Numerical Differentiation

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In this module, we will learn how numerical differentiation works and how you can solve them computationally using Python and compare each results. This module will cover:

- Forward Finite Derivatives
- Central Fininte Derivatives
- Backward Fininte Derivatives
- Taylor Series Expansion

```
import numpy as np
import math
import sympy as sy
from pandas import DataFrame
from sympy.functions import sin,cos
import matplotlib.pyplot as plt
from scipy.special import binom
from scipy.misc import derivative
plt.style.use("seaborn-poster")
```

```
## Creating Pascal Triangle
binom_coeffs = lambda n : np.asarray([binom(n,k) for k in range(n+1)])
x = 0.1
dx = 0.01
y1 = lambda x :( ( (4*x**2+2*x+1) / (x+2*np.exp(x)) )**x )
y2 = lambda x : (np.cos(2*x)) + (x**2)/20 + np.exp(-2*x)
```

```
def diff_cen(f,x,dx,degree=1):
    if degree % 2 == 0:
        f_ans = f(x+((np.arange(np.ceil(degree/2),-(np.ceil(degree/2)+1),-1))*dx))
        bin = binom_coeffs(degree)
        bin[1::2] *= -1
        diff = (bin @ f_ans) / (dx**degree)
    elif degree == 3:
        array = np.arange(np.ceil(degree/2),-(np.ceil(degree/2)+1),-1)
        f_ans = f(x+((np.delete(array,len(array)//2))*dx))
        bin = np.array([1,2,2,1])
        bin[1::2] *= -1
        diff = (bin @ f_ans) / (2*dx**degree)
    else:
        array = np.arange(np.ceil(degree/2),-(np.ceil(degree/2)+1),-1)
        f_ans = f(x+((np.delete(array,len(array)//2))*dx))
        bin = binom_coeffs(degree)
        bin[1::2] *= -1
```

```
        diff = (bin @ f_ans) / (2*dx**degree)
    return diff
```

```
def diff_bwd(f,x,dx,degree=1):
    f_ans = f(x+((np.arange(0,-(degree+1),-1))*dx))
    bin = binom_coeffs(degree)
    bin[1::2] *= -1
    diff = (bin @ f_ans) / (dx**degree)
    return diff
```

```
def diff_fwd(f,x,dx,degree=1):
    f_ans = f(x+((np.arange(degree,-1,-1))*dx))
    bin = binom_coeffs(degree)
    bin[1::2] *= -1
    diff = (bin @ f_ans) / (dx**degree)
    return diff
```

Y1

```
degree = 3
for n in range(0,degree+1):
    print(diff_fwd(y1,x,dx,n))
```

```
0.9396682312481898
-0.49250950821845274
2.129163586548355
0.32210589673287154
```

```
degree = 3
for n in range(0,degree+1):
    print(diff_bwd(y1,x,dx,n))
```

```
0.9396682312481898
-0.5137063367627537
2.103159425123069
2.44101012469855
```

```
degree = 3
for n in range(0,degree+1):
    print(diff_cen(y1,x,dx,n))
```

```
0.9396682312481898
-0.5031079224906032
2.119682854430094
1.300208071264297
```

Y2

```
degree = 3
for n in range(0,degree+1):
    print(diff_fwd(y2,x,dx,n))
```

```
1.7992973309192233
-2.0276083626139307
-0.5932774836159282
-4.532857641237341
```

```
degree = 3
for n in range(0,degree+1):
    print(diff_bwd(y2,x,dx,n))
```

```
1.7992973309192233
-2.022157328017027
-0.4940513772555555
-5.396264885604295
```

```
degree = 3
for n in range(0,degree+1):
    print(diff_cen(y2,x,dx,n))
```

```
1.7992973309192233
-2.024882845315479
-0.5451034596903881
-4.9613053180186375
```

Checking Error

Y1 TABLE

```
def error_fwd(f,n):
    return abs(derivative(y1,x,dx,n=n,order=5)-diff_fwd(y1,x,dx,n))
def error_bwd(f,n):
    return abs(derivative(y1,x,dx,n=n,order=5)-diff_bwd(y1,x,dx,n))
def error_cen(f,n):
    return abs(derivative(y1,x,dx,n=n,order=5)-diff_cen(y1,x,dx,n))

dy1 = np.array([[diff_fwd(y1,x,dx,1),diff_bwd(y1,x,dx,1),diff_cen(y1,x,dx,1)],
                [diff_fwd(y1,x,dx,2),diff_bwd(y1,x,dx,2),diff_cen(y1,x,dx,2)],
                [diff_fwd(y1,x,dx,3),diff_bwd(y1,x,dx,3),diff_cen(y1,x,dx,3)]
               ]) ## y1 differentiation result
```

```
              ]) ## y1 differentiation result

dy1_error = np.array([[error_fwd(y1,1),error_bwd(y1,1),error_cen(y1,1)],
                      [error_fwd(y1,2),error_bwd(y1,2),error_cen(y1,2)],
                      [error_fwd(y1,3),error_bwd(y1,3),error_cen(y1,3)]
                    ]) ## y1 error

dy2 = np.array([[diff_fwd(y2,x,dx,1),diff_bwd(y2,x,dx,1),diff_cen(y2,x,dx,1)],
                [diff_fwd(y2,x,dx,2),diff_bwd(y2,x,dx,2),diff_cen(y2,x,dx,2)],
                [diff_fwd(y2,x,dx,3),diff_bwd(y2,x,dx,3),diff_cen(y2,x,dx,3)]
               ]) ## y2 differentiation result

dy2_error = np.array([[error_fwd(y2,1),error_bwd(y2,1),error_cen(y2,1)],
                      [error_fwd(y2,2),error_bwd(y2,2),error_cen(y2,2)],
                      [error_fwd(y2,3),error_bwd(y2,3),error_cen(y2,3)]
                      ]) ## y2 error

dfy1 = DataFrame(data= dy1,columns= ['Forward finite','Backward finite', 'Central finite'],in
dfy1
```

	Forward finite	Backward finite	Central finite
1	-0.492510	-0.513706	-0.503108
2	2.129164	2.103159	2.119683
3	0.322106	2.441010	1.300208

```
dfy1_error = DataFrame(data= dy1_error,columns= ['FWD Error','BWD Error', 'CEN Error'],index=
dfy1_error
```

	FWD Error	BWD Error	CEN Error
1	0.010620	0.010577	0.000022
2	0.008894	0.017110	0.000587
3	0.978102	1.140802	0.000000

Y2 TABLE

```
dfy2 = DataFrame(data= dy2,columns= ['Forward finite','Backward finite', 'Central finite'],in
dfy2
```

Forward finite Backward finite Central finite

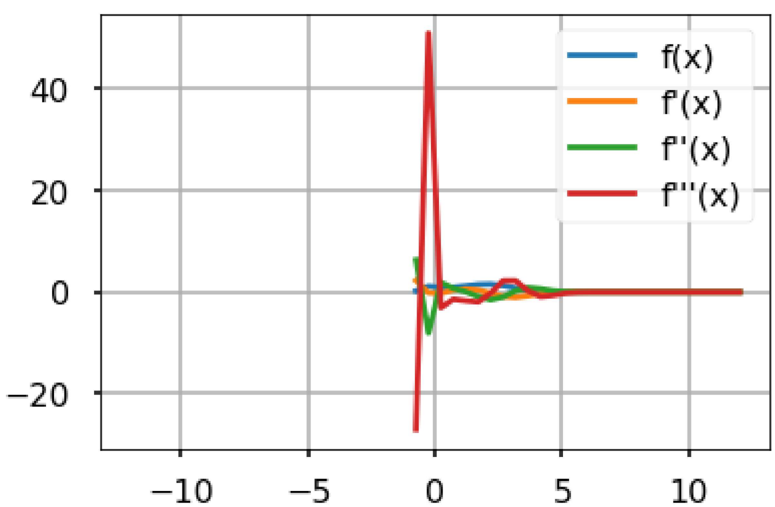
```
dfy2_error = DataFrame(data= dy2_error,columns= ['FWD Error','BWD Error', 'CEN Error'],index=
dfy2_error
```

	FWD Error	BWD Error	CEN Error
1	0.010620	0.010577	0.000022
2	0.008894	0.017110	0.000587
3	0.978102	1.140802	0.000000

Plotting

```
X = np.linspace(-12,12)
plt.plot(X,y1(X), label='f(x)')
plt.plot(X,derivative(y1,X,dx), label='f\'(x)')
plt.plot(X,derivative(y1,X,dx,n=2), label='f\'\'(x)')
plt.plot(X,derivative(y1,X,dx,n=3,order=5), label='f\'\'\'(x)')
plt.legend()
plt.grid()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning: in
  """
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning: in
  """
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning: in
  """
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning: in
  """
```



Taylor Series Expansion

```
XX = np.linspace(5,7)
a = 2*np.pi
n = 7
def taylor(f,a,n,x):
    y=0
    for i in range(n+1):
        y += ( ( (derivative(f,a,dx,n=i,order=9)) * ((x-a)**i) ) / np.math.factorial(i)  )
    return y
```

Y1 using Taylor series Expansion

```
# plt.axvline(x = 2*np.pi, c = "k")
plt.plot(XX,taylor(y1,a,n,XX),label='taylor series')
plt.plot(XX,y1(XX),label='Analytic',c="k")
plt.legend()
plt.grid()
plt.show()
```

Y2 using Taylor series Expansion

```
# plt.axvline(x = 2*np.pi, c = "k")
plt.plot(XX,taylor(y2,a,n,XX),label='taylor series')
plt.plot(XX,y2(XX),label='Analytic',c="k")
plt.grid()
plt.show()
```