CS 687 2023 Midsem

Due: March 3, 2025, 12 noon

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1. Suppose $f : \Sigma^* \to \Sigma^*$ is one-to-one (*i.e.* an injective function) and is partial computable. Show that f^{-1} is partial computable - *i.e.* there is a Turing machine $M : \Sigma^* \to \Sigma^*$ such that for for every $y \in \text{range}(f)$, $M(y)$ halts and outputs the a value x such that $f(x) = y$. **[10]**

2. Consider a string x with $K(x) \geq |x| - c$. Let y be an arbitrary string. Show that $|x| + K(y|x) \leq K(x, y) + O(1)$. (In words: if you condition on an incompressible string, the symmetry of information inequality is much simpler.) **[10]**

3. Suppose, for every $k > 1$, you can obtain the first k bits of Chaitin's Omega (described in Question 7 of Homework 1). Using this, define an algorithm to decide whether any program $p \in P$ of length $< k$ halts, where P is the prefix-free set of programs. **[10]**

4. Show that for every pair of strings x and y, we have $C(x, y) \leq C(x) + C(y) + \log C(x) + \log C(y) + O(1)$. **[10]**