The “Real” Approximation Factor of the MST Heuristic for the Minimum Energy Broadcasting

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This paper deals with one of the most studied problems in the last few years in the field of wireless communication in ad-hoc networks. The problem consists of reducing the total energy consumption of wireless radio stations distributed over a given area of interest in order to perform the basic pattern of communication by a broadcast. Recently, a tight 6-approximation of the minimum spanning tree heuristic has been proven. While such a bound is theoretically optimal if compared to the known lower bound of 6, there is an obvious gap with practical experimental results. By extensive experiments, proposing a new technique to generate input instances and supported by theoretical results, we show how the approximation ratio can be actually considered close to 4 for a “real-world” set of instances. We consider, in fact, instances more representative of common practices. Those are usually composed by considerable number of nodes uniformly and randomly distributed inside the area of interest.

Categories and Subject Descriptors: C.4 [**Performance of Systems**]: Measurement Techniques

General Terms: Algorithms, Experimentation

Additional Key Words and Phrases: Ad-hoc networks, broadcast, energy saving, spanning tree

1. INTRODUCTION

In the context of ad-hoc networking, one of the most popular studied problems is the so called *minimum energy broadcast routing* (MEBR). The problem arises from the requirements of a basic pattern of communication, such as a broadcast.

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Given a set of radio stations (or nodes) distributed over a given area of interest with one of those stations specified as the source, the problem is to assign the transmission range of each station so as to broadcast from the source with minimum overall power consumption. A communication session can be established through a series of wireless links involving any network node and, therefore, ad-hoc networks are considered *multihop* networks. To this aim, the nodes have the ability to adjust their transmission power as needed. Thus, every node is assigned a transmission range and every node inside this range receives its message. Considering the fact that nodes operate with a limited supply of energy and given the nature of the operations for which these types of networks are used, such as military operations or emergency disaster relief, a fundamental problem consists of assigning transmission ranges in such a way that the total consumed energy is minimized.

According to the mostly used power attenuation model [Rappaport 1996], when a node s transmits with power P_s, a node r can receive its message if, and only if, $\frac{P_s}{\|s,r\|^2} > 1$, where $\|s,r\|$ is the Euclidean distance between s and r.

Since the MEBR problem is *NP*-hard [Clementi et al. 2001], a lot of effort was devoted to devise good approximation algorithms. Several papers progressively reduced the estimate of the approximation ratio of the fundamental minimum spanning-tree (MST) heuristic from 40 to 6 [Clementi et al. 2001; Wan et al. 2002; Klasing et al. 2004; Cai and Zhao 2005; Flammini et al. 2004; Navarra 2005; Ambuehl 2005]. Roughly speaking the heuristic first computes the minimum spanning tree of the complete weighted graph obtained from the set of nodes in which weights are the square distances of the endpoints of the edges. Then, starting from the given source, it visits the obtained tree and directs the edges toward the leaves. Finally, for each node, the heuristic assigns a power of transmission equal to the weight of the longest outgoing edge.

Even if the 6-approximation ratio provided in Ambuehl [2005] is tight according to the lower bound of Wan et al. [2002], there is an evident gap between such a ratio and the experimental results obtained in several papers (see, for instance, Wieselthier et al. [2000], Clementi et al. [2003], Klasing et al. [2004], Martnez et al. [2004], Athanassopoulos et al. [2004], and Penna and Ventre [2004]). In particular, in Klasing et al. [2004], for a considerable number of nodes, the MST heuristic was directly compared with the optimal solution over hundreds of instances, but no more than a 2-approximation arose among all the computed experiments. This suggests that more careful investigation is needed of the possible input instances in order to better understand this phenomenon. The goal is to classify some specific family of instances according to the output of the MST heuristic. The most common method used to generate the input instances has been that of uniformly and randomly spreading the nodes inside a given area. This is the so called *uniform random model* (see Clementi et al. [2003]). Such a method, in fact, reflects a practical application of the problem, since usually the areas of interest can be inaccessible.

In this paper, we propose a new method for generating instances with an arbitrary number of nodes in order to maximize the final cost of the MST heuristic. In this way, we better grasp the intrinsic properties of the problem. Motivated by the experimental results, we provide further experiments that, combined with the theoretical results of Flammini et al. [2004], show an almost tight 4-approximation ratio for high-density instances of the MEBR problem. Roughly speaking, "high-density" means instances with a large number of nodes uniformly and randomly distributed, in which the longest computed MST edge tends to zero as the number of nodes grows. The tightness of such a ratio is interesting on its own, since the knowledge gained was of a much better performance of the MST heuristic in high-density instances. As already noted, such instances are more representative of practical environments and are theoretically interesting, since, for a small number of nodes, exhaustive algorithms can be applied (see, for instance, the integer linear programming formulation proposed in Klasing et al. [2004]).

The paper is organized as follows. In the next section, we briefly provide some basic definitions and summarize the estimation method proposed in Flammini et al. [2004] by which the 8-approximation of the MST heuristic has been proved. It will be, in fact, useful for the rest of the paper. In Section 3, we formally describe the algorithm to generate suitable instances that maximize the cost of the MST heuristic and, in Section 4, we present the related experimental results. In Section 5, we present the above-mentioned combination of further experimental and theoretical results showing the tightness of the ratio for high-density instances. Finally, in Section 6, we conclude.

2. DEFINITIONS AND NOTATION

Let us first provide a formal definition of the minimum energy broadcast routing (MEBR) problem in the 2-dimensional (2D) space (see Clementi et al. [2001], Wan et al. [2002], and Clementi et al. [2003] for a more detailed discussion). Given a set of points S in a 2D Euclidean space that represents the set of radio stations, let $G_2(S)$ be the complete weighted graph, whose nodes are the points of S, and in which the weight of each edge $\{x, y\}$ is the power consumption needed for a correct communication between x and y, that is, $\|x, y\|^2$.

A range assignment for S is a function $r : S \to \mathbb{R}^+$ such that the range $r(x)$ of a station x denotes the maximal distance from x at which signals can be correctly received. The total cost of a range assignment is then $cost(r) = \sum_{x \in S} r(x)^2$.

A range assignment r for S yields a directed communication graph $G^r = (S, A)$ such that, for each $(x, y) \in S^2$, the directed edge (x, y) belongs to A if, and only if, y is at distance at most $r(x)$ from x. In other words, (x, y) belongs to A if, and only if, the power emission of x is at least equal to the weight of $\{x, y\}$ in $G_2(S)$. In order to perform the required *minimum energy broadcast* from a given source $s \in S$, G^r must contain a directed spanning tree rooted at s and must have the minimum cost.

One fundamental algorithm, called the MST heuristic [Wieselthier et al. 2000], is based on the idea of tuning ranges so as to include a spanning tree of minimum cost. More precisely, denoted as $T_2(S)$, a minimum spanning tree of $G_2(S)$, and as $MST(G_2(S))$ its cost, let us consider $T_2(S)$ rooted at the source station s. The heuristic directs the edges of $T_2(S)$ toward the leaves and sets the range $r(x)$ of every internal station x of $T_2(S)$ with k children $x_1, \ldots, x_k$ in such a way that $r(x) = max_{i=1,\ldots,k} \|x, x_i\|^2$. In other words, r is the range assignment of minimum cost inducing the directed tree derived from $T_2(S)$ and it is such that $cost(r) \leq MST(G_2(S))$.

Let us denote by C_r a circle of radius r. From Clementi et al. [2001], Wan et al. [2002], Flammini et al. [2004], and Ambuehl [2005], it is possible to restrict the study of the performance of the MST heuristic just considering C_1, centered at the source, as area of interest to locate the radio stations. In order to provide knowledge of such a restriction, let us consider a generic optimal solution for a given instance. Such a solution is, in general, composed by several transmitting nodes, i.e., several circles. According to those papers, the estimation of the approximation ratio can then be performed considering separately each circle with its contained nodes, scaling distances so as to obtain a radius equal to 1. Therefore, we can focus just on the most expensive subinstance in a unitary circle.

In Flammini et al. [2004], an 8-approximation is proved by assigning a growing circle to each node contained in C_1 until all the circles form a unique connected area component.[1] Such an area, denoted by $a(S, \frac{r_{max}}{2})$, is related to the *MST* cost according to the following equation (see Frieze and McDiarmid [1989] and Flammini et al. [2004]):

$$MST(G_2(S)) = 2\int_0^{r_{max}} (n(S, r) - 1)r\ dr$$

where r_{max} is the size of the longest edge contained in $MST(G_2(S))$ and $n(S, r)$ is the number of connected components obtained from $G_2(S)$ by deleting all the edges between the nodes at distance greater than r in S. The following bounds are then derived

$$\frac{\pi}{4}MST(G_2(S)) + \frac{\pi}{4}r_{max}^2 \leq a\left(S, \frac{r_{max}}{2}\right) \leq \pi\left(1 + \frac{r_{max}}{2}\right)^2$$

obtaining

$$MST(G_2(S)) \leq 4(1 + r_{max})$$

The 8-approximation then holds by observing that $r_{max} \leq 1$. For further details on the correctness of the analysis, see Flammini et al. [2004].

For r_{max} tending to 0, the approximation ratio of the MST heuristic tends to 4. The property r_{max} tending to 0 can be translated as placing a large number of nodes uniformly and randomly inside C_1. When such a property holds, i.e., when the number of nodes grows, while r_{max} tends to 0, we say we are in the *high-density* case. It is worthwhile noting that, at this stage, it is not clear if such a case can really occur or not. This will be investigated in more detail in Section 5. Moreover, for any considered number of nodes, the approximation ratio of the MST heuristic is still 6. Such a ratio is, in fact, optimal and it holds for any number of nodes we place inside C_1.

[1]Two distinct connected components become just one as soon as they start to overlap each other in at least one point or if they both join to a same connected component.

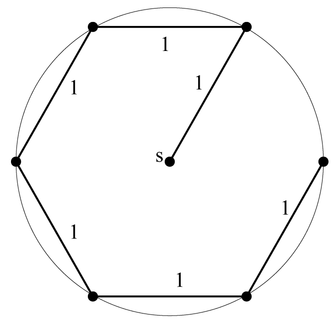

Fig. 1. The 6 lower bound for the MST heuristic.

Consider an instance composed by seven nodes disposed in the hexagonal shape given by the lower-bound instance of the MST heuristic (see Figure 1). Then, adding as many nodes as we want, surrounding those seven nodes at a distance close to zero, the cost of the obtained instances will always be close to 6, while we can vary the number of nodes as we wish. Indeed, such examples are quite trivial and those instances do not represent practical cases where usually the nodes are placed according to the uniform random model. Moreover, in such a case, r_{max} remains close to 1.

Studying the results obtained by extensive experiments, we are going to show that in the high-density case, in the meaning previously defined, the bound of 4 is almost tight.

3. AUGMENTING ALGORITHM

It is well-known that the lower bound for the MST heuristic is given by the hexagonal shape presented in Wan et al. [2002], where the instance is given by seven nodes that are the center and the vertices of a regular hexagon inscribed in C_1 (see Figure 1). In such an instance, the MST heuristic cost can be equal to 6, while the optimal solution costs just 1. It is evident that 6 is the maximum cost for instances inside a C_1 in which the source is its center and the number of nodes, is at most, seven. Performing experiments as described in Wieselthier et al. [2000], Clementi et al. [2003], Klasing et al. [2004], Martnez et al. [2004], Athanassopoulos et al. [2004], and Penna and Ventre [2004], even just placing seven nodes, in which one of them is fixed to be the center of C_1 and the other ones are randomly and uniformly distributed inside such a circle, it is really "lucky" that a similar high-cost instance appears to occur. Moreover, increasing the number of nodes involved in the experiments, on average, the cost of the performed MST decreases.

In this paper we are interested in maximizing the cost of a possible MST inside C_1 considering its center s as the source in order to better understand the actual quality of the performance of the MST heuristic over interesting instances more representative of real-world applications. Roughly speaking, starting from random instances, the maximization is because of slight movements of the nodes according to some useful properties of the MST construction. For instance, if we want to increase the cost of an edge of the MST, the easiest

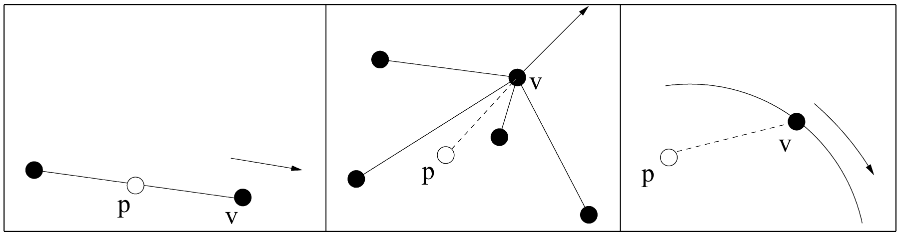

Fig. 2. Augmenting the edge costs when a node has one or more neighbors and when it is on the circumference of C_1.

idea is to increase the distance of its endpoints. Let us now consider a node $v \neq s$ of a generic instance given at input. We consider the degree of such a node in the undirected tree obtained from the MST heuristic before assigning the directions. Let $N_v = \{v_1, v_2, \ldots, v_k\}$ be the set of the neighbors of v in such a tree. We evaluate the median point $p = (x, y)$ whose coordinates are given by the average of the corresponding coordinates of the nodes in N_v, that is

$$x = \frac{1}{k}\sum_{i=1}^{k} x_{v_i}, \qquad y = \frac{1}{k}\sum_{i=1}^{k} y_{v_i}$$

The idea is then to move the node v farther from p but, of course, remaining inside the considered circle. In general, this should augment the cost of the MST on the edge connecting the node v to the rest of the tree (see Figure 2).

It can also happen that such a movement completely changes the structure of the MST, hence, reducing the initial cost. In that case, we do not validate the movement. Given an instance, the augmenting algorithm performs this computation for each node twisting over all the nodes but s until no movements are allowed. As we are going to show, the movements depend also on a random parameter *rand*. Therefore, in order to give a node a "second chance" to move, we can repeat such computations for a fixed number of rounds. Note that, when a node reaches the border, that is, the circumference of the circle, the only allowed movement is over such a circumference.

A further way to increase the cost of the MST is then to try to delete a node. We choose as a candidate the highest degree node. The idea behind this choice is that the highest degree node could be considered as the intermediary node to connect its neighbors. Thus, by removing it, a "large hole" appears. On one hand, this means that the distances to connect the remaining disjoint subtrees should increase the overall cost. On the other hand, we are creating more space for further movements. After a deletion, the algorithm again starts with the movements. Indeed, the deletion can be considered as a movement in which two nodes are overlapping. If the deletion does not increase the cost of the current MST, we do not validate it. In such a case, the next step will be the deletion of the second highest degree node, and so on. The entire procedure is repeated until no movements and no deletions are allowed. Note that the entire algorithm can be repeated several consecutive times in order to obtain more accurate results. Sometimes, in fact, it can occur that the algorithm is stuck in some local maximum. Because of the randomness of the movements, the more it is executed, the higher is the probability to exit from such a status.

We now define more precisely the algorithm roughly described above. Let $V = \{s, v_0, v_2, \ldots, v_{n-1}\}$ be a set of nodes inside C_1, centered in s, and let ϵ be the step of the movements we allow, that is, the maximum fraction of the distance from the median point p we allow to move the current node v.

Algo (s, V, ϵ)

```
flag1 = 1;  \* flag1 determines if a node movement is still allowed.
flag2 = 1;  \* flag2 determines if a node deletion is still allowed.
N = |V| − 1;  \* Number of available nodes for the augmenting methods.
i = 0;
j = 0;
Compute the MST over the complete weighted graph G induced by the set of nodes
   V in which each edge {x, y} has weight ‖x, y‖^2; save its cost in cost1;
while flag2 ≤ N do
   while flag1 ≤ N do
      Consider the node v_i = {x_i, y_i} and its k_i neighbors,
        x = (1/k_i) Σ_{i=1}^{k_i} x_{v_i};  y = (1/k_i) Σ_{i=1}^{k_i} y_{v_i};  \* Coordinates of the median point p.
      Let rand be a random number in [0, 1];
      if v_i is not on the circumference then
         Let v'_i be a point in C_1 furthest away from p on the half-line from p passing
         through v_i and such that ‖v'_i, p‖ ≤ (1 + ε · rand)‖v_i, p‖;
      else
         Let v'_i be a point on the circumference further from p with respect to v_i such
         that the arc joining v_i and v'_i has length ε · rand;
      end if
      Compute the MST over the complete weighted graph induced by the set of nodes
      (V \ v_i) ∪ v'_i; save its cost in cost2;
      if cost2 > cost1 then
         V = (V \ v_i) ∪ v'_i;
         cost1 = cost2;
         flag1 = 1;
      else
         flag1 = flag1 + 1;  \* The movement is not valid.
      end if
      i = (i + 1) mod N;
   end while
   Let v_j be the jth highest degree node of the current MST, compute the MST over
   the complete weighted graph induced by the set of nodes V \ v_j; save its cost in
   cost2.
   if cost2 > cost1 then
      V = V \ v_j;
      N = N − 1;
      cost1 = cost2;
      flag1 = 1;
      flag2 = 1;
   else
      flag2 = flag2 + 1;  \* The deletion is not valid.
      j = (j + 1) mod N;
   end if
end while
```

Table I. The Average and Maximum Costs Obtained on Standard Random Instances and Using the Previous augmenting Algorithm on Instances of 5 up to 100 Nodes and ϵ equal to 0.1 and 0.5

	Random		Augmented, $\epsilon = .5$		Augmented,$\epsilon = .1$	
n	Average	Max	Average	Max	Average	Max
5	1.301	2.8752	3.6456	4	3.6276	4
7	1.4799	2.4793	4.5454	5.7386	4.5606	5.8797
10	1.8019	3.1231	5.2848	5.7851	5.353	5.9187
15	1.8875	2.6691	4.8648	5.4803	4.777	5.7728
20	1.854	2.6187	4.2817	5.0906	4.1316	5.1222
30	1.8252	2.2328	4.137	4.45	3.991	4.1819
50	1.812	1,9718	3.7319	3,8901	3.6331	3,7598
100	1.6833	1.8829	3.5673	3.7223	3.4898	3.812

The validated movements (and deletions) imply a monotonic increasing function on the cost of the MST. Since such a cost is bounded by 6 [Ambuehl 2005], the termination of the algorithm is guaranteed. Actually this is accomplished by the minimal constant growth in each computation given by the minimum performable positive number of the working machine. A strategy to speedup the algorithm could be to modify the *if* condition of code lines 17 and 27 by $cost2 > cost1 + c$, hence, introducing a further parameter c that fixes the minimal growth at each augmenting step.

4. EXPERIMENTAL RESULTS

We run the algorithm over hundreds of instances from 5 up to 100 nodes. Table I shows the average and the maximum costs obtained on random instances, as in previous papers, and using our augmenting method for ϵ equal to 0.5 and 0.1. Such values refer to the costs of the corresponding range assignment computed by means of the MST heuristic. We repeated the execution of the algorithm two consecutive times for each instance.

Compared to the standard random-generated instances, the average costs were almost tripled, while the maximum almost doubled. The numerical results obtained are very interesting, since they show that standard random instances are not very representative in studying the bounds of the MST heuristic for the MEBR problem. Moreover, as a “side effect” of such experiments, another very interesting obtained property concerns the topologies obtained in the augmented instances. While for instances of less than 20 nodes, our method modifies the distribution of nodes collapsing into the well-known hexagon shape of Figure 1, by increasing the number of nodes, the problem becomes more interesting.

In Figure 3, an instance of 100 nodes is given before and after the movements and deletions. What follows from those experiments is an evident regularity on the final obtained instances. As shown in Figure 3, in general, after the augmentation, nodes appear to be disposed on some kind of regular grid. This strengthens the lower bound given by the regular hexagon shape.

A remarkable digression concerns the well-known problems in the field of self-organizing microrobots. Usually regular formations are preferred in order to cope with more extended areas (see Figure 4).

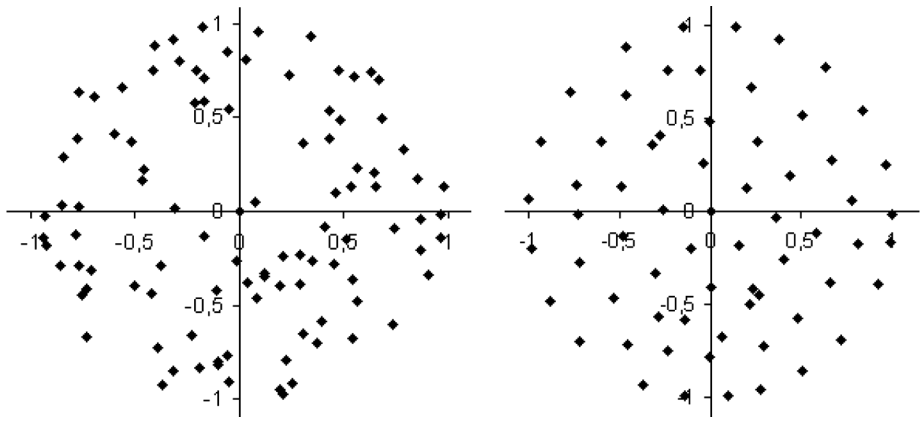

Fig. 3. A random instance of 100 nodes before and after the augmenting method. The number of nodes decreased from 100 to 65, while the cost increased from 1.8774 to 3.6809.

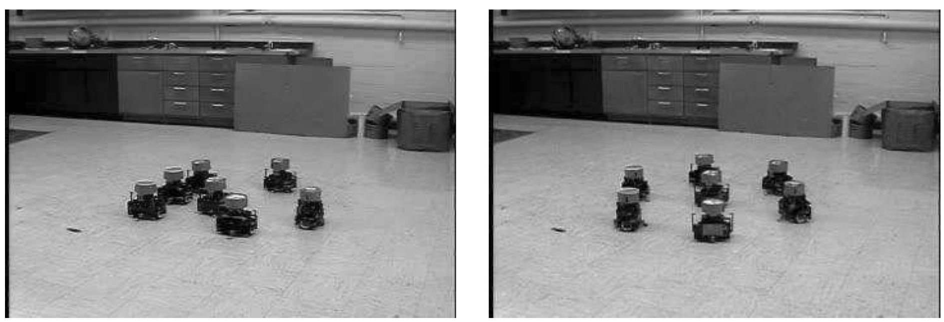

Fig. 4. Seven self-organizing robots making the hexagon shape.

Such a solution is also dictated by the requirement for which the involved robots should consume a uniform amount of energy (see, for instance, Doshi et al. [2002] and Spears et al. [2004]). Indeed, we showed by our experiments that in this way the total consumed energy is maximized. Moreover, the communications of the robots and in ad-hoc networks as well, are not uniform, in general, that is, some sensor/robot has to communicate more information than others, depending on its own application. A better trade-off depending on the communications would probably be found in order to reduce the total consumed energy. For instance, we could expect to see robots with more residual energy to cope with more extended areas and robots with less energy with associated smaller areas.

Returning to our main goal of maximizing the instances costs, it is evident that our method considerably increases the average and the maximum cost of the investigated instances. Moreover, the experiments also suggest to consider regular distributions of the nodes in order to obtain maximum cost instances. In the next section, we investigate this property by obtaining an almost tight 4-approximation upper bound for the MST heuristic in the case of high-density distributions.

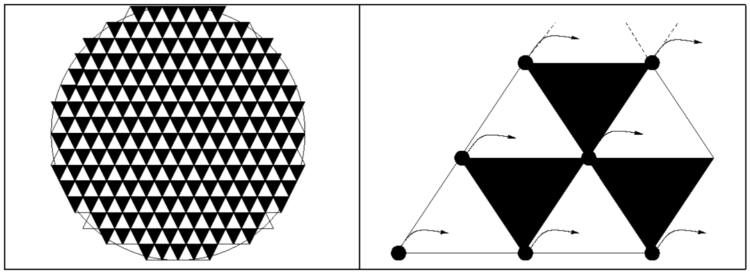

Fig. 5. The subdivision of a circle in triangles and the association of each node to a triangle.

5. HIGH-DENSITY CASE

In this section, we give experimental evidence that the upper bound of 4, provided in Section 2 for the MST heuristic, is almost tight in the case of r_{max} tending to 0.

In particular, we first provide an example of uniform distribution with the high density of the radio stations, e.g., with an arbitrarily large number of nodes, in which the cost of the solution returned by the MST heuristic is very close to 4. This is a particularly relevant result since, as noted before, it was a common idea, even supported by experimental results, that the MST heuristic is very close to the optimum for the high-density case (see Wieselthier et al. [2000], Clementi et al. [2003], Klasing et al. [2004], Martnez et al. [2004], Athanassopoulos et al. [2004], and Penna and Ventre [2004]). It is also interesting to note that its construction follows directly from the previous experimental results, which, in fact, suggests investigation of the case of equidistant nodes in order to increase the cost of the computed MST.

More precisely, we consider a high-density uniform distribution in which a huge set S of nodes is equally distributed inside C_1. In particular, nodes are located on the vertices of a grid composed by equilateral triangles, as shown in Figure 5.

Roughly speaking, in order to give a lower bound on the approximation ratio of the MST heuristic in such an instance, the idea is to estimate the cost of the heuristic[2] and to compare it with the optimal solution, whose cost is upper bounded by 1, that is, the square of the radius of C_1. Associating a triangle to each node, roughly one-half of the triangles remain "singles" (the black triangles in Figure 5). Since, for a given side l, the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}l^2$, and considering that, by construction, the number of nodes of the MST is equal to the number of its edges plus 1, $\frac{\sqrt{3}}{4}MST(G_2(S)) \simeq \frac{\pi 1^2}{2}$ and then

$$MST(G_2(S)) \simeq \frac{2\pi}{\sqrt{3}} > 3.62$$

Let us then concentrate on the determination of an upper bound on the approximation ratio of the MST heuristic. By means of an optimal algorithm,

[2]In the case of regular distribution, such as a triangular grid, there always exists an MST composed by a path that visits all the nodes, as in Figure 1. Therefore, the maximal cost of the MST heuristic coincides with the cost of the MST.

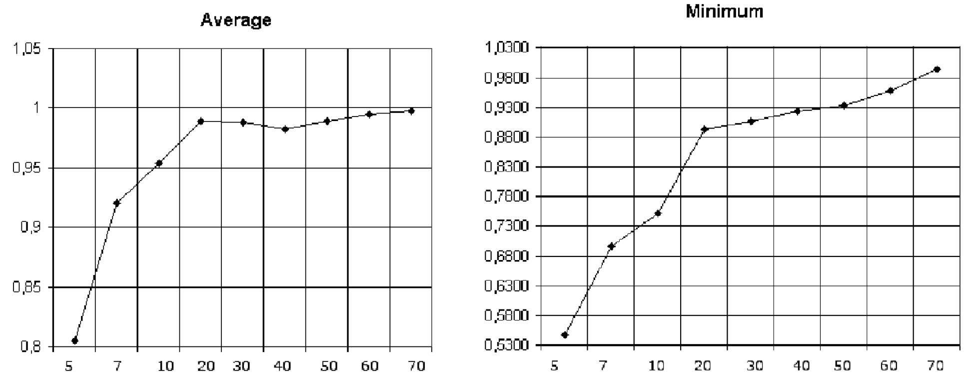

Fig. 6. The average and the minimum costs (y axis) obtained by the linear program for hundreds of instances of 5 up to 70 nodes (x axis).

we have verified that triangular grids up to 61 nodes have an optimal solution coinciding with C_1.[3] Moreover, by means of the experiments described below, we have given evidence that such a property, in general, holds for high-density distributions under the uniform random model. Namely, as the number of the nodes increases, the optimal solution tends to have cost 1, that is, to coincide with C_1.

Intuitively, because of the high density, the optimal solution has to somehow cover all the area of C_1 in order to reach all the nodes. In fact, the regions of C_1 in which there are no nodes inside are infinitesimally small. This suggests that if an optimal solution is composed by more than one circle, such circles must be infinitesimally small as well, since the overlapping of two adjacent circles is, at least, $\frac{1}{3}$ of the area of the smallest one.[4] On the other hand, the MST is actually composed by infinitesimally small circles, but its cost is much larger than 1.

Therefore, in the above distributions, as the number of the nodes increases, an upper bound converging to 4 directly holds by applying the arguments presented in Section 2 as derived by Flammini et al. [2004] as r_{max} tends to 0. This, in particular, shows that the analysis in Flammini et al. [2004], for instance, in which r_{max} tends to 0, is almost tight.

Let us describe in more detail our experiments and their outcome. We have used the integer linear programming formulation presented in Klasing et al. [2004] that computes the optimum and drawn several instances for an incremental number of nodes from 5 up to 70, following the uniform random model inside C_1. We have fixed the center of C_1 as the source and, for each instance, one node on the circumference of C_1 in such a way that the integer solution of cost 1 composed by just C_1 is candidate to be optimal. Because of efficiency considerations, we have run the program by relaxing the integer constraints, obtaining the results summarized in Figure 6.

Clearly, the relaxed optimal solutions provide lower bounds on the respective nonrelaxed ones. In this respect, it is very interesting to note that even for such

[3]Note that the next set S corresponding to the triangular grid distribution has 91 nodes.
[4]This bound is given by the superposition of two circles with the same radius.

relaxed solutions, the average cost of instances composed by more than seven nodes is already larger than, 0.95. Moreover, the minimum cost is already larger than 0.9, starting from instances of 30 nodes. We can consider those results as strong evidence of the fact that, as the number of nodes increases, the optimal (integer) solution is composed by just C_1 or at least that its cost is 1, hence, validating the discussion above.

6. CONCLUSIONS

We closely examined the MST heuristic for the MEBR problem by extensive experiments. The main goal was to find special instances in order to maximize the possible cost of the MST heuristic. Motivated by the gap between the theoretical bounds and the values observed by experimental studies, we proposed a new method to generate input instances, hence, obtaining interesting results. Those experiments, in fact, showed that the usually considered standard random instances are not particularly representative of upper bounding the cost of the MST heuristic. Moreover, they also suggested how to build expensive instances, hence validating the well-known lower bound of 6, performed by the MST heuristic for the MEBR problem and the 4-approximation factor in the high-density case. The experiments, showed that equidistant nodes instances tend to maximize the cost of the MST heuristic. The hexagon shape given in Figure 1 is then a suitable example of an instance with equidistant nodes, in the case of seven nodes. Concerning the 4-approximation factor in the high-density case, thanks to the performed experiments, we were able to build an instance for which the cost of the MST heuristic almost coincides with the upper bound of 4, directly derived from Flammini et al. [2004]. The high cost of such an instance, compared to the usually performed random instances, is interesting on its own, since the common knowledge was of a much better performance of the MST heuristic in the high-density case. Moreover, instances composed by a large number of nodes, uniformly and randomly distributed, can be considered more representative of practical environments and theoretically more interesting. For small numbers of nodes, in fact, exhaustive algorithms can be applied (see, for instance, the integer linear programming formulation proposed in Klasing et al. [2004]. Indeed, the definition of high-density still remains a bit vague and it raises directions for further investigations. Another interesting issue is of providing stronger evidence (experimental and/or theoretical) for which the optimal cost of high-density instances in C_1 is very close to 1.

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