

CS 687 Endsem exam: 2023-24 II Semester

April 30, 2025

1. Show that if there is a constant c such that for the sequence $X \in \Sigma^\infty$, we have $C(X[0 \dots (n-1)] \mid n) < c$, then X is computable. [10]
2. Show that if for any infinite binary sequence X ,

$$\sum_{n \in \mathbb{N}} 2^{n-K(X[0 \dots (n-1)])} < \infty, \quad (1)$$

then

$$\lim_{n \rightarrow \infty} K(X[0 \dots (n-1)]) = \infty. \quad (2)$$

[10]

3. Suppose $d : \Sigma^* \rightarrow [0, \infty)$ is a lower semicomputable martingale. Let X be a sequence on which d succeeds. Then show that X has infinitely many prefixes with low self-delimiting Kolmogorov complexity. [10]
4. Show that the following inequality holds for all strings x, y and z of length n .

$$2C(x, y, z) \leq C(x, y) + C(y, z) + O(\log n).$$

[10]