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ALGORITHM FOR PID CONTROLLER TUNING USING LQG COST MINIMIZATION

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ABSTRACT

There are more three-term controllers in industry than any other type of controller and a long-standing practical solution to he tuning problem has been the development of empirical or automated procedures. In this search for simplicity the optimisation of time domain cost functions to generate tuning rules has been well explored. However, there have been very few extensions of this procedure to an optimal PID trade-off from a formal time domain or frequency domain LQG or LQ cost minimisation problem. Such a link is established in the following to provide the basis of a new practical tuning algorithm for either on-line or off line use.

An LQG cost minimisation problem is defined and the PID controller coefficients which minimise the index are required. The advantages of the approach are that direct cost function optimisation is performed rather than simply matching a PID controller to an LQG solution. This enables the LQG underlying problem to be used more directly for the design process. A key idea is to choose the LQG cost function specification allowing for the constraints of the PID structure. The result is a PID design approach which is very closely integrated with the underlying optimisation problem. In the paper the algorithm is specified and some potential applications are explored.

1. INTRODUCTION

The relay based PID tuning experiment of Astrom and Hagglund (1985) ushered in a new generation of process controllers having an AUTOTUNE facility. However, recent evidence seems to indicate that process control engineers still have difficulties tuning PID controllers (Ender, 1993). The seminal contribution of Ziegler and Nichols (1942) initiated several different PID tuning rules. Surveying the literature of PID rules (now spanning over 56 years) to produce a comparative assessment is not an easy task. In the process industries there are three classes of methods.

1.1 *Time Response Tuning*

The starting point for this class of methods was work of Ziegler and Nichols (1942). The non-parametric data obtained is the ultimate gain and ultimate period, K_u, P_u respectively. The sustained oscillation method was upgraded by Astrom (1982) using the relay experiment and who gave an interpretation of the tuning rules as a re-positioning of one point of the frequency response. But the main thrust of the PID rule philosophy has been tuning to achieve an acceptable closed loop system time response. Modified Ziegler-Nichols rules permitting some overshoot or no overshoot have been published and in the early 1990's, the Z-N rules were revisited to produce a set of Refined Ziegler-Nichols rules (Hang *et al*, 1991).

1.2 *Time Domain Optimisation Methods*

The concept of choosing PID controller coefficients to minimise an integral cost functional has a long pedigree for model based methods.The contribution of Zhuang and Atherton (1993) used an integral criterion with data from a sustained oscillation or a relay experiment. The time weighted system error integral criterion was given as:

$$J_n(\theta)=\int_0^\infty \{ t^n e(\theta,t) \}^2 dt$$

where θ is a vector of (controller) parameters chosen to minimise the criterion and $e(\theta,t)$ is a system error signal. After experimentation Zhuang and Atherton decided to set $n = 1$ and the ISTE criterion resulted. Other contributions include those due to Pessen (1994) who used the Integral Absolute Error (IAE) criterion:

$$J_{IAE}(\theta)=\int_0^\infty |e(\theta,t)| dt$$

1.3 *Frequency Domain Shaping*

Astrom (1982) established the idea of using a set of rules to achieve a desired phase margin specification. More recent papers have pursued this idea in detail (Ho *et al*, 1997). Voda and Landau (1995) applied the Kessler symmetric optimum principle (Kessler, 1958) to shape the compensated system frequency response.

1.4 *A New Tuning Approach*

Today the framework for the controller is better known and must incorporate several control system performance objectives: reference tracking, rejection of supply disturbances, load disturbances and measurement noise. These criteria can readily be incorporated into a LQG optimal control and the problem is then how to minimise an LQG cost-function where the controller structure is fixed for example to a particular PID industrial form. The strategy followed is to minimise an LQG criterion in such a way that the controller is of the desired form and is causal. A simple analytic solution cannot be obtained, as in the case where the controller structure is unconstrained (*Kucera* 1979). However, a direct optimisation problem can be established which provides the desired solution. An obvious and necessary assumption is that, for the given controller structure, a stabilising controller law exists. Given a stabilising solution exists then the proposed method will enable the optimal controller to be found.

2. SYSTEM MODEL

The usual single degree of freedom controller structure is assumed with linear, continuous-time and SISO models. External white noise sources drive colouring filters which represent the reference $W_r(s)$ and disturbance $W_d(s)$ subsystems. The system equations are:

Output: $y(s) = d(s) + W(s)u(s)$ (1)

Input disturbance: $d(s) = W_d(s)\xi(s)$ (2)

Reference: $r(s) = W_r(s)\zeta(s)$ (3)

Tracking error: $e(s) = r(s) - y(s)$ (4)

Control signal: $u(s) = C_0(s)(r(s) - y(s))$ (5)

For notational simplicity the arguments in system transfer functions and the other disturbance and reference models, are omitted.

2.1 *Assumptions*

(i) The white noise sources, ξ *and* ζ are zero-mean and mutually statistically independent. The intensities for these signals are without loss of generality taken to be unity.

(ii) The plant W is assumed free of unstable hidden modes and the reference W_r and disturbance W_d subsystems are asymptotically stable.

2.2 *Signals and sensitivity*

The output, error and controls signal are:

$$y(s) = WC_0(1+WC_0)^{-1}r(s) + (1+WC_0)^{-1}d(s) \quad (6)$$

$$e(s) = r(s) - y(s) \quad (7)$$

$$= r(s) - WC_0(1+WC_0)^{-1}r(s) - (1+WC_0)^{-1}d(s) \quad (8)$$

$$u(s) = (1+WC_0)^{-1}C_0(r(s) - d(s)) \quad (9)$$

The system sensitivity operators are:

Sensitivity:

$$S = (1+WC_0)^{-1} \quad (10)$$

Complementary sensitivity :

$$T = 1 - S = WC_0 S \quad (11)$$

Control sensitivity:

$$M = C_0 S = C_0(1+WC_0)^{-1} \quad (12)$$

2.3 *Polynomial system description*

The system may be given polynomial form as:

Plant : $W = A^{-1}B$ (13)

Reference generator: $W_r = A^{-1}E$ (14)

Input disturbance: $W_d = A^{-1}C_d$ (15)

The various polynomials are not necessarily coprime but the plant transfer-function is assumed to be free of unstable hidden modes.

The spectrum of the signal $r(s)$ - $d(s)$ in equation (9) is denoted by $\Phi_{ff}(s)$ and a generalised spectral-factor Y_f may be defined from this as:

$$Y_f Y_f^* = \Phi_{ff} = \Phi_{rr} + \Phi_{dd} \quad (16)$$

In polynomial form $Y_f = A^{-1}D_f$. The disturbance model is assumed to be such that D_f is strictly Hurwitz and satisfies:

$$D_f D_f^* = EE^* + C_d C_d^* \quad (17)$$

3. THE LQG COST AND RESTRICTED STRUCTURE CONTROL

The LQG cost-function to be minimised is defined as:

$$J = \frac{1}{2\pi j}\oint_D \{Q_c(s)\Phi_{ee}(s) + R_c(s)\Phi_{uu}(s)\}ds \quad (18)$$

where Q_c, R_c represent dynamic weighting elements, acting on the spectra of the error $e(t)$ and feedback control $u(t)$ signals. The R_c weighting term is assumed to be positive definite and Q_c are assumed to be positive-semidefinite on the D contour of the s-plane. The weightings can be written in polynomial form as:

$$Q_c = Q_n / (A_q^* A_q) \text{ and } \qquad R_c = R_n / (A_r^* A_r) \quad (19)$$

where A_q is a Hurwitz polynomial and A_r is a strictly Hurwitz polynomial. The problem will be to minimise the above criterion with the controller chosen to have a specified structure; examples are:

PID: $C_0(s) = k_0 + k_1/s + k_2 s$

Lead lag: $C_0(s) = \dfrac{(c_{n0} + c_{n1}s)(c_{n2} + c_{n3}s)}{(c_{d0} + c_{d1}s)(c_{d2} + c_{d3}s)}$

Reduced order : $C_0(s) = \dfrac{c_{n0} + c_{n1}s + \ldots + c_{np}s^p}{c_{d0} + c_{d1}s + \ldots + c_{dv}s^v}$

where $v \geq p$ is less than system order (plus weightings).

A stabilising structured control law is assumed. The optimal control solution is required to be causal. The solution to this problem uses the results in Theorem 1. Note that the spectral factors introduced depend upon the stochastic inputs (through (17)) and the cost weightings. It is assumed that the controller structure is consistent with the choice of error weighting, in the sense that if A_q includes a j axis zero then the controller denominator $C_{0d}(s)$ also includes such a zero. In practice this simply recognises that when $1/A_q$ denotes an integrator the optimal controller will also include an integrator.

Theorem 1: ***Restricted Structure Single Degrees of Freedom LQG Problem***

Consider the following LQG error and control weighted criterion:

$$J = \frac{1}{2\pi j}\oint_D \{Q_c\Phi_{ee} + R_c\Phi_{uu}\}ds \qquad (20)$$

where the weightings $Q_c = Q_n/(A_q^* A_q)$ and $R_c = R_n/(A_r^* A_r)$. The LQG controller, of restricted structures, to minimise the cost function (20), for the system of §2 may be calculated as the solution of a simpler direct optimisation problem. Introduce the strictly Hurwitz filtering and control spectral factors D_f and D_c:

$$D_f D_f^* = EE^* + C_d C_d^* \qquad (21)$$

$$D_c^* D_c = B^* A_r^* Q_n A_r B + A^* A_q^* R_n A_q A \qquad (22)$$

The following diophantine equations must then be solved.

Feedback diophantine equations:

Calculate (G_0, H_0, F_0), with F_0 of minimum degree:

$$D_c^* G_0 + F_0 A A_q = B^* A_r^* Q_n D_f \qquad (23)$$

$$D_c^* H_0 - F_0 B A_r = A^* A_q^* R_n D_f \qquad (24)$$

The optimal controller then minimises the causal terms in the cost-function term:

$$J_1 = \frac{1}{2\pi}\int_{-j\omega}^{j\omega} \{T_1^+(j\omega)T_1^+(-j\omega)\}d\omega$$

where

$$T_1^+ = [C_{0n}H_0A_q - C_{0d}G_0A_r]/(A_w(AC_{0d} + BC_{0n})) \qquad (25)$$

If the controller $C_0 = C_{0n}/C_{0d}$ has a limited structure, the minimum of the cost term J_{1min} will be non-zero. For an unconstrained solution the minimum is achieved when $T_1^+ = 0$ and the minimum of J_1 (denoted J_{1min}) is zero.

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Proof: (Grimble, 1998) •

3.2 The restricted structure LQG controller

Lemma 1 gives the restricted structure LQG controller properties.

Lemma 1: ***Structured LQG Controller Properties***

The *characteristic polynomial* which determines stability and the *implied equation* are given as:

$$\rho_c = AC_{0d} + BC_{0n} \qquad (26)$$

and

$$AA_qH_0 + BA_rG_0 = D_cD_f \qquad (27)$$

The minimum of the cost-function, with controller of restricted structure, is given as:

$$J_{min} = \frac{1}{2\pi j}\oint_D \{T_1^+T_1^{+*} + T_1^-T_1^{-*} + \Phi_0\}ds$$

$$= \frac{1}{2\pi j}\oint_D \{T_1^+T_1^{+*} + \frac{F_0F_0^* + Q_nD_fD_f^*R_n}{D_c^*D_c}\}ds \qquad (28)$$

Proof: (Grimble, 1998) •

Remarks:

Equation (28) reveals the increase in cost which occurs by restricting the controller structure, namely:

$$\Delta J_{min} = \frac{1}{2\pi j}\oint_D \{T_1^+T_1^{+*}\}ds$$

4. PARAMETRIC OPTIMIZATION PROBLEM

From Theorem 1 the computation of the optimal feedback controller C_0 reduces to minimisation of the term J_1:

$$J_1 = \frac{1}{2\pi j}\oint_D T_1^+T_1^{+*}ds = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_1^+(j\omega)T_1^+(-j\omega)d\omega$$

From (25) T_1^+ can be written in the form:

$$T_1^+ = (C_{0n}L_1 - C_{0d}L_2)/(C_{0n}L_3 + C_{0d}L_4) \qquad (29)$$

where $C_0 = C_{0n}/C_{0d}$ has a specified structure.

Assume, for example, that C_0 has a modified PID structure of the form:

$$C_0 = k_0 + (k_1/s) + (k_2s/(1+s\tau)) \qquad (30)$$

so that the numerator:

$$C_{0n} = k_0(1+s\tau)s + k_1(1+s\tau) + k_2s^2 \qquad (31)$$

and the denominator:

$$C_{0d} = s(1+s\tau) \qquad (32)$$

Let the superscripts r and i denote the real and imaginary parts of a complex function, so that

$$C_{0n} = C_{0n}^r + jC_{0n}^i \quad \text{and} \quad C_{0d} = C_{0d}^r + jC_{0d}^i$$

The controller numerator term may be split into frequency dependent components, through comparison with (31):

$$C_{0n}(j\omega) = -k_0\omega^2\tau + k_1 - k_2\omega^2 + j(k_0\omega + k_1\omega\tau) \qquad (33)$$

and

$$C_{0n}^r(j\omega) = -k_0\omega^2\tau + k_1 - k_2\omega^2 \quad \text{and} \quad C_{0n}^i(j\omega) = k_0\omega + k_1\omega\tau \qquad (34)$$

Similarly, for the denominator term:

$$C_{0d}(j\omega) = -\omega^2\tau + j\omega \qquad (35)$$

and hence

$$C_{0d}^r(j\omega) = -\omega^2\tau \quad \text{and} \quad C_{0d}^i(j\omega) = \omega \qquad (36)$$

If the solution of the optimisation problem is to be found by iteration the denominator term in T_1^+ can be assumed to be known and the minimisation can then be performed on the numerator (linear terms). Thus, to set up this problem let,

$$T_1^+ = C_{0n}L_{n1} - C_{0d}L_{n2}.$$

where

$$L_{n1} = L_1 / (C_{0n} L_3 + C_{0d} L_4)$$

and

$$L_{n2} = L_2 / (C_{0n} L_3 + C_{0d} L_4) \qquad (37)$$

Substituting from (29) and (37,

$$T_1^+ = (C_{0n}^r + jC_{0n}^i)(L_{n1}^r + jL_{n1}^i) - (C_{0d}^r + jC_{0d}^i)(L_{n2}^r + jL_{n2}^i)$$
$$= C_{0n}^r L_{n1}^r - C_{0n}^i L_{n1}^i - C_{0d}^r L_{n2}^r + C_{0d}^i L_{n2}^i$$
$$+j(C_{0n}^i L_{n1}^r + C_{0n}^r L_{n1}^i - C_{0d}^r L_{n2}^i - C_{0d}^i L_{n2}^r)$$

and after substitution from (34) and (36) obtain

$$T_1^+ = k_0\left(-\omega^2 \tau L_{n1}^r - \omega L_{n1}^i + j(\omega L_{n1}^r - \omega^2 \tau L_{n1}^i)\right)$$
$$+k_1\left(L_{n1}^r - \omega\tau L_{n1}^i + j(\omega\tau L_{n1}^r + L_{n1}^i)\right) + k_2\left(-\omega^2 L_{n1}^r - j\omega^2 L_{n1}^i\right)$$
$$+\omega^2 \tau L_{n2}^r + \omega L_{n2}^i + j(\omega^2 \tau L_{n2}^i - \omega L_{n2}^r)$$

The real and imaginary part of T_1^+ may therefore be written as: $T_1^+ = T_1^{+r} + jT_1^{+i}$ and it follows that,

$$\left|T_1^+\right|^2 = \left(T_1^{+r}\right)^2 + \left(T_1^{+i}\right)^2$$

Write a vector form of the above equations as:

$$\begin{bmatrix} T_1^{+r} \\ T_1^{+i} \end{bmatrix} = F \begin{bmatrix} k_0 \\ k_1 \\ k_2 \end{bmatrix} - L = Fx - L$$

where

$$F(\omega) = \begin{bmatrix} -\omega(\omega\tau L_{n1}^r + L_{n1}^i) & (L_{n1}^r - \omega\tau L_{n1}^i) & -\omega^2 L_{n1}^r \\ \omega(L_{n1}^r - \omega\tau L_{n1}^i) & (\omega\tau L_{n1}^r + L_{n1}^i) & -\omega^2 L_{n1}^i \end{bmatrix}$$

and

$$L(\omega) = \begin{bmatrix} -\omega(L_{n2}^i + \omega\tau L_{n2}^r) \\ \omega L_{n2}^r - \omega^2 \tau L_{n2}^i \end{bmatrix}$$

The cost-function can be optimised directly but a simple iterative solution can be obtained if the integral is approximated (*Yukitomo et al* 1998) by a summation with a sufficient number of frequency points $\{\omega_1, \omega_2, \ldots, \omega_N\}$. The optimisation can then be performed by minimising the sum of squares at each of the frequency points. The minimisation of the cost term J_0 is therefore required where,

$$J_0 = \sum_{k=1}^{N} (Fx - L)^T (Fx - L) = (b - Ax)^T (b - Ax) \qquad (38)$$

where

$$A = \begin{bmatrix} F(\omega_1) \\ \vdots \\ F(\omega_N) \end{bmatrix}, \quad b = \begin{bmatrix} L(\omega_1) \\ \vdots \\ L(\omega_N) \end{bmatrix}, \quad x = \begin{bmatrix} k_0 \\ k_1 \\ k_2 \end{bmatrix} \qquad (39)$$

Assuming the matrix $A^T A$ is non-singular the least squares optimal solution follows as:

$$x = (A^T A)^{-1} A^T b \qquad (40)$$

4.1 ***Iterative Solution***

The following successive approximation algorithm can be used to compute the restricted order LQG controller.

Algorithm: *Optimal Restricted Structure Controller*

(1) Solve for the spectral factors D_c, D_f and the diophantine equations for G_0, H_0, F_0. Compute full order design by choosing appropriate weightings and setting (25) to zero.

(2) For restricted structure PID let

$$\alpha_0 = (1 + s/\theta)s, \quad \alpha_1 = (1 + s/\theta), \quad \alpha_2 = s^2$$
$$L_1 = H_0 A_q, \quad L_2 = G_0 A_r, \quad L_3 = A_w B, \quad L_4 = A_w A$$

(3) Initialise k_0, k_1, k_2.

(4) Compute $C_{0n}(s) = \alpha_0(s)k_0 + \alpha_1(s)k_1 + \alpha_2(s)k_2$ and $C_{0d}(s) = (1 + s/\sigma)s$.

(5) Compute $L_{n1}(s) = L_1 / (C_{0n} L_3 + C_{0d} L_4)$ and $L_{n2}(s) = L_2 / (C_{0n} L_3 + C_{0d} L_4)$

(6) Compute for all chosen frequencies $L_{n1}^r(\omega), L_{n1}^i(\omega), L_{n2}^r(\omega), L_{n2}^i(\omega)$, $\alpha_0^r(\omega), \alpha_0^i(\omega), \alpha_1^r(\omega), \alpha_1^i(\omega), \alpha_2^r(\omega), \alpha_2^i(\omega)$, and hence find:

$$f_{11}^r(\omega) = \alpha_0^r(\omega)L_{n1}^r(\omega) - \alpha_0^i(\omega)L_{n1}^i(\omega),$$
$$f_{12}^r(\omega) = \alpha_1^r(\omega)L_{n1}^r(\omega) - \alpha_1^i(\omega)L_{n1}^i(\omega)$$
$$f_{13}^r(\omega) = \alpha_2^r(\omega)L_{n1}^r(\omega) - \alpha_2^i(\omega)L_{n1}^i(\omega),$$
$$f_{11}^i(\omega) = \alpha_0^i(\omega)L_{n1}^r(\omega) + \alpha_0^r(\omega)L_{n1}^i(\omega)$$
$$f_{12}^i(\omega) = \alpha_1^i(\omega)L_{n1}^r(\omega) + \alpha_1^r(\omega)L_{n1}^i(\omega),$$
$$f_{13}^i(\omega) = \alpha_2^i(\omega)L_{n1}^r(\omega) + \alpha_2^r(\omega)L_{n1}^i(\omega)$$

(7) Compute for all chosen frequencies $C_{0n}^r(\omega), C_{0n}^i(\omega), C_{0d}^r(\omega), C_{0d}^i(\omega)$, and find:

$$L_{11}^r(\omega) = C_{0d}^r(\omega)L_{n2}^r(\omega) - C_{0d}^i(\omega)L_{n2}^i(\omega)$$
$$L_{11}^i(\omega) = C_{0d}^r(\omega)L_{n2}^i(\omega) + C_{0d}^i(\omega)L_{n2}^r(\omega)$$

(8) $A_a = \begin{bmatrix} f_{11}^r(\omega) & f_{12}^r(\omega) & f_{13}^r(\omega) \\ f_{11}^i(\omega) & f_{12}^i(\omega) & f_{13}^i(\omega) \end{bmatrix}, \quad B_b = \begin{bmatrix} L_{11}^r \\ L_{11}^i \end{bmatrix}$

$$\begin{bmatrix} k_0 & k_1 & k_2 \end{bmatrix}^T = \left[A_a^T A_a\right]^{-1} A_a^T B_b$$

(9) If error $> \in$ go to (4) else compute controller:

$$C_{0n}(s) = \alpha_0(s)k_0 + \alpha_1(s)k_1 + \alpha_2(s)k_2 \text{ and}$$
$$C_0(s) = C_{0n}(s) / C_{0d}(s).$$

End of Algorithm

Remarks

(i) As a matter of notation the above functions of frequency are denoted $f(\omega)$ rather than $f(j\omega)$ for simplicity.

(ii) The stopping criterion is not important and experience reveals a fixed number of steps, say 10, can be used.

(iii) More sophisticated numerical methods such as gradient algorithms would give even faster convergence.

(iv) The functions in steps (6) and (7) can be evaluated for each frequency and stored in vectors. The Hadamard product of two vectors $X^oY = [x_i y_i]$ can be utilised to form the element by element products, and a neat matrix solution generated.

4.2 *Design and Applications Issues*

The LQG controller should be designed in such a way that it is consistent with the restricted controller structure of interest. For example, A_q should approximate a differentiator if near integral action is required. Theorem 1 was derived assuming that the controller structure is compatible with the choice of error weighting. Hence, if $1/A_q$ includes an j axis pole then this will be included in the chosen controller. If the the controller is to include integral action, then the weighting ($1/A_q$) will be chosen as an integrator. The control weighting $1/A_r$ is not so critical but if for example, a PID structure is to be used, then the point at which the differential (lead term) comes in can help to determine the A_r weighting.

Clearly, there is no point in designing an LQG controller which has an ideal response but cannot be approximated by the chosen controller structure. Thus, the weightings should be selected so that the closed-loop properties are satisfactory but consider the limitations of the controller structure required.

Simple representative plant models covering many process control applications could be used with this method. A simple plant test could be conducted to obtain the plant model parameters. The choice of the disturbance model depends on the designer's skill; in many applications a simple integral model for both W_r and W_d can be used.. Once the model is obtained the LQG design is very simple, involving the choice of one, or more, scalars that determine the bandwidth/speed of response (*Grimble* 1994).

There is rather more freedom in computing the reduced-order/PID controller which minimises the chosen LQG criterion than might be expected. This freedom arises because the frequency domain version of the cost-function involves an optimisation over all frequencies. However, the most important frequency range for stability and robustness is a decade above and below the unity-gain crossover frequency. Thus, an option is to minimise the frequency domain criterion over a limited *important* frequency range. This is like multiplying the frequency domain version of the cost integrand by a window function. This facility is particularly valuable for very high order systems where the low-order controller is required, since it provides an additional tuning facility.

5. CONCLUSIONS

The paper makes the following contributions:

1. The PID control design paradigm for the process industries was reviewed. The rule based design methods use several categories of techniques. One of these exploits the optimisation of time based integral cost functions. However a major conclusion was that these approaches cannot readily produce PID tunings for more demanding control performance specifications.
2. To meet the requirements of PID control design for a more demanding specification, the design of an optimal restricted structure controller to minimise an LQG criterion was devised. A new optimality theorem and computational procedure was presented.

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