflect the average-case performance and certainly do not imply relatively poor performance of the four online algorithms on random-unit disk networks.

—Though there is no dramatic difference among the performance of the four algorithms, best-fit is superior to all other algorithms in the sense that it yields smaller $\bar{\alpha}$ and $\bar{\beta}$, produces shorter average path length, and generates higher throughput.

—The average path length is quite stable and does not depend too much on the number of connection requests.

6. CONCLUDING REMARKS

We have investigated the problem of online routing and wavelength assignment and the related throughput maximization problem in wavelength division multiplexing optical networks. It is very encouraging to find that even simple online RWA and TM algorithms can achieve excellent average-case competitive ratios. Our results also imply that the room for performance improvement by using offline algorithms is very limited.

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