University of Hamburg

Mathematical Modelling Camp

Predicting Wind Turbine Power Output Using Mathematical Models

in favour of energy generation in Germany

Authors :
Rovanos Tsafack Nzanguim
7704600
Jessie Ighogho Ifeanyi
7704546
Saidi Atwaya Nchimbi
7704562

Supervisors:
Gasser, Prof. Dr Ingenuin
Prof. Giuseppe Maria Coclite

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Modelling Camp Group 5

Rovanos Tsafack Nzanguim, Jessie Ighogho Ifeanyi, Saidi Atwaya Nchimbi

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Abstract

Wind power curves are essential for various applications such as wind power forecasting, wind turbine condition monitoring, wind energy potential estimation, and wind turbine selection. However, creating reliable wind power curves from raw wind data is challenging due to the presence of outliers resulting from unexpected conditions like wind curtailment and blade damage. This project proposes an Approximated cubic power curve model for predicting wind turbine power output, a 5PL model for best fitting the manufacturer data incorporates and a Levelized Cost of Energy (LCOE) model to assess the economic feasibility of wind energy projects. The Approximate cubic power curve and 5PL model capture the non-linear relationship between wind speed and power output, the 5PL describes sigmoidal or "S"-shaped curves that best fit a particular dataset, while the cost model considers capital costs, operation and maintenance expenses, and revenue generation. The findings contribute to improved wind power analysis and decision-making processes, enabling stakeholders to make informed choices regarding wind energy investments. By leveraging the 5PL model and cubic power curve model, this study aims to enhance the accuracy and reliability of wind power curve modeling. The findings and insights from this research contribute to the development of improved methods for wind power analysis and decision-making in the field of wind energy.

KeyWords: Wind Power curves, Approximate Cubic Power Curve, 5PLF, Levelized Cost of Energy.

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Notations

Symbols	**Meaning**
v	Wind speed
v_{ci}	Cut-in wind speed of turbine (m/s)
v_{co}	Cut-out wind speed of turbine (m/s)
v_r	rated wind speed of turbine (m/s)
A	Swept area by the rotor blades m^2
ρ	Air density (kgm^{-3})
θ	Pitch angle
$q(v)$	Approximated Cubic Power Curve function
$P(v)$	Power output
P_r	Rated Power output
$C_{P,eq}$	Power coefficient
$C_{P,max}$	maximum Power coefficient
5PLF	Five Parametric Logistic Function
I_0	Investment expenditure (I0) in EUR
A_t	Total Annual cost in EUR per year
$M_{t,el}$	Amount of electricity produced annually in kWh per year
i	Real interest rate in %
n	Economic lifetime in years

Chapter 1

Introduction and Literature Review

Demand for energy is observing constant and steady growth worldwide due to increased industrial and agricultural activities. This indeed is increasing environmental pollution and its ill effects. Thus it is a matter of concern for every developing country to focus more on renewable energy sources.

Wind energy is a valuable renewable resource, and accurately predicting the power output of wind turbines is crucial for the effective planning, operation, and maintenance of wind farms. Mathematical models provide a useful tool for estimating and forecasting the power generation capacity of these turbines, enabling better decision-making in the renewable energy sector. Predicting wind turbine power output is not only essential for day-to-day operations but also for grid integration and stability. Accurate forecasts support the seamless integration of wind energy into the existing power infrastructure, enabling grid operators to manage electricity supply and demand, balance grid operations, and optimize energy trading strategies. Furthermore, power output forecasts aid in developing reliable energy management systems, contributing to a more sustainable and resilient power supply.

Using mathematical models to predict wind turbine power output is crucial for optimizing performance, maintenance, and integration of wind energy into the power grid. Many researchers have worked on developing Mathematical Models to best approximate the Power output of Wind Turbines from the manufacturer's data.

[Habib et al., 1999] proposed that maximum possible power generation from a wind turbine assumes a mechanical to electrical conversion efficiency of 100%. This Model took C_P as 0.593. But Practically, 100% efficiency in mechanical to electrical conversion is impossible to be achieved, thus both of the above factors lead to inaccurate results.

[Yang et al., 2003] in 2003 observed that the power curve of a wind turbine is presumed to follow a typical shape and hence, for different ranges of wind speed between cut-in and cut-out, a set of characteristic equations for power prediction were developed. These models were not very accurate, since the characteristic equations evolved were more general and not specific to any turbine, hence do not replicate the performance of a specific turbine very clearly.

In 2009, [Yang et al., 2009, Thapar et al., 2011]. proposed models based on actual power curves supplied by the manufacturer. It is the mathematical procedure of fitting the best curve for the given set of data points in such a way that the sum of the squares of the offset of the points from the curve is minimum. The approximated fitting cubic spline power curve model which approximates the cubic spline model introduced by [Thapar et al., 2011] was introduced by [Thapar et al., 2011]. This model was found to best fit the Manufacturer's data by producing an overfeeding. [Wang et al., 2023] and [Aldaoudeyeh et al., 2022] proposed a new mathematical method for best fitting the manufacturer's data. This method took into consideration the shape of the power curve and logistic functions (5PLF) were studied and were found to best fit this power output curve.

In this work, we will develop and validate mathematical models that accurately predict wind turbine power output and associated costs. The aim is to estimate the power generation capacity of wind turbines by utilizing historical wind speed data and relevant parameters. By incorporating the cubic power curve model into our prediction framework, we aim to improve the accuracy and reliability of power output forecasts. The project entails analyzing historical wind speed data, deriving the cubic power curve equation, and fitting it to the data to establish the relationship between wind speed and power generation. The validation process will involve comparing the predicted power output with actual measurements obtained from wind turbines. The ultimate goal is to provide stakeholders in the renewable energy sector with a dependable tool for forecasting wind turbine power output. This tool will facilitate optimal planning, operation, and maintenance of wind farms, enabling efficient utilization of wind energy resources. The structure of this work is as follows: Chapter 2 contains some notions which are going to help us for a better understanding of various computations throughout this work. Chapter 3 contains the build-up of the Mathematical Model for the Power Curve and the Cost Model. In this chapter, we also establish the 5-parameter logistic function. In Chapter 4 we perform some numerical simulations, explain the observed output, and give conclusions and recommendations.

Chapter 2

Preliminary Studies

In this chapter, we gather all Analytical, Mathematical and Physical materials and terms which are going to help us through the write-up of our work. In Section 2.1, we talk about the main components of a wind turbine. In section 2.2 we define the variables and parameters used in our power output model. Section 2.3 contains explanations of the various terms used in our Cost Model.

2.1 Modern Wind Turbine Design

In this section, we talk about the build-up of the wind turbine, the main components, and their use. Today the most common design of wind turbines and the type which is the primary focus is that of the **horizontal axis wind turbine (HAWT)**. That is the axis of rotation is parallel to the ground. HAWT rotors are usually classified according to the rotor orientation (upwind or downwind of the tower), hub design (rigid or teetering), rotor control (pitch vs stall) number of blades (usually 2 or 3), and how they are aligned with the wind (free yaw or achieve yaw). The main components of the HAWT and their uses are given below, see [Manwell et al., 2010]

1. **Rotor:** The rotor consists of the hub and blades of the wind turbine. These are often considered to be the turbine's most important components from both a performance and overall cost standpoint. Most turbines today have upwind rotors with three blades. There are some downwind rotors and a few designs with two blades.

2. **Drive Train:** The drive train consists of the other rotating parts of the wind turbine downstream of the rotor. These typically include a low-speed shaft (on the rotor side), a gearbox, and a high-speed shaft (on the generator side). Other drive train components include support bearings, one or more couplings, a brake, and the rotating parts of the generator. The purpose of the gearbox is to speed up the rate of rotation of the rotor from a low value to a rate suitable for driving a standard

generator. Two types of gearboxes are used in wind turbines: parallel shaft and planetary.

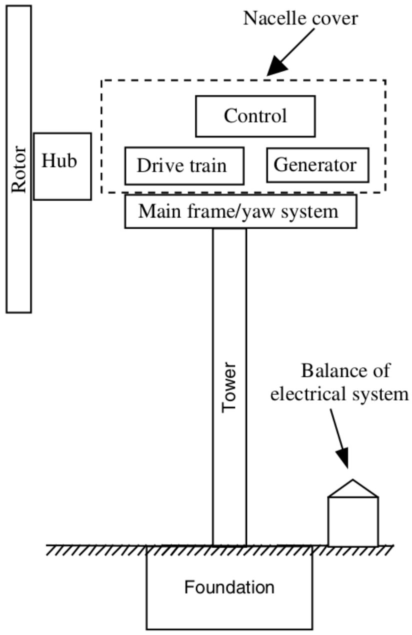

Figure 2.1: Major Components of a Horizontal Axis Wind Turbine

3. **Generator:** Nearly all wind turbines use either induction or synchronous generators. These designs entail a constant or nearly constant rotational speed when the generator is directly connected to a utility network. If the generator is used with power electronic converters, the turbine will be able to operate at variable speeds. An increasingly popular option for utility-scale electrical power generation is the variable-speed wind turbine. There are a number of benefits that such a configuration offers, including the reduction of wear and tear on the wind turbine and the potential operation of the wind turbine at maximum efficiency over a wide range of wind speeds, yielding increased energy capture.

4. **Nacelle and Yaw System:** This category includes the wind turbine housing, the machine bedplate or main frame, and the yaw orientation system. The main frame provides for the mounting and proper alignment of the drive train components. The nacelle cover protects the contents from the weather. A yaw orientation system is required to keep the rotor shaft properly aligned with the wind. Its primary component is a large bearing that connects the main frame to the tower.

5. **Tower and Foundation:** This category includes the tower itself and the supporting foundation. The principal types of tower design currently in use are the free-standing

type using steel tubes, lattice (or truss) towers, and concrete towers. For smaller turbines, guyed towers are also used. Tower height is typically about 1 to 1.5 times the rotor diameter, but in any case, is normally at least $20m$. Tower selection is greatly influenced by the characteristics of the site. The stiffness of the tower is a major factor in wind turbine system dynamics because of the possibility of coupled vibrations between the rotor and the tower.

6. **Controls:** The control system for a wind turbine is important with respect to both machine operation and power production. A wind turbine control system includes the following components: sensors, controllers, power amplifiers, actuators and intelligence (computers, microprocessors)

7. **Balance of Electrical System:** In addition to the generator, the wind turbine system utilizes a number of other electrical components. Some examples are cables, switchgear, transformers, power electronic converters, power factor correction capacitors, yaw and pitch motors.

The principal subsystems of a typical (land-based) horizontal axis wind turbine are shown in 2.1

2.1.1 Wind Farms

A wind farm is also known as a wind power plant or wind energy facility. It is a collection of wind turbines located in a specific area for the purpose of generating electricity from wind energy. Wind farms are designed to capture the kinetic energy of the wind and convert it into electrical energy through the rotation of the turbine blades. Most often when a good wind site has been found it makes sense to install a large number of wind turbines in what is often called a wind *farm or a wind park*. A lot of advantages come with the clustering of wind turbines together at a windy site. Wind farms are more cost-efficient compared to installing individual wind turbines, due to economy of scale. Costs for transportation, planning, construction of the parts, and the power electronics for the connection of wind turbines to the grid become cheaper on a large scale. The negative effect of wind farms is the loss of production due to wake effects see [Gilbert, 2004]. Wind turbines located too close together will result in upwind turbines interfering with the wind received by those located downwind. As we know, the wind is slowed as some of its energy is extracted by a rotor, which reduces the power available to downwind machines. Eventually, however, some distance downwind, the wind speed recovers. Some Theoretical studies of square arrays with uniform, equal spacing illustrate the degradation of performance when wind turbines are too close together. Wind farms are considered a renewable energy source as wind is an abundant and clean resource. They contribute to reducing greenhouse gas emissions and dependence on fossil fuels for electricity generation. The scale of wind farms can vary, ranging from a few turbines to large utility-scale installations consisting of hundreds of turbines.

2.1.2 Power Curve

The power curve is a graphical representation that describes the fluctuation of wind power output with varying wind speeds. It indicates the cut-in speed, rated speed, and cut-out speed of the turbine. The power curve helps visualize the relationship between wind speed and power output.

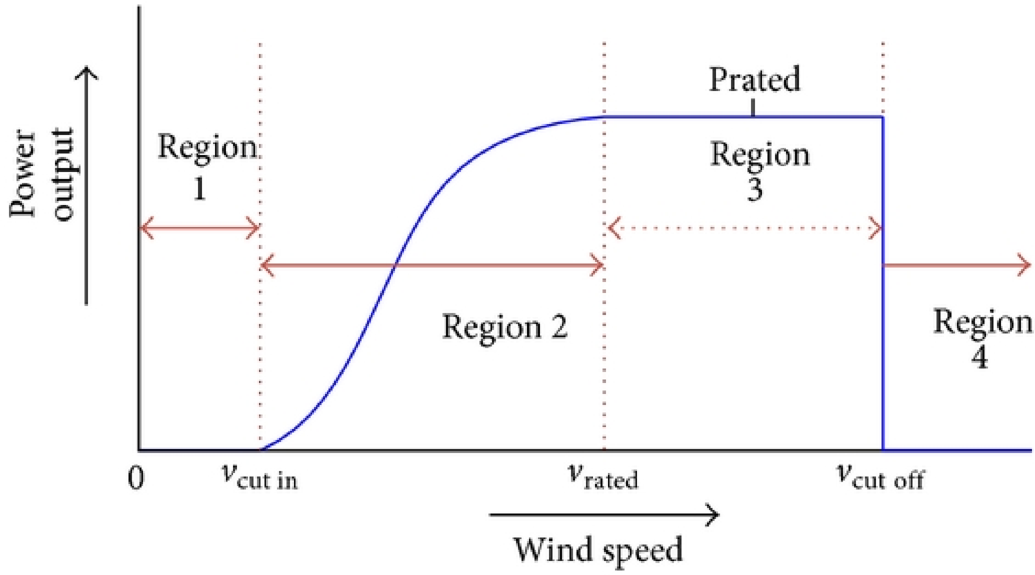

Figure 2.2: Power Curve Description

The above graph from [Sohoni et al., 2016] represents a sample power curve, where the x-axis represents wind speed, and the y-axis represents power output. The curve shows the increase in power output as wind speed rises, reaching a maximum at the rated wind speed. Below the cut-in wind speed, the turbine does not produce power, and above the cut-out wind speed, the turbine shuts down for safety reasons.

The power curve consists of different regions, including:

a) Region 1: Below the cut-in wind speed, the turbine does not produce power. The power output remains at zero or a very low level. The turbine is unable to capture sufficient energy from the wind to generate significant power.

b) Region 2: In this region, the power output of the turbine increases as the wind speed rises. The power output follows a cubic relationship with the wind speed according to the wind power equation. The power output in this region increases rapidly and reaches its maximum at the rated wind speed.

c) Region 3: Once the wind speed exceeds the rated wind speed, the power output of the turbine remains constant at the rated power output. In this region, the turbine operates at its maximum capacity and generates its rated power consistently.

d) Region 4: As the wind speed continues to increase beyond the rated wind speed, the turbine enters the cut-out region. The turbine starts to shut down or limit its power output to ensure safe operation. The power output drops to zero or a reduced level to protect the turbine from excessive stress and damage.

2.2 Variables and factors considered in the Power Output model

When developing a mathematical model for predicting wind turbine power output, several variables and factors should be considered. These include:

Definition 2.2.1. Wind Speed: Wind speed is a crucial variable that directly impacts power output. It represents the velocity at which the wind is blowing and is typically measured at hub height. The relationship between wind speed and power output is nonlinear, with an optimal range where power output increases before reaching a maximum.

Definition 2.2.2. Wind Direction: Wind direction refers to the angle or direction from which the wind is blowing. It affects power output as wind turbines are designed to extract maximum power when the wind is coming from a specific direction relative to the turbine's orientation. Deviations from the optimal wind direction can result in reduced power output.

Definition 2.2.3. Cut-in Wind Speed: The cut-in wind speed is the minimum wind speed required for a wind turbine to start producing useful power. Below this speed, the turbine may not generate significant power. see fig ??

Definition 2.2.4. Rated Wind Speed: The rated wind speed is the wind speed at which the wind turbine achieves its maximum possible power output. The turbine is designed to operate optimally at this wind speed see fig ??.

Definition 2.2.5. Cut-out Wind Speed: The cut-out wind speed is the maximum wind speed at which a wind turbine can safely operate. It takes into account various factors such as technical constraints, safety considerations, and the turbine's design limits, see fig ??.

Definition 2.2.6. Rated Power Output: P_{rated} represents the rated power output of the wind turbine. It is the maximum power output that the turbine is designed to generate at the rated wind speed. The rated power output is typically provided by the turbine manufacturer and is an important parameter for analyzing the performance of the turbine.

Definition 2.2.7. Air Density: Air density is a measure of the mass of air molecules per unit volume. It affects power output as lower air density, which occurs at higher altitudes or in warmer conditions, leads to a decrease in power production. Mathematical models often incorporate air density as a parameter to account for this effect.

Definition 2.2.8. (Wind Power Equation) The wind power equation represents the mechanical equation that describes the power derived from the kinetic energy of the wind.

Definition 2.2.9. Turbine Characteristics: The specific design and characteristics of the wind turbine itself play a crucial role in power output prediction. Parameters such as rotor diameter, hub height, blade pitch, and generator efficiency impact the relationship between wind speed and power output. These parameters are often included in mathematical models to estimate power output accurately.

Definition 2.2.10. Environmental Factors: Various environmental factors can influence wind turbine performance. These include temperature, humidity, turbulence, and terrain conditions. Temperature affects air density, while humidity and turbulence can impact wind flow characteristics. Models may incorporate empirical or analytical approaches to account for these factors and their influence on power output.

Definition 2.2.11. Wake Effects and Turbine Interactions: When multiple wind turbines are located in close proximity, the wake effects and interactions between turbines can impact power output. Turbine wake refers to the disturbed airflow downstream of a turbine, which can reduce the power potential of downstream turbines. Advanced models may consider waking effects and turbine interactions for more accurate predictions in wind farms.

Definition 2.2.12. Rotor Sweep Area of a Wind Turbine: The 'sweep area' of a wind turbine refers to the circular area covered by the rotating blades as they move through the air. It is also commonly referred to as the rotor diameter or rotor sweep. It is an important parameter for wind turbines because it directly affects their power generation potential. A larger sweep area allows the turbine to capture more of the kinetic energy present in the wind, resulting in higher power output.

Example 1 (Swept Area)**.** To calculate the swept area, you need to know the diameter or radius of the rotor. The formula for calculating the swept area (A) of a wind turbine is $A = \pi \times (\text{radius})^2$, For example, if a wind turbine has a rotor diameter of 100 meters, the radius would be half of that value (50 meters). Using the formula, the swept area would be: $A = 3.14159 \times 50^2 = 3.14159 \times 2500 \approx 7853.98$ square meters Therefore, the swept area of the wind turbine in this example is approximately 7853.98 square meters. where $\pi \approx 3.14159$.

By incorporating these elements, the model can provide valuable insights for optimal planning, operation, and maintenance of wind farms.

2.3 Variables and factors considered in the cost model

In addition to predicting wind turbine power output, We also consider the model of costs associated with wind energy generation. Some variables and factors related to costs that should be considered in the model include:

Definition 2.3.1 (Capital Costs)**.** Capital costs include the initial investment required for purchasing and installing wind turbines, including the costs of turbines, foundations, towers, and electrical infrastructure. These costs can vary based on the size and capacity of the wind turbines, project location, and other factors.

Definition 2.3.2 (Operations and Maintenance Costs)**.** Operations and maintenance (O&M) costs encompass the ongoing expenses associated with operating and maintaining wind turbines over their lifespan. This includes costs for regular maintenance, inspections, repairs, spare parts, and personnel. O&M costs can depend on factors such as turbine type, size, and site-specific conditions.

Definition 2.3.3 (Financing costs)**.** Involve the expenses related to obtaining funds for wind energy projects. This includes interest payments on loans, transaction fees, and other financing-related charges. The interest rates and terms of financing can influence the overall project costs.

Definition 2.3.4 (Grid Connection Costs)**.** Grid connection costs cover the expenses associated with connecting wind turbines to the electrical grid. This includes costs for transformers, substations, transmission lines, and any necessary upgrades to the grid infrastructure to accommodate wind energy generation.

Definition 2.3.5 (Land and Permitting Costs)**.** Land acquisition costs and permitting fees are essential factors to consider. The availability and cost of suitable land for wind farm development can vary depending on the location. Permitting costs include expenses associated with obtaining necessary permits, licenses, and environmental impact assessments.

Definition 2.3.6 (Decommissioning and End-of-Life Costs)**.** Wind turbines have a finite lifespan, and decommissioning costs involve the expenses associated with removing and disposing of turbines at the end of their operational life. Proper decommissioning planning and cost estimation are important for ensuring the long-term financial viability of wind energy projects.

Definition 2.3.7 (Annuity)**.** An annuity is a financial product or contract that pays a series of periodic payments to a person or entity over a certain length of time. These payments might be made on a monthly, quarterly, annual, or other basis.

Definition 2.3.8 (Weighted Average Cost of Capital(WACC))**.** WACC is a financial metric used to assess the average cost of financing a company's operations. It takes into account the proportion of debt and equity in a company's capital structure and calculates the weighted average cost of each component. WACC is used as a discount rate in financial analysis to evaluate investment projects or determine the present value of future cash flows. In the context of energy projects, WACC can be employed to evaluate the financial viability of renewable energy installations, such as wind farms or solar power plants.

Definition 2.3.9 (LCOE (Levelized Cost of Energy))**.** LCOE is a financial metric used to assess the cost of producing electricity from a particular energy source over its lifetime. It represents the average cost per unit of electricity generated, taking into account the initial investment, operational and maintenance costs, fuel or resource costs, and the expected energy output over the project's lifespan.

Chapter 3

Model Development and Analysis

This chapter is concerned with the energy conversion system from wind turbines, the Mathematical methods for wind power predictions, and the development of Mathematical and cost models for a yearly investment in the production of wind energy systems.

3.1 Wind Energy Conversion Systems

The following notions on wind conversion systems can be found in [Manwell et al., 2010]. The basic power stages for a wind energy conversion system are shown below, [Thapar et al., 2011]:

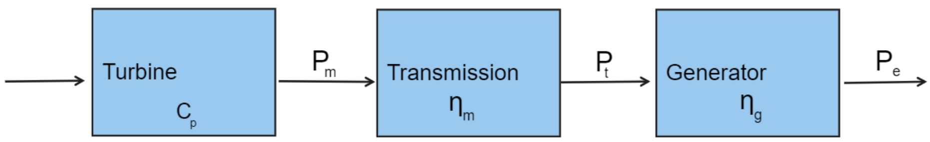

Figure 3.1: Caption

The available wind power in a cross sectional area A perpendicular to wind stream moving at speed $v(m/s)$ having air density ρ is expressed as

$$P_W = \frac{1}{2}\rho A v^3 \quad (W) \tag{3.1}$$

This power is related to power generated by a wind turbine by means of the power coefficient $\frac{P(v)}{P_W(v)}$. where $P(v)$ is the power generated by the wind turbine in W, Cp is the power coefficient that is related to the blade design, the tip angle and the relationship between rotor speed and wind speed. The maximum theoretical value of the power coefficient, known as the **Betz limit**, is $0.593(\approx \frac{16}{27})$.

Proof. If we consider a rotor disc of area A (see fig), one can determine the mass flow of air $\frac{dm}{dt}$ through the rotor disc. From the continuity equation of continuum mechanics,

$$\frac{dm}{dt} = \rho A v$$

The kinetic energy per unit of time, or power flow is given by

$$\begin{aligned} P_W &= \frac{1}{2}\frac{dm}{dt}v^2 \\ &= \frac{1}{2}\rho A v^3 \quad (W) \end{aligned}$$

□

The wind power is converted into mechanical power P_M by the wind turbine which is given by

$$P_M = C_p P_W \tag{3.2}$$

The mechanical power is then supplied to the mechanical transmission with output P_t which is then fed to the electrical generator output

$$P_t = \eta_M P_M \tag{3.3}$$

Thus the generator output is given as;

$$P_e = \eta_g P_t \tag{3.4}$$

Combining (3.2),(3.3) and (3.4), we get:

$$P_e = \eta_0 P_W \;\; \textbf{where} \;\; \eta_0 = (C_p \eta_m \eta_g) \tag{3.5}$$

However, there are factors that influence this power output of the wind turbines: some of which are wind speed distribution of the selected site where the wind turbine is installed, the tower height and power output curve of the chosen wind turbine, [Thapar et al., 2011]. Wind speed varies by minute, hour, day, season and even by year; therefore to accurately predict the energy yield from a wind turbine, wind speed distribution at the site is required. Studies have proven that the "Weibull" probability distribution function $f(v)$ describes most suitably the wind speed distribution. Thus we say that the wind speed v is distributed as Weibull distribution if it has a probability density function;

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-}\left(\frac{v}{c}\right)^{k} \;\; for \;\; k > 0, \;\; v > 0 \;\; c > 0 \tag{3.6}$$

To obtain the occurrence of a particular wind speed in terms of the number of hours in a year, the probability of occurrence can be multiplied by 8760.

3.2 Predicting wind turbine power output using Mathematical models

Predicting wind turbine power output involves modelling the relationship between various factors such as wind speed, wind direction, and turbine characteristics.

According to [Shetty et al., 2020] power output prediction can be classified based on fundamental equations of power available in the wind and models based on the concept of power curve of wind turbines. This was further classified into parametric and non-parametric techniques, [Shokrzadeh et al., 2014]; where the parametric techniques are based on Mathematical models such as the linearized segment model, the polynomial power curve, the probabilistic model, the ideal power curve, the 4-parametric logistic function and 5-parametric logistic function. The non-parametric does not impose any pre-specified model.

Modelling methods in which the actual power curve of an individual wind turbine is used for developing characteristic equations, by utilising various curve fitting techniques, accurately predict the power output of the wind turbine. The suitability of a particular model depends on the shape of the power curve of the selected wind turbine, [Thapar et al., 2011]. One commonly used mathematical model for this purpose is the power curve model. In this work, we will use the cubic power approximation model of the power curve to estimate the production of electricity.

3.2.1 Cubic Power Curve Function For Wind Power Curve

A power curve describes the power production of a wind turbine as a function of the wind speed. Power curves could either be provided by the wind turbine manufacturer or be approximated. The power delivered by a wind turbine is usually represented through its power curve, where a relationship between the wind speed and the power is established. For the variable speed wind turbines, this relationship can be expressed in the following way, [Carrillo et al., 2013]:

$$P(v) = \begin{cases} 0 & v < v_{ci} \quad \text{or} \quad v > v_{co} \\ q(v) & v_{ci} \leq v < v_r \\ P_r & v_r \leq v \leq v_{co} \end{cases} \tag{3.7}$$

where $P(v)$ is the electric power in W, v_{ci} is the cut-in wind speed in m/s, v_{co} is the cut-out wind speed in m/s, v_r is the rated wind speed in m/s, P_r is the rated power in W and $q(v)$ is the non-linear relationship between power and wind speed. The most typical mathematical equations for representing the non-linear part $q(v)$ of a power curve are; Polynomial power curve, Exponential power curve, Cubic power curve and Approximate cubic power curve. In this work we are going to use the approximate power curve which is best fitted for electricity production, see [Carrillo et al., 2013]. The approximation cubic power curve is an approximation of the cubic power curve

$$q(v) = \frac{1}{2}\rho C_{p,eq} v^3 \tag{3.8}$$

where $C_{p,eq}$ is a constant equivalent to the power coefficient. This is obtained by assuming that $C_{p,eq}$ is equal to the maximum value of effective power coefficient $C_{p,max}$. We use the term "effective" to say that mechanical and electrical losses are included in this coefficient. Thus our cubic power curve function becomes

$$q(v) = \frac{1}{2}\rho C_{p,max} v^3 \tag{3.9}$$

Thus using (3.9) in (3.7), we obtain

$$P(v) = \begin{cases} 0 & v < v_{ci} \quad \text{or} \quad v > v_{co} \\ \frac{1}{2}\rho C_{p,max} v^3 & v_{ci} \leq v < v_r \\ P_r & v_r \leq v \leq v_{co} \end{cases} \tag{3.10}$$

3.2.2 Logistic Function For Wind Power Curve

Due to aerodynamics over the surface of wind turbine blades, C_p varies with v near cut in speed (see WTPSC character in[]. The result is thus an acutely concave upwards curve near cut-in speed (region A in Figure 1). However, as v increases, C_p becomes essentially constant which means the relationship between P and v should become cubic (see Equations 2 v) and should start to resemble a cubic relationship. Eventually, as v approaches vr, C_p, starts to decrease (since the angle is increased to maintain the output power at its rated value [6]). The result is thus a concave downward curve near rated wind speed (region C in Figure 1). The overall shape of the Wind Power curve resembles the letter 'S' (Figure 1). Thus, it becomes prevalent to try to model it with logistic functions [12]. In particular, for a good fit of our Mathematical Model, we use the 5PLF for our approximated power curve. Thus our Mathematical model () becomes

$$P(v) = \begin{cases} 0 & v < v_{ci} \quad \text{or} \quad v > v_{co} \\ q(\theta, v) & v_{ci} \leq v < v_r \\ P_r & v_r \leq v \leq v_{co} \end{cases} \tag{3.11}$$

with

$$q(\theta, v) = d + \frac{a - d}{1 + (1 + (v/c)^b)^g} \tag{3.12}$$

, where $\theta = [a, b, c, d, g]$ is a parameter vector. d is the maximum (or upper) asymptote a is the minimum or lower asymptote, c is the inflexion point, b is the hill slope (which controls the rate of rise at point c) and g is the asymmetric try factor. In order to help the STIR algorithm minimize the objective function we need to derive some limits on the parameters of the 5PLF model. We define the main limits on θ as

$$\begin{array}{ll} a_{min} \leq a \leq a_{max} & b_{min} \leq b \leq b_{max} \\ c_{min} \leq c \leq c_{max} & d_{min} \leq d \leq d_{max} \\ g_{min} \leq g \leq g_{max} & \end{array}$$

In order to determine the values for these boundaries, we use the Monotonicity Condition on $q(\theta, v)$ and observe that $c_{min} = 0$, $g_{min} = 0$, $a_{max} = d_{min}$ and $b_{min} = 0$, see [Yang et al., 2023]. Also, using the asymptotes condition of the 5PLF and letting $v \to 0$ we have $a_{min} = 0$ and taking $v \to \infty$ we have d. To estimate d, we allow the value of d to fluctuate between $\pm 10\%$ of P_r, resulting to $d_{min} = 0.9P_r$, $d_max = 1.1P_r$ and $a_{max} = 0.9P_r$. The 3 limits b_{max}, c_{max} and g_{max} which were not specified in our work, we check their ratings from the power output of the Manufacturer's data or actual data being considered. Thus our new equation for $q(\theta, v)$ becomes

$$q(\theta, v) = d_n + \frac{a_n - d_n}{1 + (1 + (v/c_n)^b_n)^g_n} \tag{3.13}$$

3.3 Economics

Wind turbine economics has been changing rapidly as machines have gotten larger and more efficient and are located in sites with better wind. The average rated power of new Danish wind turbines by year of sale shows a steady rise from roughly $50kW$ in the early 1980s to $1200kW$ in 2002 (Denmark accounts for more than half of world sales), see [Gilbert, 2004]. More efficient machines located in better sites with higher hub heights have doubled the average energy productivity from around $600kWh/yr$ per square meter of blade area 20 years ago to around $1200kWh/m^2$ -yr today.

3.3.1 Cost Model

There are several key models and approaches to cost models for wind energy production and prediction that are commonly used. These models help in estimating and predicting the costs associated with wind energy projects. Some of which are: Levelized Cost of Energy (LCOE) Model, Cash Flow Model, Monte Carlo Simulation, Learning curve Models, and Cost Estimation. In this work, we are going to concentrate more on the most widely used and efficient one which is the levelized cost model. The Levelized Cost of Energy (LCOE) model calculates the average cost of generating one unit of electricity over the lifetime of a wind energy project. LCOE can be conducted either based on the net present value or the so called annuity method [Fraunhofer, 2013]. With the net present value method, the expenses for the investment, as well as the payment flows of revenues and expenditures during the power plant's lifetime, are calculated by discounting related to a shared reference date. For this purpose, the *present values of all expenses are divided by the present value of electricity generation.* The underlying idea is that the generated electricity implicitly corresponds to the revenue from the sale of this energy. Thus, the further this income is in the future, the lower the associated present value. The total annual expenditure throughout the entire operating period consists of the investment expenditure and the operating costs, which arise during the lifetime. The total annual costs consist of fixed and variable costs for the operation of the power plant, maintenance, servicing, repairs and insurance payments. The share of debt and equity can be explicitly included in the analysis by the weighted average cost of capital (WACC) over the discount factor (interest rate). The discount factor depends on the amount of the equity, the return on equity over the lifetime, the borrowing costs and the share of the contributed debt the LCOE model with net present value method is given by

$$\mathrm{LCOE} = \frac{I_0 + \sum_{t=1}^{n} \frac{A_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{M_{t,el}}{(1+r)^t}} \tag{3.14}$$

where I_0 represents the investment expenditure in EUR, A_t the annual total cost in EUR per year t, $M_{t,el}$ produced amount of electricity in kWh per year, i real interest rate in %, n the economic lifetime in years and t year of lifetime $(1, 2, \cdots, n)$. Moreover, the total annual costs in the calculation of LCOE is given by:

$$\begin{aligned}\text{Total annual costs} \quad A_t &= \text{fixed operating costs} \\ &+ \text{variable operating costs(+residual value/ disposal of the power plant)}\end{aligned}$$

The calculation of LCOE using the annuity method is a simplification of the NPV method. Here, the LCOE can be defined as the quotient of the annualized investment and operating costs and the average electricity yield. Although the calculation of LCOE based on the annuity methods offers the advantage of a lower calculation effort, but depending on the selected input parameters, significant deviations from the calculation using the NPV can occur. Since the application of the NPV method for the calculation of LCOE best reflects reality. In this work the LCOE were calculated on the basis of the NPV method.

3.4 Case Study

In this section, we perform a case study using two models. We will present a wind value study and a power system study in section 3.4.1 and the application of both the power curve and cost model in section 3.4.2. The wind value study is made on current power systems to analyse the value of produced electricity from wind power, and we will focus the scenarios on future power systems. The study is based on wind values and cost of production data from the city of Hamburg. The wind turbine configurations included have a specific power of $3000kW$

An approximated power curve was used to study the wind power output. We used Texas and Hamburg data to test this output. Moreover, We modelled power curves by modification of the approximate cubic power curve. This was made in order to better match the real (tested on Texas data for the year then onto Hamburg data for the year 2021, see [Montel, 2023]) power curves with respect to the rated speed. The C_p value is set to 0.59. Using the decay formulation of C_p, we studied the power output of the approximated cubic power curve model. An aggregation of power curves is made to account for wind farm and regional effects. Neglecting the internal and external loss the variation within (name of data source) wind data point accounted for by normal distribution of wind speed input data is plotted against the power production output. A scenario of the modelled power curve for Texas and Hamburg data is seen in (Fig 3)

A cost model for the investment and installation of wind farms was developed (see section 3.3). The parameters for this LCOE model were chosen to best fit Hamburg's data for the cost of production for onshore wind turbines (see [Fraunhofer, 2013]). According to the historical data (see section 3.4.2), the investment expenditures in EUR I_0 were considered to be 100000000 Euros, the fixed operating was given as 80000 Euros the variable operating costs, residual value and disposal of the power plant 70000, the annual electricity production $M_{t,el}$ in kWh was equivalent to $1800000kWh$, the real interest in % and with a constant economic lifetime of 0.05%. The LCOE of wind power plants is highly dependent on the local conditions with respect to both onshore and offshore power plants. In general, locations with favourable conditions have an average wind speed of more than 7m/s. A calculator for the annual computation of LCOE was developed and the annual electricity for 4 years was predicted see Table in section 4. Moreover, the average rate of return required by investors, taking into account the cost of debt and the cost of equity. It is used as the discount rate in the annuity method to calculate the present value of cash flows by incorporating the share of debt and equity into the analysis using the weighted average cost of capital (WACC). We consider the cost of debt, the cost of equity, and the respective weights of debt and equity in the capital structure. The WACC is given by

$$WACC = (W_d \times R_d) + (W_e \times R_e) \tag{3.15}$$

Where W_d is the weight of debt in the capital structure (expressed as a decimal), R_d is the cost of debt (interest rate on debt), W_e is the cost of debt (interest rate on debt), and R_e is the cost of equity (required rate of return on equity). Using this in 3.14, LCOE using the WACC and annuity method becomes

$$\text{LCOE} = \frac{I_0 + \sum_{t=1}^{n} \frac{A_t}{(1+WACC)^t}}{\sum_{t=1}^{n} \frac{M_{t,el}}{(1+WACC)^t}} \tag{3.16}$$

Chapter 4

Numerical Results and Conclusion

This chapter is concerned with the numerical findings and interpretation of Wind energy production in terms of power production and the Power curve from which a prediction over the year 2030 was made, see section 4.1. In Section 4.2 we give a brief summary output of the LCOE for establishing a wind farm and finally, section 4.3 consists of recommendations, limitations and future studies.

4.1 Historical Data Description

In this work, we used 3 data sets. German Manufacturers' wind turbines (Siemens and Nordex Manufacturers) [Openenergy, 2023], Hamburg [Montel, 2023], and Turkey Scada data [Kaggle, 2021] for our wind power curve modelling. Siemens is a multinational conglomerate based in Germany that operates in various sectors, including energy, healthcare, transportation, and industrial automation.

We filtered Siemens and Nordex Manufacturers' data from [Openenergy, 2023]. Siemens is a multinational conglomerate based in Germany that operates in various sectors, including energy, healthcare, transportation, and industrial automation. In the wind power industry, Siemens is known for its high-quality wind turbines and comprehensive solutions for renewable energy projects. They offer a wide range of onshore and offshore wind turbines with different capacities and rotor diameters to suit various wind conditions and project requirements. Siemens wind turbines are known for their advanced technology, reliability, and performance.

Nordex is a German manufacturer specializing in the production of wind turbines. With several decades of experience, Nordex has established itself as a leading player in the global wind energy market. Nordex offers a range of onshore wind turbines designed for different wind conditions and power requirements. Their turbines are known for their efficiency, robustness, and high performance. Nordex focuses on delivering sustainable and reliable solutions for renewable energy projects worldwide.

Both Siemens and Nordex have made significant contributions to the wind power industry and have a strong track record in delivering innovative and reliable wind turbines. They continue to play a vital role in advancing the adoption of renewable energy and promoting a sustainable future. They offer a wide range of onshore and offshore wind turbines with different capacities and rotor diameters to suit various wind conditions and project requirements. (See attached code for the corresponding tidied and cleaned data).

The second dataset contains Hamburg wind speed data for 2022-2023 merged with the private data set from captured hourly readings between 01.01.2022 and 16.06.2023 by [Montel, 2023] in Hamburg. Each data value belongs only to the relevant time period and the input variables transmitted in the data set for the time period. We extracted the following relevant columns for an Onshore wind farm.

The Turkey wind turbine's scada data set consists of real-time SCADA data. Each data value belongs only to the relevant time period and the input variables transmitted in the data set for the time period to be predicted are prepared to be used to predict the power generation result in the same time period. In the shared dataset, the real-time power generation amount (Power(kW)) of a wind turbine between 01.01.2019 and 14.08.2021 is given on a 10-minute basis.

4.2 Wind Power Curve

The cubic spline Mathematical Model was used to model the wind power output for a wind farm of 10 turbines. In order to check the validity of this model, a plot of fig (3.7) for one turbine was compared to one turbine of the manufacturer's data (see fig). There is little or no variance between the cubic power curve with and without C_p decay. Though it seems $C_{p,max}$ decay gives a better fit than the approximate cubic spline as shown in Fig 4.1. In contrast to this, in order to optimize our model, a 5PLF was used for the cut-in and rated wind speed. We observed that the 5PLF method produces a better fit than the approximated cubic spline see Fig 4.1.

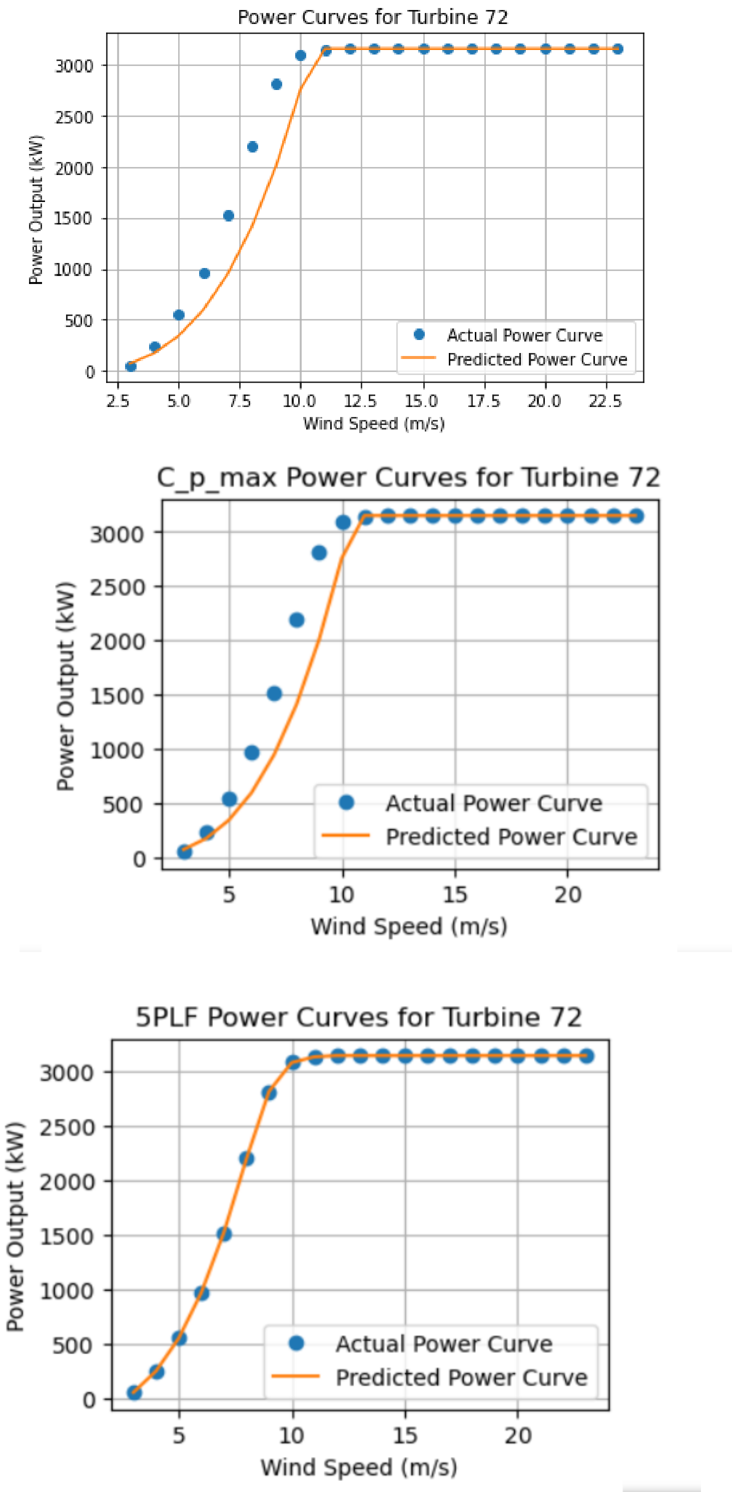

Figure 4.1: Power Curve simulation of the approximated cubic spline, the $C_{p,max}$ decay and the 5PLF Models for Wind Turbine 72 with respect to Manufacturers' data.

Moreover, considering a wind farm with respect to Turkey data, we observed that the 5PLF gives a better fit to the manufacturer data scatter plot as compared to the approximate cubic spline. This can be seen numerically in table 1 where we have a best approximation with a lower root mean square error (RSE) with the 5PLF (158.23845202712334) as compared to the approximate cubic spline (241.80473908617262) for different wind speeds recorded in time intervals of 10 minutes. This is shown in Fig 4.3.

Comparing the power curve for both models, we see that the distribution plot of the 5PL model output is similar to the actual output compared the that of the Cubic Power Output. The curve of the cubic power output in region 2 (between v cut in and v rated) does a poor prediction. This is seen to spread the distribution of the cubic power prediction output far from the actual Power (LVActivePower(KW)) Output.

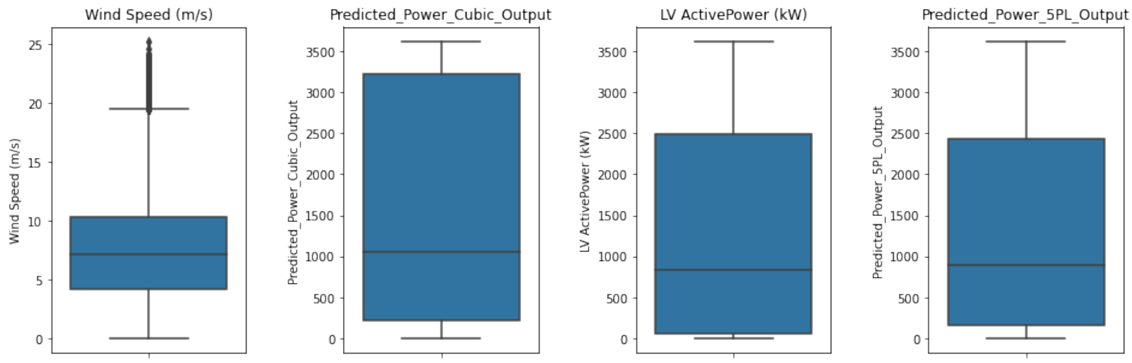

Figure 4.2: Comparing the distribution of the Power Output of Cubic and 5PL models to actual data

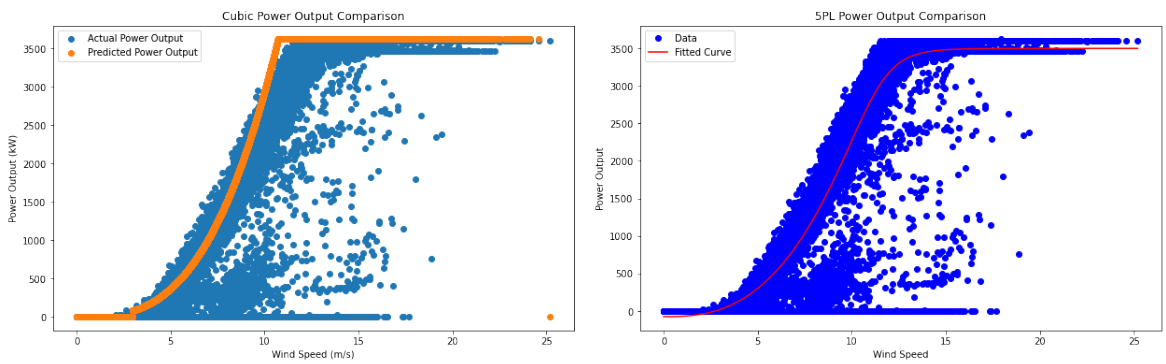

Figure 4.3: Power Curve simulation of the approximated cubic spline and the 5PLF for Turkey Manufacturer data.

Similarly considering a wind farm with respect to Hamburg data, we observed that the 5PLF gives a better fit to the manufacturer data scatter plot as compared to the approximate cubic spline. This can be seen numerically in table 2 where we have a best approximation with a lower root mean square error (RSE) with the 5PLF (70.42320122447751) as compared to the approximate cubic spline (168.9273722176038) for different wind speeds recorded in time intervals of 10 minutes as seen in Fig 4.4

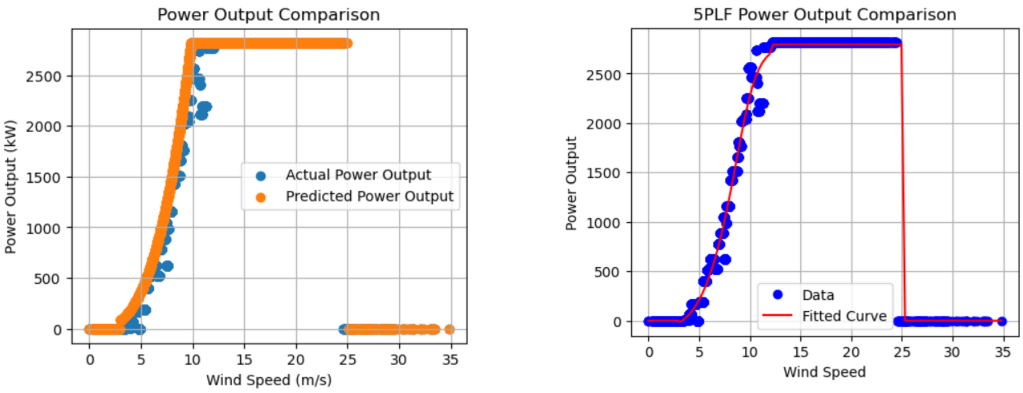

Figure 4.4: Power Curve simulation of the approximated cubic spline and the 5PLF for Hamburg Manufacturer data.

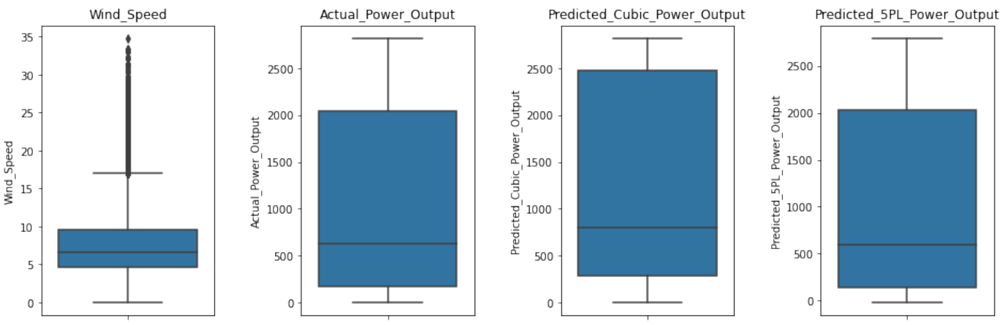

Figure 4.5: Comparing the distribution of the Power Output of Cubic and 5PL models to actual data

Furthermore, upon predicting the Power output for our wind farm with wind speed respecting the Weibull distribution over a period of $300hrs$, we find out the 5PLF gives us a more accurate approximation to the curve as compared to the Cubic Spline which overfits (see Fig 4.6).

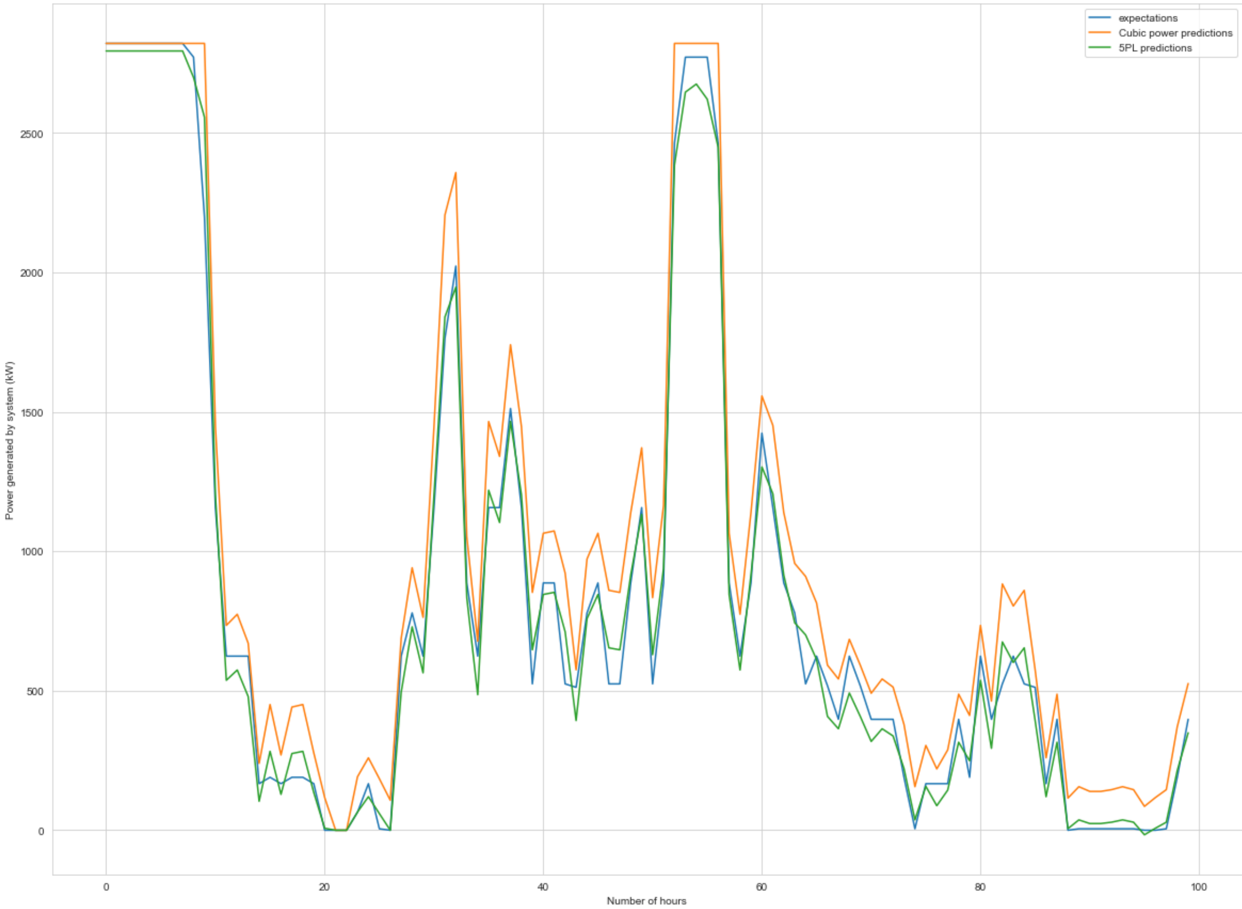

Figure 4.6: Prediction of the Power Output for Hamburg data over a period of 300hrs

4.3 Levelized cost of Electricity

Amongst all the renewable sources of energy, wind power has been demonstrating high competitiveness against conventional power generation for the longest time and its global market penetration is correspondingly strong see [Fraunhofer, 2013].

A calculator to predict the LCOE per unit output of onshore WPP generates an LCOE of 30.019 Euros per kWh with an investment expenditure of 100000000EUR, a project lifetime of 4yrs, an annual total operations and maintenance cost in years 1, 2 and 3 respectively of 50000EUR, 60000EUR and 70000EUR, and electricity yield for these years (respectively) of 1000000kWh, 1200000kWh, and 1500000kWh, and an interest rate of 0.05%.

Using the WACC to calculate the LCOE could result in underestimating or overestimating the project's viability, depending on the project's risk profile. If the project has a higher risk than the company's average operations, using the WACC could lead to underestimating the LCOE and potentially make the project appear more favourable than it actually is. Conversely, if the project has a lower risk, using the WACC could overestimate the LCOE and make the project seem less attractive.

The advantage of using the WACC method alongside the annuity method is that management can then decide the best way to finance the project that is more profitable to the company by comparing the expected cost using both methods and playing around with the capital structure.

Just as a sample, using 3 years investment portfolio, we compared the effect of the capital structure on the LCOE using an equity rate of 0.06 with a 0.6 weight on the capital structure. we see that the cost of producing 1 unit of electricity is higher for the WACC, so this capital structure plan is not a very good idea.

```
Enter the investment expenditure in EUR: 10000000
Enter the number of years for the project's lifetime (n): 3
Enter the annual total operations and maintenance cost in year 1 in EUR: 100000
Enter the electricity yield in year 1 in kWh per year: 130000
Enter the annual total operations and maintenance cost in year 2 in EUR: 120000
Enter the electricity yield in year 2 in kWh per year: 160000
Enter the annual total operations and maintenance cost in year 3 in EUR: 150000
Enter the electricity yield in year 3 in kWh per year: 200000
Enter the discount rate or interest rate (r) as a decimal: 0.05
LCOE using annuity method: 23.395100963183886
```

Figure 4.7: Prediction of the LCOE using annuity method

```
Enter the investment expenditure (I0) in EUR: 10000000
Enter the number of years for the project's lifetime (n): 3
Enter the annual total cost in year 1 in EUR: 100000
Enter the electricity yield in year 1 in kWh per year: 130000
Enter the annual total cost in year 2 in EUR: 120000
Enter the electricity yield in year 2 in kWh per year: 160000
Enter the annual total cost in year 3 in EUR: 150000
Enter the electricity yield in year 3 in kWh per year: 200000
Enter the weight of debt in the capital structure (Wd) as a decimal: 0.4
Enter the cost of debt (Rd) as a decimal: 0.05
Enter the weight of equity in the capital structure (We) as a decimal: 0.6
Enter the cost of equity (Re) as a decimal: 0.06
LCOE using annuity method and WACC: 23.668835584026148
```

Figure 4.8: Prediction of the LCOE using WACC method

In summary, while the regular annuity method's required rate of return reflects the specific risks of a project, using the WACC to calculate the LCOE may not adequately capture the project's unique risk profile. It is generally more appropriate to use the required rate of return specific to the project being evaluated when calculating the LCOE. WACC just help with financing structure plan for increasing profitability.

4.4 Conclusion

In conclusion, our study involved analyzing data using two mathematical models: the Cubic Power curve model and the 5 PL model, to investigate the relationship between wind speed and power output in wind turbines. Firstly, the Cubic Power curve model assumes a cubic relationship between wind speed and power output. It provided reasonably accurate estimates for power output, especially in the vicinity of the rated wind speed.
However, this model had limitations as it couldn't accurately capture the non-linear behaviour of power output at low and high wind speeds. To overcome these limitations, we also made use of the Model- 5PL.

It is based on a logistic function and proved to be more effective in representing the characteristics of power output over a wider range of wind speeds. By taking into account parameters like minimum and maximum asymptotes, inflexion point, hill slope, and asymmetry factor, this model provided a more holistic explanation of the power curve. The results from our analysis indicated that the Model-5PL is closely aligned with the actual power curve that is based on the actual data. This led to more accurate predictions for various wind speeds. The flexibility of Model-4PL made the model applicable to accounting for variations in power production, and accurately depicting how power output behaved - concave upward at low wind speeds and concave downward at higher wind speeds. Our study shows that careful consideration must be given to selecting an appropriate mathematical model for highly precise predictions of power output from wind turbines. In this regard, the 5PL model, with its incorporation of an S-shaped curve for power output, offers great value in wind power prediction and analysis.

The overall aim of this work was to derive a mathematical model for wind power production, find a way to optimize this model, give an idea of the cost of establishing a wind farm and give recommendations for decision-makers (management). The result shows that

- The approximated cubic spline Model gives a good wind power output prediction.
- This Model is best fitted by the use of 5 parametric logistic function which even gives a better wind power approximation over a period of time.
- Establishing a wind farm in Germany (mainly on the coastal locations) is relatively affordable and produces high power output.

- Wind Power can play a major role in future power systems with low carbon emissions.

Overall, our data analysis using the Cubic Power curve model and the 5PL model yielded useful insights into the relationship between wind speed and power output in wind turbines. These models help to improve knowledge and prediction of wind energy output, ultimately assisting in the construction and optimization of wind energy systems for long-term and efficient power generation.

Recommendations

Due to the high demand for Wind power production, the establishment of more wind farms in the coastal areas in Germany is highly encouraged.

Limitations

Our Work did not take into consideration wind directions which can also influence wind power production. Moreover, our models did not take into consideration seasonal effects on wind speed.

Future Studies

During this work, several interesting topics for further studies have appeared. Some has to do with the optimization of the cubic spline power curve using optimization algorithms such as Genetics algorithms or 6 parametric logistic function In fact a study on GA algorithm, 4PLF, 5PLF, 6PLF and higher order interpolations could be made in order to observe which model gives rise to a best power curve fit with minimum error. Also, a study of the coefficient of performance for wind turbines with low specific power, done with build dynamics and structural simulations, would be very interesting and needed in order to verify and improve the power curve model used in this report.

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Appendix A

The C_P Decay Factor and Power Output Tables

A description of the correction of C_P decay for a better approximation of the cubic power curve is given in this section. Furthermore, various tables comparing the power output of the approximate cubic spline, the approximate cubic spline with the correction of C_P decay and the 5PLF are established in this section.

The C_P Decay Factor The performance of the HAWT is overall seen to drop by $1.6 \pm 0.2\%$ according to [Iain Staffell, 2014]. This drop in performance can be associated with the decay of the C_P value annually in the power equation as a function of time. I.e., $q(v,t) = \frac{1}{2}C_P(t)\rho v^3$. Considering the maximum drop in performance of 1.8%, we have $q(v,t) = \frac{q(v,t)}{(1-0.018)}$ that is $q(v,t+1) = (0.982)q(v,t)$. Thus in terms of $C_P(t)$, at $t_0 = 0yrs$ we have $C_P(t_0) = C_{P,max}$. So for n years, we will have

$$\begin{aligned} C_P(t+1) &= (0.982)C_P(t) \\ C_P(t+1) &= (0.982)^2 C_P(t) \\ &\vdots \\ C_P(t+n) &= (0.982)^n C_P(t) \\ C_P(t_0+n) &= (0.982)^n C_P(t_0) \end{aligned}$$

Let T be the number of years elapsed from such that $T = t_0 + n = n$ and let $d = 0.018$ thus (the value of d will vary with each turbine and can be derived from observing manufacturer's data for different time spans)

$$C_P(d,T) = (1-d)^T C_{P,max} \tag{1}$$

Taking into account that in this work we consider C_P decay in less than a year, we can always find the corresponding decay of C_P in months, days, or hours in a year. We will thus have

$$C_P(d,T,\tau) = C_{P,max}\left(1 - \frac{d}{\tau}\right)^{T\tau} \tag{2}$$

where τ is the number of months or hours or days etc per year and d/τ is the decay rate of performance per month or per day etc. respectively of the choice of τ.
Furthermore, we can improve the approximation of the Decaying C_P model, by introducing a data-dependent correction factor K. This factor can simply be computed as the average of all ratios of the Actual power output to the Decaying C_P Power Output. The K Corrected Decaying C_P model now assumes the form below.

$$C_P(d,T,\tau,K) = K * C_{P,max}\left(1 - \frac{d}{\tau}\right)^{T\tau} \tag{3}$$

Appendix B

Power output Tables for Texas and Hamburg Data

Time	Wind Speed	Actual Power Output	Predicted Cubic Power Output	Predicted 5PL Power Output
2022−08−3102 : 00 : 00	4.10	64.50	203.32	74.48
2022−11−2201 : 00 : 00	9.61	2038.90	2606.00	2101.66
2023−02−0109 : 00 : 00	11.21	2198.09	2820.47	2605.47
2023−03−1620 : 00 : 00	8.16	1423.41	1598.45	1339.95
2022−10−0615 : 00 : 00	7.99	1156.62	1501.50	1252.09
2022−06−0922 : 00 : 00	5.01	189.79	370.64	213.64
2023−02−1208 : 00 : 00	4.38	166.90	246.97	109.89
2022−02−2517 : 00 : 00	13.45	2820.47	2820.47	2793.32
2022−03−1200 : 00 : 00	3.42	0.00	117.12	7.19
2023−04−1310 : 00 : 00	246.16	2820.47	2820.47	2793.32

Table 1: Comparison between Cubic, C_P decay and 5PL Power Output for Texas Data

Time	Wind Speed	Actual Power Output	Predicted Cubic Power Output	Predicted 5PL Power Output
$2023-06-1511:00:00$	3.98	5.22	184.84	59.73
$2023-06-1512:00:00$	6.29	624.07	730.64	534.32
$2023-06-1513:00:00$	5.24	189.79	423.41	259.15
$2023-06-1514:00:00$	4.38	166.90	246.97	109.89
$2023-06-1515:00:00$	5.51	397.65	491.00	318.32
$2023-06-1516:00:00$	6.92	779.03	976.24	762.26
$2023-06-1517:00:00$	7.59	624.72	1287.64	1054.23
$2023-06-1518:00:00$	8.31	1511.74	1687.90	1419.58
$2023-06-1519:00:00$	8.67	1511.74	1916.01	1614.55
$2023-06-1520:00:00$	9.23	2022.32	2309.95	1914.26
$2023-06-1521:00:00$	6.79	524.61	921.36	710.96
$2023-06-1522:00:00$	6.30	624.07	734.24	537.62
$2023-06-1523:00:00$	6.79	524.61	921.36	710.96
$2023-06-1600:00:00$	6.29	624.07	730.64	534.32
$2023-06-1601:00:00$	5.32	189.79	441.51	274.90

Table 2: Comparison between Cubic, C_P decay and 5PL Power Output for Hamburg Data