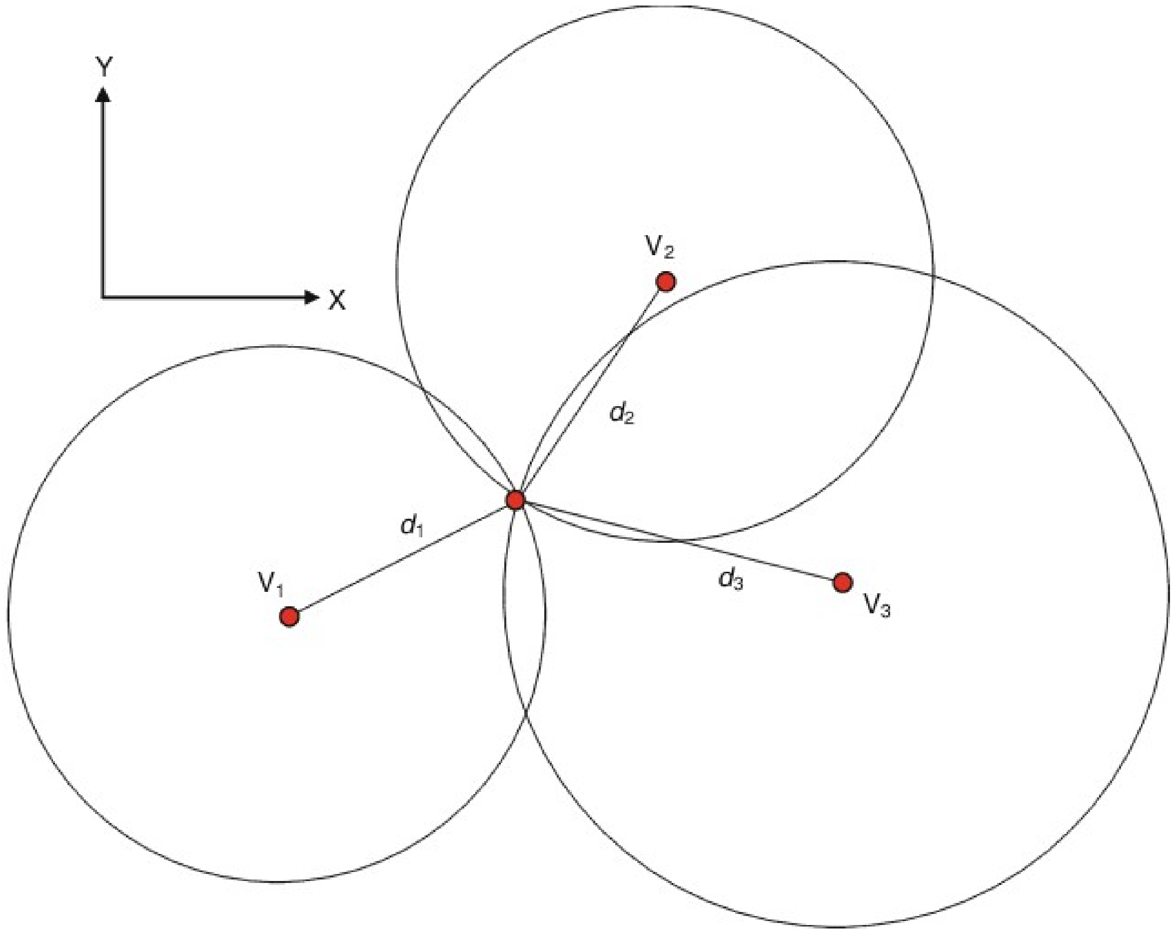
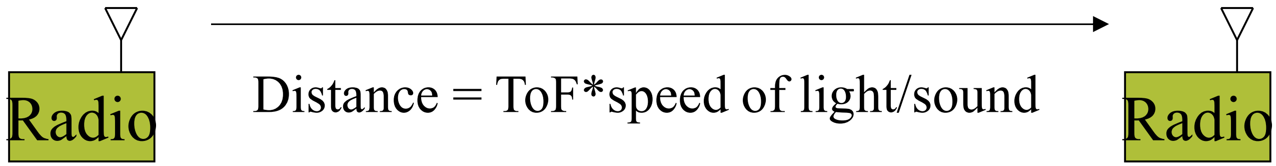
Localization Using Time of Arrival (ToA)

Amitangshu Pal

Trilateration



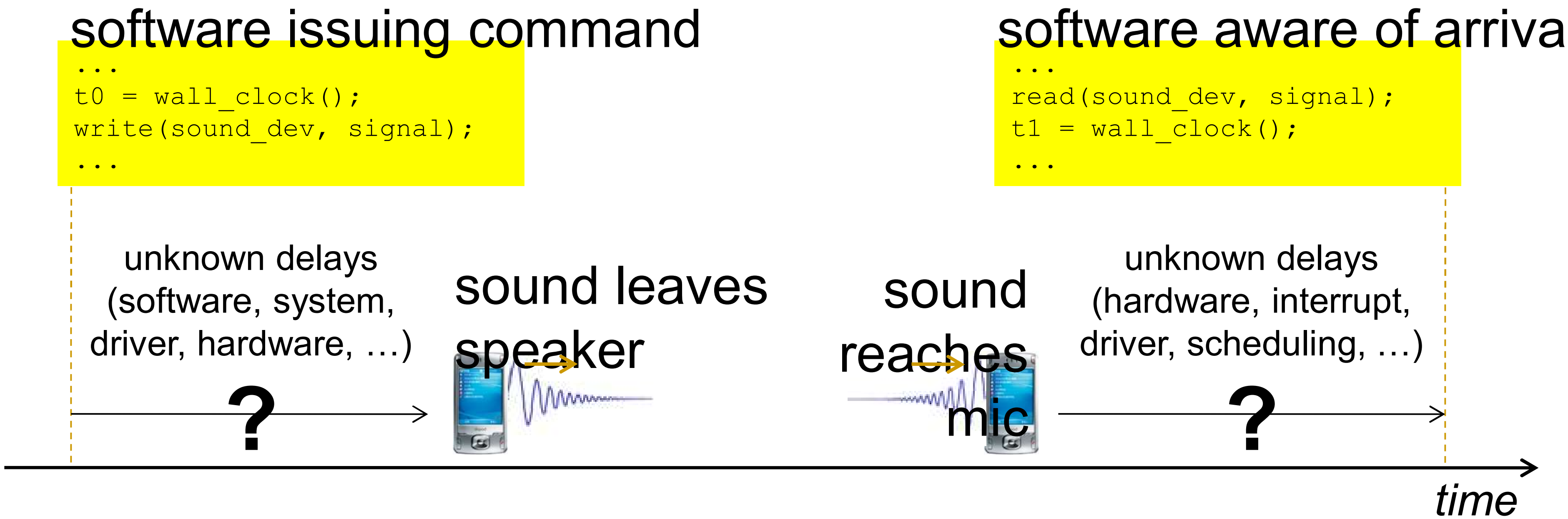
ToA Challenges



- **Transmitter end:** Processing delay between time-stamping the packet and actually sending it
- **Receiver end:** Processing time between receiving the packet and actually recording the current time
- Determining the **exact** arrival time of the packet
- Clock offset in between the transceivers

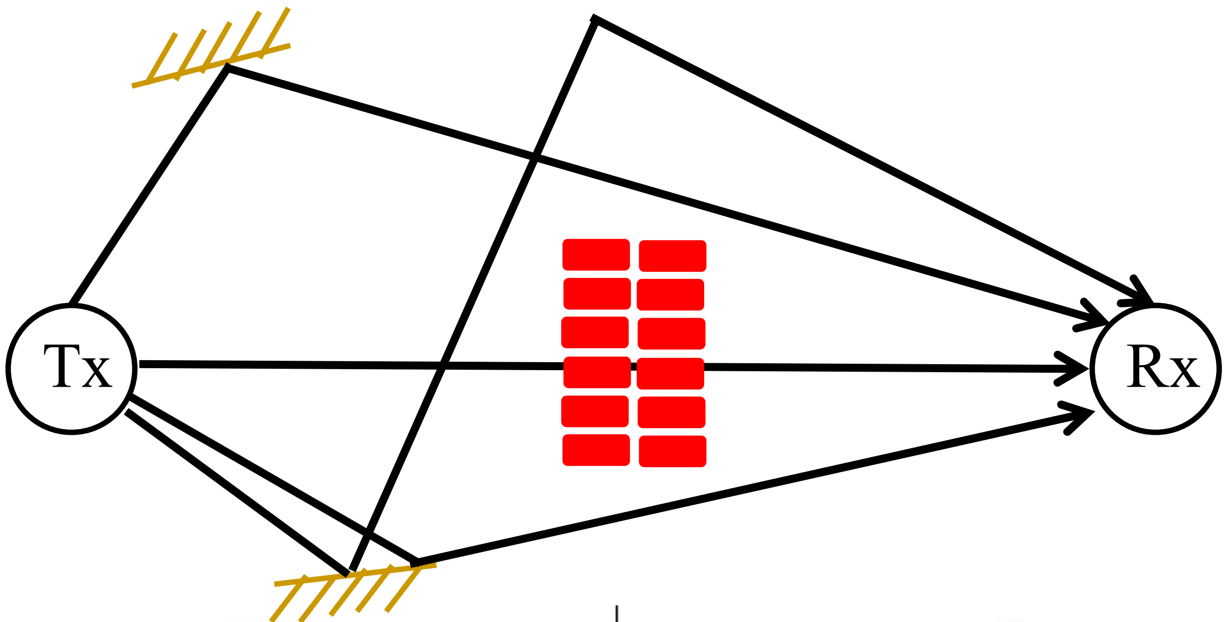
ToA Challenges

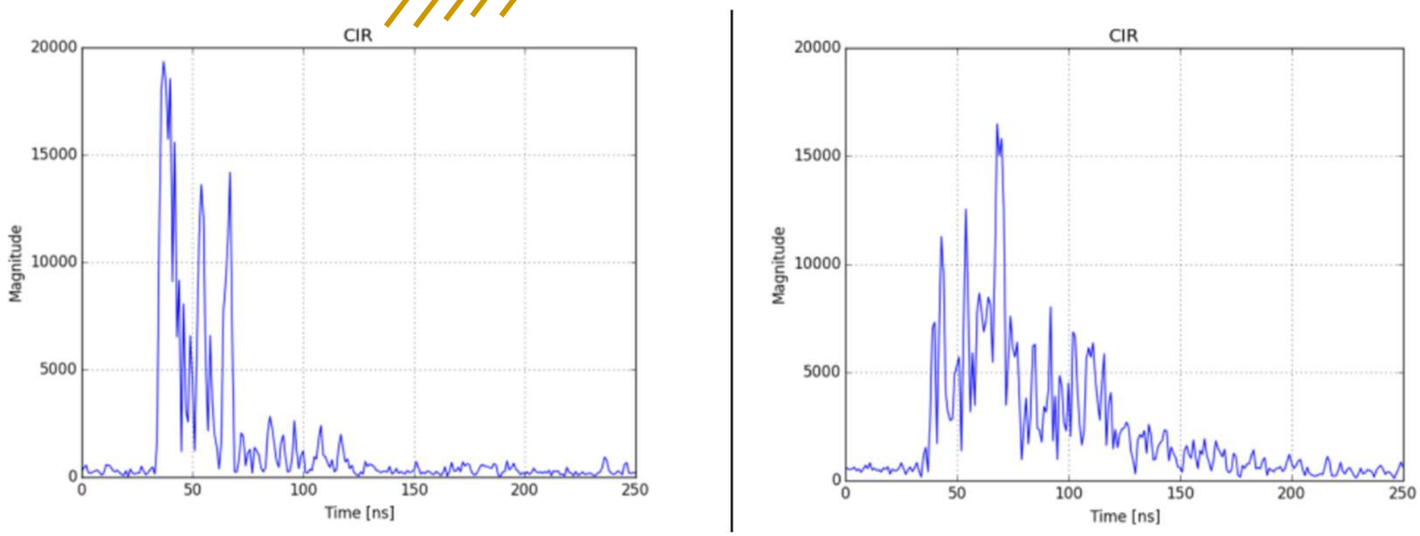
- **Transmitter end:** Processing delay between time-stamping the packet and actually sending it
- **Receiver end:** Processing time between receiving the packet and actually recording the current time



[Adapted from Peng, "BeepBeep: A High Accuracy Acoustic Ranging System using COTS Mobile Devices"

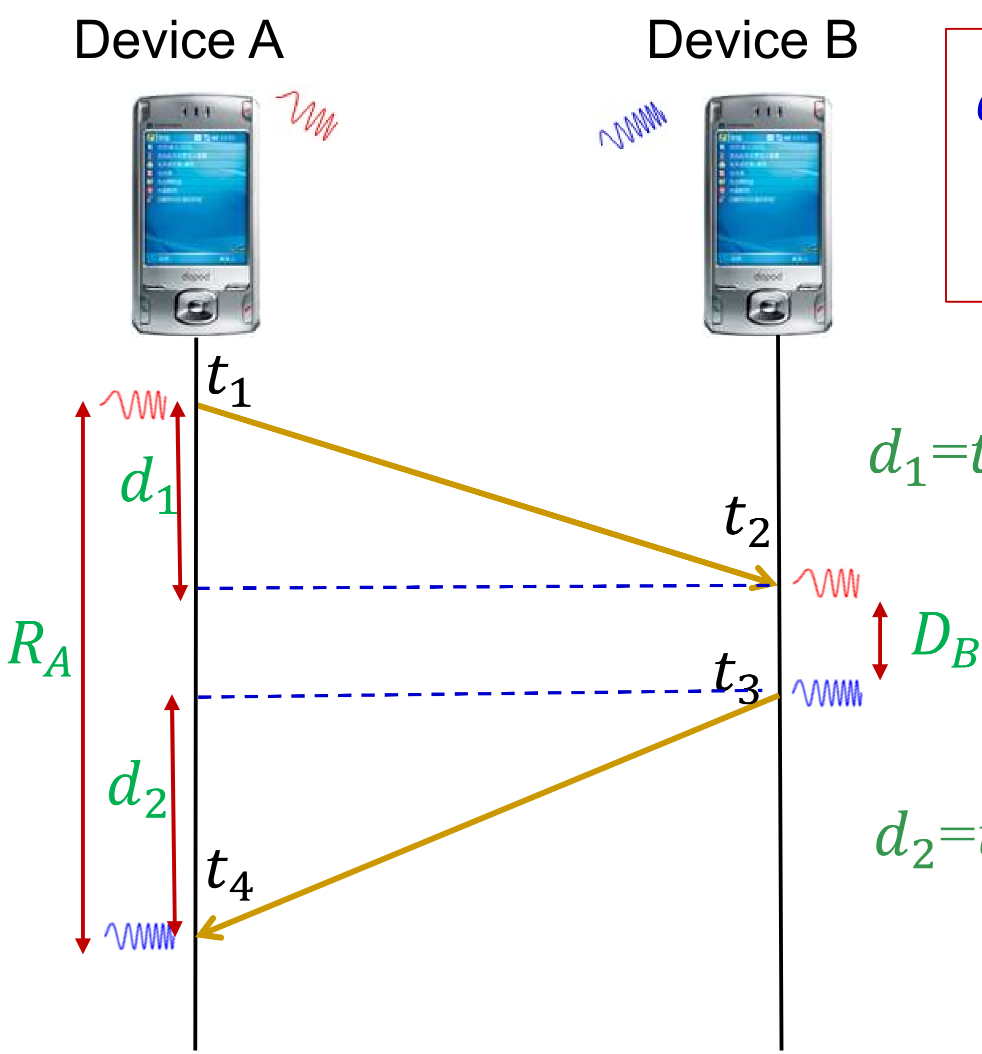
Exact Packet Arrival Time





Bregar, Klemen & Hrovat, Andrej & Mohorcic, Mihael. (2016). NLOS Channel Detection with Multilayer Perceptron in Low-Rate Personal Area Networks for Indoor Localization Accuracy Improvement.

ToA Estimation → Clock Offset Compensation



$$d = \frac{(t_2 - t_1 - \Delta clock) + (t_4 - t_3 + \Delta clock)}{2}$$

$$= \frac{(t_2 - t_1) + (t_4 - t_3)}{2} = \frac{R_A - D_B}{2}$$

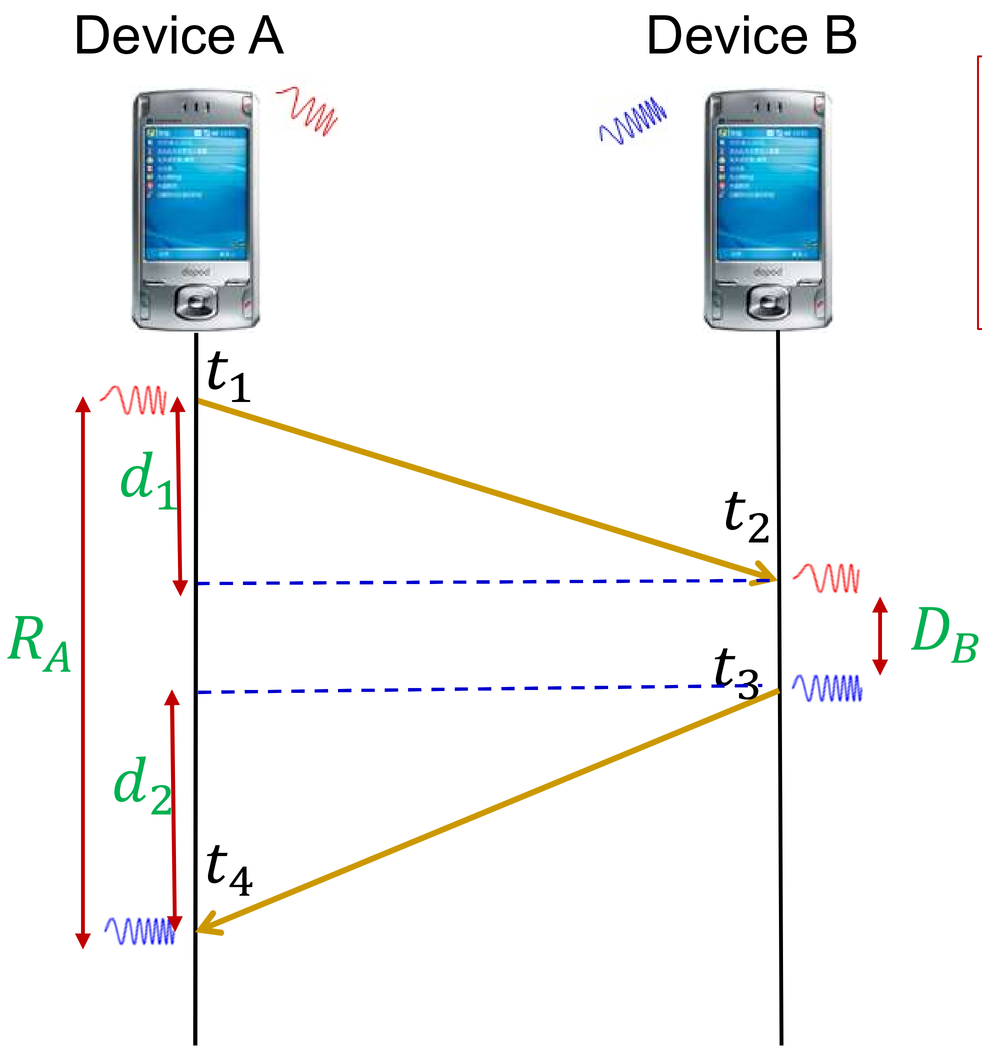
$d_1 = t_2 - t_1 - \Delta clock$

Remains same

$d_2 = t_4 - t_3 + \Delta clock$

What is the assumption?

Clock Drift Compensation → Two-way Ranging



$$d = \frac{(t_2 - t_1 - \Delta clock) + (t_4 - t_3 + \Delta clock)}{2}$$
$$= \frac{(t_2 - t_1) + (t_4 - t_3)}{2} = \frac{R_A - D_B}{2}$$

$$R_A^{est} = R_A(1 + \delta_A)$$
$$D_B^{est} = D_B(1 + \delta_B)$$

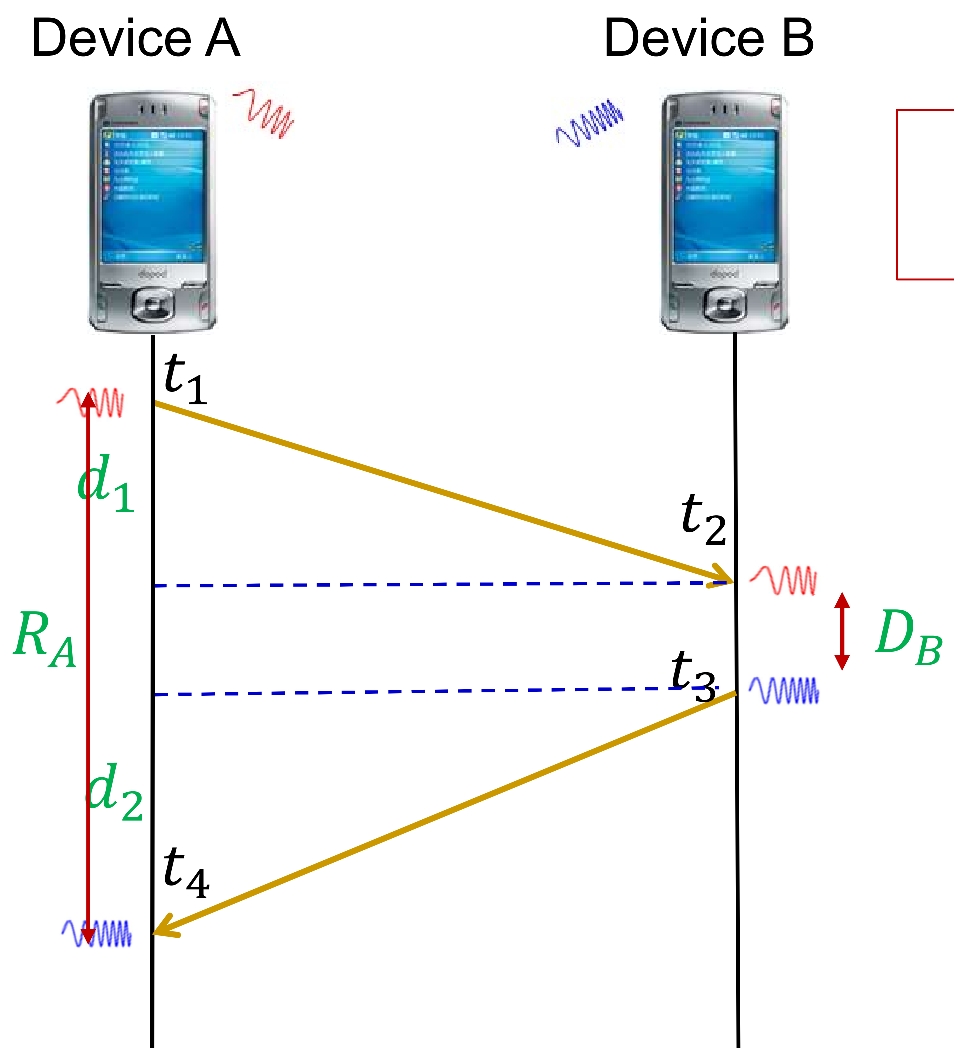
Clock Drift Compensation → Two-way Ranging

$$d = \frac{(t_2 - t_1 - \Delta clock) + (t_4 - t_3 + \Delta clock)}{2}$$
$$= \frac{(t_2 - t_1) + (t_4 - t_3)}{2} = \frac{R_A - D_B}{2}$$

$$R_A^{est} = R_A(1 + \delta_A)$$
$$D_B^{est} = D_B(1 + \delta_B)$$

$$d^{est} = \frac{R_A^{est} - D_B^{est}}{2} = \frac{R_A(1 + \delta_A) - D_B(1 + \delta_B)}{2}$$
$$= \frac{(R_A - D_B) + R_A\delta_A - D_B\delta_B}{2}$$
$$= \frac{(R_A - D_B) + \delta_A(d_1 + d_2 + D_B) - D_B\delta_B}{2}$$
$$= \frac{(R_A - D_B) + (\delta_A - \delta_B)D_B + \delta_A\cancel{(d_1 + d_2)}}{2}$$
$$= \frac{(R_A - D_B) + (\delta_A - \delta_B)D_B}{2}$$

Clock Drift Compensation → Two-way Ranging

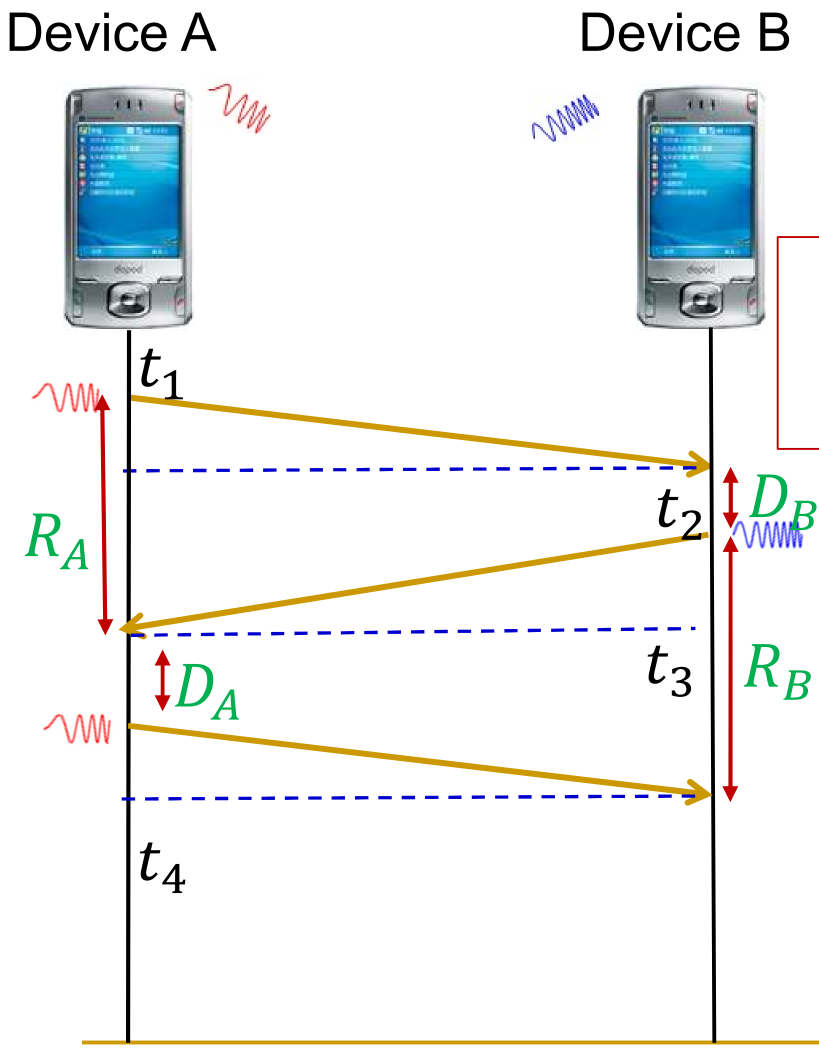


$$d^{est} = \frac{(R_A - D_B) + (\delta_A - \delta_B) D_B}{2}$$

- **Processing time**
- Radio needs to receive/decode packets → hand it over to NIC
- Kernel needs to record the packet, then form another packet and send out

- Difficult to be reduced in inexpensive hardware

Clock Drift Compensation → Symmetric Double-Sided TWR



$$d = \frac{(R_A - D_B) + (R_B - D_A)}{4}$$

$$d^{est} = \frac{R_A(1+\delta_A) - D_B(1+\delta_B) + R_B(1+\delta_B) - D_A(1+\delta_A)}{4}$$

Clock Drift Compensation → Symmetric Double-Sided TWR

$$\boxed{d = \frac{(R_A - D_B) + (R_B - D_A)}{4}}$$

$$\boxed{d^{est} = \frac{R_A(1+\delta_A) - D_B(1+\delta_B) + R_B(1+\delta_B) - D_A(1+\delta_A)}{4}}$$

$$d^{est} = \frac{R_A(1+\delta_A) - D_B(1+\delta_B) + R_B(1+\delta_B) - D_A(1+\delta_A)}{4}$$

$$= \frac{(R_A - D_B) + (R_B - D_A) + (R_A\delta_A - D_B\delta_B) + (R_B\delta_B - D_A\delta_A)}{4}$$

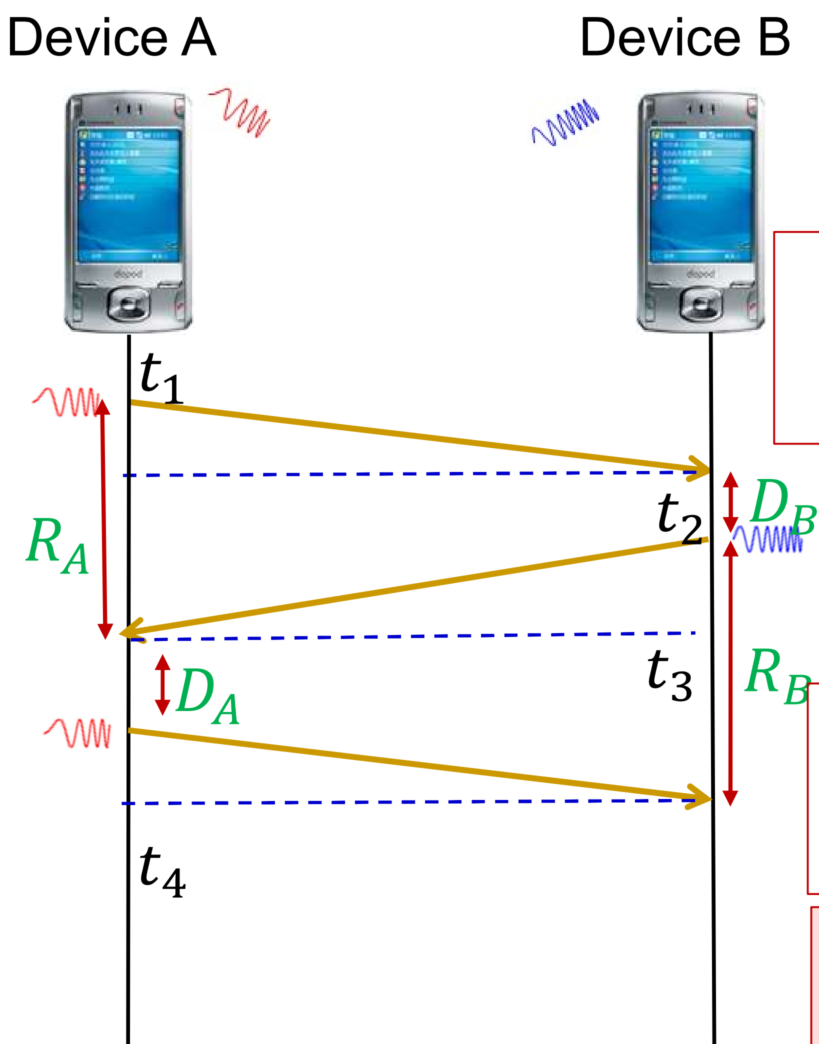
$$= \frac{(R_A - D_B) + (R_B - D_A) + (R_A\delta_A - D_A\delta_A) + (R_B\delta_B - D_B\delta_B)}{4}$$

$$= \frac{(R_A - D_B) + (R_B - D_A) + (D_B\delta_A + \cancel{2d\delta_A} - D_A\delta_A) + (D_A\delta_B + \cancel{2d\delta_B} - D_B\delta_B)}{4}$$

$$= \frac{(R_A - D_B) + (R_B - D_A) + \delta_A(D_B - D_A) + \delta_B(D_A - D_B)}{4}$$

$$= \frac{(R_A - D_B) + (R_B - D_A) + (\delta_A - \delta_B)(D_B - D_A)}{4}$$

Clock Drift Compensation → Symmetric Double-Sided TWR



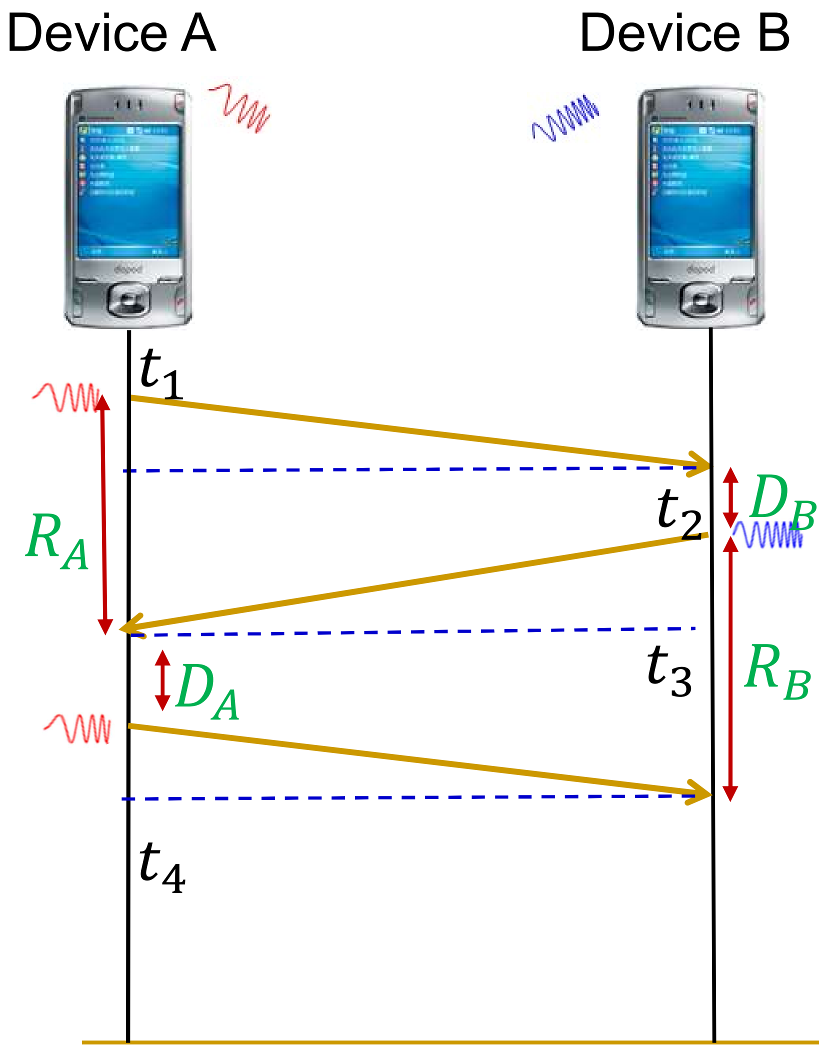
$$d = \frac{(R_A - D_B) + (R_B - D_A)}{4}$$

$$d^{est} = \frac{R_A(1+\delta_A) - D_B(1+\delta_B) + R_B(1+\delta_B) - D_A(1+\delta_A)}{4}$$

$$d^{est} = \frac{(R_A - D_B) + (R_B - D_A) + (\delta_A - \delta_B)\cancel{(D_B - D_A)}}{4}$$

Both devices can be programmed to approx. have the same turnaround time

Clock Drift Compensation → Alternative Double-Sided TWR



$$R_A = 2d + D_B$$
$$R_B = 2d + D_A$$

$$R_A R_B = (2d + D_B)(2d + D_A)$$
$$= 2d(2d + D_A + D_B) + D_A D_B$$

$$d = \frac{R_A R_B - D_A D_B}{2\,(2d + D_A + D_B)}$$
$$= \frac{R_A R_B - D_A D_B}{2\,(R_A + D_A)} = \frac{R_A R_B - D_A D_B}{2\,(R_B + D_B)}$$

Clock Drift Compensation → Alternative Double-Sided TWR

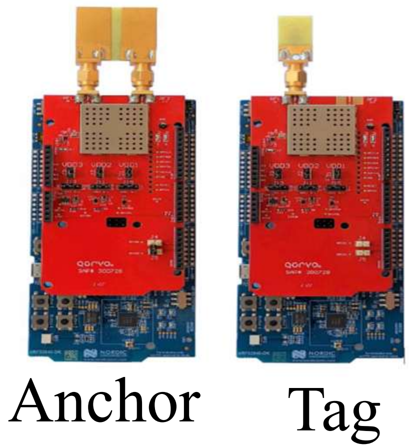
$$d = \frac{R_A R_B - D_A D_B}{2\,(\,2d + D_A + D_B)} = \frac{R_A R_B - D_A D_B}{2\,(R_A + D_A)} = \frac{R_A R_B - D_A D_B}{2\,(\,R_B + D_B)}$$

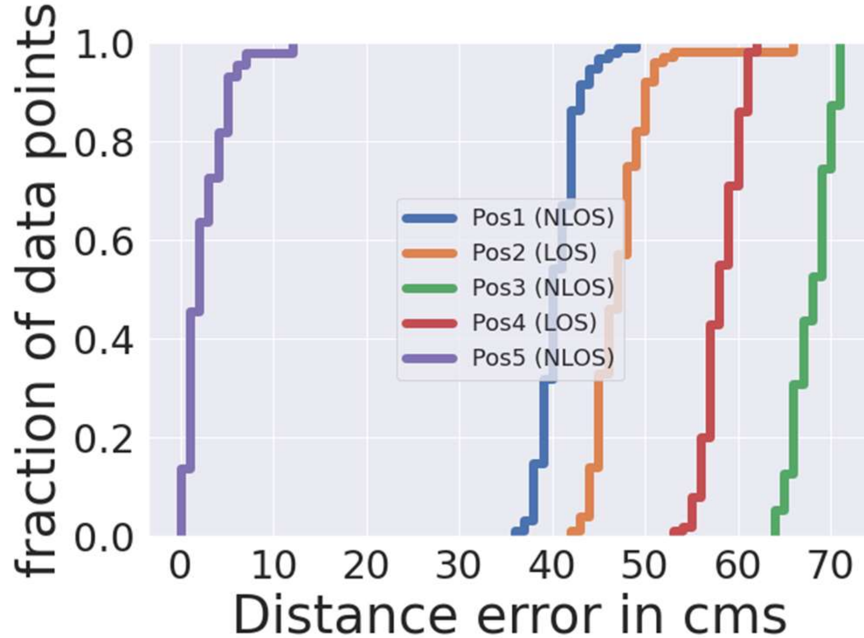
$$d^{est} = \frac{R_A(1+\delta_A) R_B(1+\delta_B) - D_A(1+\delta_A) D_B(1+\delta_B)}{2(\,R_B(1+\delta_B) + D_B(1+\delta_B)\,)}$$

$$= \frac{(1+\delta_A)\cancel{(1+\delta_B)}(R_A R_B - D_A D_B)}{2\cancel{(1+\delta_B)}(R_B + D_B)}$$

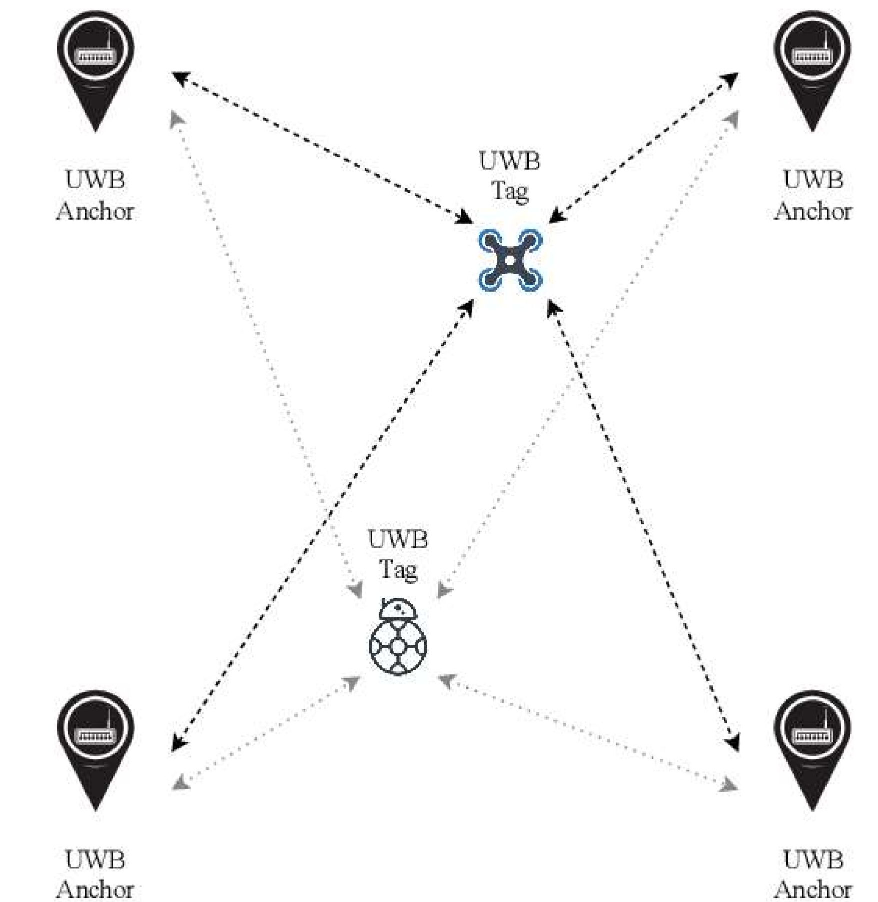
$$= (1+\delta_A)d = d + \cancel{\delta_A d}$$

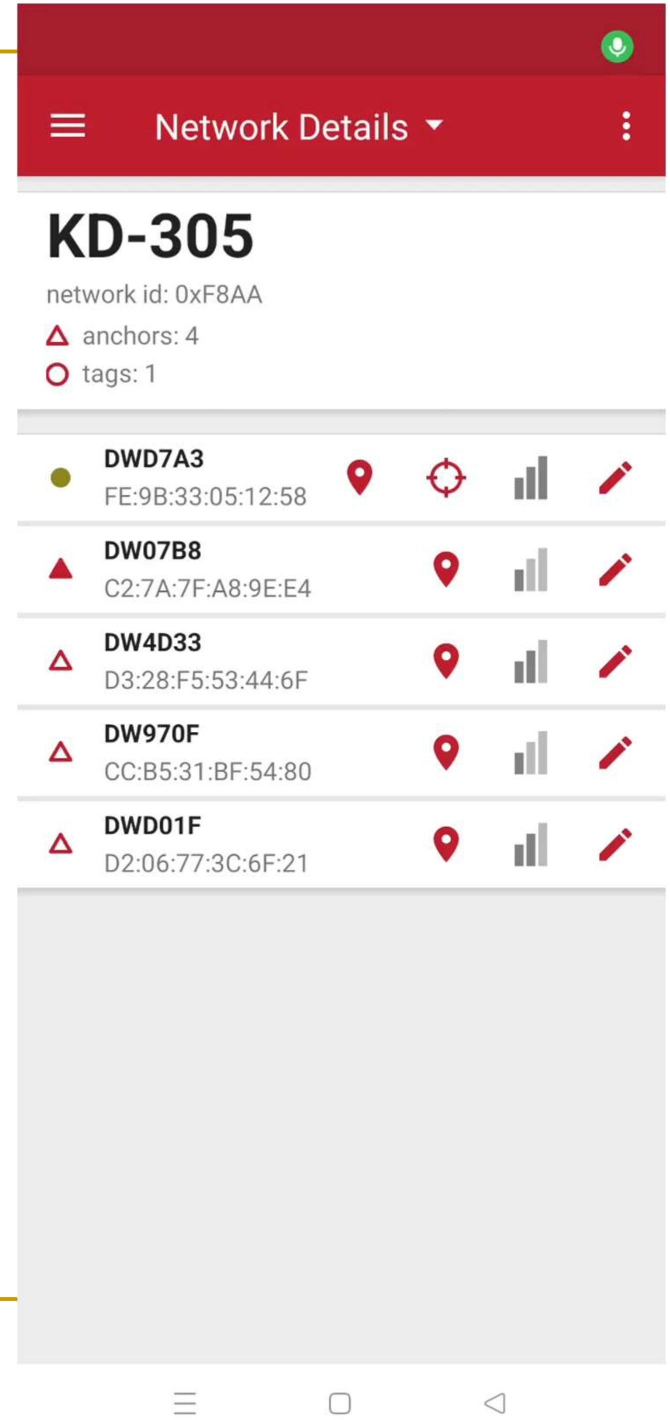
ToF For Localization


Anchor
Tag


fraction of data points
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
70
Distance error in cms
Pos1 (NLOS)
Pos2 (LOS)
Pos3 (NLOS)
Pos4 (LOS)
Pos5 (NLOS)


Collapsible
Door
Anchor
Tag


UWB Anchor
UWB Anchor
UWB Tag
UWB Tag
UWB Anchor
UWB Anchor

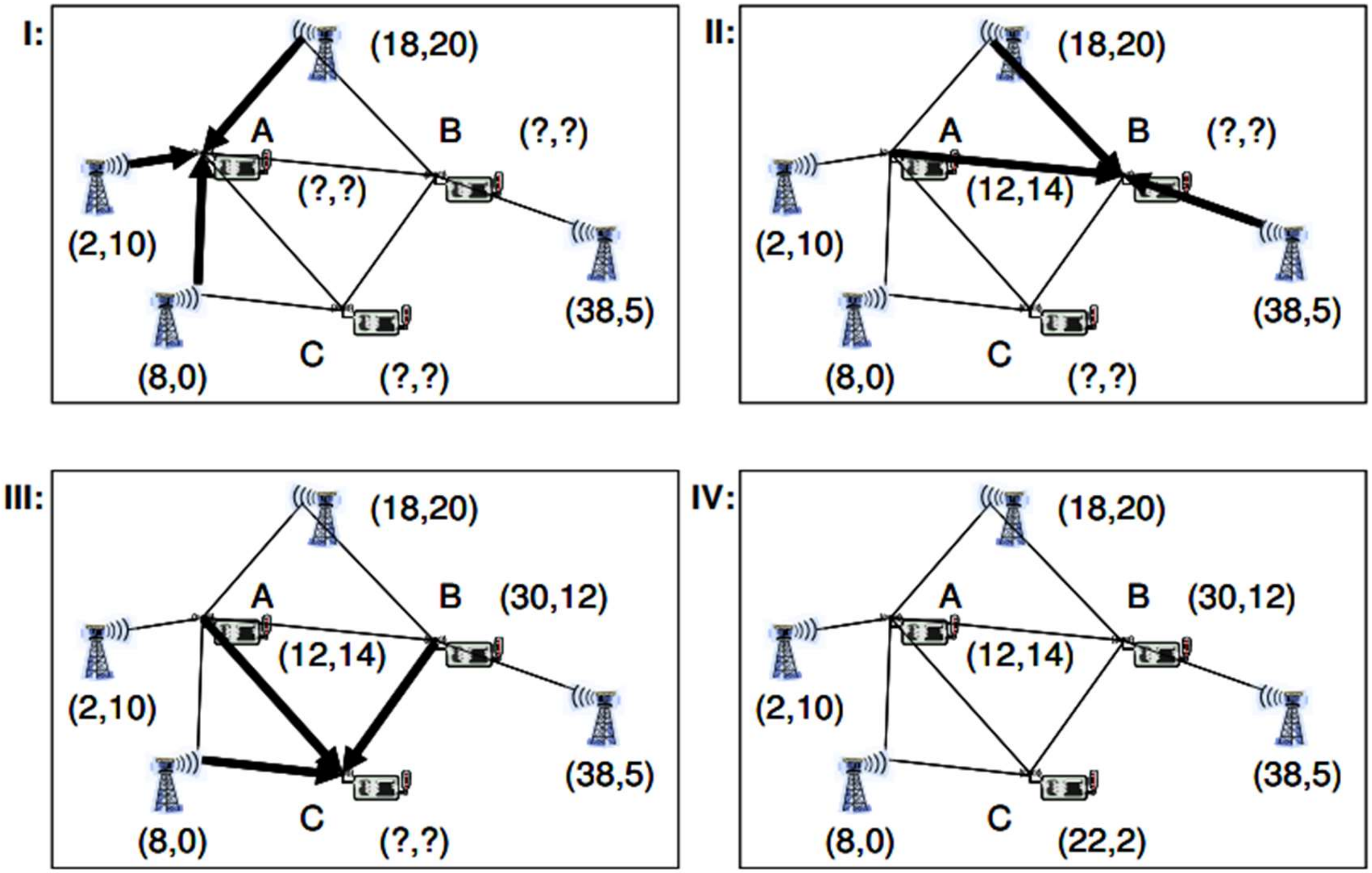

Network Details
KD-305
network id: 0xF8AA
anchors: 4
tags: 1
DWD7A3
FE:9B:33:05:12:58
DW07B8
C2:7A:7F:A8:9E:E4
DW4D33
D3:28:F5:53:44:6F
DW970F
CC:B5:31:BF:54:80
DWD01F
D2:06:77:3C:6F:21

What's About in Multi-hop Network

- How to localize nodes in multi-hop network?
  - Assume some nodes are equipped with GPS → Called beacons/anchors
  - Others localize them selves w.r.t these beacons

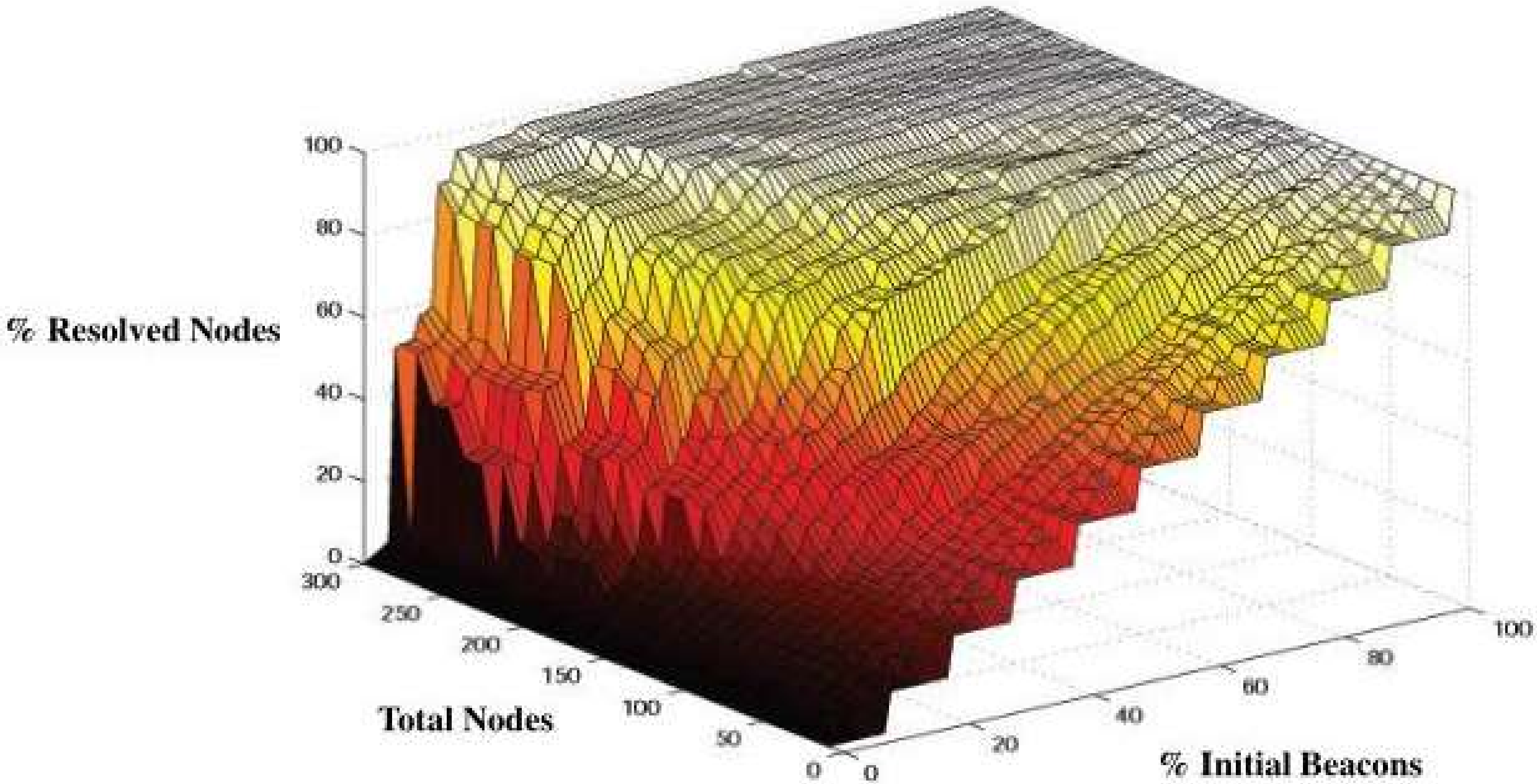
- Two key procedures
  - Iterative multilateration
  - Cooperative multilateration

Iterative Multilateration



Src: http://hscc.cs.nthu.edu.tw/~sheujp/lecture_note/10wsn/wsn07.pdf/

Iterative Multilateration



Error accumulates with the number of hops from beacons

Src: https://www.slideserve.com/dustin-gibson/fine-grained-ad-hoc-localization-in-wireless-sensor-networks/

Collaborative Multilateration

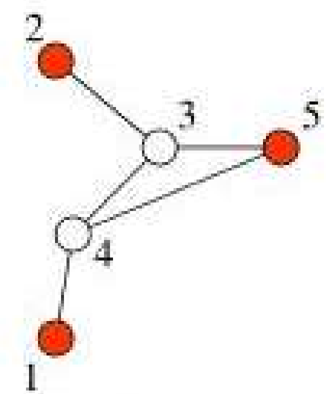
$$f_{2,3} = R_{2,3} - \sqrt{(x_2 - ex_3)^2 + (y_2 - ey_3)^2}$$

$$f_{3,5} = R_{3,5} - \sqrt{(ex_3 - x_5)^2 + (ey_3 - y_5)^2}$$

$$f_{4,3} = R_{4,3} - \sqrt{(ex_4 - ex_3)^2 + (ey_4 - ey_3)^2}$$

$$f_{4,5} = R_{4,5} - \sqrt{(ex_4 - x_5)^2 + (ey_4 - y_5)^2}$$

$$f_{4,1} = R_{4,1} - \sqrt{(ex_4 - x_1)^2 + (ey_4 - y_1)^2}$$

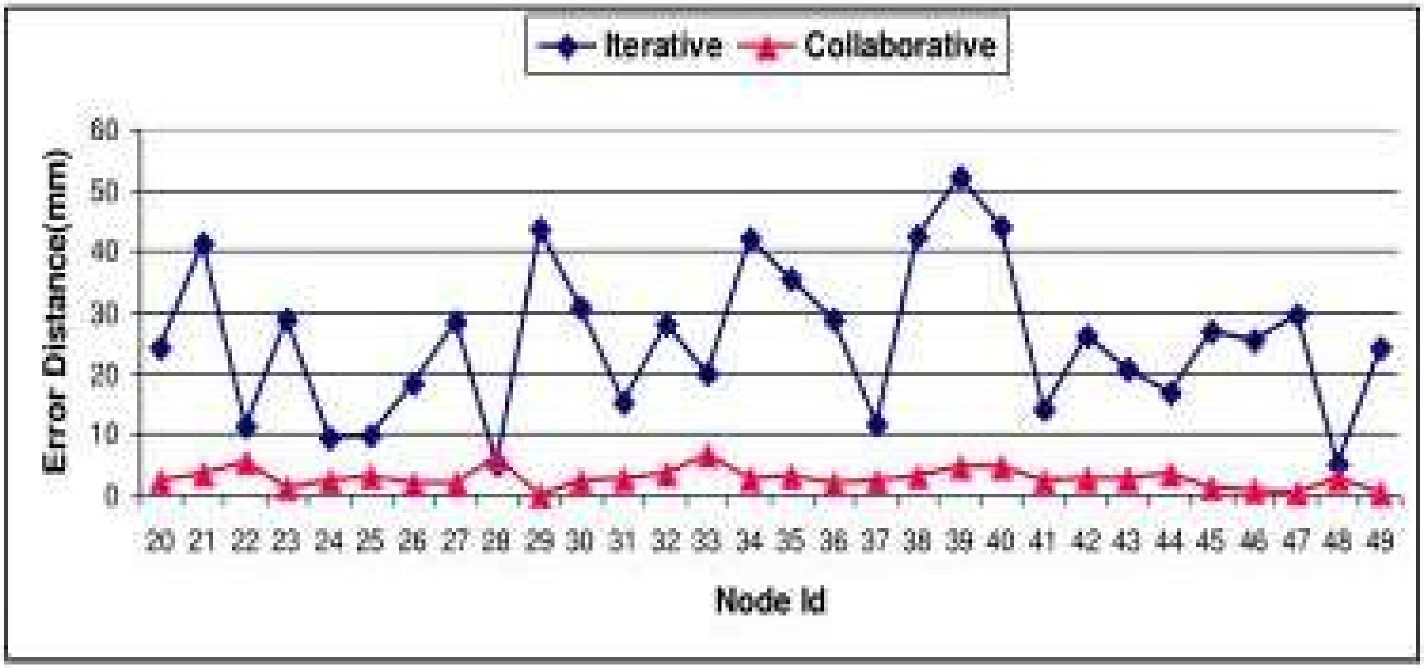


$$\boxed{f(ex_3, ey_3, ex_4, ey_4) = \min \sum f_{ij}^2}$$

- Can be solved using any iterative least square method

Src: https://www.slideserve.com/dustin-gibson/fine-grained-ad-hoc-localization-in-wireless-sensor-networks

Iterative vs Collaborative Multilateration



50 nodes, 20 beacons

Src: https://www.slideserve.com/dustin-gibson/fine-grained-ad-hoc-localization-in-wireless-sensor-networks

References

- D. Neirynck, E. Luk and M. McLaughlin, "An alternative double-sided two-way ranging method," 2016 13th Workshop on Positioning, Navigation and Communications (WPNC), Bremen, Germany, 2016, pp. 1-4, doi: 10.1109/WPNC.2016.7822844.

- https://bleesk.com/uwb.html.