Beamforming, Beam Rotation and Angle of Arrival

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DFT Properties

$$z_m = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi.m.n}{N}}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt$$

Linearity property: $x_1(t) \leftrightarrow X_1(f)$, $x_2(t) \leftrightarrow X_2(f) \equiv a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 X_1(f) + a_2 X_2(f)$

$$a_1 x_1(t) + a_2 x_2(t) \leftrightarrow \int_{-\infty}^{\infty} \{a_1 x_1(t) + a_2 x_2(t)\} e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} a_1 x_1(t) e^{-j2\pi f t}\, dt + \int_{-\infty}^{\infty} a_2 x_2(t) e^{-j2\pi f t}\, dt = a_1 X_1(f) + a_2 X_2(f)$$

DFT Properties

$$z_m = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi.m.n}{N}}$$

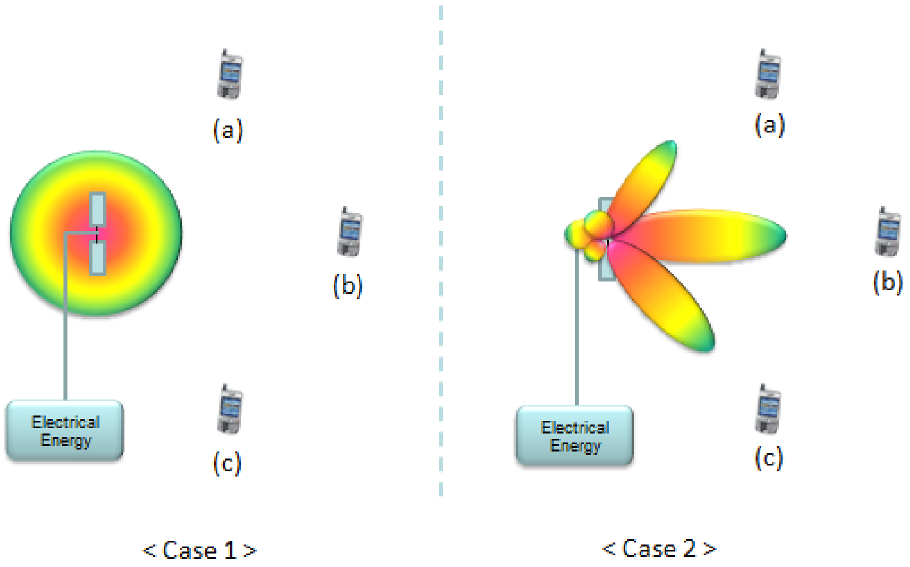
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt$$

Time shifting property: $x(t) \leftrightarrow X(f) \ \equiv \ x(t - t_0) \leftrightarrow e^{-j2\pi f t_0} X(f)$

$$x(t - t_0) \leftrightarrow \int_{-\infty}^{\infty} x(t - t_0) e^{-j2\pi f t}\, dt \qquad t - t_0 = \tau \quad \therefore t = \tau + t_0 \quad \therefore dt = d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f(\tau + t_0)}\, d\tau = e^{-j2\pi f t_0} \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f \tau}\, d\tau = e^{-j2\pi f t_0} X(f)$$

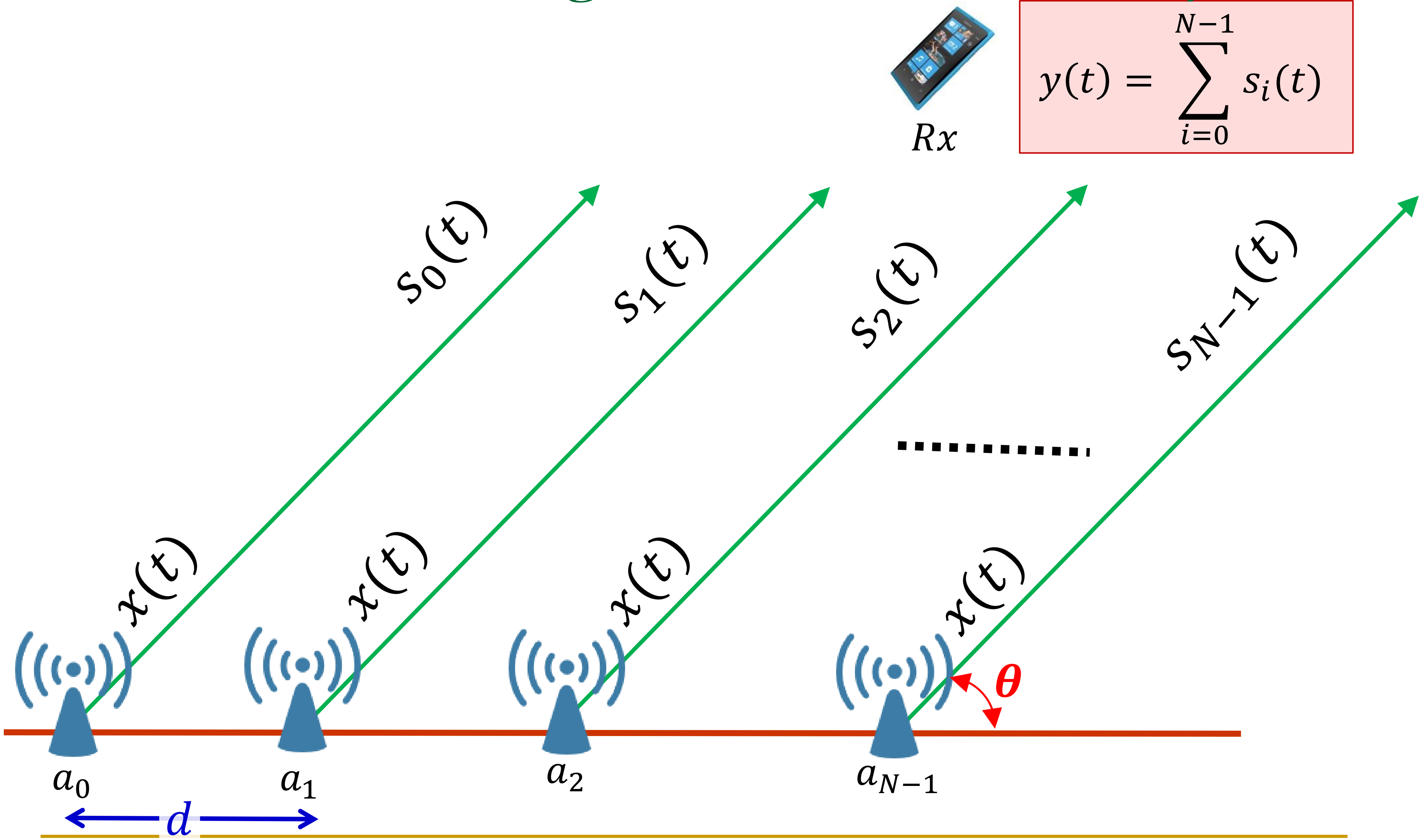
Beamforming Examples



https://www.sharetechnote.com/html/Handbook_LTE_BeamForming.html
https://voicebot.ai/2017/07/26/xiaomi-mi-ai-smart-speaker-takes-alibaba-tmall-genie-linglong-dingdong/

Beamforming and Antenna Array

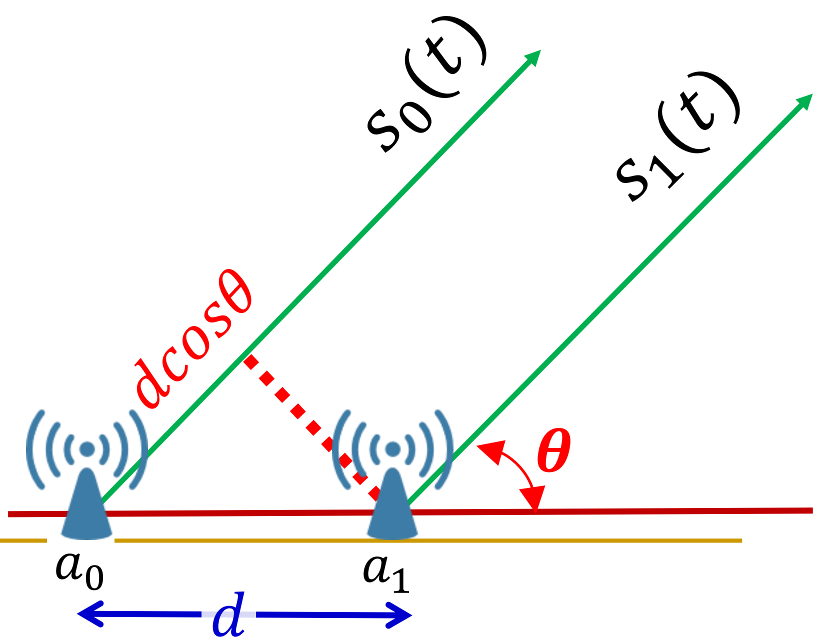
$$y(t) = \sum_{i=0}^{N-1} s_i(t)$$



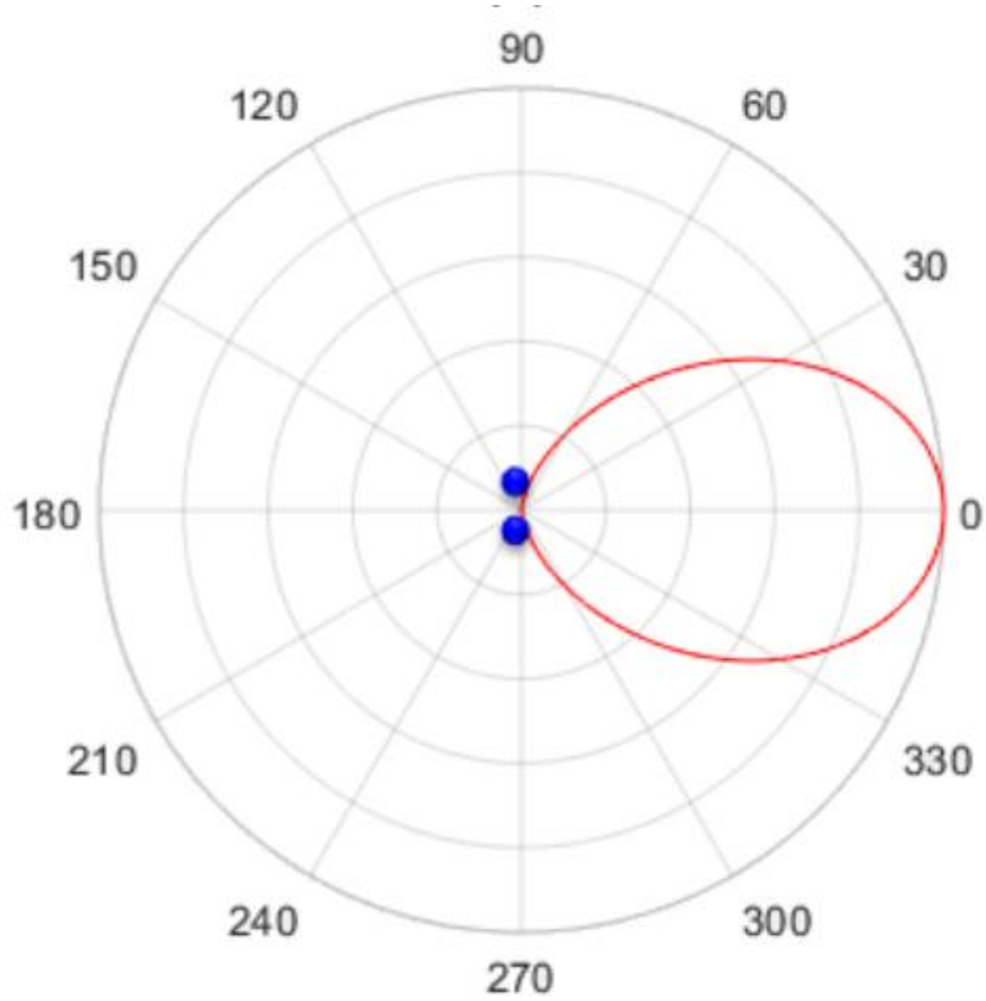
Beamforming and Antenna Array

- $s_1(t)$ travels $dcos\theta$ less than $s_0(t)$
  - How much phase shift ϕ does it cause?
- λ distance causes a phase shift of 2π
  - $dcos\theta$ distance causes a phase shift ϕ of $\frac{2\pi}{\lambda}dcos\theta$
- Assume $s_0(t) = \cos(2\pi f_0 t)$, then $s_1(t) = \cos(2\pi f_0 t + \phi)$
  - $S_1(f) = S_0(f)e^{j\phi}$
  - $S_2(f) = S_0(f)e^{2j\phi}$
  - $S_3(f) = S_0(f)e^{3j\phi}$
  - $S_{N-1}(f) = S_0(f)e^{(N-1)j\phi}$

$$Y(f) = \sum_{k=0}^{N-1} S_0(f)e^{kj\phi} = S_0(f)\left(\frac{1-e^{jN\phi}}{1-e^{j\phi}}\right)$$

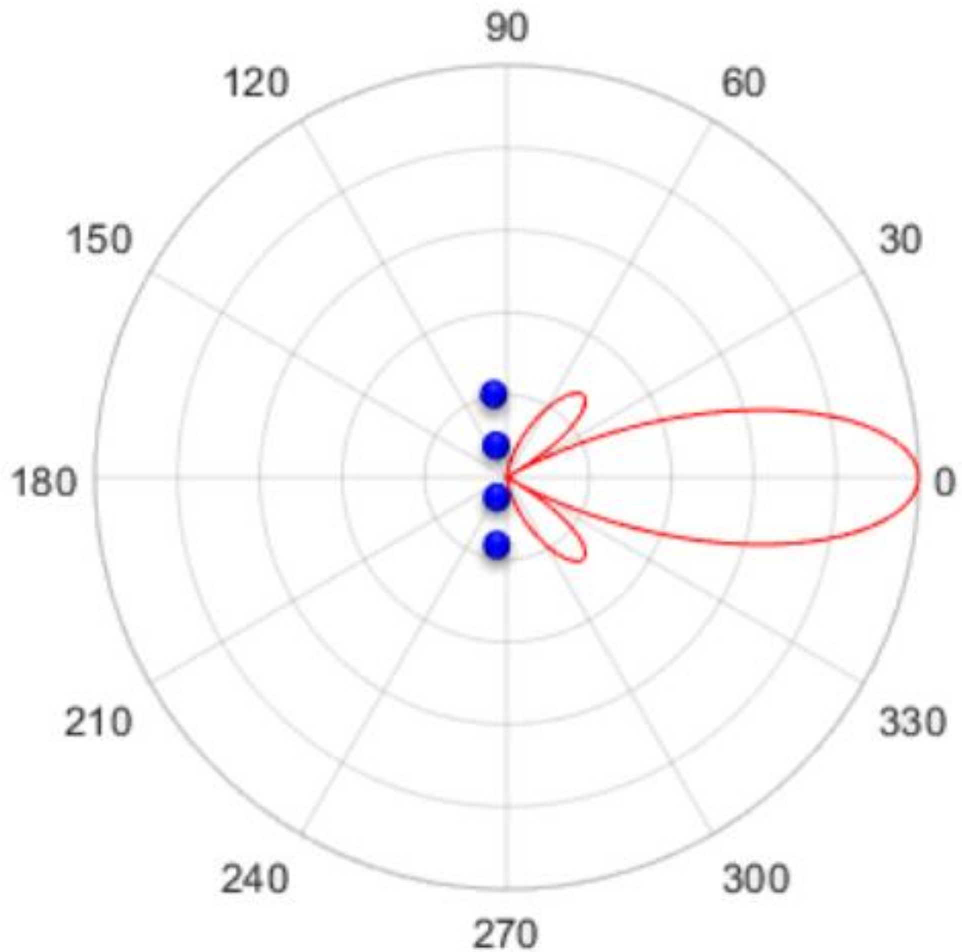


Beamforming (N=2)



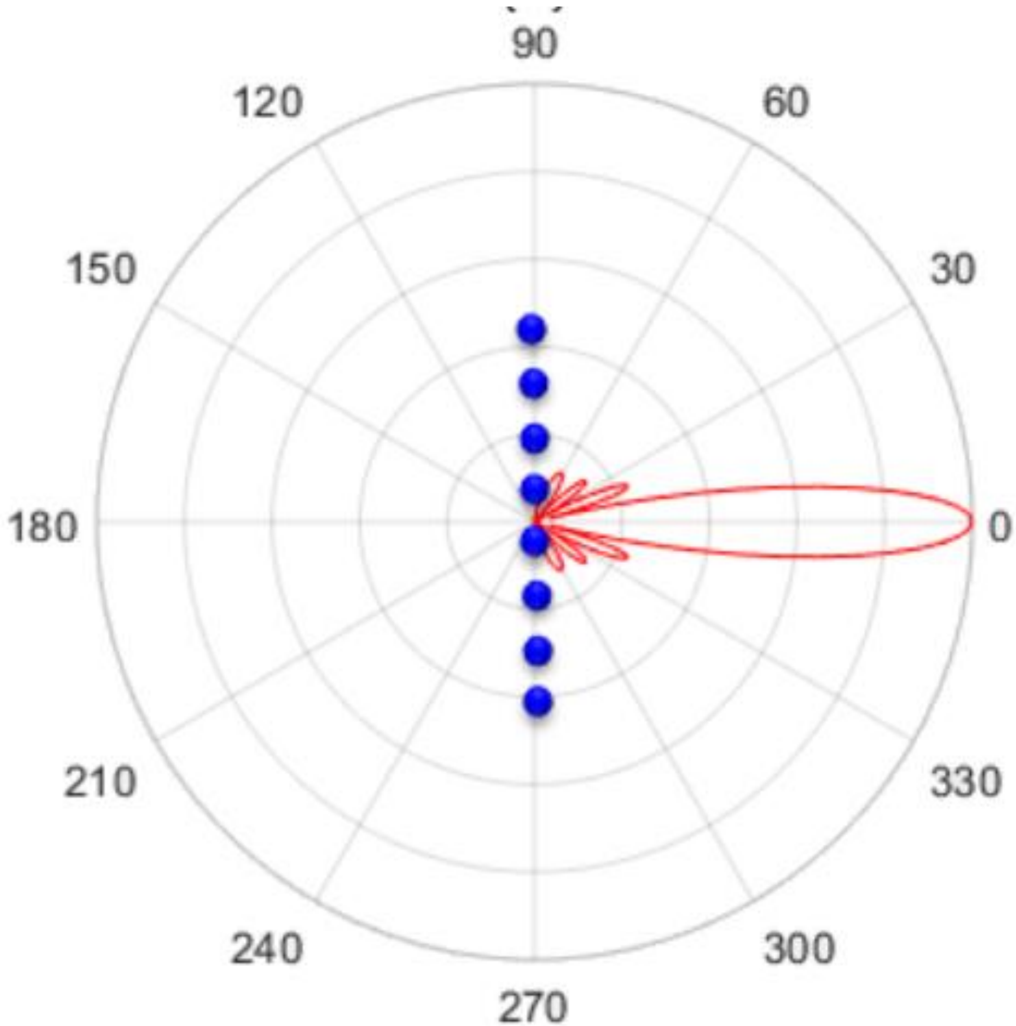
https://www.sharetechnote.com/html/Handbook_LTE_BeamForming.html

Beamforming (N=4)



https://www.sharetechnote.com/html/Handbook_LTE_BeamForming.html

Beamforming (N=8)



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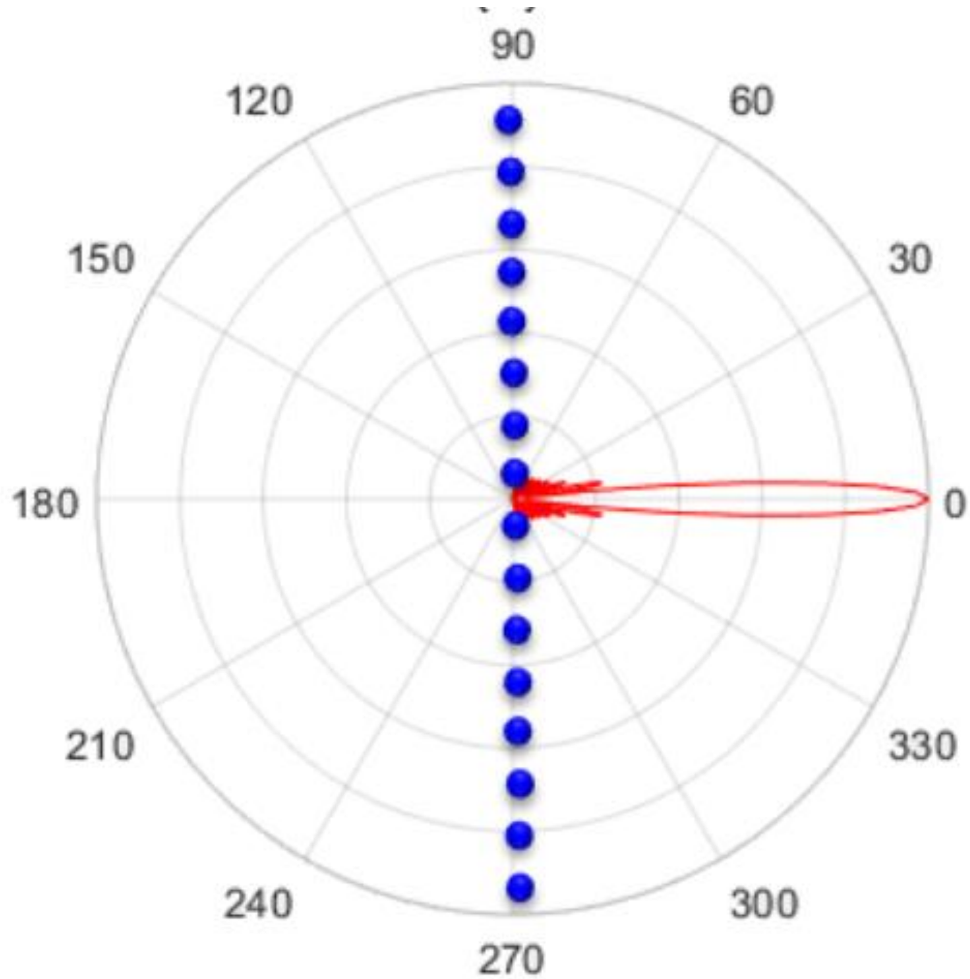
Beamforming (N=16)

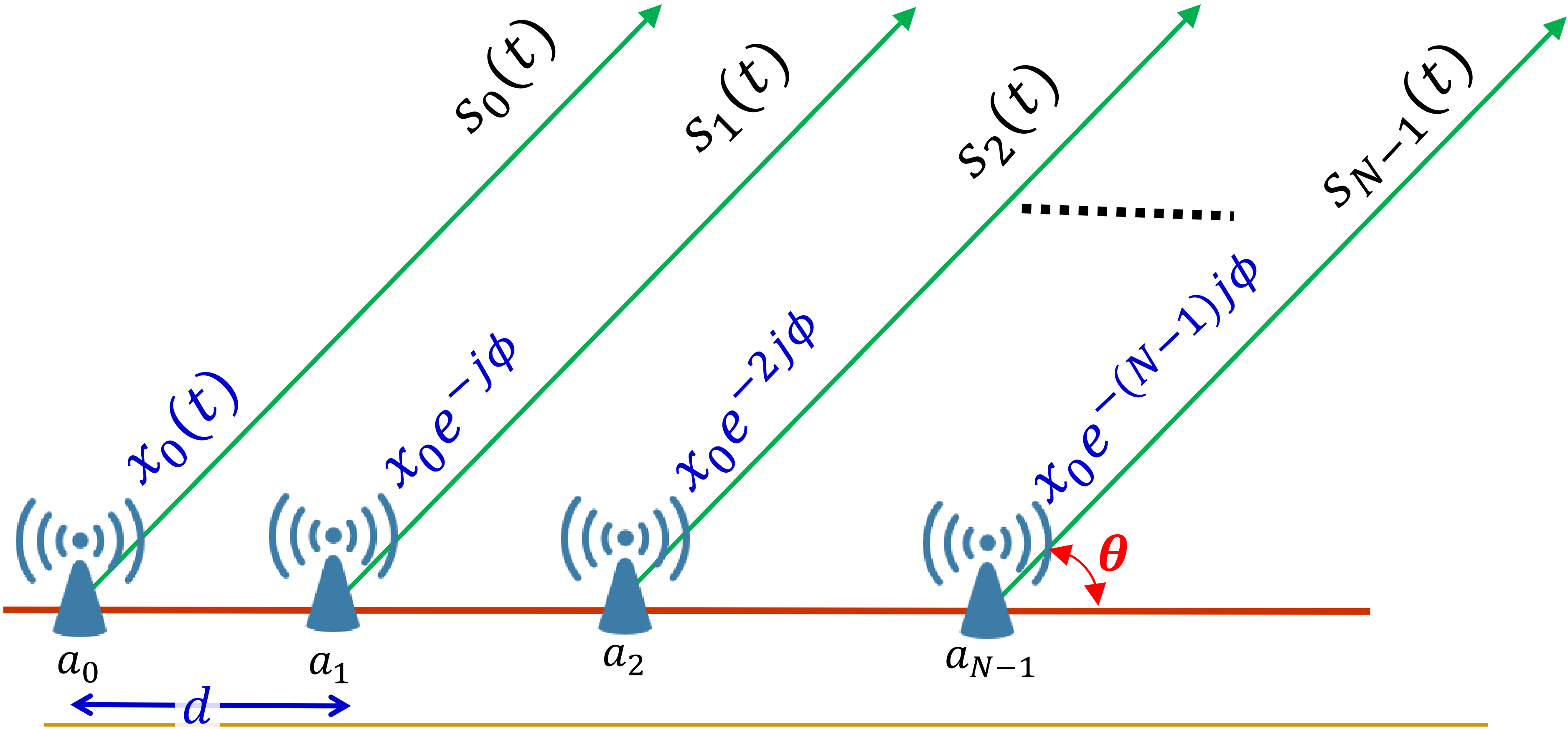


https://www.sharetechnote.com/html/Handbook_LTE_BeamForming.html

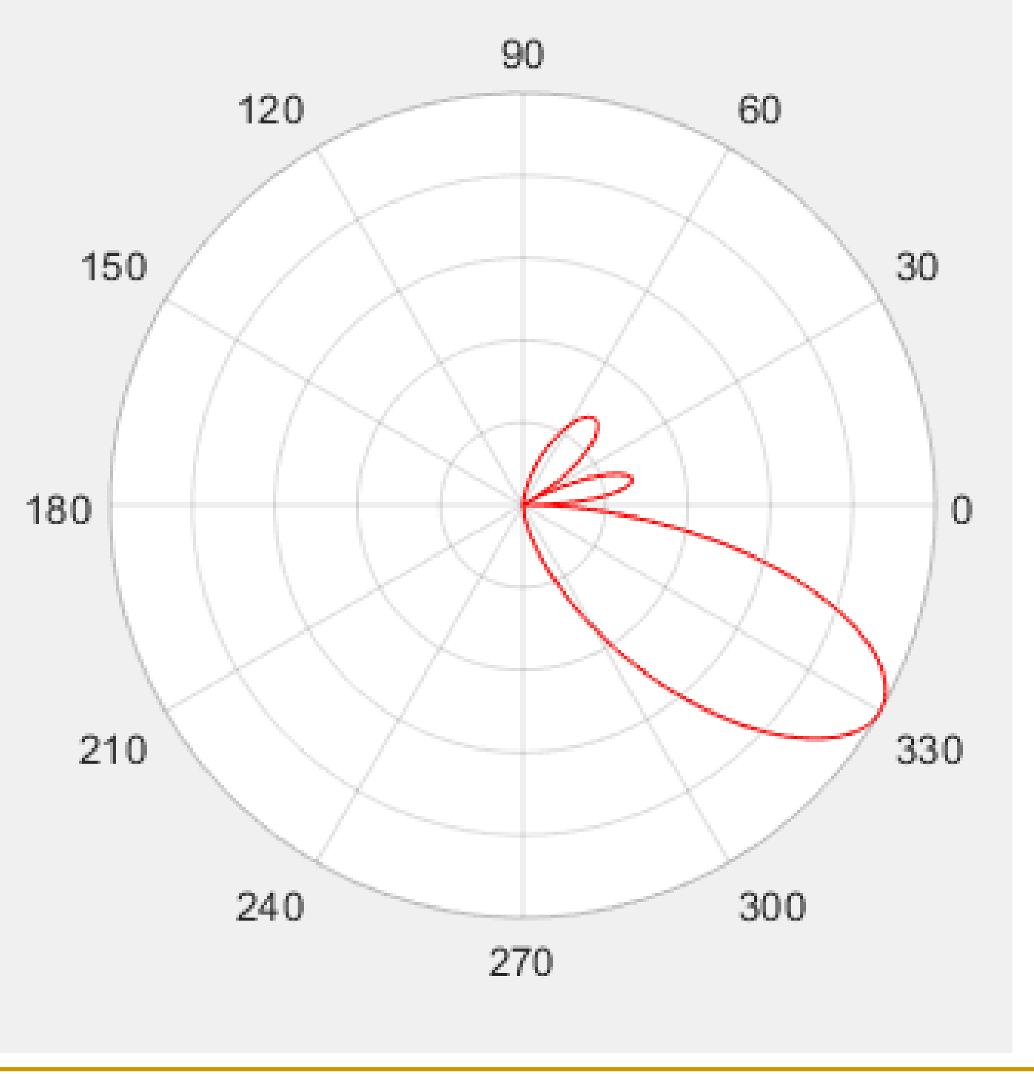
Beam Rotation

$$y(t) = \sum_{i=0}^{N-1} s_i(t)$$

$$Y(f) = \sum_{k=0}^{N-1} \left(S_0(f)e^{-kj\phi}\right)e^{kj\phi} = \sum_{k=0}^{N-1} S_0(f) = NS_0(f)$$

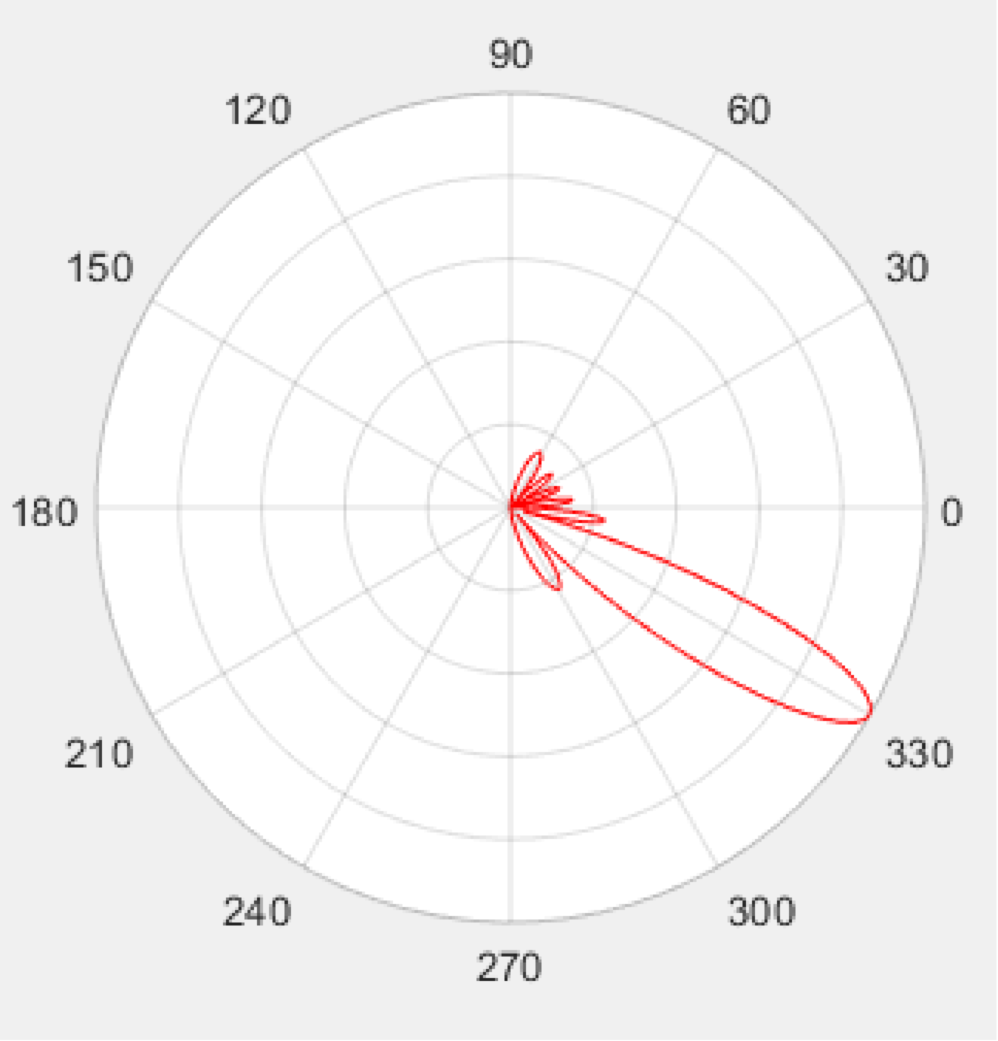


Beam Rotation (N=4)



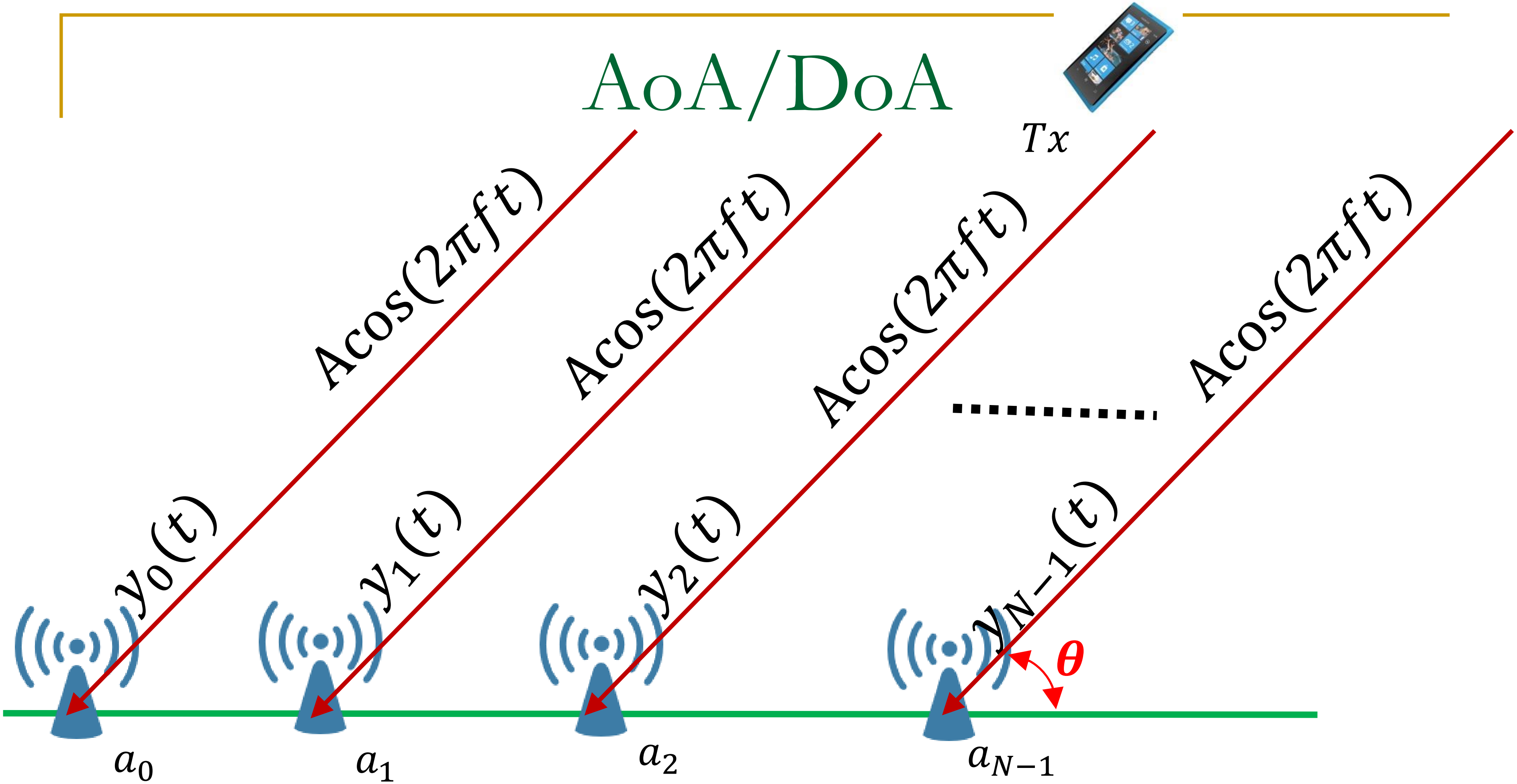
https://www.sharetechnote.com/html/lte_toolbox/Matlab_Toolbox_PhaseArraySystem_ULA_Basic.html#Plotting_Radiation_Pattern_in_2D

Beam Rotation (N=8)



https://www.sharetechnote.com/html/lte_toolbox/Matlab_Toolbox_PhaseArraySystem_ULA_Basic.html#Plotting_Radiation_Pattern_in_2D

AoA/DoA



$$y(t) = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \mathrm{B}\cos(2\pi f t) \\ \mathrm{B}\cos(2\pi f t + \phi) \\ \mathrm{B}\cos(2\pi f t + 2\phi) \\ \dots \\ \mathrm{B}\cos(2\pi f t + (N-1)\phi) \end{bmatrix} \leftrightarrow \begin{bmatrix} \mathrm{S(f)}e^{j0} \\ \mathrm{S(f)}e^{j\phi} \\ \mathrm{S(f)}e^{j2\phi} \\ \dots \\ \mathrm{S(f)}e^{j(N-1)\phi} \end{bmatrix}$$

AoA/DoA

For $\theta_i = -\pi$ to π {

$$\alpha_i = \frac{2\pi}{\lambda} d cos\theta_i;$$

$$C_{\theta_i} = \begin{bmatrix} e^{-j0} & e^{-j\alpha_i} & e^{-j2\alpha_i} & \dots & e^{-j(N-1)\alpha_i} \end{bmatrix} . \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_{N-1} \end{bmatrix}$$

}

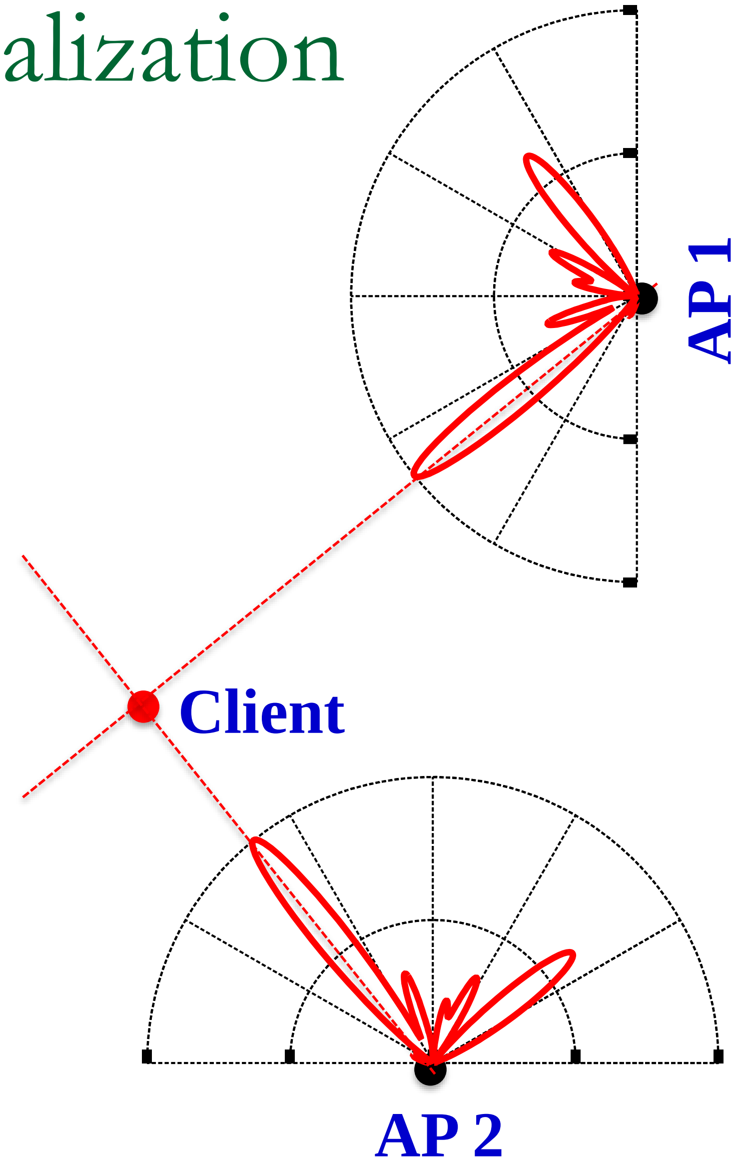
Return $\underset{\theta_i}{\operatorname{argmax}} C_{\theta_i};$

Beamforming and Antenna Array

https://hometoys.com/what-is-the-difference-between-the-amazon-echo-and-dot/

AoA Based Localization

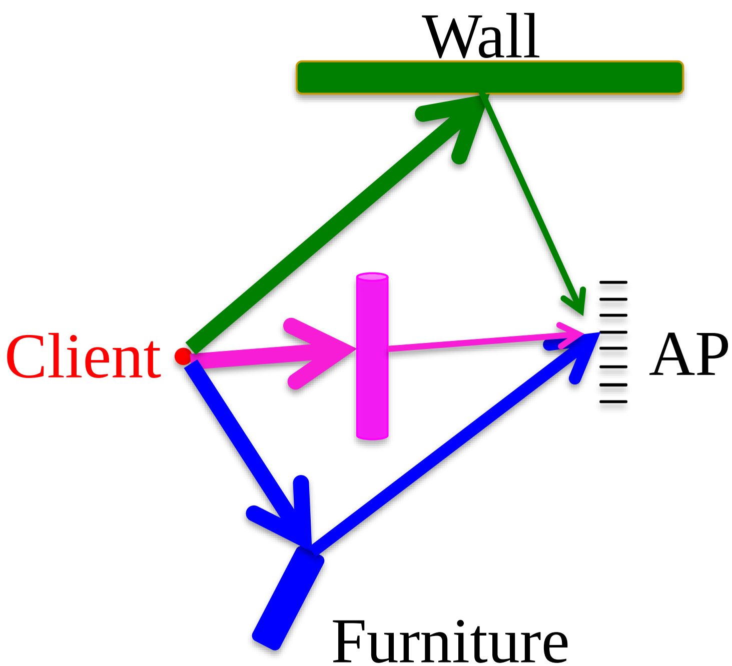
- AP overhears a client’s transmission
- APs **leverage multiple antennas** to generate angles of arrival of a client's signals:
- With multiple APs, central server **synthesizes** AoA spectra to obtain a location estimate for the client

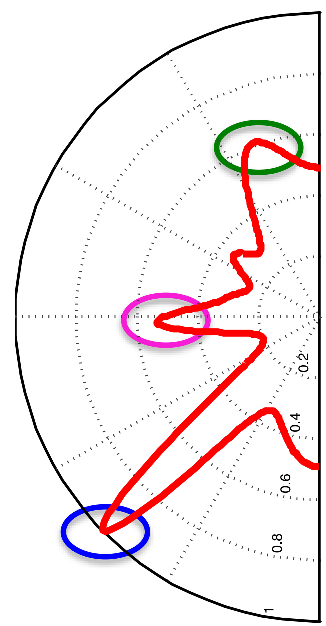


https://www.usenix.org/sites/default/files/conference/protected-files/xiong_nsdi13_slides.pdf

AoA Based Localization

- **Problem #1:** Strong multipath reflections indoors
- **Problem #2:** **Direct path** attenuated or completely blocked
  - **Direct path signal may not be the strongest**



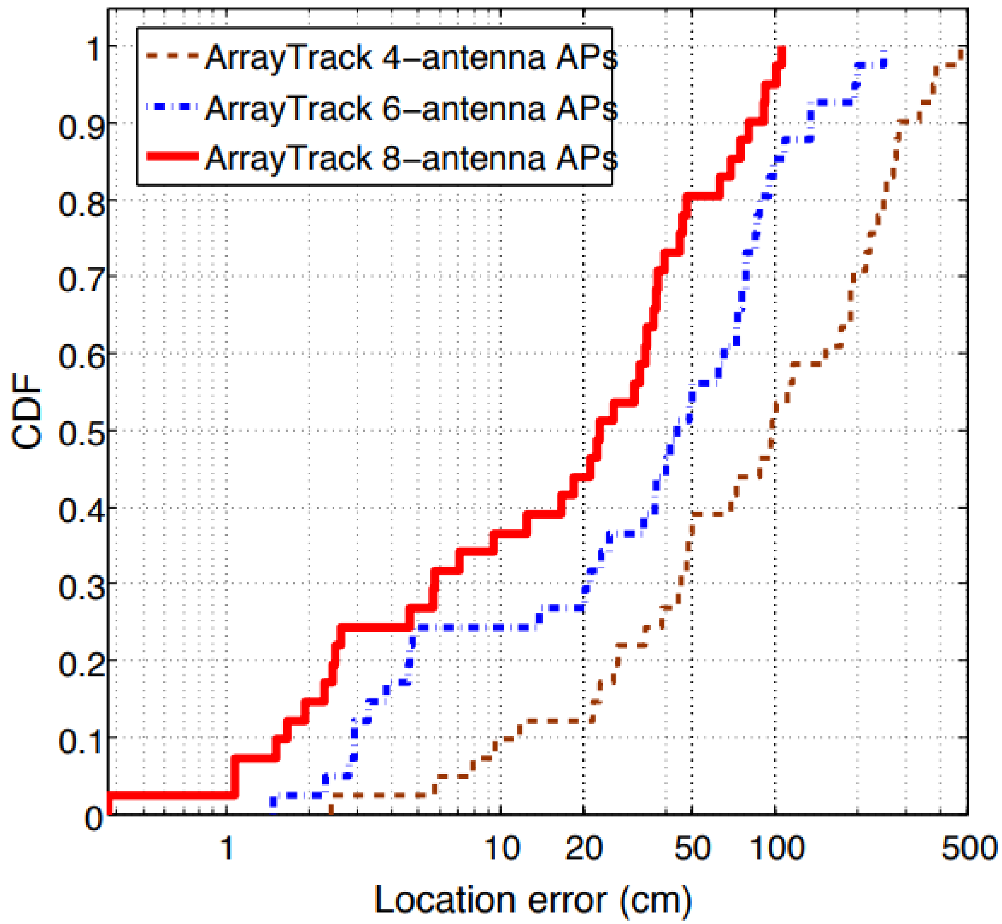


https://www.usenix.org/sites/default/files/conference/protected-files/xiong_nsdi13_slides.pdf

AoA Based Localization

- Direct path is more stable than reflection paths when client moves slightly

Median: 23 cm (with 8 APs)



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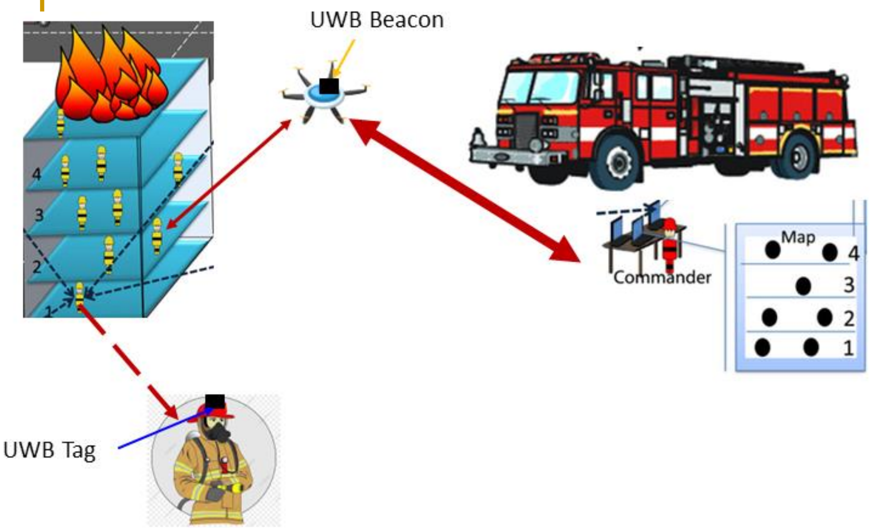
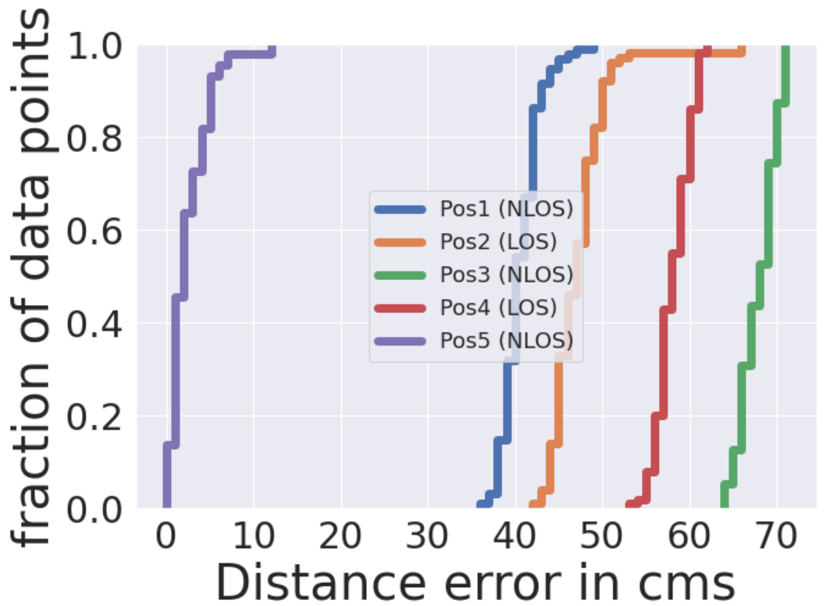
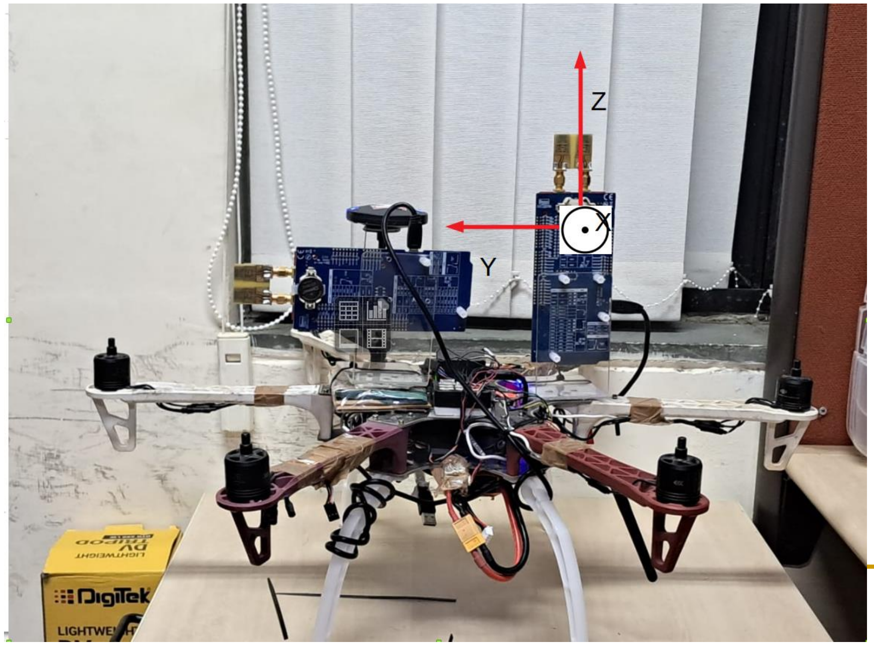
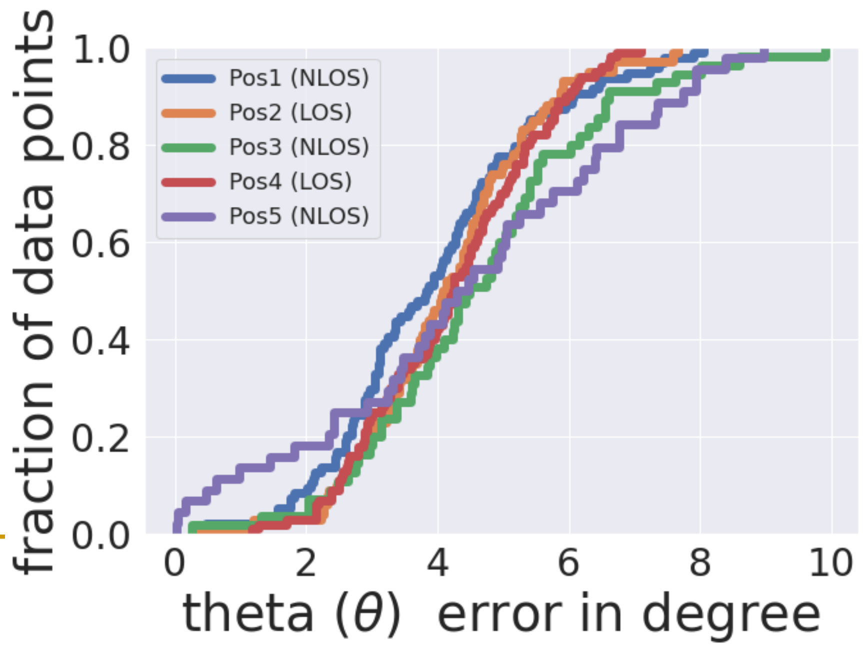

UWB Beacon
4
3
2
1
Map
4
3
2
1
Commander
UWB Tag


fraction of data points
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
70
Distance error in cms
Pos1 (NLOS)
Pos2 (LOS)
Pos3 (NLOS)
Pos4 (LOS)
Pos5 (NLOS)


Z
Y
X
DigiTek


fraction of data points
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
theta (θ) error in degree
Pos1 (NLOS)
Pos2 (LOS)
Pos3 (NLOS)
Pos4 (LOS)
Pos5 (NLOS)